

A STOCHASTIC METHOD FOR OPTIMIZING PORTFOLIOS USING A COMBINED MONTE CARLO AND MARKOWITZ MODEL: APPROACH ON PYTHON

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ABSTRACT

The main of the study is to comprehend how the mean variance efficient frontier method may be used in conjunction with Markowitz portfolio theory to produce an optimal portfolio. The study uses daily observations 8 pharma companies closing price namely Auropharma, Granules, Glaxo, Lauruslabs, Pfizer, Sanofi and Torntpharma. Further, Nifty pharma index is considered as benchmark index to check the performance of the chosen companies. The study chosen the reference period from 2020 to 2023 and required data has been extracted from the National Stock Exchange (NSE). This research is based on implementing a stochastic method for efficient portfolio optimisation employing a blended Monte Carlo and Markowitz model. In order to forecast the price of these indices in the future and to determine the likelihood of profit or loss while investing in a portfolio of stocks representing the aforementioned indices, the study also uses Monte Carlo simulation. The study involves two algorithms, namely the deterministic optimisation algorithm, which uses Markowitz Portfolio Theory, and the probabilistic optimisation algorithm, which uses Monte Carlo simulation. The study employed correlation matrix to find the exist relationship between the chosen companies and benchmark index. Also, expected return and volatility has been identified with the help of standard deviation using Python. The study found that the NIFTY Pharma index offers a higher return of 14.35. In addition to this, NIFTY Pharma portfolio's volatility is considerably higher. The study concludes that the NIFTY pharma portfolio is more suitable for those investors who have an appetite for risk.

Keywords: Return, Risk, Efficient Frontier, Markowitz Monte Carlo, Pharma index, Correlation

1. INTRODUCTION

Financial markets around the world are becoming more and more intricate. Investors must consequently consider a variety of factors and market dynamics in order to increase their earnings (Vunjak and Stojanović, 2022). Thanks to developments in financial engineering, many methods have been developed to study the behavior of financial markets. The majority of investors pick combinations of numerous stocks and connect their funds to stock exchange marketplaces because single equities have inherent dangers. Consequently, an important field for research is portfolio selection (Akhtar, 2013). An ideal portfolio is one that perfectly balances risk and

reward. The optimum portfolio aims to strike a balance between securities with the highest returns and the lowest risks, or vice versa, given a particular return (Alexeev et al., 2012). The perfect portfolio essentially argues that if one wants higher returns, they must be prepared to accept higher risk (Đukić, 2018). Greater risk does not, nonetheless, automatically imply greater return (Wang et al., 2024). The ideal portfolio is one in which risk and return have been precisely balanced. Investments with the least amount of risk for a specific return, or investments having the largest potential returns, are balanced in the optimum portfolio to achieve these goals (Bozanic et al., 2023). Therefore, risk taking needs to be analyzed, assessed and monitored. It might be difficult to assess whether a portfolio is profitable or optimum. Consideration must be given to the interests and objectives of the investor in order to ascertain whether a portfolio is indeed optimal. A portfolio of more cautious assets with slower growth would probably be unappealing to investors who are more ready to take risks (Kuncoro and Pardede, 2024). When this is the case, the best investing strategy will concentrate on buying stocks, assets, and other types of investments that offer the chance for a higher return on investment. While there is the potential for greater profits, the investments additionally are more unpredictable, which raises the risk of suffering some short-term losses (El-Morsy, 2023). In contrast to volatility, which is a negative factor for investors, getting a high profit is a positive factor (Vlasenko, 2023). How much of an investor's portfolio must be assigned to current equities, or how should investors divide their available cash across various stocks, is the subject at hand (Wang and Hu, 2024). As a result, portfolio optimization is very crucial to both investment and finance projects (Molchanova and Kovtoniuk, 2023). Investors can increase their wealth and make more informed financial choices by analyzing and assessing the basics of portfolio management (Bai, 2009). Evaluation of a set of securities from a large pool of prospective options is the aim of portfolio selection (D'Agostini, 2015). It aims to increase investment returns for investors. According to Markowitz (1952), investors must decide between return maximization and risk reduction. Investors have the option of concentrating on return maximization for the level of uncertainty that has been taken into consideration or on risk minimization for a fixed rate of return.

The Monte Carlo simulation was named after a renowned casino in Monaco because probability and unpredictable outcomes are crucial to this modelling technique, just as they are to games like cards and slots machines (Deepali, n.d.). Since its creation, the Monte Carlo model has assessed how risk affects a range of real-world scenarios, including Artificial Intelligence, stock markets, sales plans, corporate strategy, and pricing. For valuing portfolios and pricing stock options, financial professionals employ Monte Carlo simulation. A strong portfolio is one that gives an investor the ability to choose their investment strategy carefully (David, 2020). Investors can understand that there exists a risk-return trade-off because to Markowitz's methodology. He contends that investors should start their portfolio analysis by obtaining data on specific risks before concluding with data on the entire portfolio. The goal, in Markowitz's opinion, is to make it possible for investors to identify portfolios that satisfy their particular goals. Any investor can devote their resources to investments by following a procedure called portfolio selection. The method is especially important since it is based on the efficient frontier notion, which argues that a logical investor would select a portfolio with the maximum return for a certain standard deviation. MPT (Markowitz portfolio theory) advocates a portfolio with lower risk for a particular amount of return because investors can be risk-averse. This idea states that investors are willing to participate in high-risk ventures if they believe they would win more money.

