sciendo

Civil and Environmental Engineering
Vol. 17, Issue 1, 219-228, DOI: 10.2478/cee-2021-0023

MODIFIED REED EQUATION OF BLAST LOAD ON PLATE WITH STIFFENER

Haryo Koco BUWONO[1,*], Sofia W. ALISJAHBANA[2], NAJID[1]

[1] Civil Engineering, Engineering Faculty, Tarumanagara University, Indonesia.
[2] Civil Engineering, Engineering and Computer Science Faculty, Bakrie University, Indonesia.
[*] corresponding author: haryo.koco@umj.ac.id

Abstract

The purpose of this study is to analyze numerically the effect of explosions on orthotropic slabs which have partial fixity placement and stiffeners in the x direction, namely in the short span direction. The modified blast load dynamic behavior is from Reed's equation with 4th order polynomial on orthotropic plates with x-direction stiffener. The localized blast load centered in the middle of the strain, and the effects of thickness and stiffening on the vertical deflection of the plates are solved numerically using two auxiliary equations in the x and y-directions. It is found that there is vertical deflection with related to time.

Keywords:

Blast loads;
Slabs;
Stiffeners;
Reed;
4th order polynomials.

1 Introduction

1.1 Background

The floor slab system chosen for building is different, depending on the function of the space and economic limitations and architectural requirements. The reinforced concrete slab system in the SNI 2847: 2019 regulation are classified into two types, namely one way slab (one-way plate) and two way slab (two-way plate) [1]. The need for plate design in the ideal plan is to be able and effective to withstand any loads that may work on the plate, including blast load, which is very fast chemical reaction releasing pressure and heat energy that only lasts a few milliseconds. The resulting pressure is enormous through the air and fills the entire space through propagating waves that are faster than the speed of sound. The enormous pressure of the explosion presses the air around the explosion, creating the effect of the blast wave [2]. Each blast wave is characterized by the following parameters: a) positive (P_{so}) and negative pressure peaks, b) the explosion period is positive (t_{dp}) and negative (t_{dn}) duration. The blast load scenario is influenced by the equation model which is inputted as a dynamic load in the construction [3]. The Reed's equation in 1977 states that the Friedlander's equation in 1946 uses blast load equation approach as follow: (1).

$$P(t) = P_{r,max}\left(1 - \frac{t}{t_d}\right)e^{-bt/t_d}. \tag{1}$$

According to Smyth theory, an approach to the convenience of using polynomial equation [4], which is stated as follows [5].

$$P(t) = P_r\left(1 - \frac{t}{td}\right)\left[1 - \frac{7}{25}\frac{t}{td}\right]\left[1 - \left(\frac{7}{25}\frac{t}{td}\right)^2\right], \qquad 0 \le t \le \frac{25}{7}td\,. \tag{2}$$

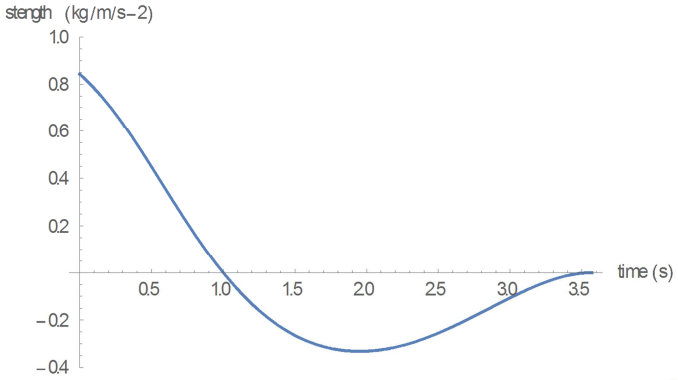


Fig. 1: Strength of reed's blast load equation in 1977 (kg/m/s^2) [5].

