

ESTIMATION OF TWO-LAYER SOIL PARAMETERS USING GRADIENT METHOD ACCELERATED WITH ATKIN'S δ^2 METHOD

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Abstract: An efficient method for evaluation of an optimal two-layer soil model from Wenner four-probe measuring method, which has been used during experimental investigations, is presented within this paper. A two-layer soil model is assumed, and this soil model is an adequate representation of nonhomogeneous soil for grounding system design. The application of optimization techniques is required to estimate the electrical parameters of the proposed soil model. In this paper, first the fast gradient-descent method to solve a given optimization problem is chosen, and then with the aim of faster calculation for accelerating the rate of convergence of an infinite sum, the application of Aitken's δ^2 method is proposed.

Keywords: grounding system, soil resistivity, Wenner four-probe method, two-layer soil model, Fast gradient-descent method, Aitken's δ^2 method

INTRODUCTION

One of the main input data for the grounding system parameters calculation, for both static and impulse analysis, is soil resistivity and soil stratification. It is a well-known fact that the soil in most cases consists of numerous layers with different chemical and biological compositions. This further affects that the soil has different layers of different electric properties. Numerous high-precision numerical methods for grounding system parameters calculation are completely useless if inadequate soil input parameters are not available [1]. Therefore, when analysing grounding systems, the data about soil electrical properties must be known [2].

Soil resistivity is a specific parameter and therefore it must be measured for each individual case. The soil resistivity measurement methods are described in [3], and some of these measurement methods are:

- Wenner four-probe method,
- Schlumberger four-probe method,
- general method,
- three-probe method and
- dipole-dipole method.

In this paper, the Wenner four-probe method has been used because this method is characterized by simple implementation on site (have symmetrical probe arrangement), relatively large depth of the measurement compared to the other methods and high immunity to potential noise caused by external sources, which could

cause a measurement error. Although the Wenner four-probe method is the simplest measuring method to implement and interpolate the results of the performed measurements, in situations of complex soil structures the interpolation of the results can be complicated. In these situations, it is necessary to apply optimization techniques in combination with numerical methods for the calculation of the current fields in the ground.

The Wenner four-probe method is the most commonly used measurement methods for soil resistivity in practice situations. The measuring depth of Wenner four-probe method is equal to the distance between two adjacent measuring probes in the measuring arrangement. Therefore, multiple measurements with different spacing between measuring probes are required to obtain a profile of soil change resistivity with depth [4]. This measuring method consists of the following:

- current (outer) probes, used to inject current into the soil,
- voltage (inner) probes, used to measure the electric potential difference in the field of current probes.

Low-frequency electric sources are mostly used with a frequency different from electric power grid frequency as the current source, in order to avoid interactions, which could cause measurement errors [5]. Also, DC current sources are nowadays avoided due to effect of polarization voltages, which could also be sources of measuring errors [2].

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Furthermore, if the measurement of soil resistivity is being conducted in the proximity of high voltage transmission lines it is recommended that the measurement arrangement is perpendicular to the transmission line route, in order to avoid the coupling effects [5]. Nowadays, most of manufactures of measuring equipment offer devices that incorporate the function of the current source, ammeter and voltmeter at same time. This way, the connection of the measuring circuit is considerably facilitated.

1. THE WENNER FOUR-PROBE METHOD

The simplified scheme of measurement circuit for soil resistivity by using the Wenner method is shown in Figure 1 [6].

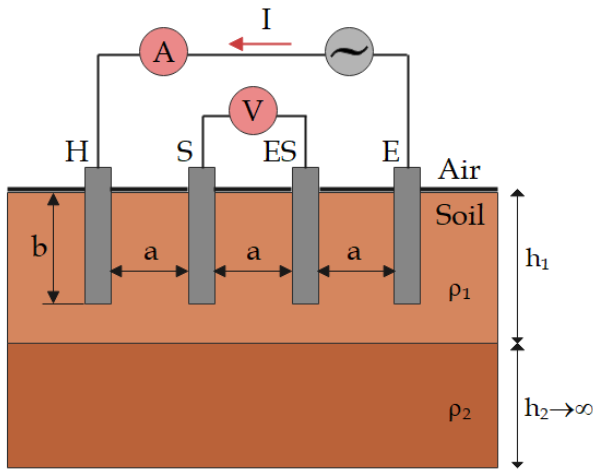


Figure 1: The Wenner four-probe soil resistivity measurement method

The apparent resistivity can be determined based on the following equation for two-layer soil [2]:

