F O U N D A T I O N S O F C O M P U T I N G A N D D E C I S I O N S C I E N C E S
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An Algorithm for Choosing, Ordering a New Criteria of a Bi-Objective Flow Problem

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Abstract. In this paper, we propose an algorithm which is based on many things: the notions well-known of the simplex network method, Ford Fulkerson's algorithm and our new idea, which is << the gain cycles >>, applied on a bi-objective minimum cost flow problem. This algorithm permits us to have a good order of many criteria in a rapid and an efficient way; because this classification permits us to structure the optimal area, in which we can choose the best action among the others which exist in the objective space. From this one, we distinguish, that the resolution of this problem comes to find an under set of good actions, among which the decider can select an action of best compromise, or make a decision, in the case where reference indications of the deciders may change. A didactic example is done to illustrate our algorithm.

Keywords: Bi-objective minimum cost flow problem, Network flows, Network mathematical programming, Efficient extreme point.

1. Introduction

In combinatorial optimization problems, the selection of the optimal solution takes into account more than one criterion. Frequently, all these criteria are always in conflict; for example, in the situation where we have a bi-objective problem already resolved, if one or other criteria appear suddenly, choosing one of them becomes difficult. In other words, can we classify all these new criteria, which appear unexpectedly, and which are applied on a bi-objective minimum cost flow problem?

In this kind of problems, the opinion of the decider needs a quantity of information, in order to permit a suitable modelling, so he search to elaborate models and results which constitute a coherent structured set, considered as the key of these objectives [26-28]. These economic

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calculations can take into account the immeasurable factor of quality but with difficulty [2-3, 11-13, 17-19]. In this case, there are many qualitative factors which we transform to one-dimensional monetary value well-defined, formulated by a linear objective function, and generally evaluated as a cost [4, 5, 13, 16]. As the most of the problems observed in the reality are difficult to reduce into one criterion only; so it is necessary to use this method of linear program and the analysis of the cost in these problems.
To do that, firstly, we invested in all these works [1-14, 18-25]. Then, we considered all the solutions of a bi-objective problem already resolved [14, 20-22, 25], where his basic optimal solution is a spanning tree; this one was found by optimizing the criterion 2 taking into account the criterion 1, using the simplex network method. From this solution, we applied in parallel the gain cycles on each cycle of each spanning tree, moving from iteration to another until reach the optimal solution.
This work is repeated for the other criteria considered, modifying every time the criterion 2 by others. Once all these cases are studied, we calculate the sum and the percentage of the gain and the loss cycles for each new criterion received. From this study, we recover the different areas corresponding to the criteria of gain and loss [1, 13]. Then, we determine the line right of equation y= ax+b associated to these areas, where a =ΔP/ΔG is the slope of the right of equation y = ax + b (ΔP is the variation between two points which are on the ordinate axis, and ΔG is the variation between two points which are on the absciss axis). The point b is an intersection point between the right of equation y = ax +b and the ordinate axis y'y (y'y is the axis of loss cycle). It is obtained by the resolution of the equation y - ax = b; by replacing x and y by one of the points chosen to calculate the slope a of equation y = ax +b. Once the set of gain criteria is well structured by the gain area, and the right of equation y = ax + b is well depolarized, then the best action that admits the highest value of gain cycles in percentage is chosen. On the other hand, the compromise solution is the one that corresponds to the two criteria at the same time [2-5, 16-18].
After this introduction, the paper is organized as follows. In Section 2, we give a mathematical formulation of this problem. In Section 3, we introduce the algorithm of Sedeno-Noda and Gonzalez-Martin [25], also, some results necessary for our method. In Section 4, we present our algorithm illustrated by a numerical example. We conclude in the Section 5.

2. Formalization of the problem

First, let us introduce the following notations:

- Let G = (V, A) be an oriented network where V = {1, 2, 3, ..., n} denotes the set of vertices and A the set of arcs.
- Parameter b_i represents the integer supply/demand of node i ∈V.
- Parameter u_{ij} and l_{ij} denote the upper bound and the lower bound of flow passing through arc (i, j)∈A, respectively.
- Parameter c_{ij}^k represents the cost per unit of flow on arc (i, j) in the k-th objective function (k = 1, 2).
- Variable x_{ij} represents the flow, to be determined and that passes through the arc (i, j);

Suc (i) = {j ∈V | (i, j) ∈A} and Pred (i) = {j ∈V | (j, i) ∈A}.