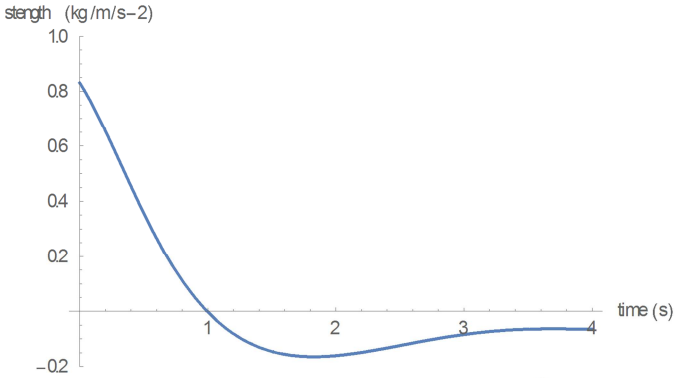


Fig. 2: Strength of polynomial blast load equation of 4th order (kg/m/s^2) [6].

In Fig. 1, Reed's polynomial equation states that the t_d limit (positive phase duration) is to solve the entire positive phase and the negative phase of the explosion at once. After the blast load passes through its positive phase, the load behavior changes to a negative pressure load (suction) [7]. This phase generally has a maximum amplitude less than the positive phase, but longer duration, roughly twice the duration of the positive phase. After going through these two phases, the explosion has an advanced phase, namely the free vibration phase, or the no-load phase. The plates are more affected by the impact after explosion or on the free vibration phase [8]. It is necessary to study the connection between Reed and modified Reed equation, to determine the percent error approach to the Friedlander equation.

1.2 Modified reed equation

Fig. 2 is modified Reed blast load equation using 4th order polynomial. That approach uses the regression method based on the smallest error factor, which the equation is as follows:

$$P_{(t)} = P_r\left(0{,}0151\left(\frac{t}{td}\right)^4 - 0{,}1946\left(\frac{t}{td}\right)^3 + 0{,}8949\left(\frac{t}{td}\right)^2 - 1{,}6898\left(\frac{t}{td}\right) + 0{,}9720\right); 0 \le t \le td. \quad (3)$$

Study of Buwono et. al in 2020, that descripted on Fig. 1, the error factor for Reed equation turns out to have a large error for Friedlander in 1946, which is 29.18 %. Based on the target of that study using the 4th order polynomial equation, it is necessary to do approach shown in equation (3). In table 1, the results of equation (3) have an error factor that is smaller than equation (2).

Table 1: Comparison of Friedlander, Reed, and modified Reed equations.

t	Friedlander *F*	Reed *R*	4th Order Polynomial *P*	*F* - *R* (%)	*F* - *P* (%)
-	1.000	1.000	0.972	0.00 %	2.80 %
0.25	0.584	0.694	0.603	-11.00 %	-1.84 %
0.50	0.303	0.422	0.327	-11.83 %	-2.42 %
0.75	0.118	0.189	0.131	-7.07 %	-1.27 %
1.00	0.000	0.000	(0.002)	0.00 %	0.23 %
1.25	-0.072	-0.143	(0.085)	7.10 %	1.33 %
1.50	-0.112	-0.239	(0.129)	12.73 %	1.74 %
1.75	-0.130	-0.291	(0.145)	16.03 %	1.46 %
2.00	-0.135	-0.302	(0.142)	16.67 %	0.63 %
2.50	-0.123	-0.230	(0.106)	10.64 %	-1.69 %
3.00	-0.100	-0.094	(0.066)	-0.54 %	-3.33 %
3.50	-0.075	-0.002	(0.042)	-7.35 %	-3.32 %
4.00	-0.055				
4.50	-0.039				
5.00	-0.027				
Error (%)				7.33 %	1.64 %

Viewing from the approach or modification of the Friedlander blast load equation, the polynomial equation is closer to the real Friedlander equation in 1946 [6].

1.3 Numerical analysis of floor slab with stiffener

In analyzing a structure, it is necessary to know how the structure behaves to various kinds of excitation or vibration. One of the quantities that reflect the behavior of a structure is its natural frequency. The natural frequency of the structure can be found by assuming that the structure has no damping and no loading acts on the structure, only initial conditions. This is also commonly known as free vibration analysis.

The damped elastic orthotropic plate differential equation is obtained by adding up all the internal forces acting on the plate system which must be balanced with the acting external loads.

$$D_x\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}\right)+2B\left(\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}\right)+D_y\left(\frac{\partial^4 w(x,y,t)}{\partial y^4}\right)+\xi h\left(\frac{\partial w(x,y,t)}{\partial t}\right)$$
$$+\rho h\left(\frac{\partial^2 w(x,y,t)}{\partial t^2}\right)=p_z(x,y,t) \quad (4)$$

In rigid rectangular orthotropic plate with stiffener, the equation (4) can be used to obtain the response of the plate to a particular load. The geometric parameters that need to be known on a rectangular orthotropic plate that is rigid with joists can be seen in Fig. 3.