$$\rho(\alpha) = \rho_1 \left(1 + 4 \sum_{n=1}^{\infty} \left(\frac{K^n}{\sqrt{A}} - \frac{K^n}{\sqrt{B}} \right) \right) \quad (1)$$

where:

- ρ_1 is resistivity of the upper layer soil,
- K represents reflection coefficient,
- n represents the number of field source mappings,
- A and B are coefficients given by the following expressions:

$$\begin{aligned} A &= 1 + \left(2n \frac{h_1}{a} \right)^2 \\ B &= 4 + \left(2n \frac{h_1}{a} \right)^2 \end{aligned} \quad (2)$$

where:

- h_1 represents the depth of the first soil layer of two-layer soil model, while it is considered that the second layer soil (lower layer) is infinitely large, i.e. $h_2 \rightarrow \infty$,
- a is a distance between two adjacent probes.

The reflection coefficient is determined by the following equation:

$$K = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad (3)$$

where:

- ρ_2 is resistivity of the lower soil layer.

Only if the soil is homogenous, it can be used that apparent soil resistivity is equal to the soil resistivity. With the aim to determine whether the soil is homogeneous, several measurements need to be conducted at different probe distances. In cases when it is necessary to assume that the soil is layered, the following assumptions must be introduced:

- there is a finite number of layers of which soil is composed,
- the thickness of the lowest layer is assumed to be infinite while the thickness of all other soil layer is finite and
- each soil layer is electrically homogeneous [7].

The greater distance between the probes means that the greater the volume of soil is surrounded by the measurement and thus the greater soil depth included in the measurement. The value of the apparent soil resistivity obtained is actually the mean value of soil resistivity for soil volume extending from surface to depth, which is equivalent to the distance between the probes.

The disadvantage of the Wenner four-probe method is that measuring with it is very time consuming. It requires constantly changing the distance between the probes, a large distance between the probes requires long cables, and all probes move with each measurement [2].

2. ESTIMATION OF TWO-LAYER SOIL PARAMETERS

The application of optimization techniques is required to estimate the electrical parameters of two-layer soil. In this case the procedure is reduced to minimizing the square error function between the measurement results and the functions defined by the equation (1). Therefore, the function to be minimized can be expressed as [2]:

$$f(\rho_1, \rho_2, h_1) = \sum_{n=1}^N \left(\frac{\rho_i^m - \rho_i(a)}{\rho_i^m} \right)^2 \quad (4)$$

where:

- N represents the number of measurement points,
- ρ_i^m represents the measured value of apparent specific resistivity at i -th measurement point and
- $\rho_i(a)$ represents the calculated value of apparent specific resistivity by the equation (1).

The optimal solution is obtained by minimizing the function (4) of the soil parameters of two-layer soil model. To solve the previous optimization problem, various approaches based on exact optimization techniques such as the fast gradient-descent [3], [8]-[10], the Quasi-Newton method [11], [12] and metaheuristic optimization techniques such as the genetic algorithm, have been proposed in the literature [13]-[15]. The function given by expression (4) is differentiable and continuous. Considering the fact that the function (4) has no local extremes and that it is an optimization problem without constraints, it seems reasonable to use the fast gradient-descent method to solve a given optimization problem.