The bi-objective minimum cost flow problem might be formulated as a linear programming problem as follows (for more details, see for, e.g., Ahuja et al) [1, 14, 21].

minimize $f_1(x) = \sum_{i \in v} \sum_{j \in Suc(i)} c_{ij}^1 \, x_{ij}$ .....(1)

minimize $f_2(x) = \sum_{i \in v} \sum_{j \in Suc(i)} c_{ij}^2 \, x_{ij}$ .....(2)

under the constraints

$\sum_{j \in Suc(i)} x_{ij} - \sum_{j \in \Pr ed(i)} x_{ji} = b_i \quad \forall i \in V$.....(3)

$l_{ij} \le x_{ij} \le u_{ij} \qquad \forall (i,j) \in A$....(4)

where

- Variable x_{ij} represents the flow on arc (i, j).
- Functions (1) and (2) represent the objective functions to minimize.
- Constraint (3) represents the flow conservation at each node. We assume $\sum_{i \in v} b_i = 0$, otherwise the problem is infeasible.
- Constraint (4) indicates the lower and upper bounds of the interval flow variable x_{ij} must belong to.

Any vector x_{ij} that satisfies the constraints (3) and (4) is called feasible solution.

We note the decision space X (composed the constraint 3 and 4) and $f(X)$ objective space of bi-objective network flow problem.

If we note $f(x) = (f_1(x), f_2(x))$, the image of X is $f(X) = \{(f_1(x), f_2(x)) / x \in X\}$ called objective space**.**

2. 1 Definitions

2.1.1 Efficient solution

A feasible solution $x \in X$ of the bi-objective network flow problem is efficient if, and only if, there does not exist another feasible solution $x' \in X$ so that $f(x') \le f(x)$ with $f(x') \ne f(x)$.

We denote by E [X] set of efficient solution of X and E [f (X)] = {f (x) / E [X]} is called the set of efficient solution of f (X). Eex [X] denotes the set efficient extreme points of X see [25].

2.1.2 Generation of efficient solutions

The following single objective problem P (λ) can be used to obtain efficient solutions (see [25]):

Minimize $(\lambda f_1(x) + (1- \lambda) f_2(x))$

under constraints $x \in X$,

where $0 \leq \lambda \leq 1$.

2.1.3 Efficient edge

The edge connecting two adjacent efficient extreme points is called an efficient edge.

2.1.4 Strong spanning tree

A spanning tree is strong and feasible if the flow is sent positively from any node to the root of the tree, without changing the flow bound [1].

2.1.5 Operation of the pivot

The operations are as follows:

- Determination of the cycle,
- Calculation of the variation of the flow,
- Modification of the flow,
- Update of the basic,
- Update of the dual variables.

2.1.6 Flow along cycles

Let w be a cycle (not necessarily directed) from a specified node s to itself, whose orientation has already been defined. Let $\overline{w}$ and $\underline{w}$ denote the sets of forward and backward arcs in this cycle, with respecting the orientation of the cycle, we define the cycle multiplier $\mu(w)$ as follows [1]:

$$\mu(w) = \frac{\prod_{(i,j)\in w} \mu_{ij}}{\prod_{(i,j)\in \underline{w}} \mu_{ij}}$$ if we send 1unit of flow along the cycle w.

If $\mu(w) > 1$, the cycle w as a gain cycle.

If $\mu(w) < 1$, the cycle w as a loss cycle.

If $\mu(w) = 1$, the flow around this cycle is conserves.

The procedure of compute_ cycle_ multiplier used in our algorithm has the following diagram**:**

Procedure compute_ cycle_ multiplier {B, L, U, π^k, μ_{ij}, $\mu(w)$, S, x^t};

Begin:

If $\mu(w)$ exist **then** store $\mu(w)$ as cycle multiplier in the objective space;

$$\mu(w) = \frac{\prod_{(i,j)\in w} \mu_{ij}}{\prod_{(i,j)\in \underline{w}} \mu_{ij}}.$$

If $\mu(w) > 1$, the cycle w as a gain cycle.

If $\mu(w) < 1$, the cycle w as a loss cycle.

If $\mu(w) = 1$, the flow around this cycle conserves.

Else stop. We cannot use compute_ cycle_ multiplier.

End.

3. Introduction of an algorithm to obtain all the efficient extreme points

In this algorithm, we introduce a procedure that is based on the method of Sedeno-Noda and Gonzalez-Martin [25]. They began by the extreme efficient point $f(x^0)$ in the objective space, which was found when they solved the problem P (λ_0) with λ_0=1. Then, they calculate the rest of the efficient points in the same space by a finite sequence of pivot. The computation of the initial extreme point $f(x^0)$ is found by optimizing the first objective using the simplex network method [1, 6]. The strong feasible spanning tree corresponding to the optimal basis (B^0, L^0, U^0) is obtained directly from the potential nodes, giving π^1 the reduced cost $\bar{c}^1_{ij} = c^1_{ij} - \pi^1_i + \pi^1_j$ for all $(i, j) \in A$ [14, 21-24].