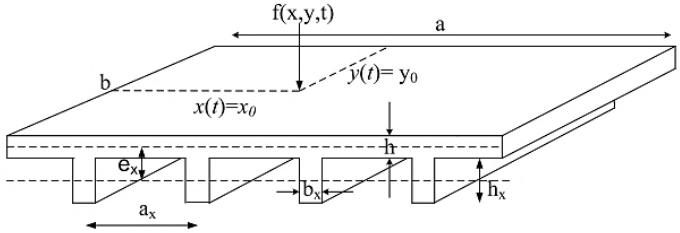


Fig. 3: Rectangular orthotropic plate with stiffener.

1.4 Levy's solution

To solve the equation of motion above, it can be done using the boundary conditions of the joint on all four sides. This condition will occur angular rotation on the edge of the plate but there will be no bending moment. Simple leaning condition can be solved with Navier's solution or Levy solution. Finding a solution to the equation of motion for the joint condition begins with selecting a trial function that satisfies the boundary conditions of the simple dependent conditions. In this solution, the damping effect on the structure is also neglected [9].

The chosen trial function equation is the multiplication of two components, namely the spatial or position function and the time function. The transverse deflection of the plates as a function of position and time can be showed as in the equation (5).

$$w(x,y,t) = W(x,y)T(t) = W(x,y) \sin \omega t \,, \tag{5}$$

where: $W(x,y)$ - position function or spatial function, ω - natural frequency of the system.

The trial function equation is substituted into the differential equation of motion for the ortrotopic plate system and algebraic operations are performed to obtain equation (6) below.

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h}\left[D_x\left(\frac{m}{a}\right)^4 + 2B\left(\frac{mn}{ab}\right)^2 + D_y\left(\frac{n}{b}\right)^4\right]. \tag{6}$$

Equation (6) is an equation for the eigenvalue for a variety of structures. By calculating the positive root of the eigenvalue, the natural frequency of the structure can be obtained. The equation for obtaining the eigenvalue above only applies to structures with simple boundary conditions on all sides, while the object of this study is a plate structure with partial fixity boundary conditions so that the equation needs to be modified.

1.5 Modified Bolotin method

Modified Bolotin Method is a way to solve the plate differential equation with partial fixity boundary conditions and can also take into account the effect of high mode.

The eigenvalue for a rectangular plate with partial fixity on all sides can be found by anologizing the plate as a simple rests on all sides. The integer index value for x direction and y direction is replaced by the p coefficient for x and q direction for y direction. The values of p and q are the real numbers obtained looking for a solution to the auxiliary problem. The eigenvalue equation for the plate with the partial fixity boundary condition is expressed as equation (7) below.

$$\omega_{pq}^2 = \frac{\pi^4}{\rho h}\left[D_x\left(\frac{p}{a}\right)^4 + 2B\left(\frac{pq}{ab}\right)^2 + D_y\left(\frac{q}{b}\right)^4\right], \tag{7}$$

where: p - the real numbers obtained from the auxiliary solution in the x direction, q - the real numbers obtained from the auxiliary solution in the y direction.

1.5.1 First auxiliary solution

The trial function for the first spatial auxiliary solution (x direction) is stated as follows:

$$W(x,y) = X(x)Sin\left[\frac{q\pi y}{b}\right]. \tag{8}$$

The equation for the solution of the position function for the x direction is as follows:

$$X(x) = A_1 e^{i\frac{p\pi}{a}x} + A_2 e^{\frac{\pi}{ab}\beta x} + A_3 e^{-i\frac{p\pi}{a}x} + A_4 e^{-i\frac{p\pi}{a}\beta x}, \tag{9}$$

so,

$$X(x) = Cosh\left[\frac{\beta\pi x}{ab}\right] + \frac{\left(b(pk_1(c_1-C_1)+a(F_1+F_2)s_1)\right)}{\left(k_1(bpS_1-\beta s_1)\right)} Sinh\left[\frac{\beta\pi x}{ab}\right] - Cos\left[\frac{p\pi x}{a}\right]$$
$$+\frac{(ab(F_1+F_2)S_1+\beta k_1(c_1-C_1))}{(k_1(\beta s_1-bpS_1))} Sin\left[\frac{p\pi x}{a}\right]. \tag{10}$$

