The Scientific Bulletin of VALAHIA University
MATERIALS and MECHANICS –Vol. 20, No. 22

DOI 10.2478/bsmm-2024-0010

NEUTRAL SURFACE POSITION OF A CIRCULAR PLANE PLATE WITH RADIAL RIBS

Carmen POPA, Veronica DESPA, Florina Violeta ANGHELINA

Valahia University of Targoviste

E-mail: dumiver@yahoo.com

Abstract: *This paper establishes the neutral surface position of a circular plane plate with radial nervures, considering anisotropic structures theory. The nervures have a constant thickness and linear variable high along the length. At the same time this paper presents the cylindrical flexural rigidity of a such structure. The obtained theoretical data represent a development of a plate study with constant thickness and height nervures, assembled on a single face. The effect of welding on global rigidity of the plate is neglected. A numerical example of such a plate is made.*

Keywords: *circular plane plate, radial ribs.*

1. INTRODUCTION

The idea of saving the construction materials, in general, respectively for those in the composition of process equipment, represents one of the basic concerns of the researchers, the designers and the users of such mechanical structures. The calculation methods with high precision allow, in the design phase, to obtain a material saving of over 25% of the final mass, which makes the mentioned problem of primary importance [1]. Among the constructive solutions adopted is the ribbing of the smooth components. The rigidity of the constructions is corresponding increased, ensuring the mechanical resistance and the prescribed service life. In this order of ideas is also the case of flat circular plates with radial ribs on one side, with constant thickness and height, placed on one side [1, 2]. The determination of the position of the neutral surface and the expression of the cylindrical bending stiffness was made for equidistant ribs, without taking into account the influence of the welding seams necessary to fix the ribs to the plate itself [3, 4].

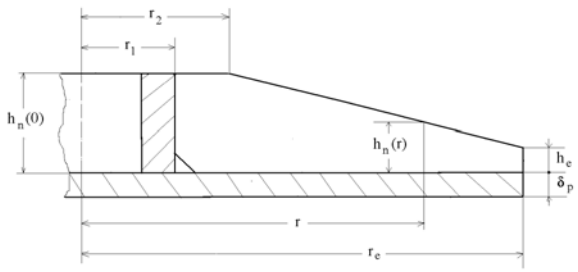


Figure 1.

2. THEORETICAL STUDY

In the present work, the two characteristics of the plates with radially equidistant ribs are also studied, but the linear variation of their height, decreasing from the center of the plate to its edge, is considered (fig.1). Next, the effect of the welding seams between the ribs and the plate is neglected. Adopting the theory of the orthotropic constructions, we consider the expressions of the filling coefficients, written in the form [2]:

$$k_r = k_\theta = 1, \tag{1}$$

for the smooth plate;

$$k_r = \frac{n \cdot \delta_n}{2 \cdot \pi \cdot r} \; ; \; k_\theta = 0 \, , \tag{2}$$

for the ribbed layer.

In the above formulas were noted: k_r, k_θ – the radial and annular filling coefficient; n – the number of radial ribs; δ_n – the thickness of the ribs, equal or different from the thickness of the base plate; r – current radius.

The position of the neutral surface of the ribbed plate is determined by canceling the function [2]:

$$f_{2r} = \int \frac{E \cdot k_r \cdot z}{1 - k_r \cdot k_\theta \cdot \nu^2} \cdot dz \, , \tag{3}$$

in which were also noted:

E – the longitudinal modulus of elasticity of the material of the base plate and of the ribs (the same or different from each other;

E_p, E_R – the longitudinal modulus of elasticity of the material of the plate, respectively of the ribs);

ν – the transverse contraction modulus of the plate material, respectively of the ribs (same or different); ν_p, ν_n – Poisson's ratio for the material of the plate, respectively of the ribs;

z – the current elevation measured from the level of the neutral surface of the plate, established with the relationship:

$$z = z_1 - z_0, \tag{4}$$

where z_1 is the current elevation measured from the smooth surface of the plate, and z_0 is the elevation of the neutral surface of the ribbed plate [1].