Further, financial market is one of the most crucial venues to help enhance asset allocation, and most people will eventually have to make some investing decisions. In their analyses, people ultimately want to obtain a suitable return with little investment risk, yet uncertainty is a key

component of the capital market. A significant concern in the financial industry is the selection of the optimal portfolio. This study uses Monte Carlo simulation and Markowitz Portfolio theory to propose a probabilistic method for determining the best portfolio.

2. REVIEW OF LITERATURE

The financial services sector does their utmost to create an effective choice of portfolio policy to maximize return on investment in the midst of an industry that is highly competitive with intricate market rivalry. Markowitz developed an official approach for building a profitable portfolio. Therefore, asset variation is crucial for building a productive investment (Dziuban, 1974). Conventional and basic investment techniques call on prospective price predictions as system components, which is a difficult issue given that such data are hard to come by in practical settings (Eero Pätäri, 2018). Moving medians, the relative stability measure, and initial sector breakout are a few examples of the arbitrary gauges that are used for creating trade signals by the majority of traditional trading systems. Nevertheless, the majority of metrics only offer confirmation of purchases and sales for long and short positions, regardless of volatility control (the applicability of equities) and position dimension (the number of equities). Therefore, asset administration is utilized to offer investors with greater oversight over the investments they make. The choice of assets, position dimension, and division of funds have all been extensively studied (Eero Pätäri, 2018). A fund supervisor's comfort in reaching close to the normal return is a further benefit of diversification. The spread between prospective investment outcomes around the profit margin of the market will determine how certain one can be. Naturally, the range of assets in the portfolio that are poorly connected determines the level of diversification. Fabozzi, (2015) based on Markowitz theory, efficient frontiers and ideal portfolios can be computed using the weekly closing prices of stocks listed on the exchanges, which serve as the data inputs. Efficient portfolios produced by the approach proposed by Markowitz can beat any domestic individual security. By selecting efficient portfolios, or those that are located on the frontier of efficiency, investors can achieve the greatest return on investment for a particular level of risk, the maximum Sharpe proportion, or the lowest risk (Frank, 2001). The strategy created by Markowitz, portrays the process of building an investment portfolio as a matter of optimization which includes the choice of prospective commodities and the distribution of resources amongst assets. Therefore, the design and implementation of a polynomial algorithm are necessary to obtain a collection of ideal portfolios. The collection of ideal mean-variance choices in the risk-return arena is known as the frontier of efficiency (Garcia, 2017).

In this article, the authors provide a quick overview of Monte Carlo simulation, including its purpose and applicability, as well as its execution, analysis, and underlying mathematical concepts. The study also demonstrates the value of Monte Carlo simulation as a methodological tool for scenario analysis and probability analysis of various circumstances. Several additional elements affect how the data characteristics for the simulations are determined. Authentic simulations are at peril because of systematic shifts of the data parameters. There are some drawbacks to this strategy. Primarily determining the ideal and least optimal cases given every data parameter could be challenging. Furthermore, not every data parameter will likely be at its greatest or poorest state simultaneously. As we are currently weighing numerous possibilities, making choices is often challenging. Additionally, system variants and archiving get challenging when a researcher raises the total amount of scenarios to take into account (Goetzmann, 2020). A portfolio's risk can be considerably impacted by the number of stocks it holds. Monte Carlo simulation also model portfolios and calculate various risk indices, including heavy headed. It is demonstrated through the simulation that at times the ideal number of stocks depends on the risk measure, the amount of confidence demanded by investors, the particular stock mar-

ket, as well as the altering correlation structure over time (Haupt, 2021). With this background, the study provides an insight about the analyzing the portfolio efficiency using Markowitz Portfolio Theory. Also, determine the probability of gain, price of NIFTY indices (Healthcare and Pharma) using Monte Carlo simulation.

2. 1. SCOPE OF THE STUDY

NIFTY Pharma, of the National Stock Exchange, India, is included in the study. The purpose of the study is to understand how the Markowitz portfolio theory may be combined with the mean variance efficient frontier method to create an ideal portfolio. The study also makes use of Monte Carlo simulation to predict the price of these indices in the years to come and to evaluate the possibility of profit or loss when investing in a portfolio of stocks that represent the aforementioned indices.

