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EFFECT OF GRAVITATION MODULATION ON VISCOELASTIC NONLINEAR FERRO-CONVECTION

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Abstract. This paper examines a nonlinear investigation of viscoelastic ferromagnetic convection under gravity modulation. A weakly nonlinear instability has been investigated using the perturbation method. Based on the Oldroyd model for ferrofluids, the constitutive equations of the flow model were developed. The coupled system of the Khayat-Lorenz model developed and was numerically solved in the presence of gravity modulation. Heat transfer results are examined for nonlinear theory and neutral stability curves for linear theory. The nonlinear PDE system is transformed into a nonlinear ODE system with similarity transformations up to the second term. The dual effect of the gravity modulation includes stabilizing and destabilizing.

Keywords: Gravity Modulation, Convection, Ferromagnetic fluid, Heat transfer, viscoelastic fluid.

1. Introduction

The early studies of gravity modulation were done by Gresham and Sani [1]. The complex spatio temporal structure of this modulation which arises from the interaction of the gravitational field and density gradients depends on the type of the density gradients as well as the spatial and frequency distribution of the vibration. There are numerous applications our model can be seen. For example under earthbound conditions it has a considerable impact on the homogeneous melt growth of semiconductor or metal crystals. Convective flow suppression is made more challenging by crew movements, mechanical vibrations (excitation of natural frequencies of spacecraft structure, pumps, and motors), atmospheric drag, the earth's gravity gradient, and space craft manoeuvres, even though the microgravity environment is helpful in this regard. There are many engineering thermal applications gain from knowledge of gravitational fluctuation in this way. The impact of gravity modulation on thermal instability is well documented in the studies of [2]-[14].

In recent studies Kiran [15] investigated the effect of gravity modulation on double-diffusive oscillatory convection in a viscoelastic fluid layer. A complex landau model was used to determine the convection finite amplitude. Kiran [16] has documented the effect of gravity modulation on weakly nonlinear thermal convection in a fluid layer with rigid boundaries. In comparison to other models, rigid boundary convective models are found to transfer less heat. Kiran et al. [19] reported on the effect of gravity modulation on double-diffusive convection in the presence of an applied magnetic field and an internal heat source. The author clarified that internal heating can either increase or decrease heat transmission depending on Nu. Kiran et al. [18, 19] examined the effects of g-jitter on double-diffusive oscillatory magneto convection in the presence of an applied magnetic field. Manjula et al. [20, 21] reported the effects of oscillatory convection generated by g-jitter and throughflow. It was observed that the effects of g-jitter and throughflow on heat transfer are dual in nature. Rotation and viscoelastic properties were also considered as additional influences on thermal instability. Pranesh and Richa [22] have examined the consequences of three-component convection in a vertically oscillating viscoelastic fluid with cross effects. It has been observed that by selecting the proper values of the parameters, the initiation of convection and heat mass transfer may be managed.

Using the weak nonlinear analysis there are several studies available for different physical models on thermal instability. Bhadauria and Kiran [23] investigated the temperature dependent viscosity effect on thermal instability in a porous media. Magneto convection under magnetic field was reported by Bhadauria and Kiran [24]. The effect of throughflow on weakly nonlinear

convection in a viscoelastic saturated porous medium was reported by Kiran et al. [25]. Rotation speed modulation effect on temperature dependent viscous fluid layer was introduced by Bhadauria and Kiran [26]. The effect of temperature modulation on chaotic convection in porous media was investigated by Kiran and Bhadauria [27]. Nanofluid convection has been studied by Bhadauria and Kiran [28] and Kiran [29] using the normal mode analysis. The effect of throughflow on thermal instability presented in the studies of [30]-[32]. The majority of the studies mentioned above addressed how inflow and outflow affect the system, stating that inflow stabilizes it while outflow destabilizes it. The effect of throughflow on porous convection was given by Homsy et al. [33]. They have drawn the results of both linear and energy theory. It was found that the fluid can lose stability by either a buoyantly driven mode or by a continuous analogue of the Saffman-Taylor mode. The limit for linear stability analysis of [33] was set by Jones et al [34].

We know that convection in ferromagnetic liquids is ferroconvection, and it has great interest because of their applications that have been proposed by Kaiser R., Miskolczy [35], Raj and Chorney [36], Scherer et al. [37]. There are many works on theoretical and experimental investigations of ferroconvection are available. Finlayson [38] noted that a fluid layer heated from below in the presence of a uniform vertical magnetic field is likely to experience convective instability of a ferromagnetic fluid. Gupta and Gupta [39] examined the convective instability of a ferromagnetic fluid layer rotating about a vertical axis. It is found that oscillatory convection is possible for Pr > 1. The extension work of Finlayson was given by Stiles and Kagan [40]. Vaidyanathan et al. [41] investigated the effect of magnetic field dependent viscosity on ferroconvection in rotating layer. It is found that the magnetic field dependent viscosity delays the onset of convection. Linear convective flow of ferromagnetic fluid between two vertical differentially heated plates placed in a uniform magnetic field perpendicular to the plates was studied by Suslov [42]. The effect of temperature modulation on ferrofluid layer subjected to vertical magnetic field was reported by Singh and Bajaj [43].

Thermo-convective instability in a rotating ferromagnetic fluid layer with temperature modulation was given by Sibanda and Noreldin [44]. The study of nonlinear thermal instability and the effect of rigid body rotation on ferroconvection were reported by Melson et al. [45, 47]. The effect of gravity modulation ferroconvection in a densely packed porous layer for linear theory was reported by Nisha et al. [46]. Thermo-rheological effects on Benard convection with rotational effects were investigated by Manjula and Kiran [48]. Kiran [49] reported double diffusive convection for viscoelastic fluids with oscillatory mode under gravity modulation. A weak

nonlinear oscillatory convection of a viscoelastic fluid layer with gravity modulation was studied by Bhadauria and Kiran [50].

