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SYNCHRONIZING ANALOG - DISCRETE NONLINEAR SYSTEMS

BY

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Abstract. Using nonlinear chaotic systems for secure communication is an application field presented in recent published research. This paper aims at improving the security of signal transmission using chaotic encryption. The proposed approach uses a combined analogue and discrete-time structure, for both chaotic emitter and synchronizing receiver. The prototype system used is similar to the Lorenz nonlinear oscillator. Our study presents the emitter dynamic properties, develops the receiver synchronization using the emitter partitioning method, and designs a linear equalizer at the receiving end of the communication chain in order to improve the demodulated signal.

Keywords: chaos synchronization, nonlinear systems, analogue circuits, discrete-time systems, chaotic systems.

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1. Introduction

The properties of nonlinear systems exhibiting complex dynamic behavior are studied with great interest due to their useful applications in secure communication, such as wide spectrum modulation (Li *et al*., 2018; Singh, 2021), chaotic encryption (Kocarev, 2002; Andreatos and Volos, 2014; Yasser *et al*., 2020; Yousif *et al*., 2020), nonlinear synchronization (Pecora and Carrroll, 2015; Eroglu *et al*., 2017; Sambas *et al*., 2015) and system parameters statistic and dynamic influence (Grigoras and Grigoras, 2015). In order to ensure a better security of the communication system, the chaotic circuit involved should have a high dynamic complexity, with larger Lyapunov exponents achievable by designing higher complexity emitter sub-systems. Both analog and discrete-time chaotic systems (Butusov *et al*., 2017; Karimov *et al*., 2018; Cuomo *et al*., 1993) were in depth studied separately to highlight their performance. The combination of both types of chaotic systems is less approached, although it has a higher complexity structure and dynamic behavior. Thus, the present contribution aims studying an emitter system composed of two order three building blocks of Lorenz-type prototypes, one analog and the other discrete and a synchronizing receiver with a similar structure. The nonlinear dynamics of the emitter is studied, the design of the receiver is presented in detail and simulation results for different modulation signals are shown, confirming the possible use of the designed six-order analog-discrete nonlinear system for secure chaotic communication.

2. The Nonlinear Emitter Design

In order to obtain a complex dynamic behavior of the proposed nonlinear emitter system, a connection of a discrete-time building block and of an analog counterpart is chosen. The two interconnected order three nonlinear building blocks are designed starting from Lorenz oscillators:

$$\begin{cases} x_a' = \sigma \cdot y_a - \sigma \cdot x_a \\ y_a' = \rho \cdot x_a - y_a - x_a \cdot z_a \\ z_a' = x_a - \beta \cdot z_a + x_a \cdot y_a \end{cases} \tag{1}$$

The state Eqs. (1) lead to the analog system implementation:

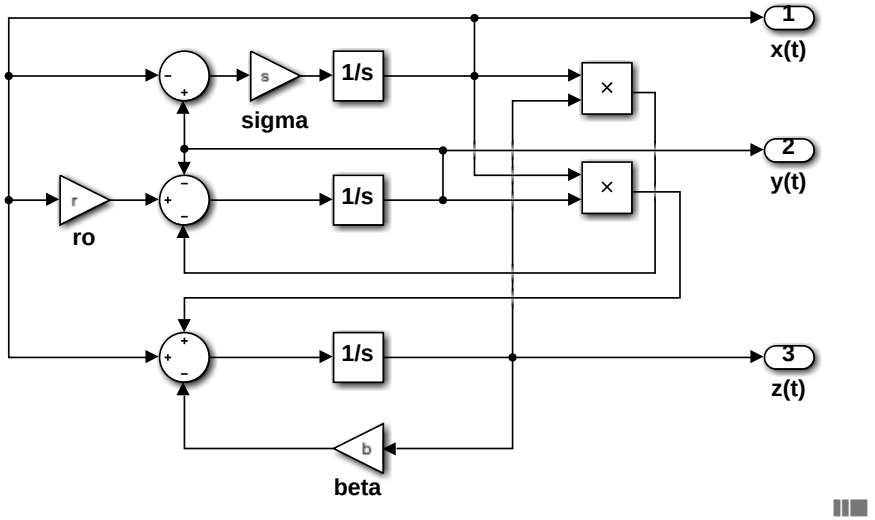


Fig. 1 – Analog Lorenz oscillator block diagram.