1.5.2 Second auxiliary solution

The solution for second auxiliary problem can be calculated in the same way as the first auxiliary problem, but the difference is in the second auxiliary is calculated only on the *y*-axis. The trial function for the second spatial auxiliary solution (*y* direction) is stated as follows:

$$W(x,y) = Y(y)Sin\left[\frac{p\pi x}{a}\right]. \tag{11}$$

The equation for the solution of the position function for the *Y* direction is as follows:

$$Y(y) = B_1 e^{\frac{\pi}{ab}\theta y} + B_2 e^{-\frac{\pi}{ab}\theta y} + B_3 e^{i\frac{q\pi}{b}x} + B_4 e^{-i\frac{q\pi}{b}x}, \tag{12}$$

so,

$$Y = Cosh\left[\frac{\theta\pi y}{ab}\right] + \frac{\left(a(qk_2(c_2-C_2)+b(F_3+F_4)s_2)\right)}{\left(k_2(aqS_2-\theta s_2)\right)} Sinh\left[\frac{\theta\pi y}{ab}\right] - Cos\left[\frac{q\pi y}{b}\right]$$
$$+\frac{(ab(F_3+F_4)S_2+\theta k_2(c_2-C_2))}{(k_2(\theta s_2-aqS_2))} Sin\left[\frac{q\pi y}{b}\right]. \tag{13}$$

1.6 Total solution of plate motion equation

The response of plate to dynamic transverse loads includes two solutions, namely the homogeneous solution (free vibration) and the particular solution (forced vibration). The homogeneous solution and the particular solution are solved separately and the two solutions add up to get the total plate response. The connection between two solutions can be expressed as the following equation:

$$w_H + w_p = w_{mnt}. \tag{14}$$

A homogeneous solution, w_H, is a solution for a structural state that does not accept excitation in the form of a load but a solution that occurs because of the deformation or initial condition that causes a structural response to occur.

The homogeneous solution of the differential equation above is linear so that it can use the superposition method for each auxiliary value of *p* and *q* [10]. Therefore, the total homogeneous solution for a given number of modes can be expressed as follows:

$$w_H = w(x,y,t) = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} W_{pq}(x,y).T_{pq}(t)$$
$$= \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} X_{pq}(x)Y_{pq}(y)e^{-\xi\omega_{pq}t}\left(a_0 cos(\omega_D t) + b_0 sin(\omega_D t)\right). \tag{15}$$

As with the completion of the homogeneous solutions, solving particular solutions also uses the variable separation method. Homogeneous solutions $\hat{T}_{pq}(t)$, still contain constants that can be obtained using the initial conditions. This function shows transient vibration conditions at no load, whereas the particular solution describes the vibrational state. Particular solutions of differential equation, $T^*_{pq}(t)$, is a combination of spatial solutions and temporal particular solutions as follows:

$$w_p = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} X_{pq}(x).Y_{pq}(y)\int_0^t [\frac{p(x,y,\tau)}{\rho h Q_{pq}}\int_{x=0}^{a} X_{pq}(x)\,dx\int_{y=0}^{b} Y_{mn}(y)\,dy\,\frac{e^{-\xi\omega_{pq}(t-\tau)}}{\omega_D} sin\left(\omega_D(t-\tau)\right)d\tau \tag{16}$$

The total solution is the real total response which takes into account the effects of both free and forced vibrations. The total solution is obtained by adding the homogeneous solution of equation (15) and the particular solution of equation (16) in order to obtain the following equation:

$$w = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} X_{pq}(x)Y_{pq}(y)e^{-\xi\omega_{pq}t}\left(a_0 cos(\omega_D t) + b_0 sin(\omega_D t)\right)$$
$$+ \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} X_{pq}(x)Y_{pq}(y)\int_0^t [\frac{p(x,y,\tau)}{\rho h Q_{pq}}\int_{x=0}^{a} X_{pq}(x)dx\int_{y=0}^{b} Y_{mn}(y)dy\,\frac{e^{-\xi\omega_{pq}(t-\tau)}}{\omega_D}\sin\left(\omega_D(t-\tau)\right)]\,d\tau \quad . \tag{17}$$

