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Mathematical Connection is at the Heart of Mathematical Creativity

Ali Bicer[1], **Aysenur Bicer**[2], **Mary Capraro**[3], and **Yujin Lee**[4]

1 University of Wyoming, United States, ORCID: 0000-0002-2147-5393
2 University of Wyoming, United States
3 Texas A&M University, United States
4 Kangwon National University, South Korea

ABSTRACT

Although teaching mathematics for creativity has been advocated by many researchers, it has not been widely adopted by many teachers because of two reasons: 1) researchers emphasized and investigated mathematical creativity in terms of product dimension by looking at what students have at the end of problem-solving or -posing activities, but they neglected the creative processes students use during mathematics classrooms, and 2) creativity is an abstract construct and it is hard for teachers to interpret what it means for students to be creative in mathematics without further guidance. These can be eliminated by employing techniques of mathematical connections as tools because using mathematical connections can help teachers make sense of how to promote the creative processes of students in mathematics. Because making mathematical connections is a process of linking ideas in mathematics to other ideas and this is a creative act for students to take to achieve creative ideas in mathematics, using the strategies of making mathematical connections has the potential for teachers to understand what it means for students to be creative in mathematics and what it means to teach mathematics for creativity. This paper has two aims to 1) illustrate strategies for making mathematical connections that can also help students' creative processes in mathematics, and 2) investigate the relationship among general mathematical ability, mathematical creative ability, and mathematical connection ability by reviewing theoretical explanations of these constructs and several predictors (e.g., inductive/deductive ability, quantitative ability) that are important for these constructs. This paper does not only provide examples and techniques of mathematical connection that can be used to foster creative processes of students in mathematics, but also suggests a potential model depicting the relationship among mathematical creativity, mathematical ability, and mathematical connection considering previously suggested theoretical models. It is important to note that the hypothesized model (see Figure 4) suggested in the present paper is not tested through statistical analyses and it is suggested that future research be conducted to show the relationship among the constructs (mathematical connection, mathematical creativity, mathematical ability, and spatial reasoning ability).

KEYWORDS:

mathematical connection, mathematical creativity, mathematical ability

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Corresponding author at:

Ali Bicer Associate Professor of Mathematics Education School of Teacher Education
University of Wyoming
E-MAIL: abicer@uwyo.edu

INTRODUCTION

Creativity has been listed as one of the most important twenty-first-century skills (Wegerif & Dawes, 2004) including critical thinking, problem-solving, decision-making, and collaboration (Maass et al., 2019). Although these identified skills, particularly creativity, are included as major educational goals, detailed descriptions of how teachers can implement practices that have the potential to support their students' creative insights in different disciplines, specifically in mathematics, are lacking (Bicer, 2021; Binkley et al., 2012). Traditional instructional practices (e.g., teacher-centered instruction, learning through repetition), which encourage rote learning and procedural knowledge have been no longer appreciated by the National Research Council (2001) and such practices have been found to stifle the creative ideas of students in mathematics (Bicer, 2021). Unfortunately, mathematics is often viewed as a discipline in which individuals need to memorize formulas, rules, or step-by-step procedures to execute problems, with either no or little connection to real life (Boaler, 2015). When students see mathematics as a discipline including a set of unconnected rules and procedures, "they cannot see the role for their own inner growth and learning" (Boaler, 2015, p. 34) and they cannot find opportunities to discover their creative potentials in mathematics (Bicer, 2021). Because mathematics is more than just a set of unconnected rules and procedures, it should be taught in a way that students comprehend its beauty and utility by observing mathematics as a discipline of patterns that require aesthetic and creative performance, and it helps us make sense of the world around us (Devlin, 2000). To convey the message to students that mathematics is a creative field in which they can be creative and enable them to both discover and manifest their creative potential in mathematics (Bicer, 2021), researchers suggested various instructional tasks and practices including employing open-ended tasks (Sullivan et al., 2000), asking students to pose mathematical problems (Silver, 1997), and converting routine tasks into creativity directed tasks (Bicer et al., 2022). Although these practices (i.e., problem-posing, and problem-solving) were found to be helpful for students to achieve creative ideas (Bicer et al., 2020), the way researchers adopted such practices generally emphasized the product dimension of mathematical creativity and mostly neglected the process dimension that leads to creative products in mathematics (Schindler & Lilienthal, 2019). Because creativity is an abstract construct and we do not have a complete picture of what processes help students achieve creative ideas in mathematics (Schindler & Lilienthal, 2019), teachers do not fully understand how to promote the creative processes of students in their mathematics classrooms while trying to cover mandated state or national standards (Bicer et al., 2022). In addition, many teachers believe that mathematics does not include much room for creativity (Boaler, 2015; Shriki, 2010), and they believe that the elementary grades are too early for creativity (Sternberg, 2014). These conceptual barriers can be eliminated when teachers are provided with concrete tools (e.g., strategies for making mathematical connections) that they are already familiar with and are guided about how to use such tools to foster the creative processes of students in mathematics. To our best knowledge, although there have been no studies investigating the relationship between mathematical creativity and mathematical connection, the tools suggested for making mathematical connections can be used to foster students' creative ideas in mathematics. Because using mathematical connections will provide teachers with a concrete tool to understand what it means for students to be creative in K-12 classrooms, they can support the creative processes of their students by applying specific techniques. Therefore, we aim to elaborate on the possibility of combining mathematical connection and mathematical creativity to promote the creative processes of students in mathematics classrooms by using theoretical and practical applications of both constructs.

LITERATURE REVIEW

Mathematical Connection Ability

A mathematical connection has been described as "the junctures, or nodes, can be thought of as pieces of presented information, and threads between them as the connections or relationships" (Hiebert & Carpenter, 1992, p. 67). Mathematical connection ability is the ability to connect mathematical ideas to other mathematical topics, other fields, and real life (Haji & Yumiati, 2018; House & Coxford, 1995; National Council of Teaching of Mathematics [NCTM], 2000). Researchers (e.g., Doleres-Flores et al., 2019; House & Coxford, 1995) identified additional ways enabling individuals to make mathematical connections. For example, House and Coxford (1995) specified that connecting conceptual and procedural knowledge of mathematics is an additional way to make a mathematical connection. However, because connecting procedural and conceptual knowledge of mathematics can be considered as one of the other three ways (i.e., connecting mathematical ideas to other ideas in mathematics and other scientific disciplines and real life), we did not consider connecting conceptual and procedural knowledge as an additional way in the present study, but we consider it as a specific tool to make a mathematical connection. For example, when students connect repeated addition to multiplication and later connect the array model of multiplication to the area formula of rectangular shapes, they connect mathematical ideas across mathematical topics by using the relationship between their conceptual knowledge of a rectangular area formula (the number of squares to cover a given rectangular space) and procedural knowledge (the formula of a rectangular area: base times height). Similarly, researchers (e.g., Doleres-Flores et al., 2019; House & Coxford, 1995) specified additional ways of making mathematical connections, such as recognizing equivalent representations of the same concepts, applying mathematical modeling to solve cognitively challenging problems, and reversibility of mathematical ideas. In the present study, we consider these as tools that enable students to make mathematical connections as these can be classified under one of making general mathematical connection ways: connecting mathematical ideas to ideas across mathematics, connecting mathematical ideas to ideas in other fields, and connecting mathematics ideas to real-life situations (Haji & Yumiati, 2018; House & Coxford, 1995; NCTM, 2000). For example, applying mathematical modeling can be classified as an ability to make a mathematical connection to real life because the mathematical problems that are designed based on the principles of mathematical modeling need to promote students' connection abilities to solve real-world problems (Lu & Kaiser, 2022). Broadly, making mathematical connections was classified as intramathematical connections (connecting mathematical ideas, models, concepts, definitions, theorems, procedures, representations, or meaning within mathematics) and inter-mathematical connections (connecting mathematical ideas, models, concepts, definitions, theorems, procedures, representations or meanings with other disciplines or real-world) (Doleres-Flores et al., 2019; Garcia-Garcia & Dolores-Flores, 2018; House & Coxford, 1995).