The Lorenz system was chosen as a prototype for both analog and discrete building blocks because it exhibits a chaotic behavior for a large range of parameters, as resulting in the bifurcation diagrams for the y(t) state variable at the variation of the state equations parameters σ, ρ and β, as presented in Fig. 2.

For the parameter values $\sigma = 10$, $\rho = 28$ and $\beta = 2.67$, the tri-dimensional representation of the strange attractor is depicted in Fig. 3, sustaining the chaotic dynamics conclusion.

The discrete-time nonlinear system counterpart is obtained by applying the Euler sampling method to the state Eqs. (1):

$$dx_a/dt \simeq \frac{x_d[k+1]-x_d[k]}{Step} \tag{2}$$

In Eq. (2), the parameter Step represents the clock period for the discrete system, equal to the sampling period of its input and output signals. The resulting state equations for the discrete-time nonlinear system is given by Eq. (3), leading to the discrete block diagram in Fig. 4:

$$\begin{cases} x_d[k+1] = x_d[k] + Step \cdot (\sigma \cdot y_d[k] - \sigma \cdot x_d[k]) \\ y_d[k+1] = y_d[k] + Step \cdot (\rho \cdot x_d[k] - y_d[k] - x_d[k] \cdot z_d[k]) \\ z_d[k+1] = z_d[k] + Step \cdot (x_d[k] - \beta \cdot z_d[k] + x_d[k] \cdot y_d[k]) \end{cases} \tag{3}$$

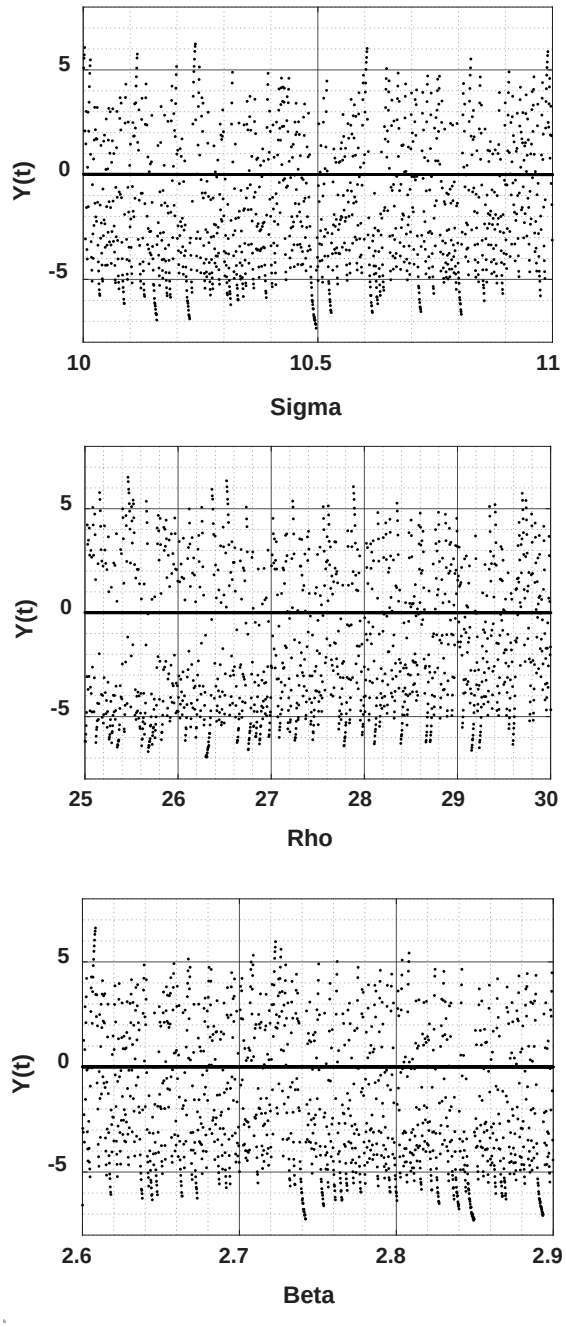


Fig. 2 – Bifurcation diagrams for the y(t) state variable of the Lorentz analogue prototype at the variation of σ, ρ and β parameters.