Fast gradient-descent method is iterative optimization method without constraints. The general form of iterative scheme of this method is given by the following expression [2]:

$$(\rho_1^{k+1}, \rho_2^{k+1}, h_1^{k+1})^T = (\rho_1^k, \rho_2^k, h_1^k)^T + \alpha_k \mathbf{d}_k \quad (5)$$

where:

- $(\rho_1^{k+1}, \rho_2^{k+1}, h_1^{k+1})^T$ represents the point in the $k+1$ -th iteration,
- $(\rho_1^k, \rho_2^k, h_1^k)^T$ represents the starting point in the k -th iteration,
- α_k represents the step size ($\alpha_k > 0$) and
- $\mathbf{d}_k$ represents the direction from point determined by function $f(\rho_1^k, \rho_2^k, h_1^k)$.

In the fast gradient-descent method the step size is selected so that in each iteration the target function has a maximum decrease. In [3] is proposed constant step size $\alpha_k = 0.005$ with the aim of faster calculation and avoiding Hessian square matrix calculations. The parameter $\mathbf{d}_k$, which represents direction from point determined by the function $f(\rho_1^k, \rho_2^k, h_1^k)$, can be expressed by the following equation:

$$\mathbf{d}_k = -\nabla f(\rho_1, \rho_2, h_1) = - \begin{Bmatrix} \frac{\partial f}{\partial \rho_1} \\ \frac{\partial f}{\partial \rho_2} \\ \frac{\partial f}{\partial h_1} \end{Bmatrix} \quad (6)$$

where first-order derivatives are given with the following expressions:

$$\frac{\partial f}{\partial h_1} = -2 \sum_{n=1}^N \left(\frac{\rho_i^m - \rho_i(a)}{\rho_i^m} \right) \cdot \left(-\frac{16\rho_1 h_1}{a^2} \sum_{n=1}^{\infty} \left[\left(\frac{K^n}{\sqrt{A}} - \frac{K^n}{\sqrt{B}} \right) n^2 \right] \right) \quad (7)$$

$$\frac{\partial f}{\partial \rho_1} = -2 \sum_{n=1}^N \left(\frac{\rho_i^m - \rho_i(a)}{\rho_i^m} \right) \cdot \left(1 + 4 \sum_{n=1}^{\infty} \left[\left(1 - \frac{n(1-K^2)}{2K} \right) \left(\frac{K^n}{\sqrt{A}} - \frac{K^n}{\sqrt{B}} \right) \right] \right) \quad (8)$$

$$\frac{\partial f}{\partial \rho_2} = -2 \sum_{n=1}^N \left(\frac{\rho_i^m - \rho_i(a)}{\rho_i^m} \right) \cdot \left(\sum_{n=1}^{\infty} \left[\left(\frac{2n(1-K^2)}{2K} \right) \left(\frac{K^n}{\sqrt{A}} - \frac{K^n}{\sqrt{B}} \right) \frac{\rho_1}{\rho_2} \right] \right) \quad (9)$$

From the equations (7)-(9) it can be noticed that in all partial derivatives there is an infinite sum that needs to be solved in each iteration, which is time consuming. There are different methods used for accelerating the rate of convergence of an infinite sum such as Aitken's δ^2 method and Wynn's ε method. With the aim of faster calculation for accelerating the rate of convergence of an infinite sum proposed Aitken's δ^2 method [16]-[17].

The iterative procedure by using Aitken's δ^2 method for accelerating the rate of convergence of an infinite sum in the equations (7)-(9) can be written by the following expression:

$$f_{k+1}^{(n)} = f_k^{(n)} - \frac{(f_k^{(n+1)} - f_k^{(n)})^2}{f_k^{(n+2)} - 2f_k^{(n+1)} + f_k^{(n)}} \quad (10)$$

where:

- k represents the number of iterations,
- n represents the number of elements in sum and
- f represents the function that is located in the sum.

The iterative procedure defined by (10) is repeated until the previously defined accuracy is achieved. In this paper accuracy for accelerating the rate of convergence is 10^{-6} . Finally, the iterative procedure for estimating of two-layer soil parameters is:

$$\begin{Bmatrix} \rho_1^{k+1} \\ \rho_2^{k+1} \\ h_1^{k+1} \end{Bmatrix} = \begin{Bmatrix} \rho_1^k \\ \rho_2^k \\ h_1^k \end{Bmatrix} - 0.005 \cdot \begin{Bmatrix} \frac{\partial f}{\partial \rho_1} \\ \frac{\partial f}{\partial \rho_2} \\ \frac{\partial f}{\partial h_1} \end{Bmatrix} \quad (11)$$

The initial conditions proposed in [3] used in this paper are required to start the iterative procedure.