The literature described above demonstrates that no study has examined the effect of gravity modulation on nonlinear ferroconvection. Therefore, the aim of the present work is to investigate the effect of gravity modulation on nonlinear ferroconvection. Here, we use truncated Fourier series to formulate a finite convective amplitude equation in terms of gravity modulation. This paper considers four distinct viscoelastic fluids–Rivlin Ericksen, Maxwell, Newtonian, and Jeffery fluids–have been made, and the results presented. To the best of the author's knowledge, no information has been published about the current problem or discovered in the literature.

2. Governing Equations

The physical configuration of the problem is given in Fig. 1.

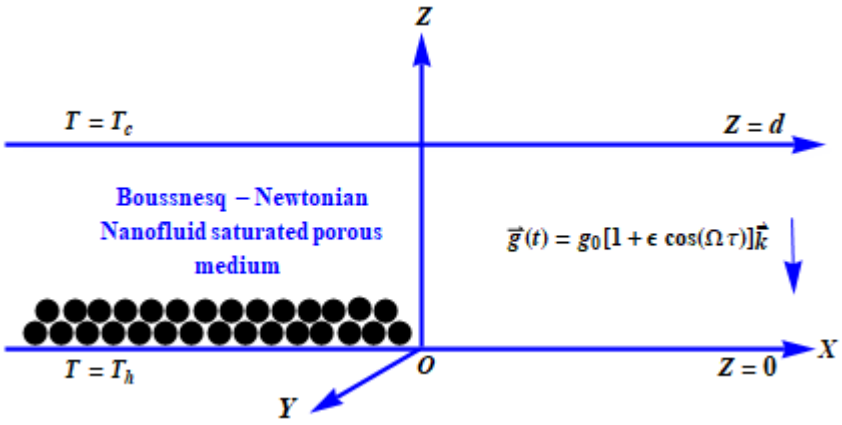


Fig. 1. Physical configuration of the problem

It is infinitely extended horizontally and departs from the depth d. This layer filled with viscoelastic ferromagnetic fluid between two stress-free parallel plates. These plates are heated from below and cooled from above has been considered. The plates are situating at z=0 and z=d. There are two components to the gravity acceleration term: a stable component and an oscillating component. The governing equations of the flow are as follows (Melson et al. [45, 47]):

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\rho_0 \frac{\partial \vec{q}}{\partial t} + \nabla p + (\vec{q} \cdot \nabla)\vec{q} = \rho \vec{g} + \nabla \cdot \left(\vec{H}\vec{B}\right) + \nabla \cdot \tilde{T} = 0, \tag{2}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\tilde{T} = \left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\mu(\nabla \vec{q} + \nabla \vec{q}^T) \tag{3}$$

$$\rho_0 C_{V,H}\left(\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla \mathrm{T})\right)=\kappa\nabla^2 T, \tag{4}$$

$$\rho=\rho_0[1-\alpha(T-T_0),] \tag{5}$$

$$\nabla\cdot\vec{B}=0,\vec{B}=\mu_0\left(\vec{H}+\vec{M}\right),\nabla\times\vec{H}=0 \tag{6}$$

$$\vec{g}=\mathrm{g}_0(1+\delta\,\mathrm{Cos}(\Omega_g \mathrm{t})\hat{\mathrm{k}} \tag{7}$$

The magnetization M is assumed to be aligned with the magnetic field H ($\vec{M}\times\vec{H}=0$), and hence, we have $\vec{M}=\frac{\vec{H}}{H}M(H,T)$ where it is given by $M(H,T)=M_0+\chi(H-H_0)-K(T-T_0)$. The momentum eq. (2) follows the Boussinesq approximation.

2.1. Conduction State

The basic state is assumed to be quiescent and represents the conduction phase of the system. The disturbances are therefore taken in the vertical directions. The basic state solution of the above system of eqs. (1-7) is results in the following equations. In this case $q_b = 0$ and we need to determine T_b; H_b and M_b as a function of z variable.

$$T_b(z)=T_0+\Delta T(1-\frac{z}{d}) \tag{8}$$

$$T_b(z)=\left(M_0-\frac{K\Delta T}{1+\chi}\left(1-\frac{z}{d}\right),H_0+\frac{K\Delta T}{1+\chi}(1-\frac{z}{d})\right) \tag{9}$$

2.2. Convection state

The perturbed state is subjected to an infinitely small disturbance that we apply as

$$X=X_b+X^{/} \tag{10}$$

where X represents the physical variables. The eqs. (1-7) are formulated after using eq. (10):

$$\nabla\cdot\overrightarrow{q'}=0, \tag{11}$$

$$\rho_0\frac{\partial\overrightarrow{q'}}{\partial t}+\nabla p'+(\overrightarrow{q'}\cdot\nabla)\vec{q}=\rho\vec{g}+\left(\overrightarrow{B_b}+\vec{B'}\right)\cdot\nabla\vec{H'}+(\vec{B'}\cdot\nabla)\overrightarrow{H_b}+\nabla\cdot\widetilde{T'}=0, \tag{12}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\widetilde{T'}=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\mu(\nabla\overrightarrow{q'}+\nabla\overrightarrow{q'}^{T}) \tag{13}$$

$$\left(\frac{\partial T'}{\partial t}+(\overrightarrow{q'}\cdot\nabla \mathrm{T_b})\right)=-\overrightarrow{q'}\cdot\nabla \mathrm{T'}+\kappa\nabla^2 T^{/}, \tag{14}$$

