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A SIMPLIFIED FRICTION FORCE MODEL OF TRANSIENT SLIDING CONTACT BETWEEN HARD AND SOFT SURFACES

I. A LITERATURE OVERVIEW OF FRICTION FORCE MODELS

BY

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Abstract. Modelling and simulation of mechanical systems is very useful to predict the system's response. Friction is important since it influence the functionality and reliability of the system. Friction force is difficult to model in transient regime when relative velocity is zero and some discontinuity appears in simulation. In this paper the authors presented a literature overview regarding the friction force models used to simulate the transient sliding frictional contact in mechanical systems. Also, the authors have proposed a simplified friction force model based on Heaviside function. Some preliminary simulation results were obtained regarding the transition of friction force from positive to negative values in a sliding frictional process.

Key words: friction force, sliding friction, transient regime, static friction models, simulation.

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1. Introduction

Friction is responsible for many different daily activities such as the sound produced by the contact between a string and a bow of a violin, striking of a match to light a candle, motion of a train running on a rail and an automobile running on a road, squeal noises generated by a wiper blade moving on a glass windshield, a chalk scratching on a blackboard, a disk sliding on a brake pad. Some of these friction-induced vibration phenomena have strong influence on the system performance and reliability (Wang *et al.*, 2024; Khonsari and Booser, 2017; Chen and Liu, 2016; Kang *et al.*, 2015; Ibrahim, 1994).

Friction has been investigated for a long time since Leonardo da Vinci (Codex Arundel, dating between 1480-1518). Two hundred years later, in 1699, Guiliam Amontons published his friction research synthesized by the two known laws: (a) the friction force (F_f) acting on the interface of two contacting bodies undergoing relative motion is proportional to the normal force (F_N) and (b) the friction force is not depended on the nominal contact area between two sliding bodies. Later, Charles–Augustin Coulomb (1785) confirms the first two friction laws of Amontons and introduced the concepts of the static (μ_s) and kinetic (μ_k) friction coefficients. Also, Coulomb elaborated the third friction law according to which the kinetic friction force does not depend on the sliding speed, during the sliding process.

In 1950, Bowden and Tabor include the role of the interactions between the roughness of the surfaces considering the micro adhesions in the real contact area. In 1966, Greenwood & Williamson elaborated the elasto-plastic theory of roughness as a solution to explain the friction between surfaces (Bhushan, 2013).

Friction force is an important parameter in every mechanical system and is influenced by a lot of parameters as: contact materials, surfaces roughness, presence, or absence of lubricant, sliding speed, contact real pressure, including the geometry of the contact surfaces, their properties, the running conditions, and type of lubricants used (Bhushan, 2013).

2. Friction Force Models

2.1. A literature overview

In reciprocating sliding friction process, the velocity and the tangential force are changing their sign with respect to the change in sliding direction. The sliding velocity have a sudden transition with a very small delay time when pass to zero due to the system's inertia, compared to tangential force that may have a greater delay when pass to zero which is influenced by the type of contacting material. According to the classical Coulomb's friction model, the friction force shows a discontinuity where the relative velocity is 0. This discontinuity is an important issue for dynamic modeling and simulations because it introduces

numerical instability in the system's response (Marques *et al.*, 2016; Kang *et al.*, 2015; Brown and McPhee, 2016).

In literature, there are a lot of static and dynamic friction models that were developed to overcome the limitations of the original Coulomb's friction model (Marques *et al.*, 2016; Andersson *et al.*, 2007; Olsson *et al.*, 1998; Pennestri *et al.*, 2016; Kang *et al.*, 2015; Pouilly-Cathelain and Feyel, 2021; Ciornei *et al.*, 2020).

The Coulomb model with stiction includes the static coefficient of friction and has similar behavior to Coulomb's except in the vicinity of zero velocity. It is also a multivalued function but can reach a higher friction force. Considering a linear relationship between the relative velocity and the friction force related to the lubricant viscosity, the combination of Coulomb's friction model with a viscous friction model and Stribeck effect could be advantageous (Marques *et al.*, 2016; Andersson *et al.*, 2007; Olsson *et al.*, 1998).