Mathematical connection is not a method, an educational theory, or a product students come up with at the end of mathematical activities, but it is a process through which students build mathematical ideas (Jawad, 2022). It is a cognitive process in which a person finds real relationships among ideas, concepts, definitions, theorems, procedures, and representations either within mathematics or with other disciplines or real-world (Dolores-Flores et al., 2019; Garcia-Garcia & Dolores-Flores, 2018). Other than considering mathematical connection as a process of doing mathematics, researchers (e.g., Evitts, 2014; Singletary, 2012) interpreted it as a product of individual mathematics understanding (Singletary, 2012). The product view of mathematics understanding can be considered as an outcome of the process of making mathematical connections (Businskas, 2008). This can be supported by Evitts' (2004) statement that students' connecting mathematical ideas can be described as their construction of mathematical knowledge through creating links between mathematical concepts and procedures within mathematics and with the concepts and procedures in other disciplines or real-life and the process of making such connection enables students to achieve deep and meaningful mathematics as an end product. This can be explained by the constructivist approach as an individual student's mathematics learning is established or strengthened by creating a relationship among the connected groups of schemes in a mental network (Eli et al., 2011). The NCTM (2000) has emphasized the importance of mathematical connection as an integral part of mathematics classroom practice by listing it as one of the five process standards (i.e., mathematical connection, mathematical communication, mathematical representations, mathematical reasoning, and problem-solving) that needs to be integrated into mathematics classrooms at every level of education. Moving from an individual level of making a mathematical connection (i.e., constructivist approach) to a classroom level as suggested by NCTM (2000) that mathematical connection should be a process in all mathematics classrooms, it is possible to explain mathematical connections from the socio-constructivist approach because students' ability to make mathematical connections can be influenced by their classroom cultures (e.g., working collaboratively to solve problems), norms (e.g., what is considered as making a mathematical connection), and others (i.e., peers, teachers) (Cobb, 2007).

Connecting mathematical ideas is an essential ability that needs to be developed in the process of students' mathematics learning and problem-solving in mathematics classrooms (NCTM, 2000). When students engage in connecting mathematical ideas during problem-solving, they can see mathematics as a connected discipline rather than a discipline that includes a set of disconnected and separated concepts and methods (Boalar, 2016; NCTM, 2000). In addition, many cognitively demanding mathematical problems require students to link mathematical concepts with other mathematical concepts, concepts in other fields, and ideas in daily life to solve such problems by recalling their previous mathematical and non-mathematical ideas and experiences, and interpreting their prior knowledge and experiences in a new context (Menanti & Sinaga-Hasratuddin, 2018). Meaningful mathematics learning occurs when students use their previous mathematical and non-mathematical knowledge and experiences to solve cognitively demanding mathematical problems, and this process often leads students to construct new mathematical knowledge (Bicer et al., 2013). From the constructivist approach (Cobb, 2007), without enabling students to connect mathematical ideas to other ideas in mathematics, other fields, and their real life, learning deep and long-lasting mathematical ideas and concepts becomes unattainable because otherwise, students must remember too many mathematical concepts and principles (Boalar, 2016; Menanti, & Sinaga-Hasratuddin, 2018, NCTM, 2000). Other than the cognitive benefits of mathematical connection, it also has affective benefits on students' mathematics learning because students who engage in mathematical connections develop confidence in their mathematical abilities and develop a higher awareness of the benefits of mathematics and its applications (Hidayah & Kurniasih, 2019).

Despite the potential of making mathematical connections in developing students' cognitive and affective outcomes, researchers in many countries (e.g., South Africa, Thailand, and Indonesia) observed that many students have difficulty connecting between mathematical concepts known by students and mathematical and non-mathematical concepts that they will learn (Jaijan & Loipha, 2012; Mhlolo, 2012; Saminanto & Kartono, 2015; Siregar & Surya, 2017). For example, the results of a sample of high school students' mathematical connection ability in Indonesia showed that only 53% of these students could connect mathematical ideas to other mathematical and non-mathematical ideas (Sugiman, 2008). The results were similar for a sample of elementary school students in Indonesia, as these students received a mean score of 10.87 from a maximum value of 24 on the mathematical connection ability test (Hermawan & Prabawanto, 2015). These results supported Sawyer (2008) who found students do not automatically connect new mathematical ideas to other mathematical and non-mathematical concepts known by themselves, but they need to be trained to make such connections.

Sugiman (2008) investigated the reasons many high school students lacked the ability to make mathematical connections by interviewing their teachers. Three reasons emerged: 1) many students have difficulties in solving problems related to real life because they memorize mathematical concepts and procedures, 2) many students are given mathematical tasks that are not connected to students' residence culture and these tasks do not require students to make mathematical and non-mathematical connections, and 3) many students are taught mathematics through direct instructions. Considering these findings, students' abilities to make mathematical connections are somewhat influenced by what kinds of mathematical tasks and instructional practices their teachers implement. Numerous researchers around the world also supported these reasons (e.g., Jaijan & Loipha, 2012; Mhlolo, 2012). For example, Mhlolo (2012) showed that a sample of students in South Africa developed their mathematical connection ability when their teachers implemented student-centered learning in their mathematics instruction. Similar results came from a sample of students in Thailand as these students demonstrated an increase in their mathematical connection ability when their teachers designed their classes according to active learning environment features (Jaijan & Loipha, 2012). Sawyer (2008) extended these findings as he worked with two novice teachers from North America to understand how their instructional practices promote students making mathematical connections and he provided several strategies for teachers to help their students develop mathematical connections in mathematics classrooms. Strategies suggested were: 1) assisting students' competencies in applying several mathematical strategies while solving problems, 2) requiring students to select the appropriate mathematical procedures and knowledge rather than providing them with solution procedures, necessary knowledge, and materials to solve mathematical problems, 3) expecting students to explain and justify their solution methods, 4) encouraging students to use ideas from other disciplines and real life while solving mathematical problems, 5) scaffolding students to re-frame mathematical ideas or information from other disciplines or real life so that they can express their reasoning by using specialized mathematical language, 6) promoting making connections by responding positively when students identify connections in mathematics, other disciplines, and real life. Sawyer (2008) also noted that not all these strategies were used by both novice teachers to the same degree, but both teachers who were encouraged to apply mathematical connections in their classrooms found that "making connections was fundamental to mathematics education influenced their teaching practice in several important ways" (p. 434). This was also confirmed by Sawyer's (2008) classroom observations as he stated that "the effectiveness of their practice was evidenced by examples of students in classes taught by these teachers who demonstrated the ability to make connections between mathematical knowledge and other forms of disciplinary knowledge, and between mathematical knowledge and real life" (p. 434).