As (B^0, L^0, U^0) is an optimal basis of the first objective, so the following optimality conditions are satisfied [1] and the conditions:

$$l_{ij} \le x^0_{ij} \le u_{ij} \quad \text{and} \quad \bar{c}^1_{ij} = 0 \quad \forall (i,j) \in B^0,$$

$$x^0_{ij} = l_{ij} \quad \text{and} \quad \bar{c}^1_{ij} \ge 0 \quad \forall (i,j) \in L^0,$$

$$x^0_{ij} = u_{ij} \quad \text{and} \quad \bar{c}^1_{ij} \le 0 \quad \forall (i,j) \in U^0.$$

Taking into account the second, the third and the fourth objective, we found the potential node, π^2, π^3 and π^4 by resolving $\bar{c}_{ij}^{k} = c_{ij}^{k} - \pi_i^k + \pi_j^k = 0$; k= 2, 3 and 4 $\forall (i, j) \in B^0$. Considering the efficient extreme point $f(x^i)$ and the strong spanning tree corresponding to the basis (B, L, U), $f(x^j)$ is an adjacent extreme point, found when we introduce the out basic variable see [1, 25].

Set $$\theta_k^t = \min\left\{ \frac{\bar{c}_{ij}^{k}}{c_{ij}^{-1}} : \bar{c}_{ij}^{k} < 0; \forall (i,j) \in L;\ \frac{\bar{c}_{ij}^{k}}{c_{ij}^{-1}} : \bar{c}_{ij}^{k} > 0;\ \forall (i,j) \in U \right\}.$$

After that, a new method is proposed using gain and loss cycles, applied throughout a spanning tree. This spanning tree was already resolved by the simplex network method see [1, 6, 14]. These gain and loss cycles, are applied at each cycle of the spanning tree from iteration to another until the end of the optimum. At each step, the modification of the spanning tree is done by the flow value, which is considered by Ford Fulkerson's algorithm [1, 6-10, 22]. When all these cycles are applied at each step of the spanning tree, considering the several criteria of this problem, we calculate the sum of gain and loss cycles for each step. Then, we classify all these criteria following the percentage of the sum of gain cycles. So, we can choose the best action which corresponds to a criterion which corresponds to the compromise solution. Without forget to take into account the satisfaction of the optimality conditions see Figure 2.
The algorithm and the procedures have the following diagram.

4. Algorithm

Begin:
1: **Input**: Resolving $P(\lambda)$ problem with λ=1 and start with the strong spanning tree (B^0, L^0, U^0) corresponding to the first criterion and three criteria c_{ij}^{k}, k = 2, 3, 4,

2: compute x_{ij}^0 optimizing the first objective by simplex network method [1],

3: compute reduced costs $\bar{c}$ for x_{ij}^0.

While S $\neq \phi$ do

4: compute _ entering _ arcs {L, U, π^1, π^k, S, $\bar{c}_{ij}^{k}$, θ_k^t, x^j},

5: compute _ cycle_ multiplier {B, L, U, π^k, μ_{ij}, $\mu(w)$, S, x^j},

6: compute _ new_ point {B, L, U, π^1, π^k, S, $\bar{c}_{ij}^{k}$, x^j},

7: compute _ entering _ arcs {L, U, π^1, π^k, S, $\bar{c}_{ij}^{k}$, θ_k^t, x^{j+1}},

8: **Output**: complete set of extreme efficient solution Eex = { x^1, x^2,..., x^{S-1}, x^S},

End.

We repeat the same processes of the procedures: compute _ entering _ arcs {*L*, *U*, π^1, π^k, *S*, $\bar{c}_{ij}^{-k}$, θ_k^t, x^j}, compute _ new _ point {*B*, *L*, *U*, π^1, π^k, *S*, $\bar{c}_{ij}^{-k}$, x^j} and compute_ cycle_ multiplier {*B*, *L*, *U*, π^k, μ_{ij}, $\mu(w)$, *S*, x^j} at each iteration.

