The Scientific Bulletin of VALAHIA University
MATERIALS and MECHANICS –Vol. 18, No. 18

DOI 10.2478/bsmm-2022-0010

RIGIDITY OF CIRCULAR PLATES WITH RADIAL RIBS, PLACED ON ONE SIDE, NEGLECTING THE INFLUENCE OF WELDING SEAMS. NERVES WITH CONSTANT THICKNESS AND HEIGHT ALONG THE ENTIRE LENGTH

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Abstract: *The present study refers to a plane circular plate, provided with equidistant radial ribs, placed on one side (fig. 1), these having the same geometry of the cross section (thickness and height) along their entire length (fig.2). There is a change in the position of the neutral fiber of the ribbed plate and also in its bending rigidity, along the radius of the plate, in different cases.*

Keywords: *rigidity, circular plates, radial ribs*

1. INTRODUCTION

The ribbed constructions have always given satisfaction, in the terms of the material consumption, to both designers and their users. As the saving of the metallic materials is imperative, it is necessary to adopt appropriate methods for calculating sizing or checking such structures. The papers published in the literature confirm the consistency of existing concerns in the field, even in the sense of finding structures with optimal geometry [1, 2, 3, 4, 5, 6].

2. THE CALCULATION OF THE ELEMENTS CHARACTERISTIC OF THE PLATE

In the present study a reference is made to a plane circular plate, provided with equidistant radial ribs located on one side (fig. 1), which have the same cross-sectional geometry (thickness and height) along their entire length (fig. 2) [7].

In the following, the influence of the welding seams between the ribs and the base plate on the analyzed sizes is not discussed. As simplifying calculation hypotheses are considered: the construction material is isotropic; the plate deformations are small; the load is axially symmetrical, and the geometric configuration of the structure, too; the solicitation is in the elastic field, characteristic of the material used. The plane circular plate with radial ribs is an orthotropic structure.

In such a situation, among other calculation variants, one can use the "smoothing method" (by using of some characteristic filling coefficients), especially in the area with ribs, and which, in turn, will affect the values of the functions that characterize elastic behavior of the cross sections of the ribbed areas [8]:

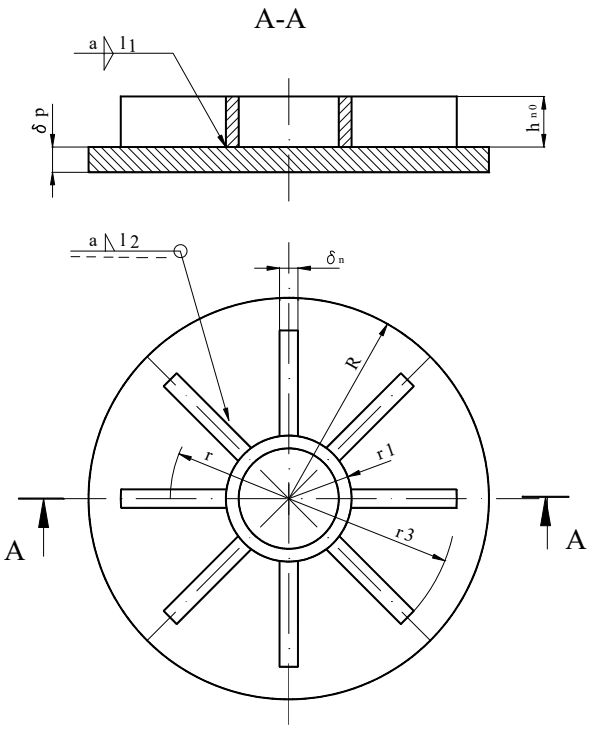


Figure 1. The plane circular plate, provided with ribs

$f_{1r}, f_2, f_{3r}, f_4, f_{5r}, f_6, f_{30}, f_{50}$,

respectively the functions that reflect the thermal regime:

$T_{1r}, T_{10}, T_{2r}, T_{20}, S_{1r}, S_{10}, S_{2r}, S_{20}$,

existing in the expressions of the efforts [8].