3. METHODOLOGY

The study is based on an exploratory research design. The daily data on pharmaceutical stocks was collected from 2020–2023. The total sample size was 8 pharma companies and the Pharma Index. The stocks comprising these pharma companies and indices are sourced from National Stock Exchange (NSE)

Holding period return:

$$\text{Total Stock Return} = (P_1 - P_0) + D/P_0 \quad (1)$$

Where, P_0 – Initial Stock Price, P_1 – Ending Stock Price (Period 1), D - Dividends

Markowitz Mean Variance Portfolio

$$\sigma^2 = \sum (x_i - \bar{x})^2 / N \quad (2)$$

The study has been conducted on the Python platform, starting with analyzing the daily closing price, computing the expected returns and volatility, and finally drawing an efficient frontier for building an optimal portfolio. This approach is famously called the Markowitz Mean Variance Portfolio Theory, which enables investors to quantitatively balance out their tolerance for risk and return anticipation.

Portfolio Return

$$E(R_p) = W_1 E(R_1) + W_2 E(R_2) + W_3 E(R_3) \dots \dots \dots W_n E(R_n) \quad (3)$$

Portfolio Risk (Portfolio with two securities)

Portfolio Variance

$$\sigma_p^2 = W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + 2(W_1 W_2 \sigma_1 \sigma_2 r_{12}) \quad (4)$$

Portfolio Standard Deviation

$$\sigma_p = \sqrt{W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + 2(W_1 W_2 \sigma_1 \sigma_2 r_{12})} \quad (5)$$

The study further computes the expected future prices of these stocks and predicts the probability of loss or gain from investing in the portfolio through the simulation approach of Monte Carlo, using which the foreseeable future is repeatedly predicted using a Monte Carlo simulation (Murtha, 1997). Several or hundreds of millions of “random tests” result in an assortment of results towards the completion of the modeling, which may then be analyzed (Etessami, K., & Yannakakis (2000).

3. 1. PLAN OF ANALYSIS

Portfolio theory-based deterministic optimization algorithm concept, which is a way for increasing earnings while decreasing predicted volatility (Shuai Liu and Chenglin Xiao, 2021). The paper also presents a probabilistic approach based on Monte Carlo simulation for estimating the likelihood of various options in a process that is difficult to predict due to the presence of random variables. It is a technique for understanding how uncertainty and risk affect things. The Python script implements Markowitz portfolio optimization theory and Monte Carlo simulation

3. 2. ALGORITHMS

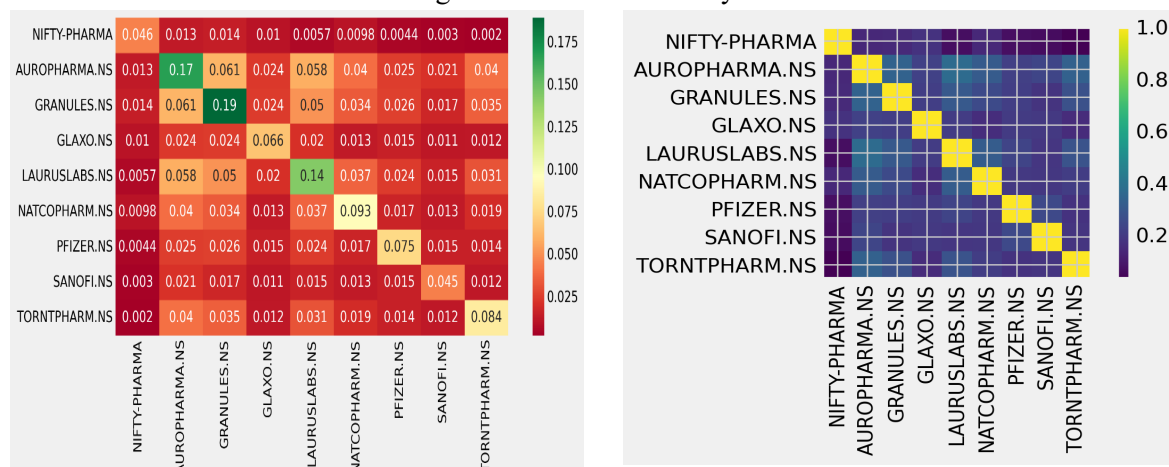
The two algorithms used in the study are the Markowitz Portfolio Theory-based deterministic optimization algorithm concept which is a method for raising earnings while lowering calculated volatility. The paper also develops a probabilistic approach using Monte Carlo simulation, and it is used to estimate the probability of various possibilities in a process that is difficult to anticipate since it involves random variables. It is a method for comprehending how uncertainty and risk impact things. Markowitz Portfolio Optimization Theory and Monte Carlo simulation is run on the Python script.