We have already mentioned that our method is based on the method of Sedeno-Noda and Gonzalez-Martin see [24-25]. We have only applied the multiplier cycles on the different criteria. Once the efficient extreme points of the objective space are obtained, it is possible to determine all the gain and the loss cycles of this space. Using them in all the iterations of this problem already resolved by the simplex network method, we can determine the sum value of the gain and loss cycles corresponding to each criterion. This is always obtained when we optimise the criterion taking into account the strong spanning tree. This strong spanning tree is constructed according to the first criterion. After that, we can reorder the criteria to choose the best of them, who will represent the good action in the reality [5, 13 and 18].

4.1. Numerical example

We consider the example given in Figure1, *V*= {1, 2, 3, 4, 5, 6}, where b_1=10, b_6 =-10 and b_i =0 with i = 2,…, 5. Each arc has a lower bound, an upper bound (l_{ij}, u_{ij}) and four criteria [$c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4$]. Firstly we consider two criteria [c_{ij}^1, c_{ij}^2] and we use this formula:

$$\theta_2^t = \min\left\{ \frac{\bar{c}_{ij}^{-2}}{\bar{c}_{ij}^{-1}} : \bar{c}_{ij}^{-2} < 0; \forall (i,j) \in L; \ \frac{\bar{c}_{ij}^{-2}}{\bar{c}_{ij}^{-1}} : \bar{c}_{ij}^{-2} > 0; \forall (i,j) \in U \right\}$$

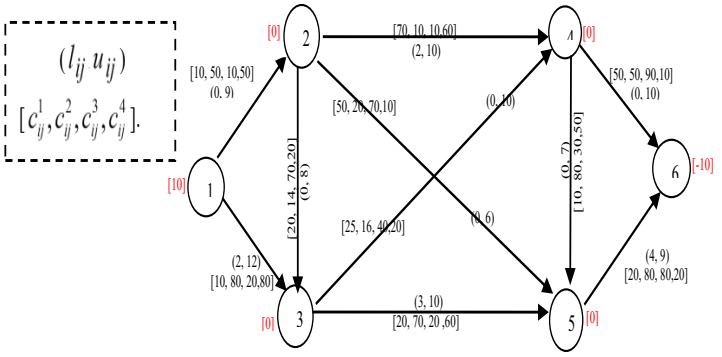


Figure 1: Network of the example

This example has eight efficient extreme points in the decision space, four of them correspond to the efficient extreme points of the objective space, these points (x^1, x^2, x^3 and x^8) can be seen in Table 1, [21, 25].

Table 1: The efficient extreme solutions for the second objective in the objective space

x^t	(1,2)	(1,3)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)	(4,6)	(5,6)	f_1	f_2
x^1	2	8	0	2	0	0	8	1	1	9	640	2170
x^2	2	8	0	2	0	0	8	0	2	8	660	2060
x^3	7	3	0	2	5	0	3	0	2	8	810	1660
x^4	8	2	1	2	5	0	3	0	2	8	830	1644
x^5	3	7	0	2	1	4	3	0	6	4	830	1644
x^6	7	3	0	6	1	0	3	0	6	4	1010	1500
x^7	8	2	5	2	1	4	3	0	6	4	930	1564
x^8	8	2	1	6	1	0	3	0	6	4	1030	1484

Starting by applying the multiplier cycle on each efficient point for each pivot that corresponds to each iteration. The pivot process is determined by the simplex network method.

Secondly, as we have already indicated that the arcs that do not fulfil the optimality conditions according to the second, the third and the fourth objectives are the candidate arcs that will enter into the basis. The (i, j) non-basic arcs are such: $\{\bar{c}_{ij}^{k} < 0 \ /(i,j) \in L$ or $\bar{c}_{ij}^{k} > 0 \ /(i,j) \in U;\ k = 2, 3, 4\}$see [24, 25]. Once, all the non-basic arcs fulfil the optimality conditions according to the second objective, we compute the gain cycles and the loss cycles. Then, we classify the best criterion, which has an important value of the gain cycles Table 1. We repeat the same process for the third criterion $[c_{ij}^1, c_{ij}^3]$ and for the fourth criterion $[c_{ij}^1, c_{ij}^4]$ see Tables 2 and 3**;** the dispersion of the solutions of the criteria: $[c_{ij}^1, c_{ij}^2]$, $[c_{ij}^1, c_{ij}^3]$ and $[c_{ij}^1, c_{ij}^4]$ is represented in Figure 2.