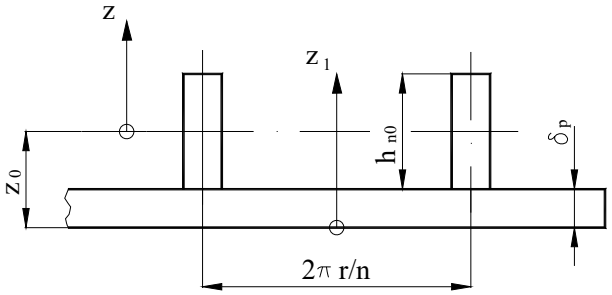


Figure 2. The equidistant radial ribs having the same section geometry

The following study considers the k_r, k_θ functions being the filling coefficients, in the radial and annular direction which, in this case, having the expressions:

$$k_r = k_\theta = 1, \text{ for the plate itself} \quad (1)$$

$$k_r = \frac{n \cdot \delta_n}{2 \cdot \pi \cdot r}, \; k_\theta = 0, \text{ for the ribbed area.} \quad (2)$$

The specific extension in radial direction will be:

$$f_{3r} = \int_0^{\delta_p} \frac{E_p \cdot (z_1 - z_0)}{1 - \nu_p^2} dz_1 + \int_{\delta_p}^{\delta_p + h_{n0}} \frac{E_n \cdot \frac{n \cdot \delta_n}{2 \cdot \pi \cdot r} \cdot (z_1 - z_0)}{1 - \frac{n \cdot \delta_n}{2 \cdot \pi \cdot r} \cdot \nu_p^2} dz_1 . \quad (3)$$

The z coordinate is determined by the relation:

$$z = z_1 - z_0, \quad (4)$$

where E_p, E_n represents the modulus of transversal elasticity of the plate, respectively of the ribs, υ_p - Poisson's ratios for plate and ribs, respectively z_1 represents the current elevation, measured from the lower smooth surface of the plate.

Taking into account the previous observations, after calculating the (3) integral and setting the condition $f_{3r} = 0$, the z_0 expression is determined, in the form:

$$z_0 = \frac{F_1}{F_2}, \quad (5)$$

where:

$$F_1 = \frac{1}{\delta_p} + \frac{n}{2 \cdot \pi \cdot \delta_p} \cdot (1 - \nu_p^2) \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \frac{h_{n0}}{\delta_p} \cdot \frac{2 \cdot \delta_p + h_{n0}}{r}; \quad (6)$$

$$F_2 = 2 \cdot \left[1 + \frac{n}{2 \cdot \pi} \cdot (1 - \nu_p^2) \cdot \frac{E_{nr}}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \frac{h_{n0}}{r} \right] \quad (7)$$

If the ribs and the base plate are made of same material, and have same thickness, then the (6) and (7) expressions become:

$$F_1 = \frac{1}{\delta_p} + \frac{n}{2 \cdot \pi \cdot \delta_p} \cdot (1 - \nu_p^2) \cdot \frac{h_{n0}}{\delta_p} \cdot \frac{2 \cdot \delta_p + h_{n0}}{r}; \quad (8)$$

$$F_2 = 2 \cdot \left[1 + \frac{n}{2 \cdot \pi} \cdot (1 - \nu_p^2) \cdot \frac{h_{n0}}{r} \right].$$

Considering that for a given circumference the position of the neutral fiber of the ribbed plate is constant, the integral that characterizes the geometric-elastic characteristics of the section is:

$$f_{5r} = \int_0^{\delta_p} \frac{E_p \cdot (z_1 - z_0)^2}{1 - \nu_p^2} dz_1 + \int_{\delta_p}^{\delta_p + h_{n0}} \frac{E_n \cdot \frac{n \cdot \delta_n}{2 \cdot \pi \cdot r} \cdot (z_1 - z_0)^2}{1 - \frac{n \cdot \delta_n}{2 \cdot \pi \cdot r} \cdot \nu_p^2} dz_1 \quad (10)$$

This relation can be written as:

$$f_{5r} = \frac{E_p \cdot \delta_p^3}{12 \cdot (1 - \nu_p^2)} \cdot k_D = \Re \cdot k_D, \quad (11)$$

Where $\Re$ is the cylindrical bending rigidity of the plane circular plate, and k_D- the rigidity multiplication factor:

$$k_D = 4 \cdot \left[1 - 3 \cdot \frac{z_0}{\delta_p} + 3 \cdot \left(\frac{z_0}{\delta_p} \right)^2 \right] + \frac{6 \cdot n \cdot (1 - \nu_p^2)}{\pi} \cdot \frac{E_n}{E_p} \cdot \frac{\delta_n}{\delta_p} \cdot \left[1 + \frac{h_{n0}}{\delta_p} + \frac{1}{3} \cdot \left(\frac{h_{n0}}{\delta_p} \right)^2 - \frac{z_0}{\delta_p} \cdot \left(2 + \frac{h_{n0}}{\delta_p} \right) + \left(\frac{z_0}{\delta_p} \right)^2 \right] \cdot \frac{h_{n0}}{r} \quad (12)$$

Due to the bending rigidity of the plate without ribs, the k_D factor indicates the overall effect on the rigidity of the ribbed plate.