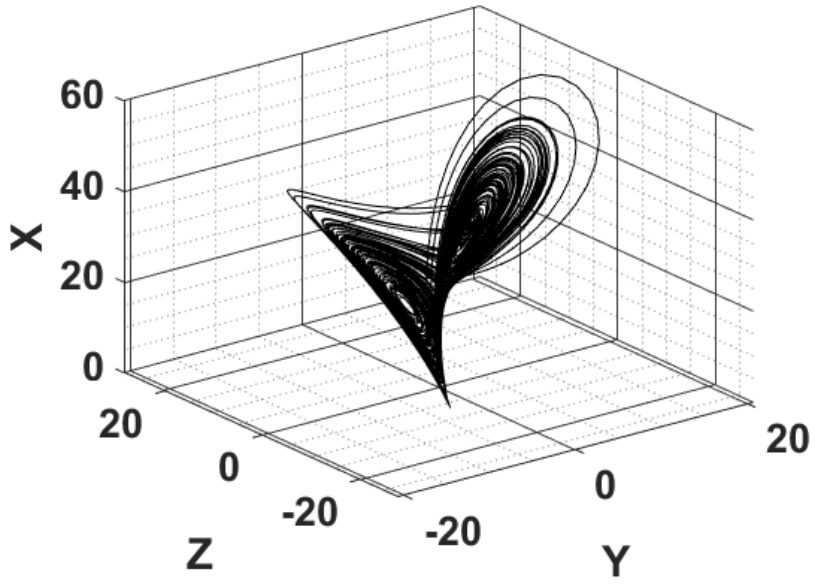


Fig. 3 – Analogue Lorenz strange attractor.

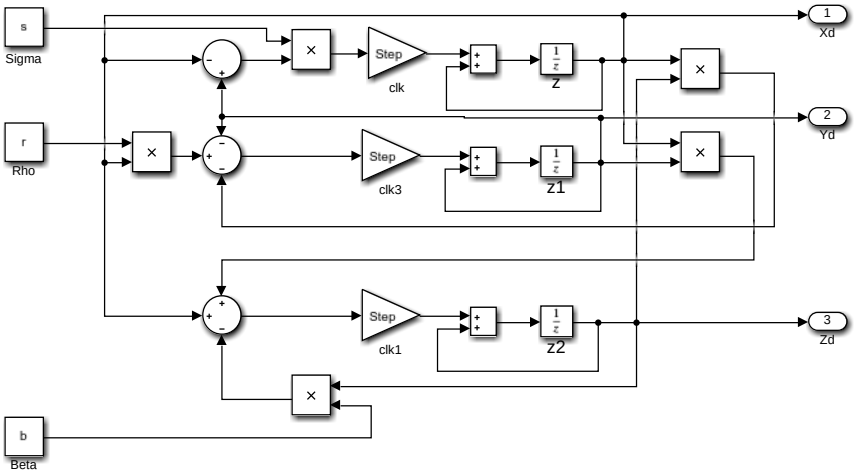


Fig. 4 – Discrete-time sub-system block diagram.

The Lorenz discrete system obtained exhibits a chaotic behavior too, as seen in the strange attractor depicted in Fig. 5 and the power spectra of the state variables $x_d[k]$ and $z_d[k]$, shown in Figs. 6 and 7.