Table 2: The efficient extreme solutions for the third objective in the objective space

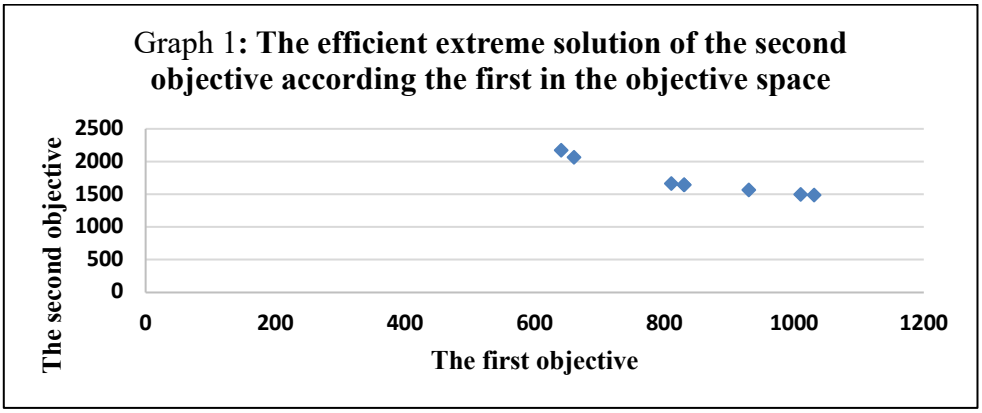
x^t	(1,2)	(1,3)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)	(4,6)	(5,6)	f_1	f_3
x^1	2	8	0	2	0	0	8	1	1	9	640	1200
x^2	2	8	0	2	0	0	8	0	2	8	660	1180
x^3	8	2	6	2	0	0	8	0	2	8	780	1540
x^4	8	2	1	2	5	0	3	0	2	8	830	1440
x^5	8	2	1	6	1	0	3	0	6	4	1030	1240
x^6	8	2	1	7	0	0	3	1	6	4	1060	1210
x^7	7	3	0	7	0	0	3	1	6	4	1040	1150
x^8	6	4	0	6	0	0	4	0	6	4	980	1140

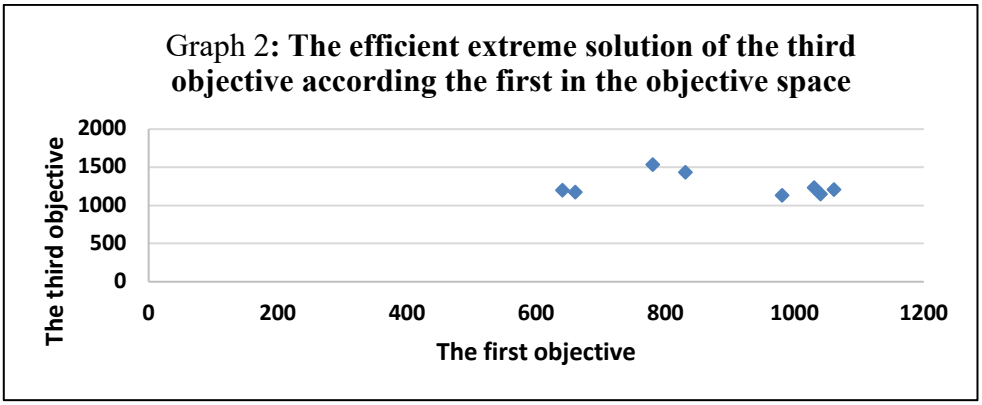
Table 3: The efficient extreme solutions for the fourth objective in the objective space

x^t	(1,2)	(1,3)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,5)	(4,6)	(5,6)	f_1	f_4
x^1	2	8	0	2	0	0	8	1	1	9	640	1580
x^2	2	8	0	2	0	0	8	0	2	8	660	1520
x^3	7	3	0	2	5	0	3	0	2	8	810	1120
x^4	8	2	1	2	5	0	3	0	2	8	830	1110

4.1.1 Interpretation of the tables

The Tables 1, 2 and 3 represent the calculations of all the efficient extreme solutions which exist in the objective space, for a minimum cost flow network problem.
In Table 1, we gave the optimization of the criterion 2 taking into account the strong spanning tree of the network, which is constructed according to the first criterion. In Table 2, we optimized the criterion 3 taking into account the criterion 1. And in the third Table, we optimized the criterion 4 taking into account the same criterion.
This calculation permits us in a side, to have the different spanning trees used to apply the different gain cycles, and in the other side, to have a vision well-structured on the dispersal of all these efficient solutions, which exist in the objective space, as well as, the position of all the extreme points which exist in the objective space (the position of the extreme points is done according to the criteria [24]).
The vision on the dispersal of the solutions will have many advantages for our project [5]. Moreover, it is permit us to guide it on the good road and to manage the budget [17]. By applying these precautions (it means: to pay attention) we can make resist our project during its functioning, and make it last in the long term and finally ameliorate the situation in the case of a rupture [1, 3, 7, 9, 10, 17].