$$\nabla\cdot\vec{B'}=0, \tag{15}$$

with the following relations:

$$M_i' = \frac{M_0}{H_0} H_i', \quad i = 1,2. \tag{16}$$

$$M_3' = \chi H_3' - KT' \tag{17}$$

The above equations are then non-dimensionalised with the dimensionless variables $q^* = \frac{q}{\kappa}, p^* = \frac{pK}{\mu\kappa}$ and $T^* = \frac{T'-T_0}{\Delta T}$, $\phi^* = \frac{1+\chi}{K\Delta T}\phi'$, $\Gamma_1 = \frac{\lambda_1\kappa}{d^2}$, $\Gamma_2 = \frac{\lambda_2\kappa}{d^2}$, $\Gamma = \frac{\Gamma_2}{\Gamma_1}$. For the two-dimensional convective mode, stream function is taken into consideration. The non-dimensionless governing equations are (after dropping the asterisk for simplicity).

$$\left(1 + \Gamma_1 \frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2\psi - Ra(1+M_1)\frac{\partial T}{\partial x} + Ra\, M_1\left[\frac{\partial^2\phi}{\partial x \partial z} - J(\phi', T')\right)T' = \left(1 + \Gamma\Gamma_1\frac{\partial}{\partial t}\right)\nabla^2\psi \tag{18}$$

$$-\frac{\partial T_b}{\partial z}\frac{\partial \psi}{\partial x} - \nabla^2 T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi, T)}{\partial(x, z)} \tag{19}$$

where Ra = R g_m, R is the Rayleigh number and $g_m = 1 + \delta\, Cos\, \Omega t$ modulation term. Here M1 is the measure of Kelvin force, M3 is the measure of the departure of magnetization from linearity with M3=1. The non-dimensionlized parameters in the above equations have their usual meanings given in nomenclature. The eq. (18) can be expressed as two first-order partial differential equations by taking a suitable substitution (Siddheshwar et al. [30]). As a result we have.

$$Pr^{-1}\frac{\partial}{\partial t}\nabla^2\psi - M - \nabla^2\Gamma\psi = Ra\left(\frac{\partial T}{\partial x} - \left(\frac{\partial^2\phi}{\partial x \partial z} - J(\phi^/, T)\right)T' \right. \tag{20}$$

$$\Gamma_1\frac{\partial M}{\partial t} + M = (1-\Gamma)\nabla^4 T \tag{21}$$

Applying the normal mode analysis to the eqs. (17, 18) we get critical Rayleigh number for stationary and oscillatory case.

Following the studies of Chandrasekhar [31] we get $Ra = R_0 + \omega^2 R_2$, for $\omega^2 = 0_2$ the stationary Rayleigh number given by:

$$R_0 = \frac{\delta 2^3\ (\pi^2 + M_3 a^2)}{a^2(1+M_1)(\pi^2 + M_3 a^2) - a^2 M_1 \pi^2} \tag{22}$$

Also if $R_2 = 0$ gives the expression for oscillatory frequency:

$$\omega^2 = \frac{Pr\ \delta 2\ (1-\Gamma)-1-Pr}{\Gamma_1^2(1+\Gamma\ Pr)}. \tag{23}$$

We know that an oscillatory mode of onset convection sets in when $\omega^2 > 0$. This is possible for $0 < \Gamma < 1$. In this case the oscillatory Rayleigh number is given by:

$$R_0^{osc} = \frac{\delta 2\ (\pi^2 + M_3 a^2)[Q_1 \delta 2^2 Pr + \omega^2(Q_2 \delta 2\ Pr - 1)]}{a^2 Pr(1+M_1)(\pi^2 + M_3 a^2) - a^2 Pr M_1 \pi^2} \tag{24}$$

where $Q_1 = \frac{1+\Gamma\Gamma_1^2\omega^2}{1+\Gamma_1^2\omega^2}$ and $Q_2 = \frac{(1-\Gamma)\Gamma_1}{1+\Gamma_1^2\omega^2}$ and $\delta 2 = \pi^2 + a^2$.

The Eq. (24) is minimized with respect to 'a' and obtain values for critical Raylegh number. The corresponding results presented the paper.

2.3. Magnetic Khayat–Lorenz model for gravity modulation

The stream function ψ, the temperature distribution T, the magnetization M (the form of M is determined by the form of ψ), and the magnetic potential ϕ can be represented in the form of a minimal Fourier series (Kiran et al. [29, 32]). A local nonlinear stability analysis shall be performed and hence we will take the following Fourier expressions:

$$\psi = \sum_{n=1,m=1}^{\infty,\infty} Amn(\tau)\sin(m\alpha x)\sin(n\pi z)$$

$$T = \sum_{n=1,m=1}^{\infty,\infty} Bmn(\tau)\cos(m\alpha x)\sin(n\pi z)$$

$$M = \sum_{n=1,m=1}^{\infty,\infty} Dmn(\tau)\sin(\mathrm{m}\alpha x)\sin(\mathrm{n}\pi z)$$

$$\phi = \sum_{n=1,m=1}^{\infty,\infty} Cmn(\tau)\cos(\mathrm{m}\alpha x)\sin(\mathrm{n}\pi z) \tag{25}$$