1.7 Moment

The plate with a partial fixity limit on the sides will have a bending moment on the sides, in which the bending moment value depends on the rotational stiffness on these sides. The boundary conditions for partial fixity conditions are obtained by equations (18) and (19) [11].

$$M_x(x,y) = -D_x\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \nu_y\frac{\partial^2 w(x,y)}{\partial y^2}\right) = k_1\frac{\partial w(x,y)}{\partial x} \text{ di } x = 0 \text{ dan } x = a\,, \tag{18}$$

$$M_y(x,y) = -D_y\left(\frac{\partial^2 w(x,y)}{\partial y^2} + \nu_x\frac{\partial^2 w(x,y)}{\partial x^2}\right) = k_2\frac{\partial w(x,y)}{\partial x} \text{ di } y = 0 \text{ dan } y = b\,. \tag{19}$$

1.8 Main stress of bending moment

The maximum principal stress is the maximum normal stress when the orientation of the particles is changed so that the shear stress value is zero. Meanwhile, the minimum main stress is the minimum normal stress when the particle orientation is changed so that the shear stress value is zero. The maximum and minimum principal stresses are calculated by the equation obtained by Mohr's theory [12].

2 Localized blast load

Localized blast load is related to explosive propagation, however the blast wave is in fact not supported by the explosive gases, so it spreads separately. In the ideal case of a spherical explosive charge, the blast wave is theoretically propagated with a constant velocity *D* and has the form of a hemisphere (when the explosion occurs on the ground) or the form of a sphere (in the case of an explosive event above the ground (in the air). In the case of ground explosions, moreover, the weight of explosion is theoretically assumed as twice the nominal value, but the real ratio of the weight strongly depends on the characteristics of the contact environment. It is also known that in the typical blast wave, the air particles are pressed and moved. When the blast wave impacts on a rigid barrier, the reflected overpressure is much greater than the overpressure of the blast wave before the impact [13].

To get a total system solution, it is must define about the dynamic load function that acts on the system. This study uses a model of local blast load to find the behavior of plate with stiffner using 10 kg TNT, that the position shows in Fig. 4. The amplitude against the duration respectively with the value of P_{so} = 4000 N and duration of t_{total} = 0.4448 s with expressed Fig. 5. The steps for determining the blast load parameters are as follows:

1) Determine the explosion scenario, the mass of the explosive material *W*, the distance from the center of the explosion to the structure under consideration *R* in equation (20) [3, 13].

$$Z = \frac{R}{\sqrt[3]{W}}\,. \tag{20}$$

2) Determining the part of the structure that is affected by the explosion. This study uses the part of the plate structure against the position of the explosion which is shown in Fig. 4 below. The assumption of the height of the blast load is at an altitude of 0.5 m as the localized blast load so that it does not consider the reflected waves.

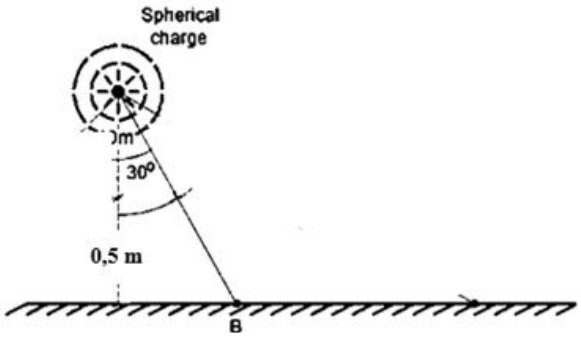


Fig. 4: Distance of explosion to the affected viewing point [3].

3) Calculating the Distance Scale Z using equation (20).

4) Explosion scenario using 1 variants of TNT explosion magnitude: 5 kg, described in Fig. 5 to get the maximum amplitude parameter of the positive phase, P_{so} which in the polynomial equation is called $P_{r,max}$.

5) $P_{r,min}$, obtained through the 4th order polynomial equation.

6) Determining the positive phase t_d parameter using Fig. 5 for the calculated z value, then the positive phase duration, t_0.

7) t_d^- negative phase is generated from the polynomial equation model of 4th order.