Researchers stated teachers should consider several different techniques or tools for making mathematical connections to encourage students to make mathematical connections in different contexts (Doleres-Flores et al., 2019). This is because while some students were able to make mathematical connections of the procedural type, they rarely made mathematical connections of the generalization type (Doleres-Flores et al., 2019). The types of tools for making mathematical connections were reported as representational, procedural, reversible, and meaningful (Garcia-Garcia & Dolores-Flores, 2018). Later, Rodriguez-Nieto et al. (2022) extended these tools for making mathematical connections into nine categories as different representations, implications, part-whole, procedural, instruction-oriented, feature, meaning, reversibility, and metaphorical. Although these tools were categorized under intra-mathematical connections, they can be used to connect ideas in mathematics to ideas in other scientific disciplines and real life. In addition, because mathematical modeling is commonly used in classrooms, it is important to state its importance one more time to make inter-mathematical connections (connection mathematical ideas to other ideas in real life and other disciplines). Students apply mathematical connections while engaging in mathematical tasks that are convenient to apply these tools (e.g., different representations, features, mathematical modeling). Using such tools has the potential to let teachers convert routine mathematical tasks into creativity-directed tasks. Creativity-directed tasks are tasks that enable students to manifest and develop their creative thinking skills while engaging in solving mathematical tasks (Bicer, 2021), and the process of making mathematical connections is an integral part of achieving creative ideas in mathematics. Before providing some examples showing how tools for making mathematical connections have the potential to foster the process of achieving creative ideas in K-12 students, providing the definition of mathematical creativity enables us to comprehend the theoretical interconnectedness of making mathematical connections and achieving creative ideas in mathematics.

Mathematical Creativity in the Context of Multiple Solution Tasks

There has been no universally accepted definition of mathematical creativity (Mann, 2006). This is because the definitions of mathematical creativity were derived from the definitions of general creativity and general creativity is a multifaceted construct (Rhodes, 1961). The 4-P framework suggested by Rhodes (1961) includes person (who is creative), process (what environment stifles or fosters creative ideas), press (how someone can be creative), and product (what outcomes are considered as creative) demonstrating the multidimensionality of the creativity construct. Because the definitions of mathematical creativity are derived from the features of general creativity, having no universally accepted definition of mathematical creativity is not surprising as different researchers focused on different dimensions of creativity (Bicer, 2021). For example, while Haylock (1987) emphasized pattern recognition when defining mathematical creativity as an "ability to see a new relationship between techniques and areas of applications and make associations between possibly unrelated ideas" (p.60), Eryvnck (1991) emphasized deductive reasoning ability when defining it as an "ability to solve problems and/or develop thinking in structures, taking account of the peculiar logico-deductive nature of the discipline, and of the fitness of the generated concepts to integrate into the core of what is important in mathematics" (p. 47). Although there is no consensus among researchers on the definition of mathematical creativity, there have been common elements among the provided definitions. Bicer (2021) conducted a systematic literature review to synthesize the provided definitions of mathematical creativity from the previous studies (e.g., Chamberlin & Moon, 2005; Eryvnck, 1991; Sriraman, 2005) to identify what common elements of mathematical creativity are covered in those provided definitions. This review revealed that mathematical creativity in the field is considered as generating "novel mathematical ideas or products, which are new to the person but may not necessarily be new to others" (Bicer, 2021, p. 3). From this definition, we understood that mathematical creativity in the field is perceived as discipline-specific (Kattou et al., 2013), product-centered (Schindler & Lilienthal, 2019; Silver, 1997), and an active rather than a static characteristic of students (Bicer, 2021; Leikin, 2009). Because researchers in the field mostly considered mathematical creativity from the product dimension, they heavily focused on how to measure students' creative outcomes (Bicer et al., 2022; Schindler & Lilienthal, 2019). Researchers mostly applied problem-posing (Bicer et al., 2020; Silver, 1997) and problem-solving (Levav-Wayberg & Leikin, 2012; Silver, 1997) tasks both as psychometric tools to measure mathematical thinking of students and as interventions to foster creative ideas during mathematics instruction.

Through problem-posing and problem-solving tasks, researchers (e.g., Bicer et al., 2020; Leikin, 2018; Leikin & Elgrably, 2019) aimed to measure three indicators (i.e., fluency, flexibility, and originality) as adopted from Torrance's (1974) general creativity framework. Posing problems and solving mathematical problems in multiple ways are also listed as creativity-directed tasks as students develop their creative ideas in mathematics through these tasks (Leikin & Elgrably, 2019). Bicer et al. (2020) stated that a task in the context of problem-solving should be open-ended to be considered a creativity-directed task. Open-ended tasks are tasks that have more than one solution path and more than one correct answer (Sullivan et al., 2000). A task does not have to have more than one correct answer to be used in the context of mathematical creativity, but it is necessary and sufficient that a task has more than one solution path (Leikin, 2009). Multiple solution tasks explicitly require students to solve mathematical problems in different ways and these tasks can be used both as a psychometric tool measuring students' creative thinking and as an intervention to develop students' creative thinking in mathematics (Leikin, 2009). Students' creative products in the context of multiple solutions tasks are measured through the three indicators: fluency (the number of solution methods for a given problem), flexibility (the number of different solution methods for a given problem), and originality (the rareness of a solution path in a group or finding unique solution methods after examining several ones) (Levav-Waynberg & Leikin, 2012). These indicators can be observable in the context of multiple solutions tasks as students solve problems by 1) using different representations of mathematical objects, 2) different definitions or theorems of mathematical concepts, and 3) different properties of mathematical objects in different disciplines and real life (Leikin, 2009).