In what follows we take the modes (1, 1) for stream function and (0, 2) and (1, 1) for other quantities. A detailed study and discussion on the number of modes can be found in Siddheshwar et al. [30]. The quantities are expanded as following:

$$\psi = A_{11}(\tau)\sin(\alpha x)\sin(\pi z) \tag{26}$$

$$T = B_{11}(\tau)\cos(\alpha x)\sin(\pi z) + \mathrm{B}_{02}(\mathrm{t})\sin 2\pi \mathrm{z} \tag{27}$$

$$M = D_{11}(\tau)\sin(\alpha x)\sin(\pi z) \tag{28}$$

$$\phi = E_{11}(\tau)\cos(\alpha x)\cos(\pi z) + \mathrm{E}_{02}(\mathrm{t})\cos 2\pi \mathrm{z} \tag{29}$$

where the amplitudes $A_{11}(t)$, $B_{11}(t)$, $B_{02}(t)$, $C_{11}(t)$, $C_{02}(t)$, $E_{11}(t)$ and $E_{02}(t)$ are functions of time and to be determined later. The solutions given eqs. (28-29) are assumed in order to satisfy the free-free boundary conditions for the eqs. (18-19).

$$\psi = T = \frac{\partial \phi}{\partial z} = 0 \;\; at \;\; z = 0 \;\; \& \;\; z = 1 \tag{30}$$

Substituting the eqs. (28-29) in eqs. (20-28) and taking the orthogonality condition with the Eigen functions associated with the considered region of convection cell given by $0 \leq x \leq 2\pi/a$ and $0 \leq z \leq 1$ as result the following system of differential equations are obtained:

$$\frac{1}{Pr}\frac{dA_{11}(t)}{dt} + \Gamma\delta 2\, A_{11}(t) = \frac{R\delta 2^2 B_{11}(t)}{a}\big(1 + M_{13}B_{02}(t)\big)g_m - D_{11}(t)\delta 2^{-1} \tag{31}$$

$$\frac{dB_{11}(t)}{dt} = aA_{11}(t) - \delta 2\, B_{11}(t) + a\pi A_{11}(t)B_{02}(t) \tag{32}$$

$$\frac{dB_{02}(t)}{dt} = \frac{a\pi}{2}A_{11}(t)B_{11}(t) - 4\pi^2 B_{02}(t) \tag{33}$$

$$\frac{dD_{11}(t)}{dt} = -\Gamma_1^{-1}D_{11}(t) + \frac{1-\Gamma}{\Gamma_1} - \delta 2^2 A_{11}(t) \tag{34}$$

where $M_{13} = \frac{\pi M_1 M_2 a^2}{\pi^2 + M_3(1+M_1)a^2}$ and $r = \frac{R}{R_0}$. The above system of simultaneous autonomous ordinary differential equations are solved numerically using Mathematica. The amplitudes E11(t) and E02(t) do not appear explicitly in eqs. (35-38) being independent of derivatives of time and the results are $E_{11} = \frac{-\pi B_{11}}{\pi^2 + M_3 a^2}$, and $E_{02} = \frac{-B_{11}}{2\pi}$. The above set of magnetic Khayat-Lorenz equations for gravity modulation are scaled by $\tau = \delta 2\, t$, $X = \frac{A_{11}a\pi}{\sqrt{2}\delta 2}$, $Y = \frac{B_{11}\pi R}{\sqrt{2}}$, $N = \frac{D_{11}a\pi}{\sqrt{2}(1-\Gamma)\delta 2^3}$, and $Z = -\pi R\, C_{11}$. The scaled magnetic Khayat-Lorenz equations for gravity modulation:

$$\frac{1}{Pr}\frac{dX(\tau)}{d\tau} = -\Gamma X(\tau) + Y(\tau)\left(1 - \frac{M_{13}Z_{11}}{\pi R}\right)g_m - (1-\Gamma)N \tag{35}$$

$$\frac{dY(\tau)}{d\tau} = RX(\tau) - Y(\tau) - X(\tau)Z(\tau) \tag{36}$$

$$\frac{dZ(\tau)}{d\tau} = -bZ(\tau) + X(\tau)Y(\tau) \tag{37}$$

$$\frac{dN(\tau)}{d\tau} = \frac{1}{\delta 2\Gamma_1}(-N(\tau) + X(\tau)) \tag{38}$$

where $b = \frac{4\pi^2}{\delta 2}$. The above system is solved numerically subject to the initial conditions:

$$Nu(\tau) = 1 + \left[\frac{\int_0^{\frac{2\pi}{a_c}} \frac{dT}{dz}\, dx}{\int_0^{\frac{2\pi}{a_c}} \frac{dT_b}{dz}\, dx}\right]_{z=0} \tag{40}$$

Substituting the expressions of T_b, T in the eq. (40) and simplifying, we get

$$Nu(\tau) = 1 - 2\pi B_{02}(\tau) = 1 + \frac{2}{R}\, Z(\tau) \tag{41}$$

3. Results and Discussion

The effect of viscoelasticity on ferroconvection under gravity modulation has been investigated using the Oldroyd model. The results corresponding to Maxwell fluids, Newtonian fluids, Jeffrey fluids and Rivlin–Ericksen fluids are discussed. The effect of various viscoelastic and modulation parameters have been obtained and depicted graphically. Our attention is to drawn to stability curves by critical Rayleigh number and heat transfer analysis by weak nonlinear theory. Neutral stability curves are depicted for oscillatory convection and compared with stationary case. For nonlinear case the effect of gravity modulation will be discussed on heat transfer. For neutral stability oscillatory curves the condition imposed ($\omega^2>0$) and it is possible only when $\Gamma>0$. The corresponding values of Γ_1 and Γ_2 are taken. The critical value of the Rayleigh number given by eq. (28) produces onset results *i.e.* neutral curves for stationary mode. The eq. (29) gives oscillatory critical Rayleigh number and produces onset neutral curves. If M1=0 we obtain the results of the usual viscoelastic fluids. However, we check our results for different fluids. In this paper we consider Newtonian fluid (Γ_1=Γ_2), Maxwell fluid $(\Gamma_1 \neq 0, \Gamma_2 = 0)$ Rivlin–Ericksen fluid $(\Gamma_1 = 0, \Gamma_2 \neq 0)$ and Jeffrey fluid $(\Gamma_1 \neq 0, \Gamma_2 \neq 0)$.