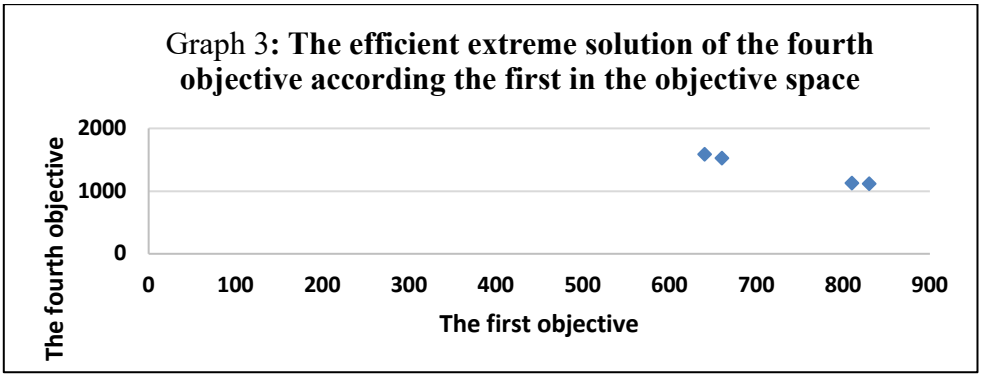


Figure 2: All the solutions of the criteria c_{ij}^2, c_{ij}^3 and c_{ij}^4, taking into account the first criterion.

After that, we calculate the percentages of the gain and the loss cycles, also we calculate the third and the fourth criterion taking into account the first one. Next, we order them by comparing their gain cycles (see Table 4, 5 and 6).

Table 4: The classification of the second criterion according to the sum of the gain cycles

Second criterion / to first	1	2	3	4
The number of gain cycle	5	2	4	4
The number of loss cycle	2	5	3	3
The total of gain cycle	52.25	78.7	83.03	36.6
The total of loss cycle	0.39	1.33	0.20	0.26

Table 5: The classification of the third criterion according to the sum of the gain cycles

Third criterion / to first	1	2	3	4
The number of gain cycle	3	2	1	4
The number of loss cycle	4	5	6	3
The total of gain cycle	37.5	48.75	8.75	317.1
The total of loss cycle	0.44	0.58	0.40	0.17

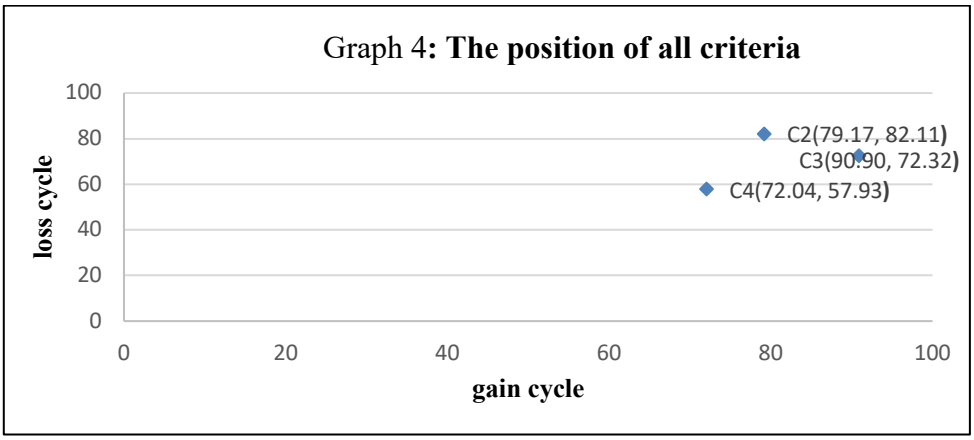
Table 6: The classification of the fourth criterion according to the sum of the gain cycles