Several researchers have proposed different mathematical expressions to describe the Stribeck curve (Hess and Soom, 1990; Tustin, 1947; Popp and Stelzer, 1990; Armstrong-Hélouvry, 1991; Makkar *et al.*, 2005), although the exponential approximation presented by Bo and Pavelescu (1982) showed greater acceptance.

The friction model proposed by Karnopp (1985) suggested that the relative velocity can be considered to be null within a given range. This model eliminates the discontinuity at zero velocity, and it is capable of capturing stick-slip motion.

Leine *et al.* (1998) proposed the "switch model" with linear transition of the friction force through the point where velocity is zero, with a certain inclination. Also, Andersson *et al.* (2007) proposed to use a linear friction-velocity dependence for low velocities, to replace the discontinuity at zero velocity by a finite slope model.

Threlfall model (1978) reveal that for high velocities return a friction force slightly lower than Coulomb's friction. Bengius and Akay (1994) propose a model that consists by two equations, one for slope and other to describe the Stribeck effect.

Ambrósio (2003) propose a modified Coulomb's friction model which includes both the linear variation of friction force and the null friction force for low velocities.

Armstrong model (1994) includes several frictional effects such as the pre-sliding displacement, Coulomb and viscous friction, Stribeck effect and frictional lag. It consists of two separate equations, one for adhesion and another for the sliding regime.

Awrejcewicz *et al.* (1999) and Wojewoda *et al.* (2008) proposed a static friction model for dry contact which is dependent of both tangential force and relative velocity. This model is governed by four different equations, one for sliding, two for the transition from stick to slip and one for sticking.

The dynamic friction models emerged along with the need of capturing some frictional characteristics that the static models were not able to describe. Marques *et al.* (2016) present a selection of the most dynamic friction models that are based on the physical interaction between the contacting surfaces asperities, adhesion, Stribeck effect, viscous effect, frictional lag. Some examples are Dahl model, bristle model, reset integrator model, Bliman and Sorine model, the LuGre, the elasto-plastic, Leuven model, Gonthier model, Liang model. Marques *et al.* (2016) examine the influence of using different friction force models on the dynamic response of a single DOF mass-spring system with sliding friction. Recently, Marques *et al.* (2021) proposed a modified LuGre model with application to a unidirectional oscillator.

In the regarding of the discontinuity of the friction force when the sliding velocity is zero, the authors proposed an approach to model the transient regime of the tangential force by Heaviside approximation step function. The proposed model allows the transition from negative to positive values of the friction force, with a certain delay given by the presence of viscoelastic hysteresis, specific to deformable materials sliding on rigid surfaces (Wang *et al.*, 2024).

2.2. A Simplified Friction Force Model

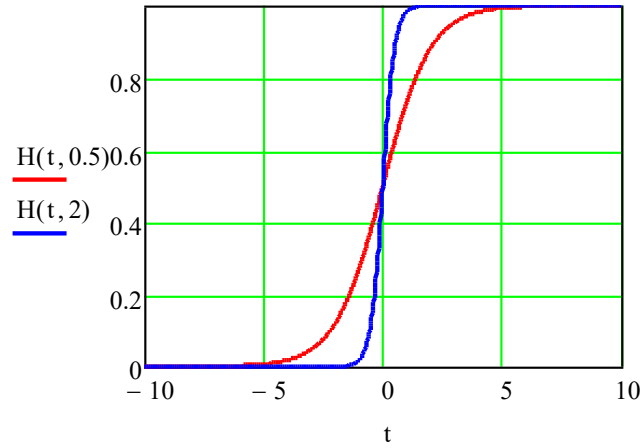
In this paragraph the authors present a friction model for transient sliding contact running under dry conditions. The friction was modeled by using Heaviside step function for modeling the transition of tangential force at the direction changes at small velocities.

The Heaviside step function, or the unit step function, the value of which is 0 for negative arguments and 1 for positive arguments. The function was originally developed to find the solution of differential equations.