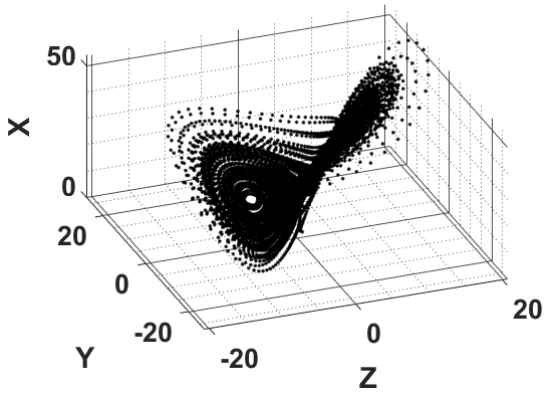


Fig. 5 – Discrete-time Lorenz system attractor.

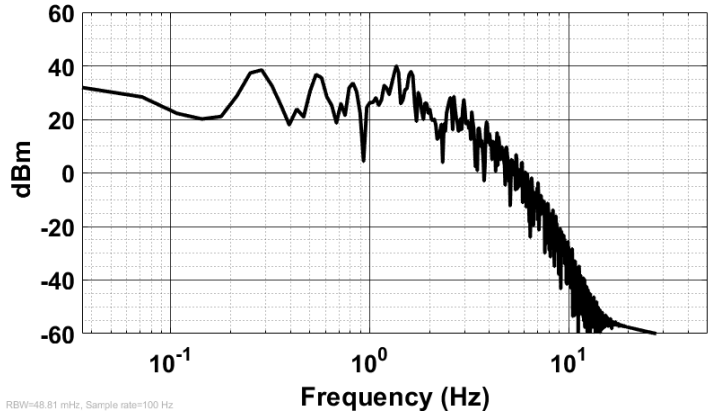


Fig. 6 – Power spectrum of the state variable x_d for the discrete-time Lorenz system.

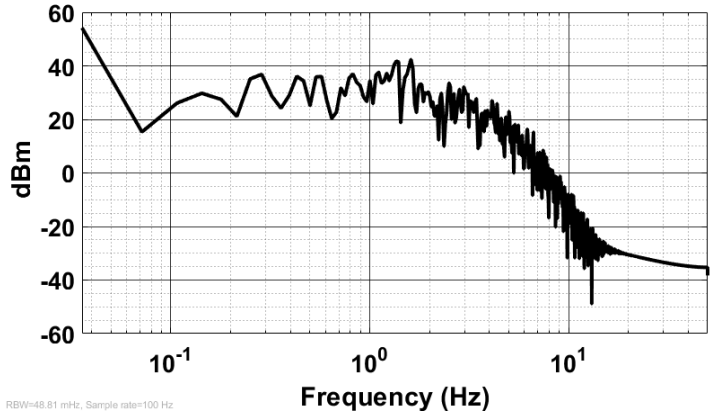


Fig. 7 – Power spectrum of the state variable z_d for the discrete-time Lorenz system.

The analog and the discrete sub-systems are connected in a cascade topology, by considering the state variable $y_a(t)$ as the analog output and applying the modulating signal, $m(t)$ to the second analog state equation:

$$\begin{cases} x_a'(t) = \sigma \cdot y_a(t) - \sigma \cdot x_a(t) \\ y_a'(t) = \rho \cdot x_a(t) - y_a(t) - x_a(t) \cdot z_a(t) + m(t) \\ z_a'(t) = x_a(t) - \beta \cdot z_a(t) + x_a(t) \cdot y_a(t) \end{cases} \quad (4)$$
$$Out_a(t) = y_a(t)$$

Connecting the sampled and A/D converted analog output to the second state equation as a discrete input, $In_d[k]$:

$$\begin{cases} x_d[k+1] = x_d[k] + Step \cdot (\sigma \cdot y_d[k] - \sigma \cdot x_d[k]) \\ y_d[k+1] = y_d[k] + Step \cdot (\rho \cdot x_d[k] - y_d[k] - x_d[k] \cdot z_d[k]) + In_d[k] \\ z_d[k+1] = z_d[k] + Step \cdot (x_d[k] - \beta \cdot z_d[k] + x_d[k] \cdot y_d[k]) \end{cases} \quad (5)$$
$$Out_d[k] = y_d[k]$$

The sampling period, T_{samp}, used to ensure the connection of the analog building block to the discrete-time one is equal to the discrete sub-system clock period.

$$In_d[k] = f_{A/D}(Out_a(t_k)); \quad t_k = k \cdot T_{samp} \quad (6)$$

This leads to the state Eqs. (4), for the analog Lorenz system and (5) for the discrete counterpart and the resulting block diagram of the cascade connection presented in Fig. 8.