Researchers who applied multiple solutions tasks to measure students' creative thinking mostly employed geometrical problems as they heavily focused on multiple representations as one technique of multiple solution tasks (Leikin, 2013; Levav-Waynberg & Leikin, 2012; Levenson et al., 2018). Because geometry was heavily emphasized over other subfields of mathematics, Schindler et al. (2018) suggested that future research should investigate whether creativity in mathematics should be considered as a subspecific construct in mathematics (e.g., geometrical creativity, algebraic creativity, numerical creativity). Later, Bicer (2021a) explained this by suggesting that multiple representations are heavily used over the other two techniques of multiple solution tasks because most researchers applied geometry over other subdisciplines of mathematics as geometry provides ample opportunities for students to manifest their creative thinking through multiple representations. In addition, it was noted that multiple representations can be used as a standalone technique to enable students to manifest their creative thinking skills to others in mathematics (Bicer, 2021a). For example, Schindler and Lilienthal (2020) observed a student's creative thinking process in mathematics through an eye-tracking technique as their sample student was engaging in solving a geometrical task through multiple representations. They suggested that future research should investigate whether eye-tracking in the context of applying multiple representations to solve mathematical problems can be used to understand students' creative thinking processes in all subdisciplines of mathematics other than geometry. Later, Bicer et al. (2021a) showed that multiple representations can be used to measure students' mathematical creative thinking processes through the eye-tracking technique because multiple representations as one component of multiple solution tasks provide students ample opportunities to not only manifest their creative ideas to others in geometry, but also to develop and manifest their creative potentials in all subfields of mathematics. This is based on the idea that there have been no mathematical ideas or concepts that cannot be presented in multiple ways (Boaler et al., 2016). Although the creative thinking process dimension is very important to arrange mathematics classrooms in ways that lead students to reach creative products, it is not as heavily focused on as a creative product dimension in the field (Bicer & Bicer, 2022; Schindler & Lilienthal, 2020). "The aim to find measurement tools has led to a product-view on students' solutions, in which creative problem-solving processes are neglected" (Schindler et al., 2016, p. 1). Recently, there have been attempts to understand how creative ideas emerge in K-12 students as Schindler and Lilienthal (2020) and Bicer and Bicer (2022) conducted case studies to deeply understand the mathematical creative processes of K-12 students in mathematics and whether a sample of K-12 students' creative processes is similar to professional mathematicians' creative process. However, there is still a need for studies with large sample sizes identifying similarities and differences between professional mathematicians and K-12 students' creative processes in mathematics. Studies investigating what specific features of mathematical tasks and instructional practices foster creative processes in young students that lead them to achieve creative ideas in mathematics are necessary (Bicer & Bicer, 2022).

Professional mathematicians' creative process occurs in a few stages (Sriraman, 2004). A four-stage Gestalt model was proposed by Poincaré (1948): *preparation, incubation, illumination, and verification*. The first stage comprises putting great effort into getting insights into the problem at hand. Poincaré called this preliminary process "conscious work," and Hadamard (1954) also referred to it as the preliminary stage. The second stage, *incubation*, refers to putting the problem aside for a while and engaging the mind in solving other problems. The third stage, *illumination*, describes when the solution emerges suddenly while the individual is doing other unrelated activities. Finally, the last stage comprises *verifying* the result by making it precise and expressing these ideas in writing or oral form. During the last stage, verification, individuals seek other alternatives, extensions, or solutions (Hadamard, 1945). Sriraman (2004) noted that this model is still applicable today to describe professional mathematicians' creative process after interviewing professional mathematicians. The studies conducted by Schindler and Lilienthal (2020) and Bicer et al. (2022) revealed that although the creative processes of a sample of young students are like professional mathematicians' creative process (i.e., incubation, illumination, and verification), K-12 students' creative processes involve some vital differences. These differences are noteworthy to provide an optimal educational environment for students in which they can have plenty of opportunities to develop and manifest their creative potential. For example, Bicer & Bicer (2022) found out that inspiration is a vital stage in their sample of four young students' creative processes to manifest and achieve creative ideas in mathematics, but this was not reported as a stage of professional mathematicians' creative process. This particularly necessitates researchers to consider the mathematical creativity processes of K-12 students in a sociocultural context. The processes that lead students to achieve creative products in mathematics classrooms can be explained by both constructivist and socio-constructivist approaches (Bicer et al., 2020). When students individually engage in solving problems by applying multiple solution paths, they can construct new mathematical knowledge (Bicer et al., 2020). Mathematical creativity can be supported by the constructivist view; Vygotsky (2004) noted that "any human act that gives rise to something new is a creative act, regardless of whether what is created is a physical object or some mental or emotional construct that lives within the person who created it and is known only to him" (p. 7). When students collaboratively engage in solving problems creatively as a group or entire class, their peers' and teachers' input can influence their construction of mathematical knowledge and ideas (Bicer & Bicer, 2022). Creativity results from interactions of a system that involves three vital elements: "a culture that contains symbolic rules, a person who brings novelty into the symbolic domain, and a field of experts who recognize and validate the innovation" (Csikszentmihalyi, 1997, p. 6). In this regard, "creativity takes place through the interaction between a person's thoughts and a sociocultural context" (Young, 2021, p. 103).

Both product and process dimensions of mathematical creativity in K-12 classrooms are important because our aim is to provide an optimal educational environment in which young students find several opportunities to develop and manifest their creative insights. For this, we should ensure that our teachers know what mathematical creativity in K-12 mathematics classrooms is and be able to implement creativity-directed mathematical tasks and instructional practices throughout their mathematics instruction (Bicer, 2021; Bicer et al., 2021). Because mathematical creativity is an abstract construct, teachers should be provided with concrete examples and tools along with training about how to effectively integrate these examples and tools into their instructions to foster the creative processes of young students. When teachers effectively implement such tools in their instruction, their students can engage in creative processes that can lead them to reach creative products in mathematics. Although barely emphasized in the literature, investigating the association between mathematical connection and mathematical creativity in the context of multiple solution tasks can enable mathematics teachers and mathematics teacher educators to be better equipped with tools to promote the creative processes of students in mathematics. Using mathematical connection as a tool can help teachers both understand what mathematical creativity is in K-12 mathematics and enable them to integrate specific techniques of mathematical connections into their instructions ensuring the potential for creating students' mathematical learning.

Mathematical Connection and Mathematical Creativity

Although the relationship between mathematical connection and mathematical creativity in the context of multiple solution tasks has not been emphasized in the literature, the vitality of mathematical connection on mathematical creativity is obvious by the definitions of both constructs.

It is quite interesting that there is almost one-to-one matching with the criteria of mathematical connection and the criteria of mathematical creativity in the context of multiple solution tasks (See Table 1). For example, when students solve mathematical problems in multiple ways by employing different representations of mathematical objects or using different definitions or concepts, they make intra-mathematical connections as students connect mathematical ideas to other ideas across mathematics by using their multiple representations skills and/or prior mathematical knowledge. Similarly, when students solve mathematical problems in multiple ways by using tools from different fields and real life, this equates to making intra-mathematical connections as students connect mathematical ideas to other ideas in other disciplines and real life by using their prior knowledge and experiences in other disciplines and real life. Each tool for making mathematical connections (e.g., different representations, and implications) (Garcia-Garcia & Dolores-Flores, 2018) can be also used to foster the creative processes of students in mathematics.