Heaviside approximation step function can be described as:

$$H(t,k)=\frac{1}{1+e^{-2\cdot k\cdot t}} \quad (1)$$

As function of exponent k , the step function can have various configurations as in Fig. 1.

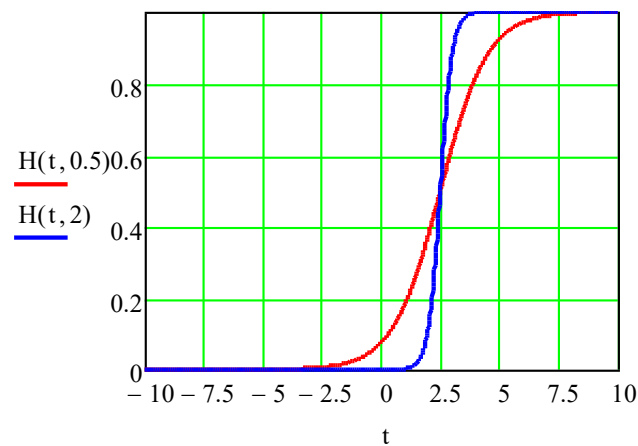
Fig. 1 – Step function simulation for $k=0.5$ and $k=2$.

As an essential problem this function is symmetrically to zero and have values between 0 and 1. To obtain a deviation from zero it was proposed following modified Heaviside function:

$$Hs(t, k, \tau) = \frac{1}{1 + e^{-2 \cdot k \cdot (t - \tau)}} \quad (2)$$

where τ is a deviation time.

For $\tau=2.5$ a positive deviation with 2.5 seconds has been obtained reported to zero vertical position, as in Fig. 2.

Fig. 2 – Modified Step function simulation for $k=0.5$ and $k=2$.

To obtain a symmetrical distribution positive and negative it propose a combination from two modified Heaviside function. So, for a continuum step of the friction force from positive to negative value and vice versa we propose following relations:

$$HF1(t, kf, \tau f) = \frac{-F}{1 + e^{-2 \cdot kf \cdot (t - \tau f)}} - \frac{-F}{1 + e^{+2 \cdot kf \cdot (t - \tau f)}} \quad (3)$$

$$HF2(t, kf, \tau f) = \frac{-F}{1 + e^{2 \cdot kf \cdot (t - \tau f)}} - \frac{-F}{1 + e^{-2 \cdot kf \cdot (t - \tau f)}} \quad (4)$$

If $F = 1.5$ N, the representation of the functions $HF(t, kf, \tau f)$ represented in Fig. 3 where $kf = 0.5$ and $\tau f = 2.5$.

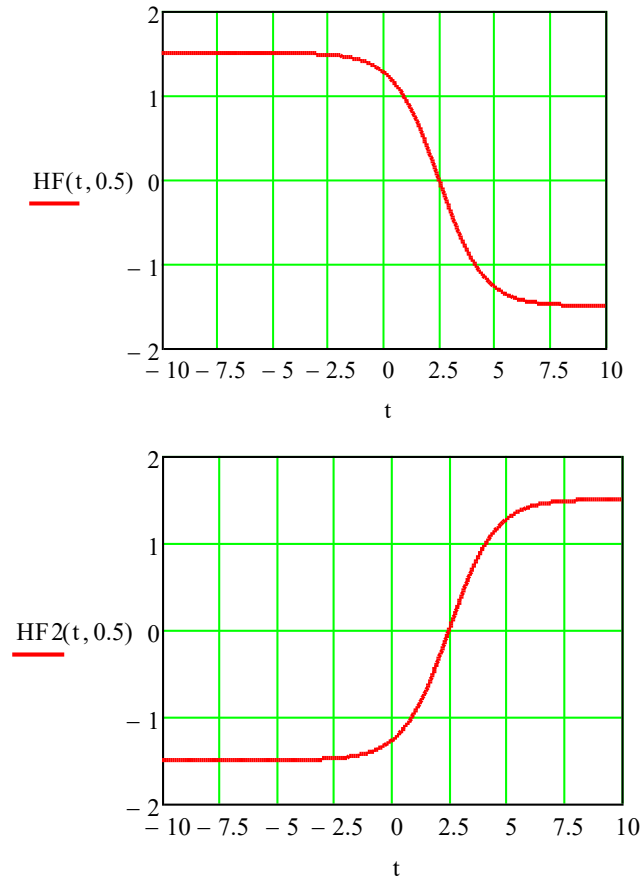


Fig. 3 – Modified step function simulation for tangential friction force.