It is well known that for oscillatory mode of convection $\omega^2 \neq 0$ given by eq. (23). Therefore, we observe an oscillatory mode of instability for Jeffrey and the Maxwell type when classical heat flux model considered. The current model is unable to predict the oscillatory instability in Rivlin–Ericksen ferroliquid and hence a non classical heat flux model may be required as suggested by Melson et al. [45,47].

Fig. 2.1 shows how the critical Rayleigh number varies for stationary and oscillatory convection. It is observed that the critical Rayleigh number for oscillatory sets in before stationary mode which concludes that oscillatory convection takes place before stationary

mode. The reason is due to the presence of incremental growth rate of oscillatory frequency (ω^2). The effect of stress relaxation parameter Γ_1 on the oscillatory neutral curves is shown in Fig. 2.2. It is observed that stress relaxation parameter results in decrease of the oscillatory Rayleigh number. Therefore, the stress relaxation parameter destabilizes the system. It is important to note that when $\Gamma_1 = \Gamma_2$ the oscillatory neutral curves concurs with Newtonian case. The stability of the system in the oscillatory mode becomes independent of the viscoelastic parameters. Fig. 2.3 represents the effect of strain retardation parameter Γ_2 on the oscillatory neutral curves. It is found that critical Rayleigh number decreases with Γ_2. Therefore, strain retardation parameter Γ_2 diminish onset convection. This case indicates that the neutral curve is identical with that of Newtonian case. Further, from the Figs. 2.1 and 2.3 it is found that the oscillatory neutral curves of viscoelastic fluids lies below that of a Newtonian case. This justifies the fact that effect of viscoelasticity destabilizes the system towards for oscillatory mode as compared to Newtonian fluid.

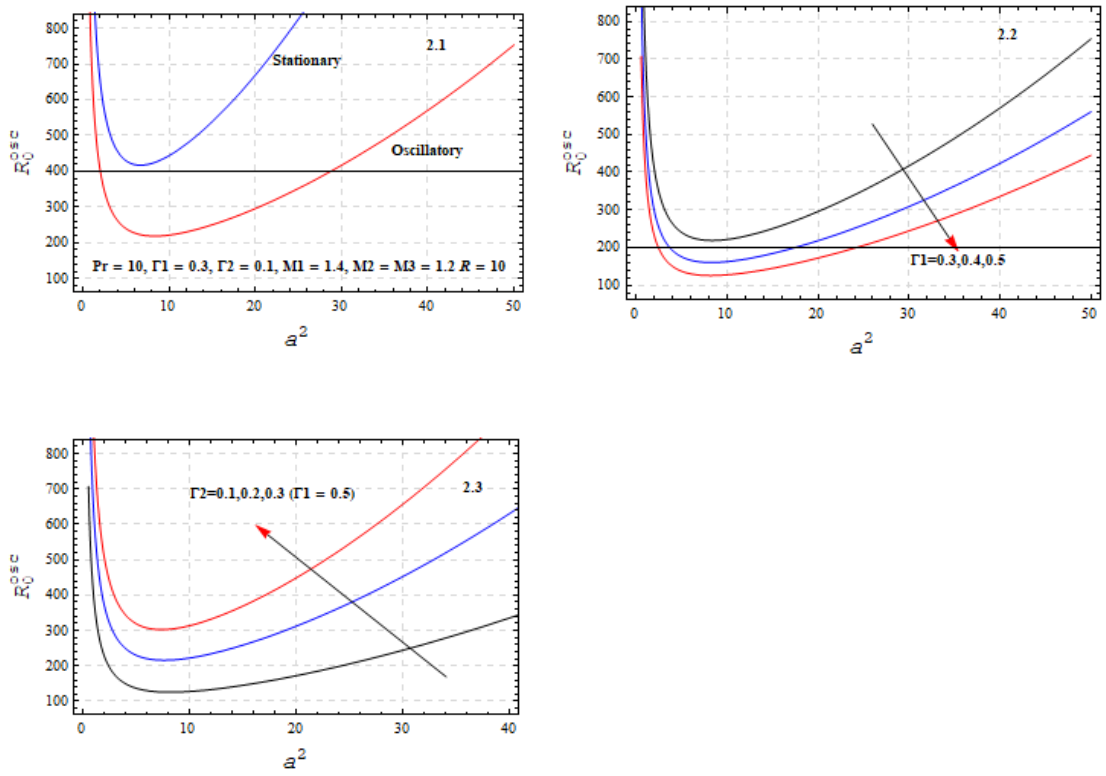


Fig. 2. Neutral stability curves; 2.1. Comparison; 2.2. Effect of Γ1; 2.2. effect of Γ2

The influence of Prandtl number Pr on the oscillatory neutral curves is similar to Fig. 2.2 and for fixed values of other parameters. From the results we observe that for small and moderate values of Pr the critical value of oscillatory R0c decreases with Pr. The figure

corresponding to Pr is not presented due to repetition. It is observed that the results of Newtonian, Maxwell, Rivlin–Ericksen and Jeffery ferro-liquids and observed the results as follows.

$$R^{osc}(\text{Rivlin}) > R^{osc}(\text{Maxwell}) > R^{osc}(\text{Newtonian}) > R^{osc}(\text{Jeffery}) \tag{42}$$

The thresholds of the onset of convection for both stationary and oscillatory modes were found and presented. These results match with the results available in the literature of Melson et al. [47].