Table 1. Matching Criteria of Mathematical Connection and Mathematical Creativity

	Mathematical Connection	Mathematical Creativity in the Context of Multiple Solution
Intra-mathematical connections	1) Connecting mathematical ideas to other ideas in mathematics.	1) Solving problems by using different representations of mathematical objects. 2) Solving problems by using different definitions, theorems, or mathematical concepts.
Inter-mathematical connections	2) Connecting mathematical ideas to ideas in other disciplines.	3) Solving problems by using tools from *different fields and real life*.
	3) Connecting mathematical ideas to ideas in real or daily life.	

Mathematical Connection Tools

We adopted mathematical connection tools by Garcia-Garcia and Dolores-Flores (2018). In the present study, these tools were adopted to emphasize how students can use these mathematical connection tools to manifest and develop their creative insights in mathematics. The ways we adopted these mathematical connection tools in the present study are somewhat different than how these were exactly identified in the literature (e.g., Garcia-Garcia & Dolores-Flores, 2018). For example, Garcia-Garcia and Dolores-Flores (2018) stated that using the formula for a derivative to find an answer is an example of making the procedural mathematical connection as students find the derivative of $f(x)=x^n$ by using the formula $df=n\times x^{n-1}$). However, in the present study, we defined procedural mathematical connection as creating connections of conceptual and procedural knowledge or understanding of mathematics because our purpose is to illustrate how students can use making mathematical connection tools to develop their conceptual understanding, mathematical reasoning, and creative insights in mathematics rather than solely developing a procedural understanding of mathematical concepts. In the present study, we particularly aim to illustrate how the processes of making mathematical connections can be used as a creative act leading students to generate new mathematical ideas, perspectives, and knowledge. It is also important to note the tools adopted by Garcia-Garcia and Dolores-Flores (2018) for making mathematical connections are not mutually exclusive from each other, but can be used interchangeably for one another. The reason these are classified under different categories is to emphasize different aims while making mathematical connections and how the creative processes of students can be supported through specific mathematical connection tools. Although these tools are reported specifically for making intra-mathematical connections (see Table 1), some of these can be also used to make inter-mathematical connections. For example, new mathematical ideas can not only be linked with mathematical ideas covered in previous mathematics instruction but also linked with ideas or knowledge covered in other disciplines (e.g., science lessons).

ILLUSTRATIONS

The results of the present study show how the 9 mathematical connection tools adopted by Garcia-Garcia and Dolores-Flores (2018) can be used to promote the creative ideas of students in mathematics. We provide possible ways or examples showing how these tools have the potential to foster creative insights in students in mathematics. Here are the 9 tools and examples describing how mathematical connection is at the heart of mathematical creativity:

1) **Instruction-oriented connections**: It refers to the understanding of a new concept in the present based on other concepts previously covered in mathematics (Busisnskas, 2008; Garcia-Garcia, 2018; Mhlolo, 2012) or non-mathematics classrooms. This can be classified as both intra- and inter-mathematical connections because teachers can link what students learn not only with what they learned in their previous mathematics instruction, but also with what they learned in previous non-mathematics instructions. For the intra-mathematical connection example, teaching the concept of a volume of a rectangular prism based on the concepts of an area of rectangular shape is an instruction-oriented connection that requires students to understand the area concept of a rectangular shape to understand the volume concept of a prism by linking area and volume concepts or formulas. The instruction-oriented technique was suggested by Bicer (2022) to foster students' creative thinking in mathematics classrooms by deliberately linking what students learned in previous instruction with what mathematical concepts they are learning in their current mathematics instruction. The aim is to provide authentic and meaningful learning instructions in which students generate new knowledge by using what they learned in their previous lessons. For example, in his book titled *Mathematical Creativity in 5th-grade Common Core Classrooms*, Bicer (2022) stated that students' understanding of the connection of decimals and fractions enables them to "create meaning of decimal

multiplication by using their previous knowledge of fraction multiplication." (p. 42). Through this book, the instruction-oriented connection was suggested many times as one of the main aims of creativity-directed mathematics lessons.

2) **Different representations**: Using different representations is another technique that allows students to apply tools, such as drawings, graphs, tables, and written or verbal symbols to discern connections among several mathematical concepts and create new mathematical knowledge (Ervynck, 1991; Levav-Waynberg & Leikin, 2012; Silver 1997). This can be easily classified under the category of connecting mathematical ideas with other ideas in mathematics, as it is an already common practice in mathematics classrooms. Using multiple representations to solve mathematical problems was also suggested not only as an intervention to foster students' creative thinking in mathematics but also as a psychometric tool to measure students' creative thinking abilities in mathematics (Bicer, 2021). For example, using multiple representations allows students to create the meaning of equivalent fractions as they connect several representations of 1/2 (see Figure 1) and observe that the same area of same size squares is shaded, and each represents a half although each can be written in different forms (e.g., 1/2 = 2/4 = 4/8 =… = 50/100). As can be seen in this example, students create new mathematical knowledge (i.e., equivalent fraction concepts) by connecting several representations of a fraction. Later, they can also generate knowledge of percentage and decimal concepts as they observe that 1/2 is equivalent to 50/100, 50%, or 0.50 through multiple representations in Figure 1.

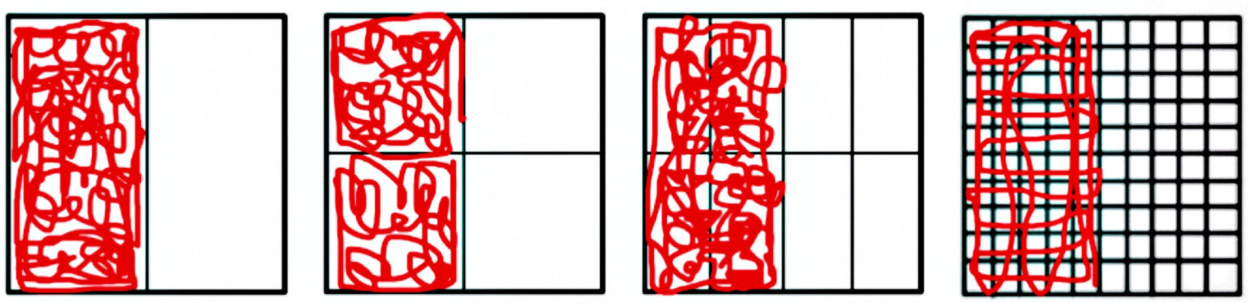

Figure 1. Several representations of ½ in same-size squares

3) **Reversibility connection**: It occurs when students connect the reverse relationship between mathematical concepts. It is defined as an ability to find a bi-directional relationship between mathematical concepts (Haciomergolu et al., 2009). For example, Garcia-Garcia and Dolores-Flores (2018) noted that recognizing the reverse relationship between a derivative and an integral is an example of making a reversibility connection in mathematics. In elementary grades, this can be considered as recognizing the reverse relationship between addition and subtraction or multiplication and division. For example, when students recognize 60 divided by 5 in terms of multiplication tasks and think about