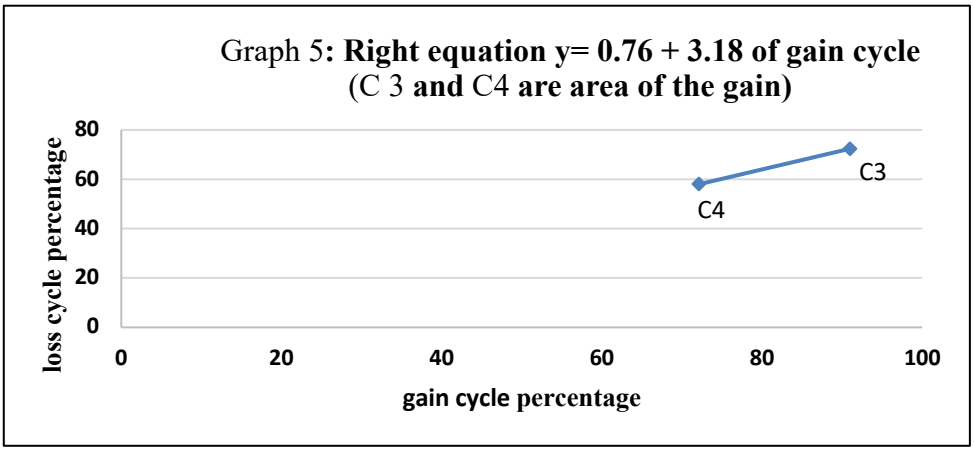
Fourth criterion / to first	1	2	3	4
The number of gain cycle	2	1	2	1
The number of loss cycle	1	2	1	2
The total of gain cycle	22.5	8.75	36.75	12.5
The total of loss cycle	0.25	0.18	0.03	0.11

Finally, we order these global percentages of all the criteria to select the best and the favourite criterion, which corresponds to the gain cycles and it's the good action. We can say that choosing the best criterion represents really a good action [1-5, 13, 16-18]. The value of loss cycles is mentioned. You can see Table 7 and Figure 3 and 4.

Table 7 : The position of all the criteria according to the first criterion.

2, 3 and 4 criteria / 1er	2/1er	3/1er	4/1er
The sum of gain cycle for all criteria	250.5	412.1	80.5
The sum of loss cycle for all criteria	2.18	1.59	0.58
% of gain cycle (Sg%)	79.17	90.90	72.04
% of loss cycle (Sp%)	82.11	72.32	57.93





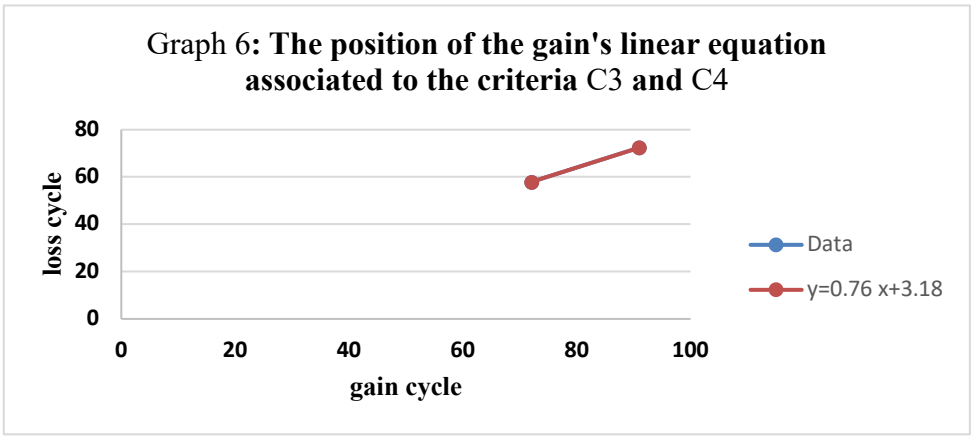


Figure3: The dispersion of all criteria and the position of the gain's linear equation

4.1.2 Interpretation of the graphs

In this graph, we represent the losses in function of the gains. After the representation of the coordinates of the gain and loss cycles, we obtain two areas, the area of loss which is represented by the criterion 2 because we have more loss than gain, and the other area which is represented by the criterion 3 and 4 because we have only gains for these two criteria. With the help of these two criteria 3 and 4, we determine the line right of equation $y = ax + b$, then $y = 0.76x+3.18$.The right will contain the best actions between the criterion 4 and the criterion 3, where the best action is the one which corresponds to the highest value of the gain cycles, in percentage [23, 29], where
$a = \Delta P/\Delta G = 72.32–57.93/90.90–72.04$, then $a = 0.76$,
and $b = y–ax = 57.93 – (0.76)\ (72.04)$ then $b = 3.18$.
Finally, $y = 0.76\ x + 3.18$.