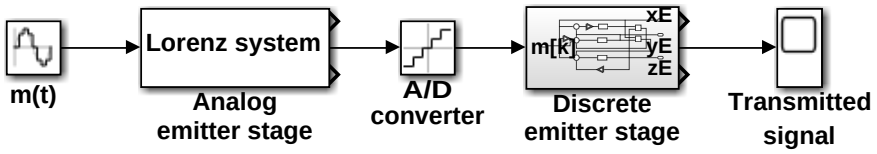


Fig. 8 – Cascade block diagram of the analogue and discrete-time emitter.

Although the nonlinear characteristics of the A/D connecting converter might influence the nonlinear dynamics of the resulting emitter system, multiple simulation tests have shown that the simulation results obtained are quite similar

for different data converters with ten to sixteen bits precision, so conversion resolution is not further taken into account.

3. Receiver Design and Synchronization Results

Starting from the combined analog-discrete emitter, the corresponding receiver is designed as an inverse cascade building blocks connection: first a discrete synchronizing receiver sub-system followed by an analog counterpart. The analog receiver is designed using the emitter partitioning approach, as first introduced in (Pecora and Carroll, 1990).

$$\begin{cases} x_{ra}'(t) = \sigma \cdot In_{ra}(t) - \sigma \cdot x_{ra}(t) \\ y_{ra}'(t) = \rho \cdot x_{ra}(t) - y_{ra}(t) - x_{ra}(t) \cdot z_{ra}(t) \\ z_{ra}'(t) = x_{ra}(t) - \beta \cdot z_{ra}(t) + x_{ra}(t) \cdot In_{ra}(t) \end{cases} \quad (7)$$
$$Out_{ra}(t) = In_{ra}(t) - y_{ra}(t)$$

In order to compensate for the low-pass characteristic of the analog emitter-receiver processing chain, with the normalized cut-off frequency $\omega_{cut} = 1$, we introduced an analog equalizer that compensates the ω_{cut} pole with an equal zero and has a higher frequency pole $\omega_{eq} >> \omega_{cut}$ resulting in an equalizer transfer function:

$$H_{a,eq}(s) = \omega_{eq} \cdot \frac{s+1}{s+\omega_{eq}} \quad (8)$$

This leads to a modulation/demodulation block diagram for the analog part of the system as presented in Fig. 9.

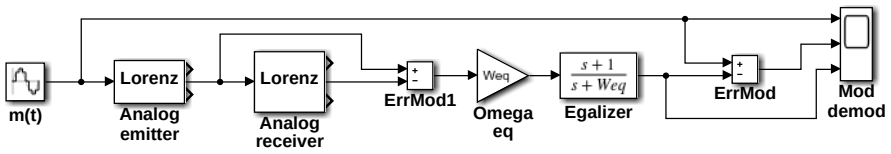


Fig. 9 – Analog modulating-demodulating system block diagram.

By simulating the analog emitter-receiver chain and comparing the two-state vectors of the corresponding building blocks, we obtained the state error depicted in Fig. 10.

In a similar way, we can deduce that the discrete-time cut-off frequency is due to cut-off pole, $z_{cut} = T_{samp}$ pole of the discrete emitter-receiver connection, leading to a compensating FIR-type equalizer transfer function:

$$H_{d,eq}(z)=\frac{1}{T_{samp}}\left[1+\left(T_{samp}-1\right)\cdot z^{-1}\right] \tag{9}$$

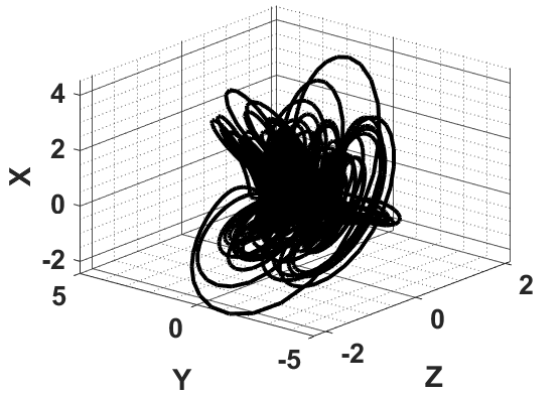


Fig. 10 – Analog modulating-demodulating state error.