4. RESULTS AND DISCUSSIONS

4. 1. COVARIANCE MATRIX

A covariance matrix map is a graphic depiction of a collection of variables' covariance matrix. The covariance between groups of data points in a set of data, or covariance matrix, is symbolized by a square matrix. Every differential's variance is represented by the matrix's diagonal, whereas the entries off the outer edge are the covariances among the parameters. In this stock market system, covariance evaluates how closely the share values of two or more stocks move in lockstep. The presence of a positive covariance suggests that the prices of the two stocks are likely moving in the exact same orientation, whilst an adverse covariance suggests that the prices of the two stocks are likely moving in the other way. With the use of the closing price data and the covariance matrix, it is possible to conclude that all the stocks in the Nifty Pharma Index move in a manner that is similar to one another because of the positive covariance among them. This indicates that the stocks do well collectively, and that they may even decline collectively. Let's look at Sanofi and Torrent pharmaceuticals in figure 1. These two businesses have a lot of parallels in the way their closing prices have moved. Similar behavior is also displayed by other businesses including Granules, Laurus labs and NATCO Pharm.

Figure 1. Covariance Analysis

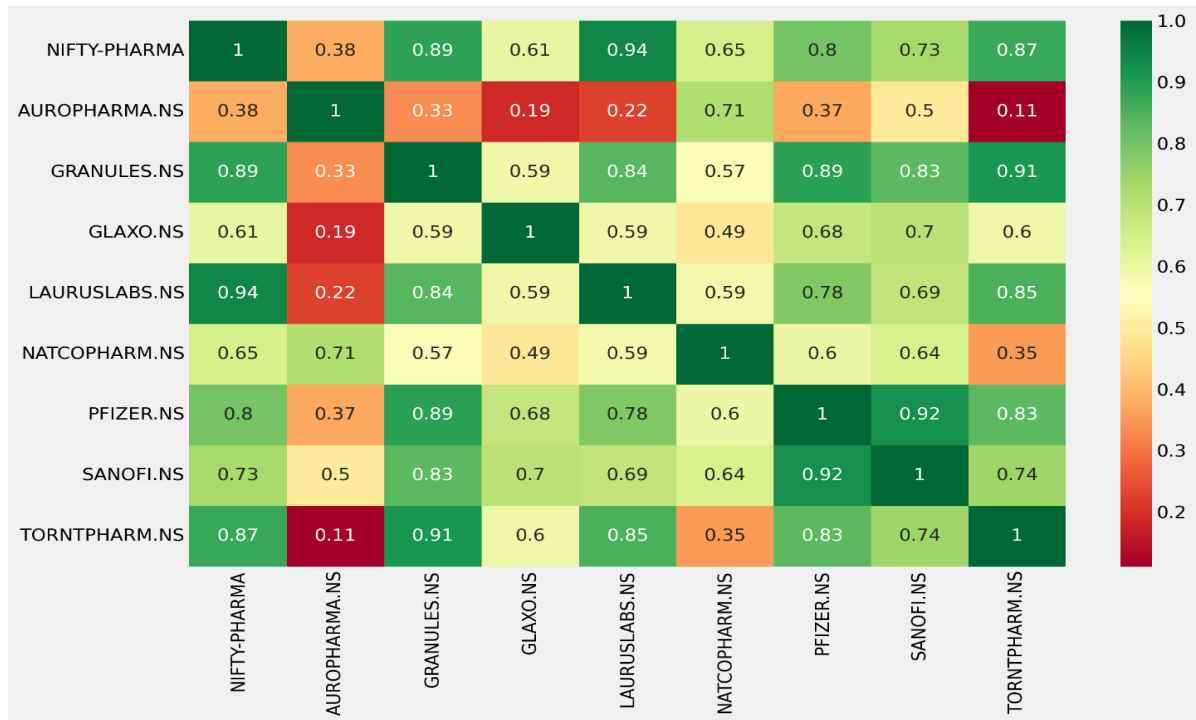


Source: Author's Calculation in Python

4. 2. CORRELATION MATRIX

Correlation coefficients have a range of -1 to +1. If the two stocks frequently rise and fall together, the correlation is good. If one stock increases while the other falls, the Pearson correlation coefficient is negative. Stocks are assessing the same quality when their correlation coefficients are strong and positive. In the event that there is little correlation between them, the stocks may measure distinct traits or not be properly defined.

Figure 2. Correlation Matrix through heatmap



Source: Author's Calculation in Python

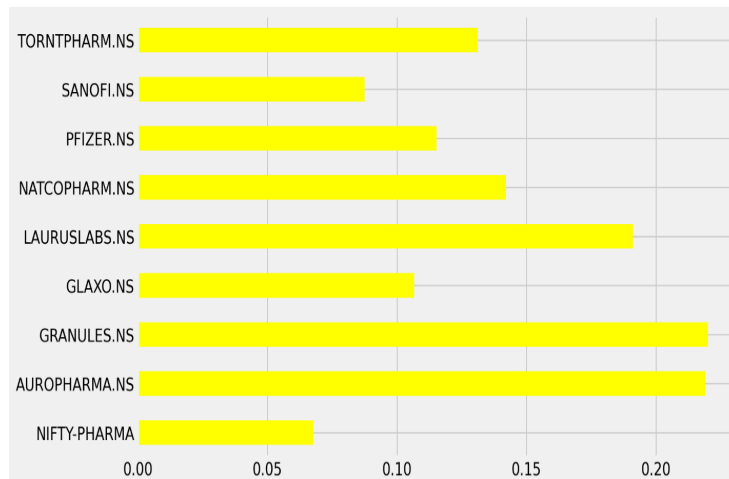
Only positive correlation values are shown in the correlation matrix which is given in figure 2. This suggests that the stocks in this index move in unison rather than counter-clockwise.