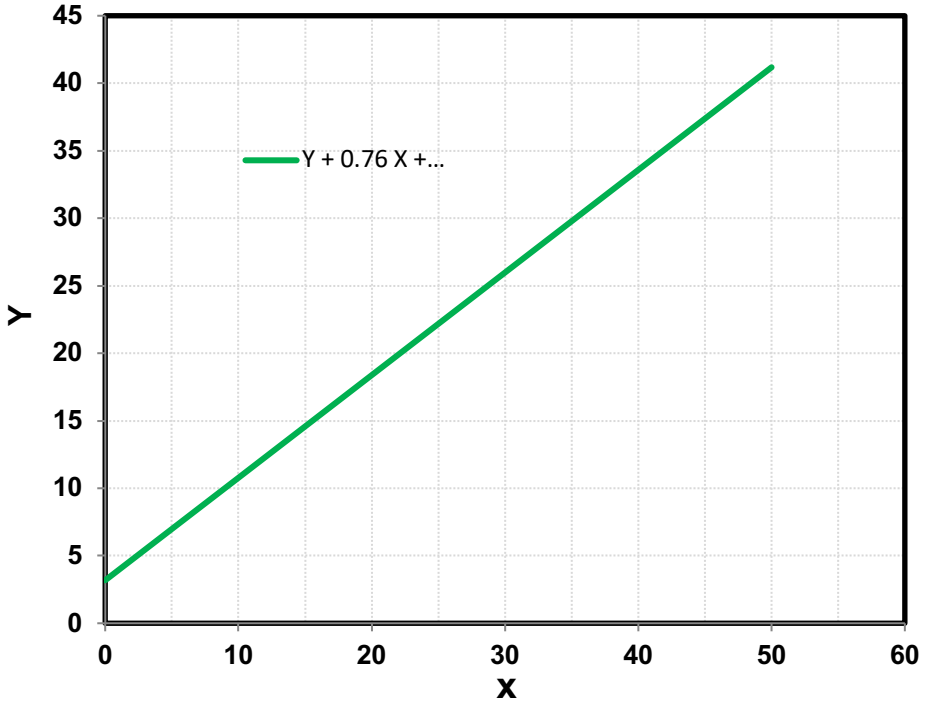


Figure 4: The recapitulation of all the results of the numerical example

4.1.3 Discussion and comparison of the results

The numerical example which we have considered is a general network, constituted of 6 vertices and 10 arcs, the strong spanning tree is determined taking into account the first criterion using the simplex network method [1, 6].
We started in Tables 1, 2 and 3, by research all the efficient solutions in the objective space for the three criteria. Then, we represented the dispersion of these solutions found in the Graphs 1, 2 and 3. In Tables 4, 5 and 6, we gave the gain and loss cycles for each criterion. Furthermore, in the 7 th Table**,** we calculated the percentages of the gain and loss values. The Graph 4 shows us the position of the criteria, while the Graph 5 detects the criteria 3 and 4 which constitute the gain area. Finally, in the 6 th and last Graph**,** we traced the linear right of equation $y = ax + b$ associated to this gain area [11, 15-19].
In another words, talking about the study of the 3 graphs (1, 2 and 3): each criterion has his graph, and each graph gives us the different results to compare between the criteria. These results of the computations of the multiple cycles detect us the gain and loss areas.
Therefore, our work brings a novelty to the resolution of the bi-objective flow problems. This novelty appears in the management of the methods applied on the spanning tree of the given network. This doesn't earn us only the time, but it permits us to have more information about the detection of the areas too. All of that was for approach the desired decision, and to choose the compromise solution to keep the good relations between the companies [2, 3, 15-19].
The association of Pareto's tools to this work, structures in the plan $R^2$the best vision on the desired decision making (see Graph 4, 5 [18, 23, 29]); with keeping all the propriety of the initial network and avoiding the risks which may produce in the future.
In conclusion, this fabulous linear right is a theory to apply in the real case in the future.

5. Conclusion

In this paper, a new method is presented to solve the bi-objective minimum cost flow problem. We apply the fascinating idea of the gain cycles in this kind of problems already resolved, modifying the criterion 2 by other criteria received unexpectedly. The gain cycles are applied on each spanning tree, moving from iteration to another till reach the optimum. Then, we study the results of these different cases. This combination of gain cycles applied on this kind of problems by the simplex network method and Ford Fulkerson's algorithm, allows us to build an algorithm capable to classify and to select the best action among the others, taking into account the hypotheses which exist in the network. Effectively, this idea permits us in a side, to guide the intuition at the moment of our reasoning, and in the other side, to get closer to the economic profit that appeared unexpectedly in a difficult situation [11, 13, 15-18]. Finally, this pushes us to modify the decision making and to make the good choice at the good moment.
For the future researches, this work facilitates us the decision making of the best project simply and rapidly, because the areas of the optimal solutions found, are well structured by the good criteria which exist in the objective space. Additionally, we want to use this contribution to resolve energy, economy and finance problems. Eventually, why not, we may program our work using different software such as Matlab and others.

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