The results corresponding to weak nonlinear analysis shall be discussed. The effect of gravity modulation on Nu as a function of slow is now discussed. The Jocobian term $\frac{\partial(\psi,T)}{\partial(x,z)}$ given in eq. (18) revel the relation between ψ and T which leads to nonlinear thermal convection. It is know that linear theory reports stability analysis and fails to report nonlinear theory. This is the reason to investigate nonlinear study with the help of eq. (29). Heat transfer results are presented in Figs. 4 and 5 in terms of Nu with slow time.

The effect of relaxation parameter Γ_1 on heat transfer is shown in Fig. 4.1. From the figure it is found that relaxation parameter enhances the amplitude of heat transfer. Further, Fig. 4.2 shows that the effect of Γ_2 is to decrease the amplitude of heat transfer. Thus, the effect of viscoelastic fluid has dual role on heat transfer. These results are competing with Bhadauria et al. [11, 12, 49] and Kiran [15, 16]. The effect of Prandtl number Pr on heat transfer is shown in Figs. 4.3. It is found that heat transfer increases with increasing Pr. The moderate values of Pr are taken for heat transfer results. The similar nature of the results obtained by Bhadauria et al. [11, 12]. It is observed that effect of scaled Rayleigh number shows its natural behaviour on Nu *i.e.* reduces heat transfer (Fig. 4.4). We know that enhancement in R0c shows more energy to promote onset convection in the system.

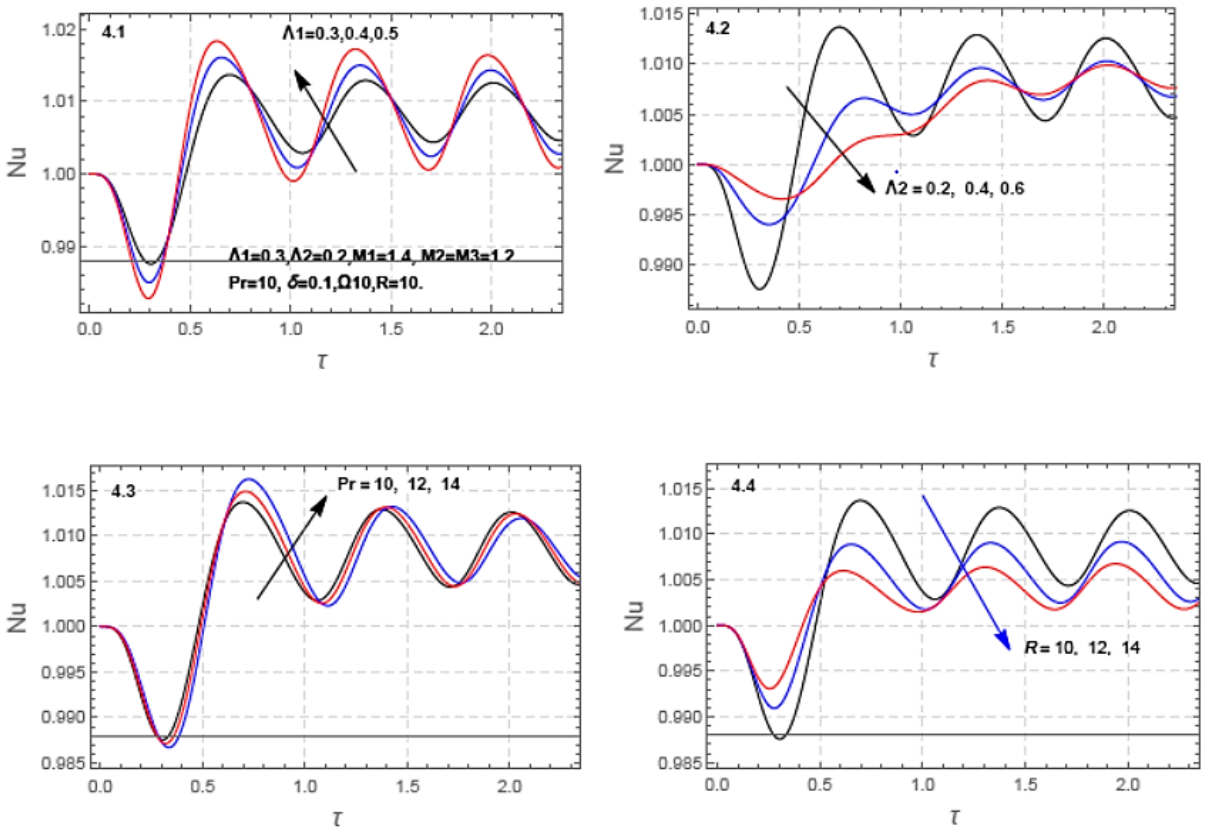


Fig. 4. Heat transfer results for (Λ1, Λ2, Pr and R)

Fig. 5 compares the effects of different fluids on heat transfer. Fig. 5 (1, 2, 3) shows the results of comparison, here Ericksen fluids transfer better heat transfer than others. Jeffrey fluids are transfer less heat transfer than others. Maxwell and Newtonian fluids transfers in between other two fluids. In Fig. 5(3) all four fluids comparison made. The effect of gravity modulation presented in Fig. 6. In Fig. 6.1 the results of amplitude of modulation shown upon increasing δ, Nu increases thus heat transfer enhances. Similarly, in Fig. 6.2 the effect of frequency of modulation shows diminishing heat transfer. With this one may conclude that gravity modulation has dual role on heat transfer. These results may confirm with Kiran et al. [18, 19], Kiran [30, 31] and Bhadauria and Kiran [49, 50].