When the ribs and the base plate have the same construction material, respectively the same thickness of the plate and the ribs, the (12) formula is converted into:

$$k_D = 4\cdot\left[1-3\cdot\frac{z_0}{\delta_p}+3\cdot\left(\frac{z_0}{\delta_p}\right)^2\right]+\frac{6\cdot n\cdot\left(1-\nu_p^2\right)}{\pi}\cdot$$
$$\left[1+\frac{h_{n0}}{\delta_p}+\frac{1}{3}\cdot\left(\frac{h_{n0}}{\delta_p}\right)^2-\frac{z_0}{\delta_p}\cdot\left(2+\frac{h_{n0}}{\delta_p}\right)^2+\right.$$
$$\left.+\left(\frac{z_0}{\delta_p}\right)^2\right]\cdot\frac{h_{n0}}{r} \qquad (13)$$

3. THE EXEMPLIFICATION OF THE CALCULATION OF THE ELEMENTS CHARACTERISTIC OF THE PLATE

In order to observe the change of the position of the neutral fiber of the ribbed plate, and also of its rigidity at bending, along the radius of the plate, a plate with the following geometric characteristics was adopted for example:

δ_p = 10 mm; r_1 = 80 mm; R = 325 mm; r_3 = 260 mm; δ_n = 5 mm; h_{n0} = 10, 20, 30, respectively 40 mm; n = 6, 8, 10, 12 pieces.

The plate was considered to be made of steel ($E_p = E_n = 2{,}1\cdot10^5$ N/mm^2; $\nu_p = 0{,}3$).

Table 1. The neutral fiber position depending on the number of ribs and the current radius

r [mm]	The number of ribs [pieces]			
	6	8	10	12
80	9,4812	10,6464	11,6897	12,6294
100	8,7152	9,7253	10,6464	11,4898
120	8,1726	9,0625	9,8846	10,6464
140	7,7683	8,5628	9,3040	9,9970
160	7,4554	8,1726	8,8467	9,4816
180	7,2061	7,8594	8,4773	9,0625
200	7,0027	7,6025	8,1726	8,7152
220	6,8336	7,3879	7,9170	8,4225
240	6,6909	7,2061	7,6995	8,1726
260	6,5688	7,0499	7,5122	7,9567

Figure 3 shows the variation of the position of the neutral fiber of the ribbed plate, z_0 , depending on the r current radius and the n number of ribs, which takes the values 6, 8, 10, 12 pieces. The height of the ribs considered in calculation is h_{n0} = 40 mm. The results obtained with the help of the Mathematica program are presented in table 1, based on them being drawn the graphs from figure 3.

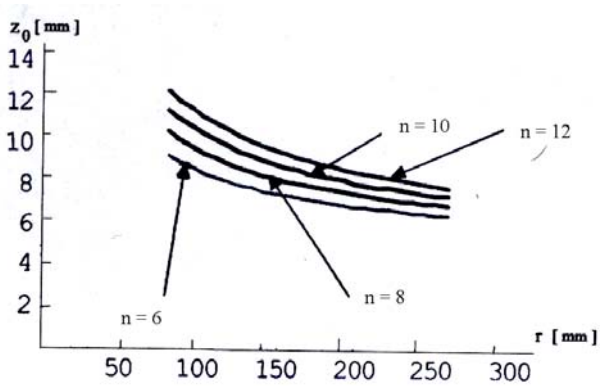


Figure 3. The variation of the position of the neutral fiber of the ribbed plate according to the number of ribs and the current radius

The figure 4 illustrates the change in the position of the neutral fiber of the ribbed plate z_0, depending on the r current radius and the h_{n0} height of the rib, for a number of eight ribs.

The results obtained are presented in Table 2.

Both the values in Table 2 and the corresponding graphs in figure 4 were obtained using the Mathematica program.