As a result, the discrete-time structure depicted in Fig. 11 performs the modulation and demodulation of the discrete signal.

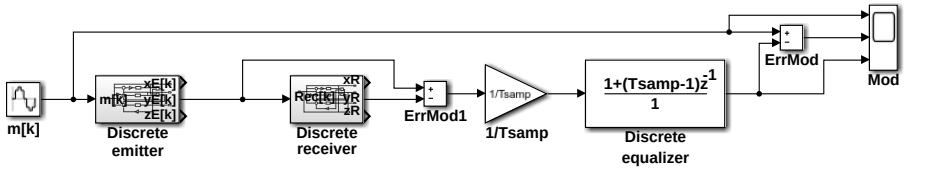


Fig. 11 – Discrete modulating-demodulating system block diagram.

The RMS discrete state synchronization error obtained by simulating the modulating-demodulating system in Fig. 11 is presented in Fig. 12.

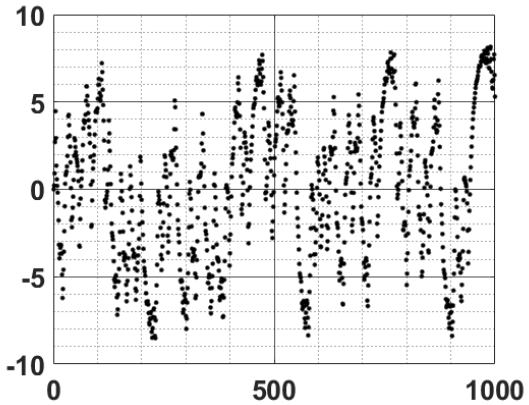


Fig. 12 – Discrete modulating-demodulating RMS state error.

By using the designed synchronizing receiver blocks and the linear equalizers presented and connecting them to the chaotic transmitted signal, we obtain the communication system with the block diagram depicted in Fig. 13.

Several simulations on the analog-discrete transmission system were performed to verify its performance. If a simple sine-wave was used as a modulating signal, as in Fig. 14, the chaotic transmitted signal in Fig. 15 shows good hiding of the sine signal, the same way its spectrum depicted in Fig. 16 hides the line of the sine frequency spectrum.

The demodulated signal, depicted in Fig. 17, highlights a short synchronization transient at the beginning, followed by a well-demodulated signal. The demodulation error shown in Fig. 18, has a similar begin transient and a regular error similar to the modulating signal due to the equalization approximation.

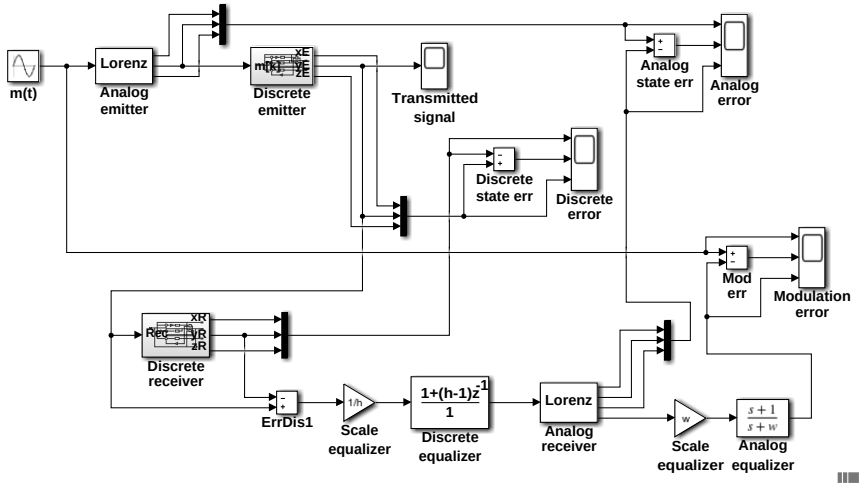


Fig. 13 – The chaotic communication modulation/demodulation connection block diagram.