The integral (3) breaks down in the form:

$$f_{3r} = \int_0^{\delta_p} \frac{E_p \cdot (z_1 - z_0)}{1 - \nu_p^2} dz_1 + \int_{\delta_p}^{\delta_p + h_n(r)} \frac{E_n \cdot n \cdot \delta_n \cdot (z_1 - z_0)}{2 \cdot \pi \cdot r} dz_1 \tag{5}$$

Putting the $f_{3r} = 0$ condition and taking into account that the ribs have a constant height on the portion between the outside of the pipe section installed in the center of the plate and a circumference characterized by the r_2 radius, after which their height decreases linearly to the h_e value, the following equalities are deduced for the position of the neutral surface of the plate [5, 6]:

- For $r \in \lfloor r_1, r_2 \rfloor$ [1]: $z_0 = \frac{F_1(1)}{F_2(2)} \cdot \delta_p$, (6)

with:

$$F_1(1) = 1 + \frac{n}{2 \cdot \pi} \cdot (1 - \nu_p^2) \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \frac{h_n(0)}{\delta_p} \cdot \frac{2 \cdot \delta_p + h_n(0)}{r}; \tag{7}$$

$$F_2(1) = 2 \cdot \left[1 + \frac{n}{2 \cdot \pi} \cdot (1 - \nu_p^2) \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \frac{h_n(0)}{r} \right], \tag{8}$$

where $h_n(0)$ is the constant height of the ribs on the respective area.

3. For $r \in \lfloor r_2, r_e \rfloor$: $z_0 = \frac{F_1(2)}{F_2(2)} \cdot \delta_p$, (9)

where:

$$F_1(2) = 1 + \frac{n}{2 \cdot \pi} \cdot (1 - \nu_p^2) \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \frac{h_n(r)}{\delta_p} \cdot \frac{2 \cdot \delta_p + h_n(r)}{r}; \tag{10}$$

$$F_2(2) = 2 \cdot \left[1 + \frac{n}{2 \cdot \pi} \cdot (1 - \nu_p^2) \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \frac{h_n(r)}{r} \right] \tag{11}$$

where:

$$h_n(r) = h_c + \frac{r_e - r}{r_e - r_2} \cdot [h_n(0) - h_e] \tag{12}$$

The stiffness of the ribbed plate is determined using the formula:

$$\int_{-\delta_1}^{\delta_2} \frac{E \cdot k_r \cdot z^2}{1 - k_r \cdot k_\theta \cdot \nu^2} \cdot dz, \tag{13}$$

Metamorphosing into:

$$f_{5r} = \int_0^{\delta_p} \frac{E_p \cdot (z_1 - z_0)^2}{1 - \nu_p^2} \cdot dz_1 + \int_{\delta_p}^{\delta_p + h_n(r)} \frac{E_n \cdot n \cdot \delta_n \cdot (z_1 - z_0)^2}{2 \cdot \pi \cdot r} \cdot dz_1. \tag{14}$$

Following the necessary calculations, we obtain:

4. For $r \in [r_1, r_2]$: $f_{5r} = D \cdot k_{D1}$, (15)

where:

$$k_{D1} = 4 \cdot \left[1 - 3 \cdot \frac{z_0}{\delta_p} + 3 \cdot \left(\frac{z_0}{\delta_p} \right)^2 \right] + \frac{6 \cdot n \cdot (1 - \nu_p^2)}{\pi} \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \left[1 + \frac{h_n(0)}{\delta_p} + \frac{1}{3} \cdot \left(\frac{h_n(0)}{\delta_p} \right)^2 - \frac{z_0}{\delta_p} \cdot \left(2 + \frac{h_n(0)}{\delta_p} \right) + \left(\frac{z_0}{\delta_p} \right)^2 \right] \cdot \frac{h_n(0)}{r}; \tag{16}$$

5. For $r \in \lfloor r_2, r_e \rfloor$: $f_{5r} = D \cdot k_{D2}$ (17)

where:

$$k_{D2} = 4 \cdot \left[1 - 3 \frac{z_0}{\delta_p} + 3 + \frac{6 \cdot n \cdot (1 - \nu_p^2)}{\pi} \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \left| 1 + \frac{h_n(r)}{\delta_p} + \frac{1}{3} \cdot \left(\frac{h_n(r)}{\delta_p} \right)^2 - \frac{z_0}{\delta_p} \cdot \left(2 + \frac{h_n(r)}{\delta_p} \right) + \left(\frac{z_0}{\delta_p} \right)^2 \right| \cdot \frac{h_n(r)}{r} \right] \tag{18}$$

In the relations (15) and (17) the notation was used:

$$D = \frac{E_p \cdot \delta_p^3}{12 \cdot (1 - \nu_p^2)} \tag{19}$$

representing the cylindrical bending stiffness characteristic of the base plate.