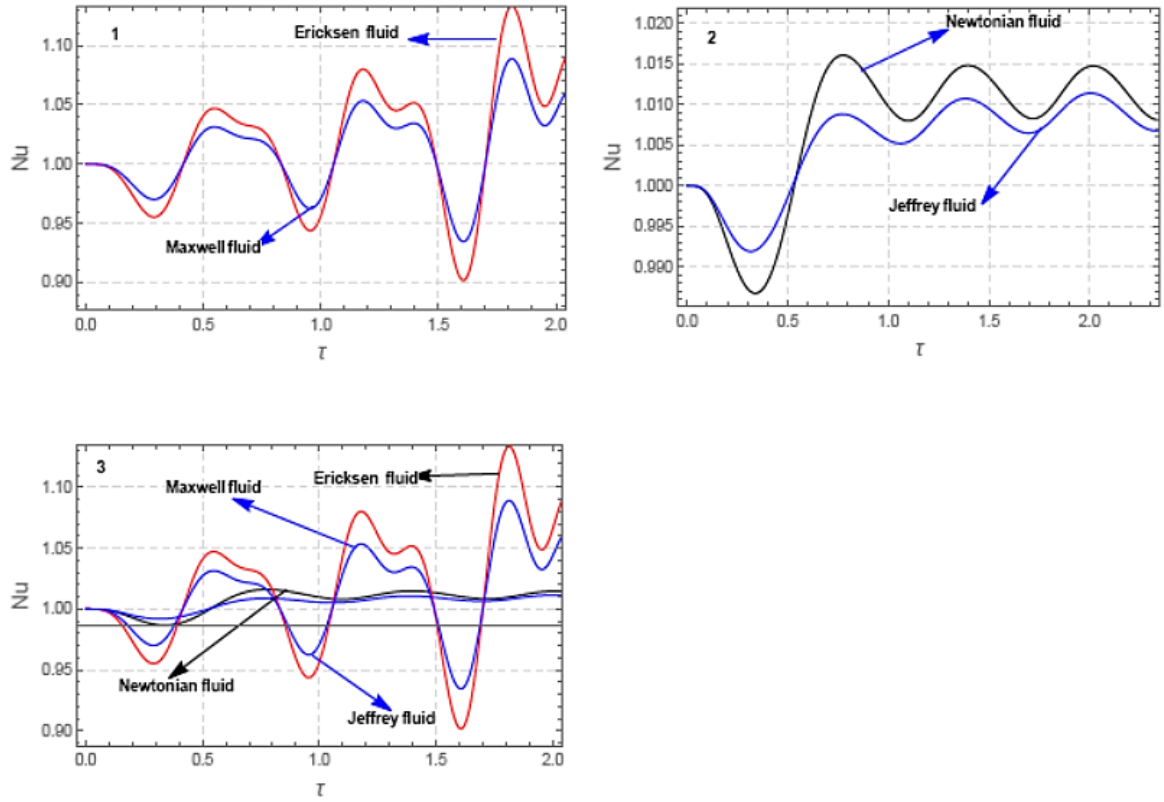


Fig. 5. Comparison of different fluids

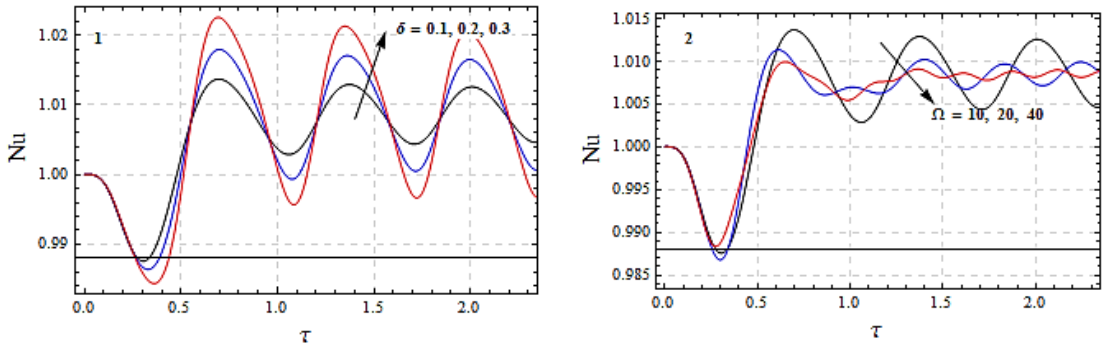


Fig. 6. Effect of modulation amplitude and frequency on Nu

In Figs. 7(1) to 7(12) the plots are depicted for streamlines and isotherms to see the convective nature. As slow time vary from τ=0.0 to τ =4.0 the stable flow becomes more vibrant and its effect of convective flow reveal. It is clear that with time the magnitudes of streamlines and isotherms are varying from the initial state (7(1),7(2),7(7),7(8)). From the streamlines one may observe that there is an enhancement in the cell size. Similarly isotherms are more vibrant as time goes on. In all the Figs. 7(3,4) to 7(9,10) there will be a rigorous change in streamlines and

isotherms. In the case of isotherms convection in the form of conduction which approaches to convection later state. The magnitude of isotherms increasing with slow time and achieves its unchangeable state after $\tau = 4$ (see in 7(5,6) to 7(11,12)). Our results are quite agree with the results of Kiran [15, 16], Kiran et al. [29] and Manjula et al. [17, 20, 21]).