4. 3. RETURNS OF THE NIFTY PHARMA INDEX-LISTED COMPANIES

Expected returns of NIFTY Pharma stocks

A detailed picture of the anticipated returns for the stocks in the NIFTY Pharma index is provided by in figure 3. Auropharma and Granules are found to have the highest predicted returns, both at 22%. The lowest predicted returns are 7% and 8%, respectively, for NIFTY Pharma and Sanofi. Average projected returns for businesses like Glaxo, Laurus Labs, NATCO Pharma, Pfizer, and Torrent Pharma are 10%, 19%, 14%, 12%, and 13%, respectively.

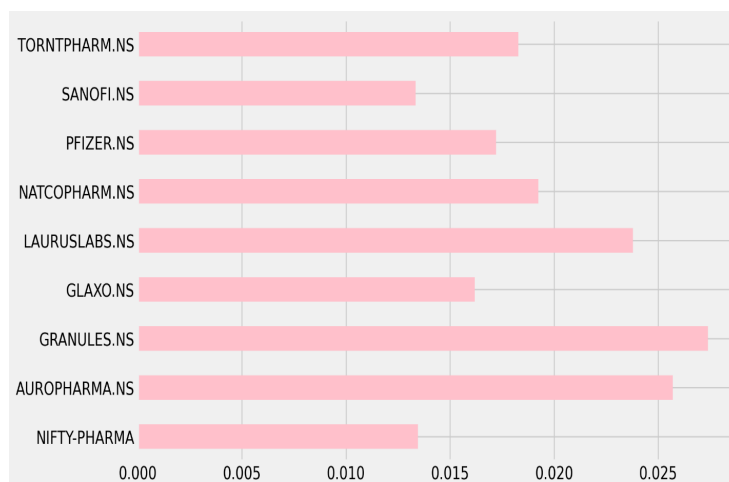
Figure 3. Expected returns of NIFTY Pharma stocks



Source: Author's Calculation in Python

Volatility is the rate of change in a stock's selling price over a predetermined period of time which is provided in figure 4. Risk is usually correlated with stock price volatility, which helps investors predict probable swings in the future. It is an important determinant of stock market risk and is essential in determining the value of assets. Investors generally avoid volatile equities because they incur the risk of experiencing significant losses if the price of the stock drops sharply.

Figure 4. Standard Deviation NIFTY Pharma stocks



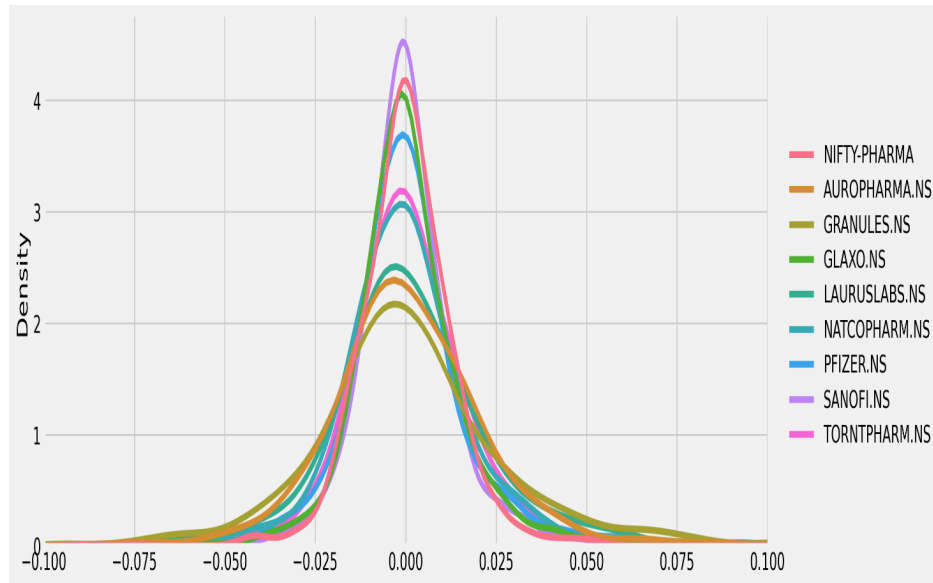
Source: Author's Calculation in Python

It is very evident that Granules is the riskiest company with the highest rate of volatility. As a result, the above stocks experience significant short-term volatility, making new spikes and crashes

4. 4. DENSITY PLOTS OF DAILY RETURNS OF STOCKS DEPICTING STANDARD DEVIATION AND VOLATILITY

When we compare the density plots of the daily returns of NIFTY Pharma stocks, we can see that the NIFTY Pharma index itself along with Sanofi has a narrower curve with a higher peak which indicates less risk or less volatility. On the other hand, Granules has a wider curve indicating higher standard deviation and hence volatility which is provided in figure 5.