3. MEASUREMENT

3.1. Measurement results

The measurement of the soil resistivity was performed using the Wenner four-probe measurement method. Using four probes set to the soil resistivity was measured at the same distance from each other and the distance was gradually increased between the probes. A total of 6 measurement points were performed on the site. Table I shows the measurement results.

Table I: Soil resistivity measurement results

Distance between probes (m)	4	6	8	10	12	14
Measured soil resistivity (Ωm)	439	174	118	102	95.4	99.2

During the measurement, the instrument couldn't measure (couldn't show the value) the specific soil resistivity at the distance between the probes $a=1$ m and $a=2$ m, which indicates a very high value of soil resistivity at these distances (over 1000 Ωm). The interpretation of a two-layer soil model is shown in Figure 2.

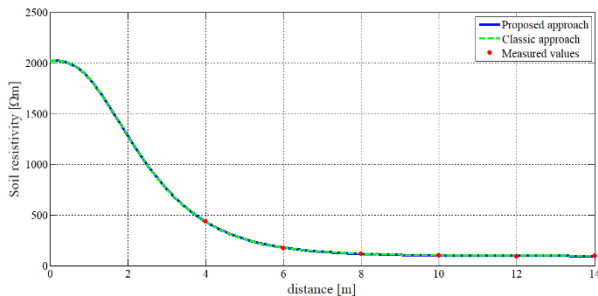


Figure 2: Interpretation of a two-layer soil model

From the results presented in Figure 2, it can be seen that the results obtained using the gradient method and the proposed modified method are in good agreement. Further, by applying the Aitken's δ^2 method, the calculation time was reduced by 22.3 %.

3.2. Measurement processing

After obtaining measurement results, the results were processed in order to determine the soil type. A software package intended for this purpose was used to process the results, which was developed for the purposes of this paper. This software package is typically used to determine equivalent soil structure models based on measured soil resistivity data.

The process of processing the measurement results begins with entering the results into the program and selecting the method by which the measurements were done. In our case the Wenner method was performed

while the depth of the probe was neglected. After entering the measurement results and selecting the data processing method, the soil model is selected. Table II shows the results of analyses for two-layer soil model. Also, the root mean square error (RMSE) is given in Table II. Calculated values of RMSE indicates that both soil layers model results closely match with measurement results. The depth of the first layer of two-layer soil model is $h_1 = 1.7574$ m.

Table II: Soil resistivity measurement

Soil model	Resistivity (Ωm)		RMSE (Ωm)
Two-layer	Top layer	2026.6	10^{-6}
	Bottom layer	92.91	

4. CONCLUSION

The Wenner four-probe method has been used during experimental investigations, and a total of six measurement points were performed on the site. A two-layer soil is assumed, so the application of optimization techniques is required to estimate the electrical parameters of the proposed soil model. In this case, the fast gradient-descent method to solve a given optimization problem is chosen. Furthermore, with the aim of faster calculation for accelerating the rate of convergence of an infinite sum, the application of Aitken's δ^2 method is proposed. The parameters of the assumed two-layer soil model were obtained with satisfactory accuracy. Finally, the calculation time is reduced by applying the proposed modified method.

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BIOGRAPHY

Nedis Dautbašić received his B.Sc. and M.Sc. degrees in electrical engineering from the Faculty of Electrical Engineering, University of Sarajevo (Bosnia and Herzegovina), in 2011 and 2013, respectively. Since 2014 he has been with the same faculty as a teaching assistant. He received his Ph.D. degrees in International Burch University in 2022. His interest is in numerical calculations of electromagnetic fields, grounding systems and electric power system.

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