Table 2. The neutral fiber position depending on rib height and current radius for 8 ribs

r [mm]	The rib height[mm]			
	10	20	30	40
80	5,6798	6,9095	8,5906	10,6464
100	5,5505	5,5655	7,9758	9,7258
120	5,4626	5,3255	7,5407	9,0525
140	5,3989	6,1508	7,2167	8,5628
160	5,3505	6,0152	6,9659	8,1725
180	5,3127	5,9098	6,7661	7,8594
200	5,2822	5,8236	6,6032	7,6025
220	5,2572	5,7523	6,4678	7,3879
240	5,2362	5,6923	6,3535	7,2061
260	5,2184	5,6412	6,2557	7,0499

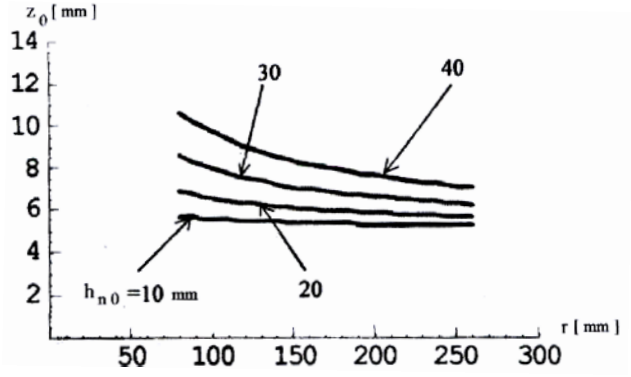


Figure 4. The changing the position of the neutral fiber of the ribbed plate depending on the height of the ribs and the current radius for 8 ribs

The figure 5 shows the variation of the k_D rigidity multiplication factor, depending on the r current radius and the n number of ribs (which takes the values 6, 8, 10, 12 pieces).

The rib height is $h_{n0} = 40$ mm.

The obtained results were introduced in the Mathematica program and are presented in table 3 and in the form of graphs in figure 5.

The graphical representation in the figure 6, based on the data in the table 4, reflects the variation of the k_D rigidity multiplication factor, depending on the r current radius and the h_{n0} rib height, for a number of eight ribs.

Table 3. The rigidity multiplication factor values as function of height nerves and current radius

r [mm]	The rib height[mm]			
	10	20	30	40
80	1,8886	5,0207	11,5868	22,6073
100	1,7189	4,2841	9,7151	18,9050
120	1,6036	3,7758	8,4075	16,2923
140	1,5202	3,4039	7,4419	14,3479
160	1,4571	3,1200	6,6994	12,8435
180	1,4076	2,8960	6,1105	11,6447
200	1,3678	2,7149	5,6321	10,6668
220	1,3350	2,5654	5,2356	9,8536
240	1,3076	2,4398	4,9017	9,1668
260	1,2844	2,3329	4,6166	8,5791

The tables 1 and 2 show the discrete values of 10, 20, 30, 40 mm for the height of the ribs, respectively 6, 8, 10 and 12 pieces for their number. It is found, from the analysis of the data contained in the mentioned tables, that the more pronounced variation of z_0 and k_D is manifested especially up to the value of 200 mm of the current radius.

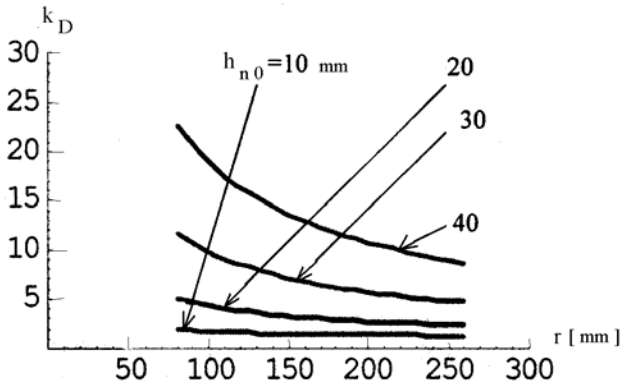


Figure 6. The rigidity multiplication factor values as function of height nerves and current radius

Towards the periphery of the plate, the values of the mentioned quantities have a much slower decrease. This situation is largely due to the low number of ribs in this area.

In order to increase the rigidity in the marginal part of the plate, it is possible to resort to a thickening of the ribs, obviously with the supplementation of the weight of the plate, as a whole.

Reducing this weight can be achieved by chamfering the ribs from the center to the periphery. On the other hand, it is also mentioned that the influence of the rib height, towards the outer contour of the plate, is less significant.

4. CONCLUSIONS

From the analysis of the expressions established in the present study, but especially from the configuration of the graphs contained in the exposed figures, the following observations can be deduced, regarding the position of the neutral surface of the plate with radial ribs:

- with the increase of the number of ribs the value of the z_0 quota is higher, moving away from that of the smooth plate;

- the height of the ribs positively influences the position of the neutral surface;

- for the same geometry of the ribbed plate - number of ribs, respectively rib height - with the increase of the radius there is a decrease of the neutral surface, with a tendency to approach that of the median surface of the base plate.

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