Figure 5. Density plots of daily returns of stocks depicting standard deviation and volatility



Source: Author's Calculation in Python

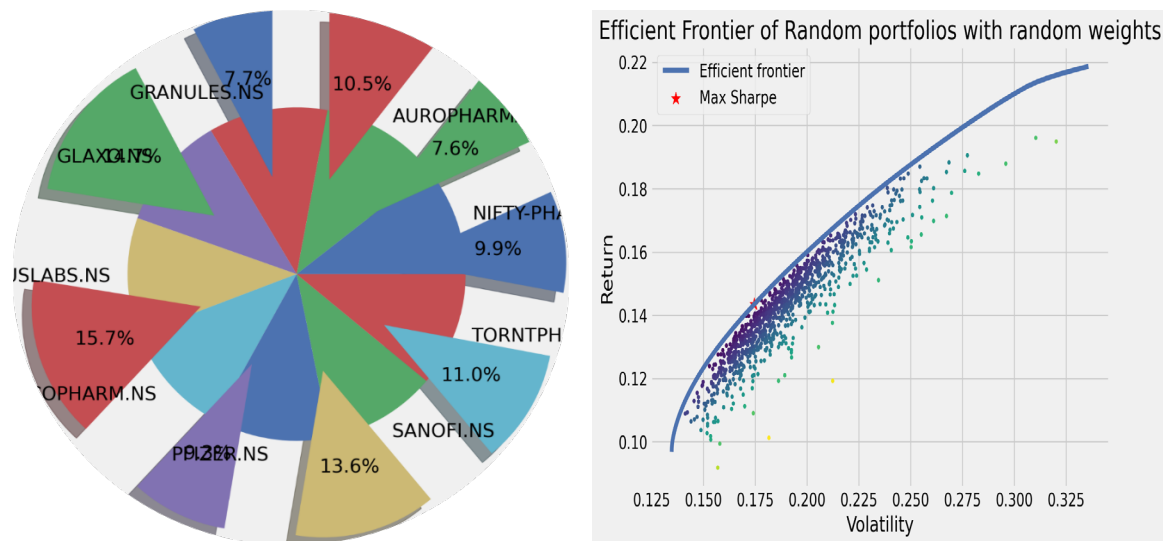
4. 5. MARKOWITZ MEAN VARIANCE PORTFOLIO

The Efficient Frontier curve plot:

A collection of investment portfolios known as an efficient frontier is supposed to offer the best returns at a specific risk threshold. If nothing else in the portfolio provides greater earnings for less or comparable degree of risk, then a portfolio is said to be efficient. The degree of risk tolerance of the investor determines where portfolios fall on the efficient frontier. A curving line is used to symbolize the efficient frontier. This is such that any rise in risk is accompanied by proportionally lower profits. In a nutshell, the diminishing optimum yield to risk causes the curvature.

An efficient frontier curve can be seen when projected returns from a portfolio are plotted against return standard deviation. On the y-axis, the portfolio's expected earnings are depicted. The x-axis displays the volatility of returns, a risk indicator. Here we have constructed the optimal frontier curve for 1000 randomized portfolios of the NIFTY Pharma index, with different weights assigned to each asset in the portfolio, taking into consideration additional components such as daily returns, expected returns, covariance, correlation, and volatility. The graph below illustrates how the efficient frontier resembles a border. Because higher projected returns but the same amount of volatility as on the frontier would result, portfolios cannot be above the frontier. Furthermore, portfolios cannot be located on the left side of the border since they would have lower volatility but the same expected returns as those located on the frontier. This suggests that a portfolio that includes NIFTY pharma index companies will be the best possible portfolio.

Figure 6. Efficient Frontier Curve



Source: Author's Calculation in Python

The link between an equity's return and risk is assessed using the Sharpe ratio. This is a mathematical expression of the idea that excess yields over a period of time may suggest increased risk and uncertainty rather than investment ability. The Sharpe ratio is one of the most widely used methods for determining risk-adjusted relative returns. It compares the portfolio's historical or predicted performance in regard to a financial benchmark with the past or prospective unpredictability of such returns. Typically, a Sharpe ratio of one to two is thought to be ideal. A ratio over 2 between two and 3 is very good, and everything greater than three is outstanding.

In the graph above, 1000 randomly chosen portfolios have a maximum Sharpe ratio that ranges between an 14% return and a volatility of 17%. We now choose one of the 1000 random portfolios with the optimum weights applied for each firm to make it an optimal portfolio: Here, we can see that some weights, such as those for Granules, Glaxo and Torrent pharma, tend to be populated by well-performing companies (high returns with acceptable volatility), whilst other weights tend to be lower in terms of returns and volatility.