Similar functions can be used for the steps of the sliding velocity. So, for a continuum step of the linear speed from positive to negative value and vice versa we propose following relations:

$$Hv1(t, kv, \tau v) = \frac{-v}{1 + e^{-2 \cdot kv \cdot (t - \tau v)}} - \frac{-v}{1 + e^{+2 \cdot kv \cdot (t - \tau v)}} \quad (5)$$

$$Hv2(t, kv, \tau v) = \frac{-v}{1 + e^{-2 \cdot kv \cdot (t - \tau v)}} - \frac{-v}{1 + e^{-2 \cdot kv \cdot (t - \tau v)}} \quad (6)$$

The values of parameters k_f and k_v can have different values. Also, the deviation times τ_f and τ_v can have different values.

3. Conclusions

The present paper is a literature review regarding the transient sliding friction that occur in many mechanical systems. In Section 2, are presented a lot of friction force models used in literature to model the transient sliding regime. Static friction models had some limitations due to the discontinuity of the friction force when the relative velocity is zero. To model the transition of the tangential force that goes from positive to negative and vice versa during the sliding friction, the authors proposed a simplified model based on Heaviside function. Some preliminary results were presented for modelling the tangential friction force transient regime. Further, the authors will use this simplified model to experimentally validate the transitions of the friction force in a dry sliding friction between hard and soft material surfaces.

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MODEL SIMPLIFICAT PENTRU REGIMUL TRANZITORIU AL FORȚEI DE FRECARĂ LA CONTACTUL DINTRE SUPRAFEȚE DURE ȘI MOI

I. Studiu din literatură cu privire la modelarea forței de frecare în regim tranzitoriu

(Rezumat)

Fenomenele de frecare sunt prezente în toate sistemele mecanice care, fie realizează deplasări sau mișcări pe diferite direcții, fie transmit puterea sau turația.

În această prima parte a lucrării se prezintă un studiu de sinteză cuprinzând diversele modele matematice, utilizate în fazele de tranziție, la schimbarea sensului vitezei de alunecare (paragraful 2.1.) și modelul teoretic propus pentru tranziția forței de frecare la schimbarea sensului vitezei pentru contacte de alunecare între materiale rigide și materiale puternic deformabile (paragraful 2.2.).

Pornind de la modelul clasic Coulombian, când schimbarea sensului vitezei presupune un salt brusc prin zero a forței de frecare, considerată constantă ca valoare, dar cu sensuri negative și pozitive, sunt prezentate în paragraful 2.1. numeroase modele în care trecerea de la o viteză negativă la una pozitivă se face cu luarea în considerare a forței de frecare statică și a forței de frecare cinetică, cu diferențe mai mari sau mai mici, cu includerea efectului Stribeck, cu înclinări diferite ale curbei de variație a forței de frecare. Majoritatea modelelor, deși prezintă continuitate din punct de vedere matematic la nivelul saltului, atunci când viteza devine nulă, modelele au valoarea zero. În paragraful 2.2. se prezintă un model original care permite trecerea de la valori negative la valori pozitive ale forței de frecare, cu o anumită întârziere dată de prezența histerezisului vâscoelastic, specific materialelor deformabile care alunecă pe suprafețe rigide. Modelul propus are la bază însumarea a două funcții treaptă Heaviside, cu includerea unor timpi de decalare stabiliți pe baza experimentelor, astfel încât forța de frecare nu mai trece prin zero la momentul schimbării sensului vitezei.