3 Numerical result and discussion

Numerical calculations are operated on various problem parameters with the number of modes in the x and y directions taken as m = 1, 2, .., 5 and n = 1, 2, .., 5 taking into account the convergence of eigenvalues. Slab parameters are plate length a = 8 m, plate width b = 5 m, plate thickness h = 0.23 m, using a stiffener with size of 30 x 50 cm^2, which the stiffener height includes the slab thickness. The density for concrete is 2400 kg/m^3. The plate is orthotropic, so the modulus of elasticity and poison ratio are in the x and y directions, namely E_{cx} = 23400 MPa, E_{cy} = 22200 MPa [10], ν_x = 0.2 and ν_y = 0.15. The rotational stiffness takes a reference from Gibigaye's journal, namely k_1 and k_2 are equal to 1.0x10^6 Nm/rad [14].

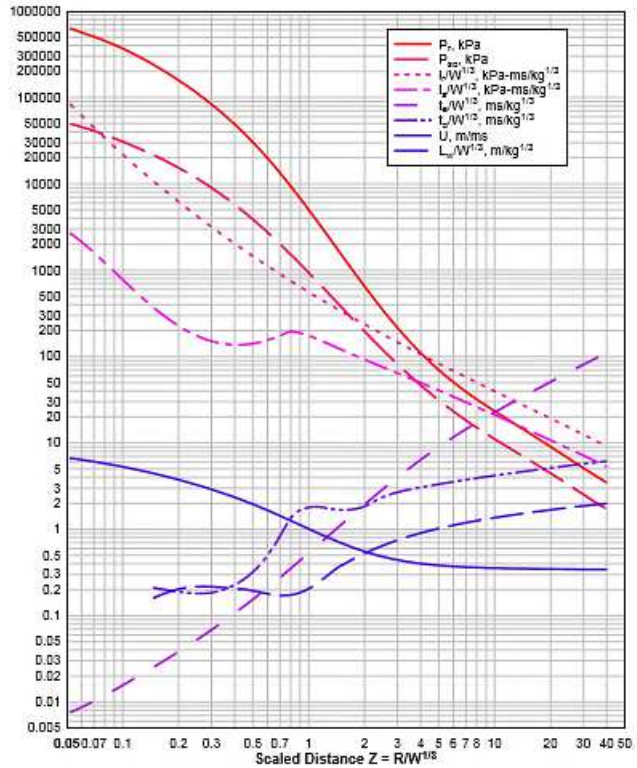


Fig. 5: Positive phase parameters of spherical shock wave for TNT free air [15].

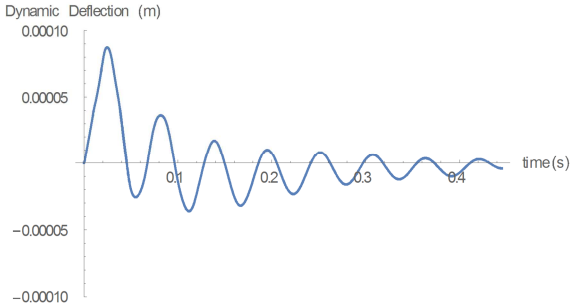


Fig. 6: Time history of explosion phase deflection.

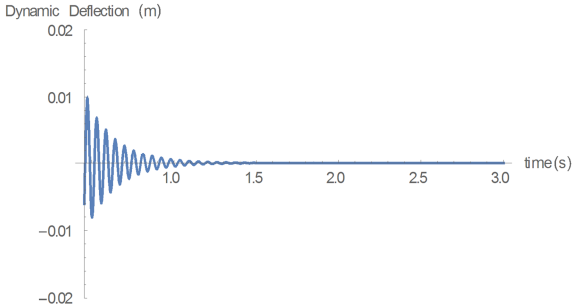


Fig. 7: Time history of free vibration phase deflection.

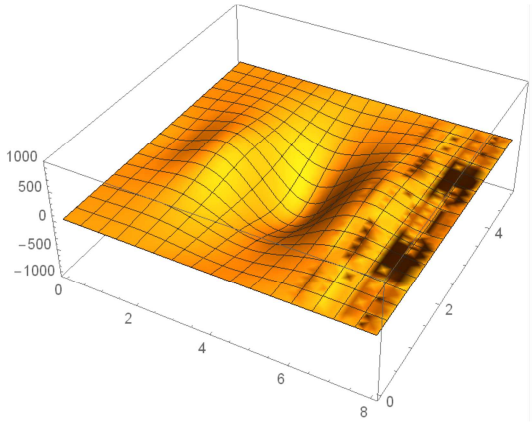


Fig. 8: Maximum deflection on free vibration phase.