Table 1. Performance Analysis of Selected Companies

Expected annual return: 14.34%
Annual volatility: 17.45%
Sharpe ratio: 0.71

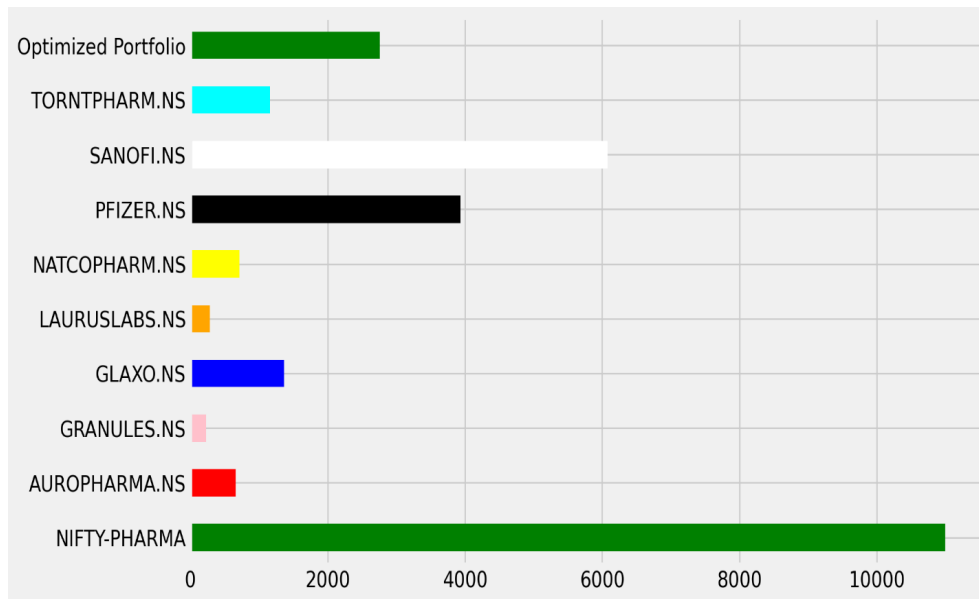
Source: Author's Calculation

The computed anticipated annual return, yearly volatility, and maximum Sharpe ratio for a portfolio with the aforementioned weights assigned to it will be 14.34%, 17.25%, and 0.7, respectively. The graph below shows the total returns of this arbitrary portfolio that we constructed using arbitrary weights.

Monte Carlo Simulation for Probability of Gain and Predicting Future Stock Prices:

With an initial investment of INR 100000, we ran 5000 Monte Carlo simulations in order to simulate the NIFTY Pharma portfolio. Figure 7 below shows the results of 5000 simulations of portfolio returns.

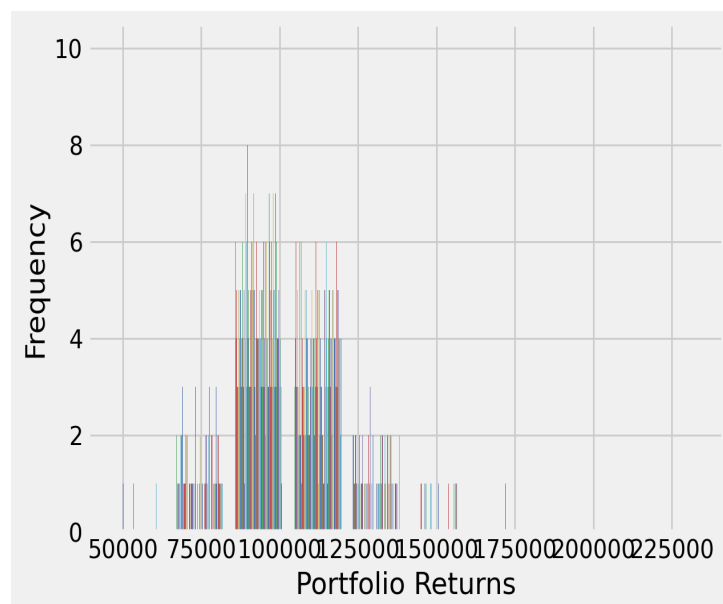
Figure 7. Simulated Portfolio Returns



Source: Author's Calculation in Python

Here is another visual representation of the portfolio returns based on 5000 simulations. The graphs demonstrate that portfolio returns typically range from 75000 to 125000 and are extremely unlikely to exceed 150,000. In order to forecast the anticipated mean end price, we first determined the portfolio returns for 5000 simulations. Here is a bar graph in figure 8 showing the same information.