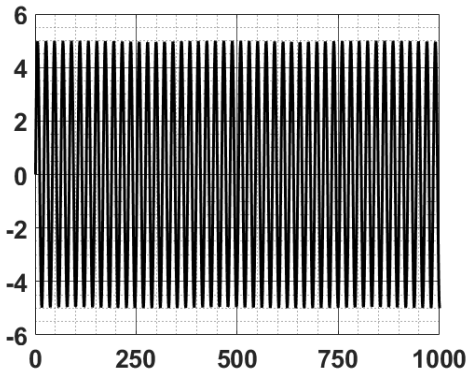


Fig. 14 – The sine modulating signal.

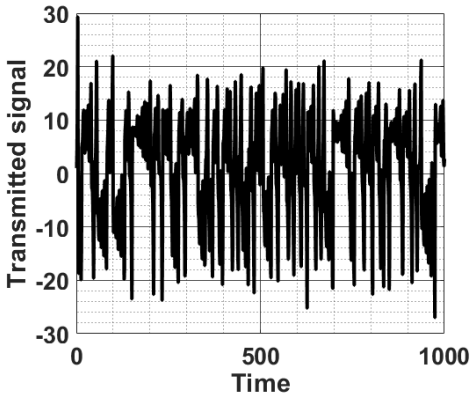


Fig. 15 – The chaotic transmitted signal.

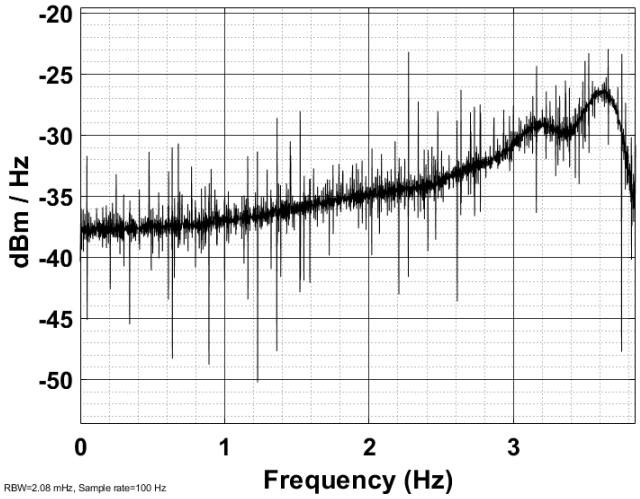


Fig. 16 – The frequency spectrum of the chaotic transmitted signal.

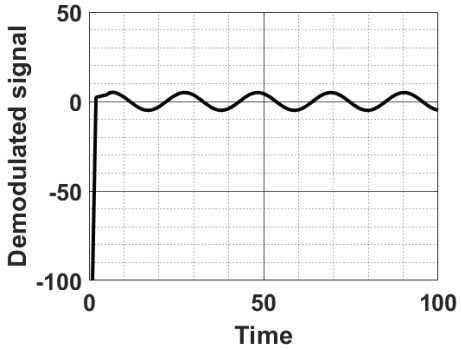


Fig. 17 – The demodulated signal.

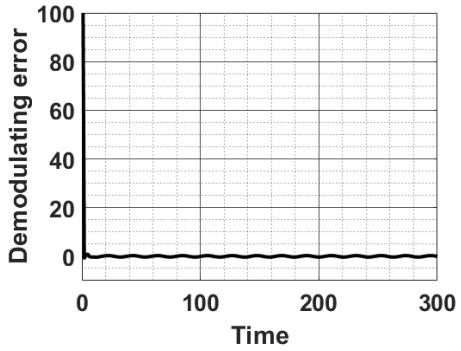


Fig. 18 – The demodulation error signal.