what times 5 is 60, they recognize the reverse connection of multiplication and division. In upper elementary or middle school grades, this can be extended to fractions and decimals. For example, students can find what is 6 divided by 2/3 by using the relationship between division and multiplication without knowing the procedure of fraction division (Bicer, 2022). Students can use the reverse connection between multiplication and division and write 2/3 times what makes 6. Once they write its equation (i.e., 6 = 2/3 x (what number)), it can be read as 6 is 2/3 of what number. Then, they can see that if 2/3 of that number is 6, 1/3 of it is 3. Then, 3 x 3 = 9 (see Figure 2). Recognizing the reverse connections between mathematical concepts is a creative act that helps students creatively use their existing mathematical knowledge to solve mathematical problems in a new context without learning new mathematical facts, procedures, or methods.

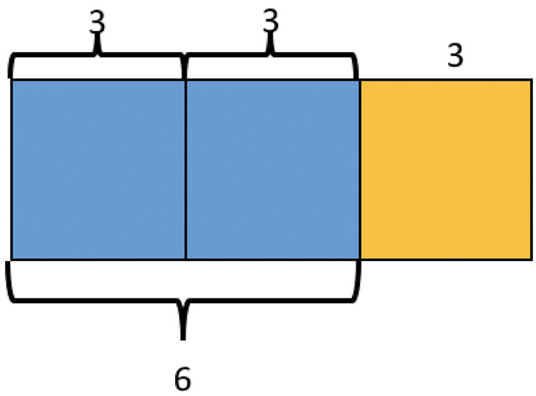

Figure 2. Solving fraction division by using multiplication for 6÷ (2/3)

4) **Procedural**: This tool is considered as making mathematical connections by identifying mathematical rules, algorithms, and procedures to solve mathematical problems (Garcia-Garcia & Doleres-Flores, 2019). In the present study, we consider this as making connections between mathematical concepts and their procedures. This can be also viewed as a generalization of mathematical procedures by connecting several methods, examples, or ways to find general procedures of mathematical algorithms. Bicer (2022) provided an example: students can find the answer to the question "How many 1/2-unit pieces of ribbon match with the length of the 12-unit piece of ribbon?" by using hands-on materials or manipulatives (see Figure 3).

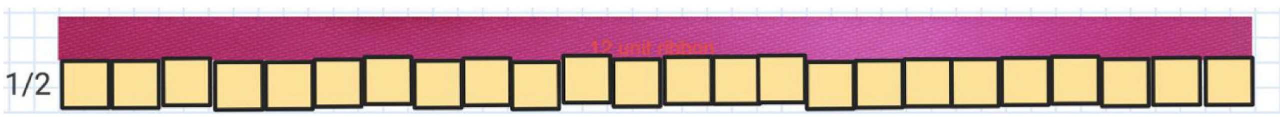

Figure 3. How many ½ units are needed?

This can be repeated with 1/3-unit, 1/4-unit, and 2/3-unit pieces of ribbon. When students find the answers as 12 ÷ (1/2) = 24, 12 ÷ (1/3) = 36, 12 ÷ (1/4) = 48, 12 ÷ (2/3) = 18, teachers can then ask students to examine the equations and attempt to come up with methods or procedures that work for each equation. Students will observe that they flip the fractions and multiply them by 12 to obtain the answer. Rather than following what their teachers tell them, students are allowed to conceptually develop what it means when a whole number is divided by a fraction and then generate its procedures by observing several examples individually or as an entire class. Here, students make intra-mathematical connections by connecting several examples and creatively generating procedures of fraction division.

5) **Part-whole**: refers to finding a connection between general and specific cases/forms/shapes (Garcia-Garcia & Dolores-Flores, 2019). Students are expected to use inductive and deductive reasoning to connect general and specific forms of mathematical objects. For example, after students are given the attributes or properties of parallelograms, rectangles, squares, and rhombus, they can creatively deduce the relationship among them. This allows them to make deductions by noticing "all of them are parallelograms because these given shapes have four sides, and their opposite sides are parallel". They can also create other statements like "all squares are rectangles, but not all rectangles are squares" or "all squares are rhombuses, but not all rhombuses are squares". In these examples, teachers can observe how students creatively find several conjectures by considering and connecting different properties of the shapes.

6) **Implications**: Making a mathematical connection through implications is described as one mathematical concept leading to another mathematical concept through mathematical reasoning (Businskas, 2008). An example was provided by Rodriguez-Nieto et al. (2022): if df (x) > 0 over an interval *I*, then f is an increasing function in that same interval *I*. This technique can be considered as making indirect mathematical reasoning. Indirect mathematical reasoning is a commonly applied technique to solve mathematics problems as mathematics teachers want students to think deeply to connect mathematical ideas by considering the implications of how to use the knowns in mathematical problems. For example, if students need to use the knowledge $a < c$ to solve a mathematical problem, this information is often indirectly provided as $a < b$ and $b < c$. Another example, if students need to know the interval of "a" to solve a mathematical problem, this information can be provided indirectly like $a^2 < 1$, and students are expected to connect this information (i.e., $a^2 < 1$) with what it implies that the value of "a" is between -1 and 1 (e.g., $-1 < a < 1$). Finding indirect reasoning in mathematical problems is a creative act as students need to connect what is directly provided in the problems with the implications of the knowns and see how they can use the implications of the knowns to solve the problems.

7) **Meaning**: Making mathematical connections through meaning refers to how a student attributes a sense to a mathematical concept in a specific context (Garcia-Garcia &

Dolores-Flores, 2019). This type of connection that students make might be limited by what a mathematical concept means in a specific context to them (Kilpatrick et al., 2005). For example, when teachers introduce multiplication concepts to students, they can connect it with their understanding of the addition concept and construct the knowledge of multiplication as repeated addition, but at first, they may not think of multiplication as equal groups. Although making this type of connection is limited to what their background knowledge is, it is important to provide meaningful mathematics learning to students. It is especially important for elementary school students to manifest their creative ideas in mathematics as their mathematical background knowledge is still emerging.