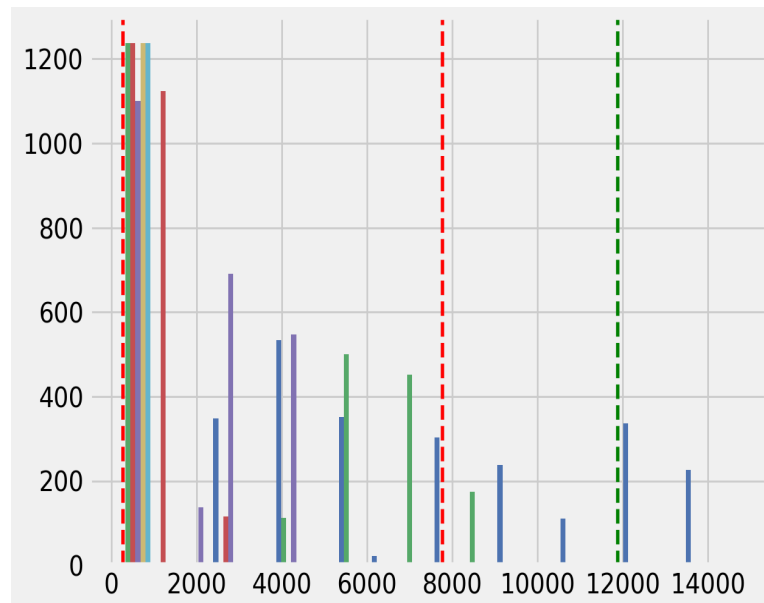
Figure 8. Portfolio Returns



Source: Author's Calculation in Python

By dividing the distribution into percentiles and selecting the top and bottom 10% of possible outcomes, the histogram below figure 9 represents the projected future end price of the NIFTY Pharma index:

Figure 9. The projected future end price of the NIFTY Pharma index



Source: Author's Calculation in Python

Here, we can see that, with the predicted price hovering around 11000 and just a 30% possibility of the price falling below 8000, there appears to be more downside than an upside.

We will now calculate the cumulative returns for our INR 100000 investment. The stock portfolio's daily returns and initial investment are used to determine cumulative returns. The below graph illustrates the movements of cumulative returns of all the stocks

5. FINDINGS

The study extracts a few highlights utilizing a stochastic approach that combines Markowitz Portfolio theory and Monte Carlo simulations, aiding in the creation of an ideal portfolio. Through the stochastic approach of using both Markowitz Portfolio theory as well as the Monte Carlo simulations, the study extracts few highlights which thereby helps in forming an optimal portfolio. The study of a portfolio containing NIFTY Pharma stocks, we can observe that companies like Auro pharma, Granules and Laurus labs have the highest expected returns and hence they also come with relative volatility. They can be called the good performing stocks of NIFTY Pharma index. The Markowitz efficient frontier curve also gave a picture of random portfolios consisting of NIFTY Pharma stocks which have the potential to be an optimal portfolio if they would be assigned with appropriate weights. For this portfolio the maximum Sharpe ratio lied between 14% return and 17% volatility. After assigning weights to the stocks in the portfolio, which was purely based on the expected return and volatility of each stock, the study found out that, this portfolio gave an expected annual return of 14.34% with annual volatility of 17.45% and a Sharpe ratio of 0.7. This is quite a good return and like the efficient frontier curve depicted. Given that the ideal Sharpe ratio is between 1 and 2, and that the study's determined Sharpe ratio was 0.7, the portfolio fails to meet the Sharpe ratio criteria for the optimal portfolio.

Moving on to the Monte Carlo simulation, the study has predicted the mean end price of the NIFTY Pharma stocks in next 1-5 years. The simulation has predicted the end price of NIFTY Pharma index to be 10996.94 in the next 1-5years. It can also be observed that with the predicted price hovering around 11000 and just a 30% possibility of the price falling below 8000 over the next five years, there appears to be more downside than an upside.

6. CONCLUSION

The study's above-extracted conclusion draws a framework for the NIFTY Pharma index computations, which allows investors to determine whether the Nifty Pharma index would be able to offer better returns for lesser risk. Overall, the study concludes that NIFTY Pharma index offers a higher return of 14.35. Additionally, NIFTY Pharma portfolio's volatility is considerably higher. The study draws the conclusion that the NIFTY pharma portfolio is more suitable for those investors who have an appetite for risk. Also, the study has identified good performing companies' stock in terms of higher return and less volatility such as Auro pharma, Granules and Laurus labs. Hence, the outcome would be helpful to the investors before making an investment decision in the pharmaceutical sector.

7. LIMITATIONS AND FUTURE SCOPE FOR THE STUDY

The current study analyzing the efficiency of portfolio using Markowitz Portfolio theory and also determine the probability of gain, price of NIFTY indices (Healthcare and Pharma) using Monte Carlo simulation. Data on the Nifty Pharma companies share price is restricted only for four years. Further, researcher may extent time period to check the efficiency of Nifty pharma portfolio by taking larger time period in the long run aspect. Also, the study is only restricts to pharmaceutical companies in Indian stock market. Further, the researcher may also analyze portfolio efficiency in the other sector such as FMCG, Bank, Automobile etc. by considering other emerging countries.

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