To verify the system behavior if larger frequency bandwidth modulating signal is applied, several simulations were performed with the periodic rectangular modulating signal, as the one shown in Fig. 19. Demodulation was performed well, with small front distortion due to the equalizer finite low pass-band, as highlighted in Fig. 20 and in Fig. 21.

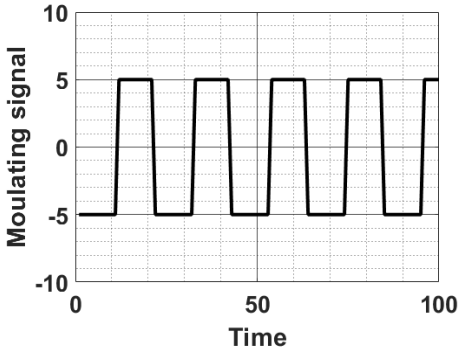


Fig. 19 – The modulating rectangular signal.

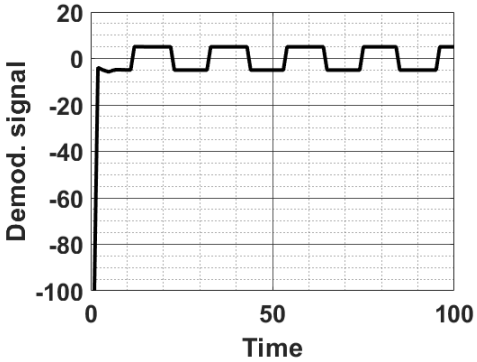


Fig. 20 – The demodulate rectangular signal.

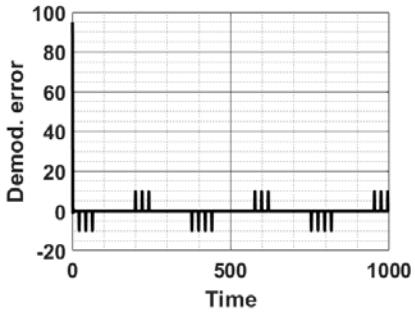


Fig. 21 – The demodulation error for rectangular modulating signal.

4. Conclusions

The present contribution presents a secure communication method based on combining analog and discrete systems connected in order to obtain a more complex emitter that can be more difficult to crack by an unauthorized receiver. We have developed an order six nonlinear emitter system based on two order three Lorenz type prototypes and designed a synchronizing receiver that demodulates the transmitted signal. The proposed design is based on the feedforward interconnection of the two chaotic oscillators. In order to obtain a more unpredictable behavior of the resulting system, the two basic blocks are dynamically different: one analog and the other discrete-time. By performing several simulations, we highlighted the complex behavior of the designed emitter system for a wide range of parameter values. Simulation performed on the nonlinear emitter-receiver pair for several sets of parameters and modulating signals confirm their complex behavior and the secure transmission possibility based on chaos synchronization. Linear equalizers were also used at the demodulating stage, to reduce the influence of the low-pass behavior of the communication system.

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SINCRONIZAREA SISTEMELOR NELINIARE ANALOG-DISCRETE

(Rezumat)

Utilizarea sistemelor neliniare haotice pentru securizarea comunicaţiilor este un domeniu aplicativ prezentat în publicaţii de cercetare recente. Acest articol vizează îmbunătăţirea securităţii transmiterii semnalelor folosind criptarea haotică. Abordarea propusă foloseşte o structură combinată analogică şi discretă, atât pentru emiţătorul haotic cât şi pentru receptorul care se sincronizează cu acesta. Sistemul prototip utilizat este similar oscilatorului neliniar Lorenz. Studiul nostru prezintă proprietăţile dinamice ale emiţătorului, dezvoltă un receptor care să se sincronizeze cu acesta, utilizând metoda partiţionării emiţătorului şi proiectează un egalizor liniar la capătul receptor al lanţului de comunicaţii, pentru a îmbunătăţi semnalul demodulat.