Based on SNI 2847: 2019 that the deflection requirement is $\delta \leq l/360$, that l = 8 m, the maximum deflection value is δ = 0.022 m [1]. Fig. 6 and 8 show the maximum deflection that occurs at -0.013, so that it still meets the requirements for plate bending due to the dynamic load of 10 kg TNT explosion.

The deflection factor is very influential on the magnitude of the moment and the shear force that occurs on the plate. The moment and shear observed at the time of the explosion (phase 1), are the moments and shear at the maximum and minimum conditions. The magnitude of the moment and shear obtained from the analysis results are as follows:

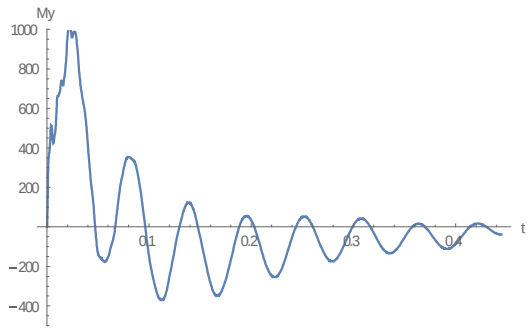


Fig. 9: Time history of moment of *y* direction.

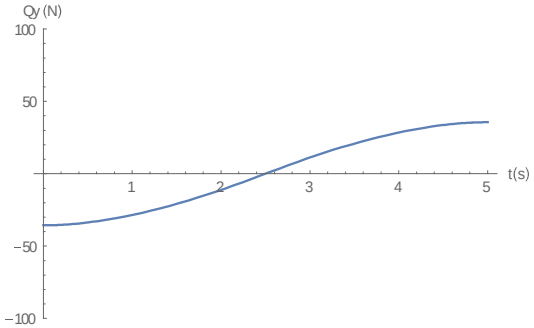


Fig. 10: Shear force on partial fixity.

Based on those moments and shear forces, the absolute moment in the middle of the maximum span occurs in the explosion phase of 964.4 Nm which occurs on the *y* axis. Meanwhile, the shear force on the support is 35.68 N.

The stress contour pattern of each load phase shows that the explosion position in the middle of the span is as Fig. 11. The stress in the free vibration phase produces the largest stress, which is $2.5x10^7$ N/m^2.

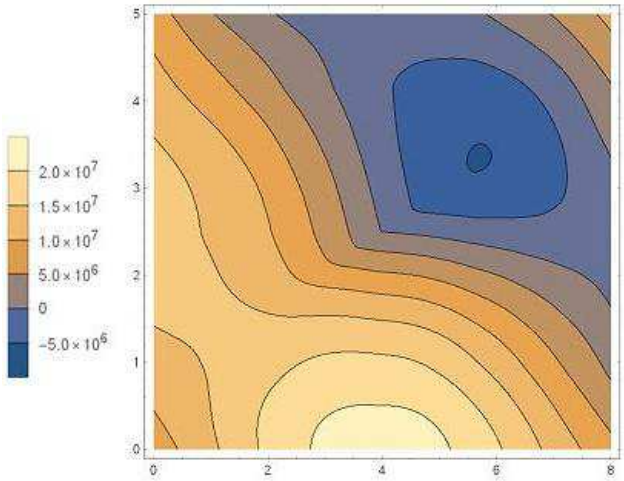


Fig. 11: Maximum stress on the free vibration phase.

4 Conclusion

This study resulted in the finding that using the modified Reed's equation is better than the Reed's equation itself, considering that the resulting error factor is smaller for the Friedlander's equation. Modified Reed's equation is enhanced with the 4th order polynomial equation. The error for the Friedlander equation is 1.64 %. From that result, the dynamic behavior of orthotropic floor slabs with a thickness of 23 cm with a stiffener of 30 x 50 cm^2 fulfills the deflection requirements standardized by SNI 2847: 2019. The condition for the greatest deflection is when the vibration is free and it is equivalent to a moment. Different from the shear force, the moment reaches an absolute maximum value when it is at the center of the span, whereas the shear is along the side of the plate with partial fixity. The absolute stress that is reviewed is the same as the moment, so that the maximum value is in the middle of the span at 0.3424 s. This study also states that the free vibration phase produces a greater deflection than the positive and negative explosion phases.