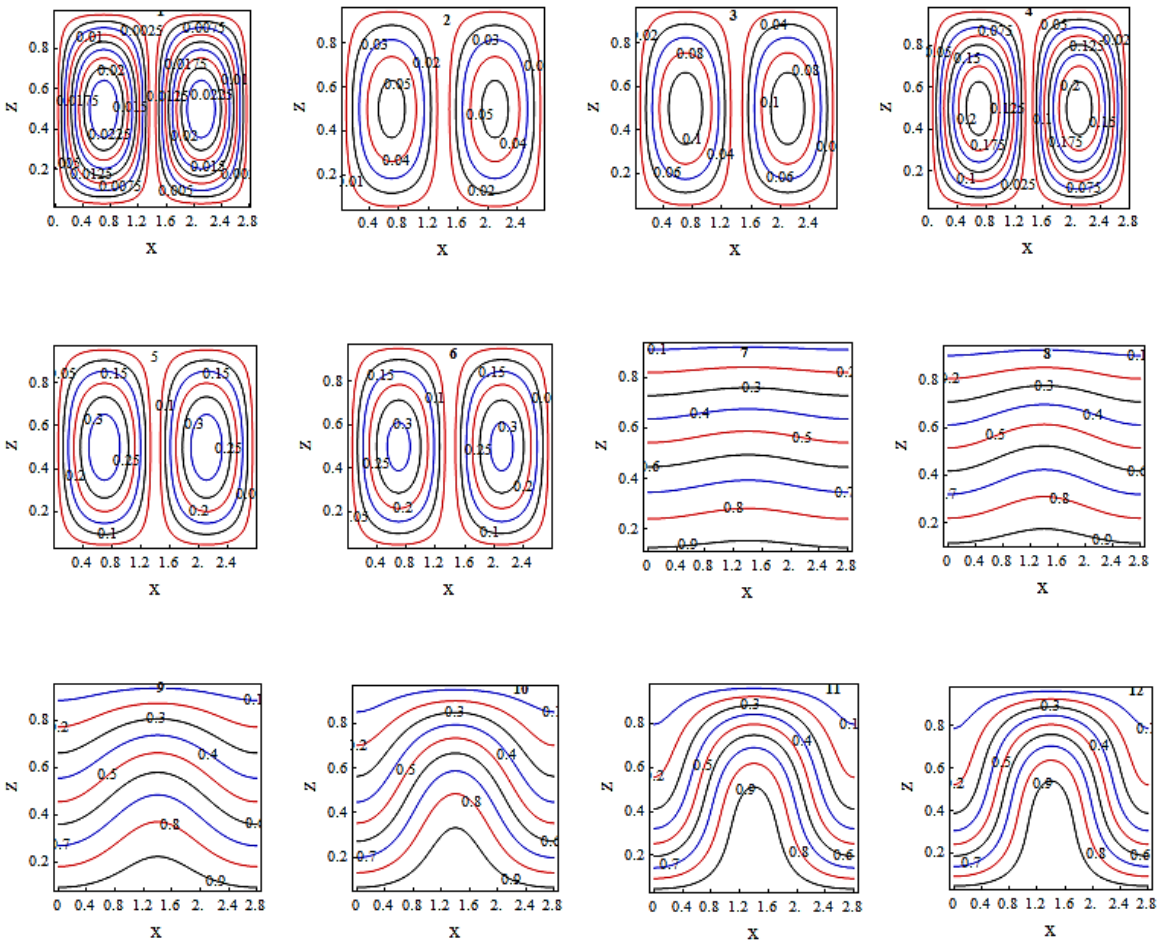


Fig. 7. Streamlines (1 to 6) and Isotherms (7 to 12)

4. Conclusions

The effect of viscoelasticity on ferro-convection under gravity modulation has been investigated using the Oldroyd model. The results are drawn corresponding to Maxwell fluids, Newtonian fluids and Rivlin–Ericksen fluids. The effect of various viscoelastic and modulation parameter has been depicted graphically on Nu and R0c. The neutral stability curves are drawn for oscillatory convection and compared with stationary curves. For nonlinear case the heat

transfer results are obtained by Nusselt number. It is concluded that gravity modulation can be used to regulate heat transport effectively. Rivlin–Ericksen fluids shows better heat transfer results than other fluids. Also Jeffrey fluids obtained less heat transfer results. The comparison of different fluids makes the following relation.

$$Nu^{Rivlin} > Nu^{Maxwell} > Nu^{Newtonian} > Nu^{Jeffery} \tag{43}$$

It is observed that effect of amplitude of modulation enhances heat transfer and frequency of modulation diminishes heat transfer. The modulation shows dual nature of heat transfer rather than the system properties or fluid properties. This gravity modulation may feel while moving the layer z = 0 to z = 1. It is concluded that modulation mostly applied to control transport analysis and onset convection.

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Latin Symbols

Pr	Pradtl number
d	Dimensional layer depth
p	Effective pressure
$\vec{q}$	velocity
T˜	liquid stress tensor
$\vec{g}$	Modulated gravity field
Ra	Thermal Rayleigh-Darcy number
τ	Time
T	Temperature
(x^*,y^*,z^*)	Cartesian coordinates
M	magnetization vector

Greek symbols

Ω	Frequency of modulation
α	Coefficient of thermal expansion
χ	Magnetic susceptibility
κ	Thermal diffusivity

λ_1 Stress relaxation parameter

λ_2 Strain retardation parameter

ρ Fluid density

φ Scalar magnetic potential

ψ Stream function

Subscripts

b Basic solution

Superscripts

* Dimensional variable

, Perturbation variable

Operators

Laplace operator $\nabla^2 = \frac{\partial^2}{dx^2} + \frac{\partial^2}{dy^2} + \frac{\partial^2}{dz^2}$

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