8) **Feature**: Mathematical connection through feature occurs when students consider some invariant or specific attribute of a mathematical idea or concept that distinguishes it from others (Rodriguez-Nieto et al., 2022). For example, when students know the Pythagorean theorem formula to find a hypotenuse for a given general right triangle as $a^2 = b^2 + c^2$, they can regenerate the formula for isosceles triangles and write that $a^2 = 2b^2$. Later, they can use this information to find the hypotenuse of an isosceles triangle by multiplying the length of one of the sides by $\sqrt{2}$. This is one example showing how students make the mathematical connection by linking the features of general (e.g., right triangle) and specific (e.g., isosceles triangles) cases to solve mathematical problems or to develop procedural fluency while solving problems. Students can have opportunities to manifest their creative thinking by making such connections because they need to recall a general case, identify a specific case and its mathematical attributes, and connect the features or attributes of a general case to a specific case.

9) **Metaphorical**: This is "understood as the projection of properties, characteristics, etc., belonging to a known domain to structure another lesser-known (abstract)" (Rodriguez-Nieto et al., 2022, p. 2367). Using metaphors to teach mathematics is strongly suggested (Shuell, 1990). "If a picture is worth 100 words, a metaphor is worth 1,000 pictures!... A metaphor provides a conceptual framework for thinking about something" (Shuell, 1990, p. 102). Rodriguez-Nieto et al. (2020) provided an example: a teacher may say *we travel through the path (the graph)* when describing the graph as students are familiar with a path more than a specific name of graphs in mathematics. In this case, teachers connect students' daily life knowledge with what new concept they are learning in mathematics. This kind of connection can be also seen in mathematical problems like "what is the length of the shortest path from point A to point B (the shortest path is often the hypotenuse)." Making mathematical connections through using metaphors can be considered as connecting mathematical ideas to ideas that are relevant to students' cultures, experiences, and daily life. Using metaphors not only enables students to understand the meaning of new mathematical ideas or concepts, but it also enables them to learn mathematics

meaningfully as they see analogies or applications of new mathematical ideas or concepts in their life. When students learn mathematics by making such connections, they can be better equipped to generate creative ideas as they will seek to see the analogies or application of mathematical concepts in real life. This type of mathematical connection can also be considered as the mathematical modeling process that includes mathematizing real-world problems, finding appropriate models, and validating the results in a real-world situation. Because mathematical modeling tasks were also considered creativity-directed tasks, it was suggested to use such tasks to foster students' creative ideas in mathematics (Lu & Kaiser, 2022). These researchers further reported that engaging in mathematical tasks has the potential of developing students' creative ideas in mathematics as students develop insights into connections between real-world problems and mathematics.

These tools were mostly suggested to allow for intra-mathematical connections while students engage in mathematical tasks. It is important to note again that these tools are not mutually exclusive and there might be many other tools and/or teachers can generate more techniques to make mathematical connections. Making intra-mathematical connections through these techniques is considered a criterion for achieving mathematical creativity in the context of multiple solution tasks as students are expected to solve problems by connecting ideas to other ideas across mathematics. To provide an example of an inter-mathematical connection that links ideas in mathematics to ideas in other disciplines (e.g., physics) and ideas in real life, teachers can consider a derivative as the velocity of an object as it travels. This velocity is the rate of change in the position of the object, meaning the velocity is considered the derivative of the position of the object. In this example, through appropriate tasks and instructional practices, teachers enable students to see how they can use their physics knowledge to understand what a derivative is and how they can solve mathematical problems creatively by recalling their ideas in physics and/or in other disciplines. That is why solving problems by recalling ideas from other disciplines is considered one of the criteria for achieving mathematical creativity in the context of multiple solution tasks. Lastly and most importantly, considering intra- or inter-mathematical connections in the context of mathematical creativity is important because it not only helps teachers understand students' creativity in mathematics from the product dimension like other commonly suggested techniques (problem-posing and multiple solution tasks), but it also helps teachers understand the creative processes of students in mathematics.

DISCUSSION

The aim of this paper is to illustrate how using tools suggested for making mathematical connections can be used to develop students' creative insights by providing teachers with practical examples showing what it means for students to be creative in K-12 mathematics classrooms and by providing teacher educators with the theoretical similarities of both constructs (i.e., mathematical connection and mathematical creativity). This aim is both theoretically and practically important.

Investigating the vitality of making mathematical connections in developing students' creative processes is theoretically important because it can provide a better understanding of the relationship between mathematical ability and mathematical creativity and whether the drawn models (e.g., Kattou et al., 2013) depicting this relationship need to be revised. This is an important approach because a current meta-analytical study revealed that students' mathematics achievement and mathematical creativity are highly correlated (Bicer et al., 2020). Schindler et al. (2018) stated that it is very hard to distinguish clearly between students' mathematical abilities and their creative abilities in mathematics. Therefore, researchers should ponder additional factors (i.e., mathematical connection ability) potentially influencing the relationship between mathematical abilities and mathematical creativity if their goal is to measure students' true creative abilities in mathematics. Krutetskii (1976) stated that mathematical ability is a multidimensional construct and later Kattou et al. (2013) investigated how mathematical creativity and mathematical ability as multidimensional constructs are related. They found out that mathematical creativity is a subfactor of mathematical ability along with other sub-factors of spatial ability, quantitative ability, qualitative ability, causal ability, and inductive/deductive reasoning ability. When we examine the definitions of the other sub-factors, we can see that causal ability, inductive/deductive ability, and qualitative ability explicitly require students to employ their mathematical connection ability to find the relationship between mathematical or non-mathematical ideas. For example, while the tasks identified to measure causal ability require students to find cause/effect connections, the tasks identified to measure qualitative ability require students to find connections by representing and processing similarity and difference relations of mathematical and non-mathematical ideas (Kattou et al., 2013). We can also consider these abilities as the tools mentioned above for making mathematical connections. For example, the causal ability can be considered as making the implication type of mathematical connection as students are expected to find what the knowns in problems imply or cause to find a specific idea, knowledge, or concept that they can use strategically to solve mathematical problems. These explanations can be supported by Anggoro et al.'s (2022) findings that mathematical reasoning was found to be an important predictor of explaining students' mathematical connection ability in the context of problem-solving. The other factor: "quantitative ability" is defined as an ability to focus on quantitative properties, such as number sense and pre-algebraic reasoning. Kattou et al. (2013) provided the following task as an example to show what task can measure students' quantitative ability: "George's marbles are three times as many as Spyros' marbles. Andreas has 20 marbles less than George. If Sypros has S marbles, how many marbles does Andreas have?" In this task, students are expected to connect the known quantities as the relationship between the number of George's marbles and the number of Sypros' marbles and then relate and represent the number of Andreas' marbles in terms of the number of Sypros' marbles. This example also illustrates that students use their mathematical connection ability to solve the problems as they connect the knowns in the problem to find the solution in terms of S. Therefore, future models depicting the relationship between mathematical ability and mathematical creativity should consider the mathematical connection as an important factor influencing that relationship. For example, the model suggested by Kattou et al. (2013) can be retested by considering mathematical connection as a factor including causal ability, inductive/deductive ability, qualitative ability, and quantitative ability. The model can also show how mathematical creativity is connected to mathematical connection ability and spatial ability. This can be also supported by the current finding that there is a moderate to high correlation between students' mathematical creative ability and their spatial thinking ability (Bicer et al., 2023). Therefore, we suggest this model (Figure 4) that can be tested using Confirmatory Factor Analysis (CFA).