3. EXEMPLIFICATION

In order to quantify the theoretical results deduced in the preceding, the following geometric and material characteristics are taken into consideration: r_1 =80 mm; r_2 = 100 mm; r_e = 2000 mm; $E_n = E_p = 2{,}1 \cdot 10^5 \frac{N}{mm^2}$

$\delta_n = \delta_p = 10$ mm; $\nu_n = \nu_p = 0{,}3$.

In order to notice the influence of the height of the ribs, on the one hand, and the number of ribs, on the other

hand, on the position of the median surface of the ribbed plate and its cylindrical bending stiffness, ribs from 6 to 18 were considered (varying by two units), respectively heights from 10 mm to 50 mm.

In this paper, reference will be made only to the influence of the number of ribs on z_0 and on the k_D stiffness coefficient, on the portion between the r_2 and r_e radii, for different values of the r current radius. The obtained results are presented in tables 1 and 2.

Tabel 1

z_0 values

r	Ribs number		
	6	8	10
100	14.09	16.01	17.60
400	7.94	8.80	9.60
700	6.75	7.29	7.81
1000	6.25	6.64	7.03
1300	5.97	6.28	6.58
1600	5.79	6.05	6.30
1900	5.67	5.89	6.10
2000	5.64	5.84	6.05
r	**Ribs number**		
	12	**14**	**16**
100	18.95	20.11	21.11
400	10.36	11.07	11.74
700	8.31	8.80	9.26
1000	7.40	7.76	8.12
1300	6.88	7.17	7.46
1600	6.55	6.79	7.03
1900	6.31	6.52	6.73
2000	6.25	6.45	6.64

Tabel 2

$k_{D\,2}$ values

r	Ribs number		
	6	8	10
100	116.66	139.82	159.55
400	40.55	51.59	61.79
700	24.91	32.08	38.90
1000	18.14	23.44	28.54
1300	14.36	18.55	22.63
1600	11.94	15.42	18.81
1900	10.27	13.23	16.14
2000	9.82	12.64	15.42
r	**Ribs number**		
	12	**14**	**16**
100	176.98	192.79	207.45
400	71.24	80.05	88.28
700	45.40	51.59	57.52
1000	33.47	38.23	42.83
1300	26.60	30.46	34.22
1600	22.13	25.37	28.54
1900	18.99	21.78	24.53
2000	18.14	20.81	23.44

4. CONCLUSION

From the analysis of the data presented above, the following conclusions can be noted:

- both z_0 and $k_{D\,2}$ (obviously the stiffness of the ribbed plate) decrease strongly in the area between 100 and 700 mm, and much more slowly in the rest.
- the decrease in the values of the two sizes is also influenced by the reduced number of ribs.

REFERENCES

[1] Popa C, Anghelina F.V., Despa V., *Regarding at the circular plates rigidity with radial ribs, neglecting the welding seams influence. Ribs with constant thickness and lineal variable highness on length, placed just on a face,* The Scientific Bulletin of Valahia University Materials and Mechanics, 2022.

[2] Öchsner AM, Öchsner M. *The finite element analysis program MSC Marc/Mentat*, Singapore: Springer, 2016.

[3] Couchaux M., Hjiaj M., Ryan I., Bureau A., *Journal of Constructional Steel Research,* Vol.157/2019, Pp. 314-336.

[4] Sarou L, Couchaux M., Hjiaj M., Sire S., Douroux J.-F., Desbordes A., *Compressive stiffness of L-stubs in contact with rigid and flexible foundations,* Journal of Constructional Steel Research, Volume 201/2023.

[5] Khedr M.A., *Design of steel transmission pole base plates using yield line method*, Journal of Constructional Steel Research, 2022.

[6] Chen Y., Mou W., Guo Y., Wang J., Xue B, *Experimental /numerical study of a circular rib-stiffened flange connection with inner and outer flange plates under combined bending and tensile loading*, Scientific Reports, 2022.