References

[1] SNI 03-2847:2019, SNI 03-2847:2019 Persyaratan Beton Struktural Untuk Bangunan Gedung Dan Penjelasan Sebagai Revisi Dari Standar Nasional Indonesia 2847 : 2013, Badan Standarisasi Nas., Vol. 03-2847:20, No. 8, 2019 pp. 1–695.

[2] RIGBY, S. E. - TYAS, A. - BENNETT, T. - CLARKE, S. D. - FAY, S. D.: The negative phase of the blast load. Int. J. Prot. Struct., Vol. 5, No. 1, 2014, pp. 1–19, doi: 10.1260/2041-4196.5.1.1.

[3] KARLOS, V. - SOLOMON, G.: Calculation of blast loads for application to structural components. 2013.
[4] SMYTH, B. G. K.: Polynomial approximation. North-holl. Math. Stud., Vol. 11, No. C, 2014, pp. 67–75, doi: 10.1016/S0304-0208(08)71986-8.
[5] GARCES, M.: Explosion Source Models. Springer International Publishing, 2019.
[6] BUWONO, H. K. - ALISJAHBANA, S. W. - NAJID: Modification Modeling of the Friedlander's Blast Wave Equation Using the 6th Order of Polynomial Equation. Int. J. Civ. Eng. Technol., Vol. 11, No. 2, 2020, pp. 183–191.
[7] BRYANT, L. M. - EREKSON, J. M. - HERRLE, K. W.: Are you positive about negative phase? Struct. Congr. 2013 Bridg. your Passion with your Prof. - Proc. 2013 Struct. Congr., 2013, pp. 103–114, doi: 10.1061/9780784412848.010.
[8] AHMET TUĞRUL - SEVIM, B.: Numerically and empirically determination of blasting response of a RC retaining wall under TNT explosive. Adv. Concr. Constr., Vol. 5, No. 5, 2017, pp. 493–512, doi: 10.12989/acc.2017.5.5.493.
[9] PEVZNER, P. - WELLER, T. - BERKOVITS, A.: Further modification of Bolotin method in vibration analysis of rectangular plates. AIAA J., Vol. 38, No. 9, 2000, pp. 1725–1729, doi: 10.2514/2.1159.
[10] SHAHSAVAR, V. L. - TOFIGHI, S.: Uncertainties Concerning the Free Vibration of Inhomogeneous Orthotropic Reinforced Concrete Plates. Slovak J. Civ. Eng., Vol. 22, No. 3, 2014, pp. 21–30, doi: 10.2478/sjce-2014-0014.
[11] SHABANA, A. A.: Theory of vibration - An introduction. Vol. XXXIII, No. 2, 2014.
[12] AHMAD SAFI, W. - HIBINO, Y.: Correlation between shear and normal strength for brittle reinforced concrete member considering internal stress condition of concrete. MATEC Web Conf., Vol. 270, 2019, p. 01007, doi: 10.1051/matecconf/201927001007.
[13] FIGULI, L. - CEKEREVAC, D. - BEDON, C. - LEITNER, B.: Numerical analysis of the blast wave propagation due to various explosive charges. Adv. Civ. Eng., Vol. 2020, No. i, 2020, doi: 10.1155/2020/8871412.
[14] GIBIGAYE, M. - YABI, C. P. - DEGAN, G.: Free vibration analysis of dowelled rectangular isotropic thin plate on a modified Vlasov soil type by using discrete singular convolution method. Appl. Math. Model, Vol. 61, 2018, pp. 618–633, doi: 10.1016/j.apm.2018.05.019.
[15] KARLOS, V. - SOLOMOS, G. - LARCHER, M.: Analysis of the blast wave decay coefficient using the Kingery–Bulmash data. Int. J. Prot. Struct., Vol. 7, No. 3, pp. 409–429, doi: 10.1177/2041419616659572.