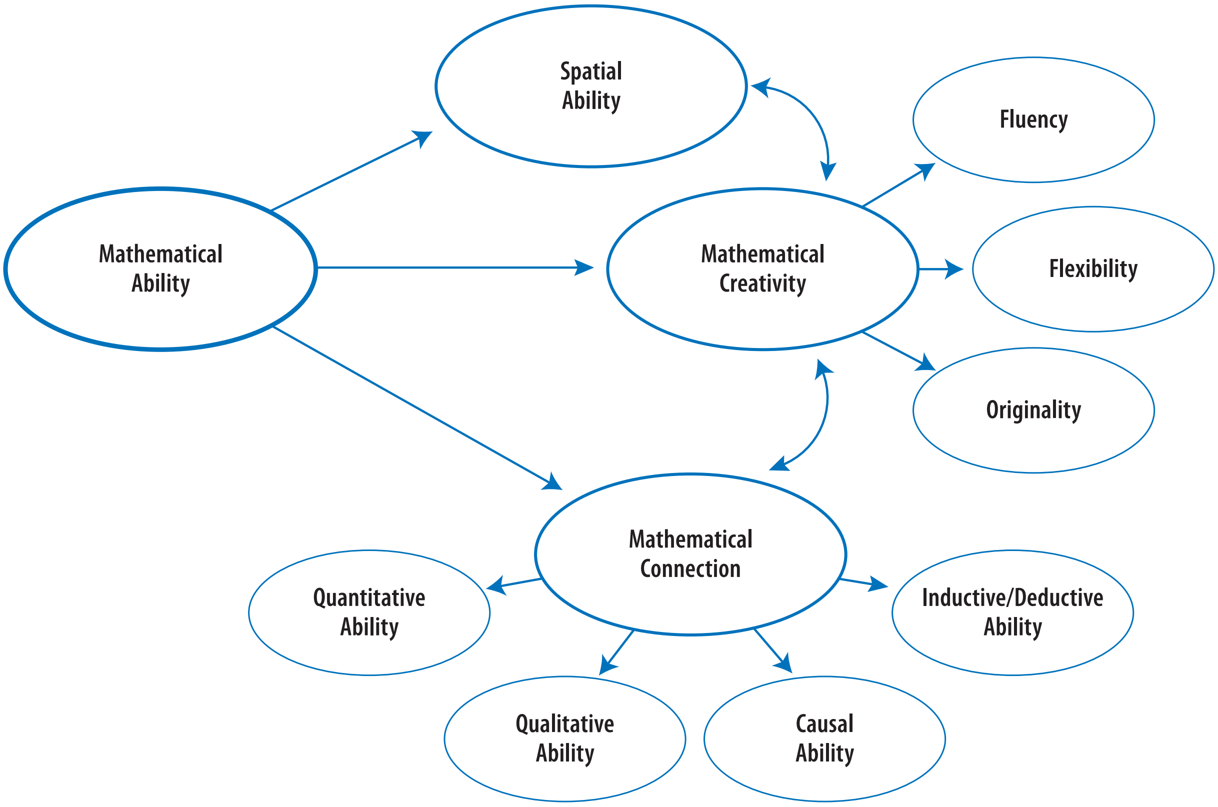

Figure 4. The relationship among mathematical ability, mathematical creativity, and mathematical connection

It is important to note that this model is not tested in the present study, but it is designed based on previous research findings and the ways each construct or variable was defined in the literature. Therefore, future research should test this model to determine whether this observation develops the fitness of the model depicting the relationship among mathematical ability, mathematical creativity, and mathematical connections compared to the model that does not include mathematical connection. Since the present paper suggests a model, it is recommended that future researchers test the model by applying CFA to confirm whether the empirical data related to mathematical connection, spatial ability, mathematical creativity, and mathematics achievement supports the suggested model.

Investigating the vitality of making a mathematical connection in developing students' creative thinking in mathematics has practical benefits, as it provides teachers with concrete tools that can help them understand what it means for students to be creative in K-12 mathematics classrooms. Previous studies emphasized and investigated mathematical creativity in terms of students' products at the end, but they mostly disregarded the creative processes of students in mathematics classrooms (Bicer & Bicer, 2022; Schindler & Lilienthal, 2019). Although there have been some current attempts to understand students' creative processes, teachers and researchers still do not fully know what important actions they should take to promote creative insights in students in mathematics classrooms (Bicer & Bicer, 2022; Schindler & Lilienthal, 2019). When the lack of knowledge of the creative processes of students in mathematics combined with the abstract nature of the construct of creativity, it is almost impossible for teachers to understand what it means for K-12 students to be creative in mathematics and how they can foster their students' creative development. However, because making mathematical connections is not only a product, but it is a process of connecting ideas in mathematics to ideas in mathematics (Dolores-Flores et al., 2019; Garcia-Garcia & Dolores-Flores, 2018), other disciplines, and real life and this requires creative thinking, teachers can use making mathematical connection strategies and principles to foster students' creative processes in mathematics. Using mathematical connections as a tool can enable teachers to understand what creativity in K-12 mathematics classrooms is and how to support their students' creative processes as they have tangible strategies and examples of mathematical connections illustrated above that can be used to encourage creative ideas in mathematics. When teachers encourage students to make mathematical connections, students engage in creative processes to find possible relationships among mathematical ideas. Students' engagement in making mathematical connections is vital because each time teachers prematurely teach students something they could have discovered for themselves, they are keeping them from inventing it and consequently from understanding it completely (Piaget, 1970). The essential message here is that teachers should empower students to make mathematical connections by integrating appropriate mathematical tasks and instructional practices into their classrooms in which their students find the courage and inspiration to generate new mathematical knowledge or find creative ideas in mathematics by using the tools of mathematical connections. *The Program for International Student Assessment* [PISA] (2012) stated that successful students in mathematics are the ones who see and learn mathematics as a series of connected ideas, but not the ones who see and learn mathematics as a series of disconnected ideas. Teachers should use the techniques of making mathematical connections mentioned above to inspire students to find creative ideas by linking mathematical ideas to other ideas across mathematics. This will enable students to see that mathematics is a creative discipline in which they can connect several mathematical ideas to create new mathematical ideas, knowledge, and concepts. Making mathematical connections is only one way among many to promote the creative processes of students in mathematics. In the present article, we do not try to equate mathematical connections to mathematical creativity, but we reinforce the idea that making mathematical connections is a creative process and its tools or techniques should be effectively used to promote students' creative ideas in mathematics. Further research is necessary to further understand how mathematical creativity can be supported by other factors (e.g., communication) potentially influencing students' creative processes in mathematics.

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