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A COMPREHENSIVE STUDY OF SHAPE ORIENTATION METHODS FOR EXTRACTION OF PHASE-PRESERVING FOURIER DESCRIPTORS

SISTEMATIČNA USPOREDBA METODA ODREĐIVANJA ORIJENTACIJE OBLIKA ZA EKSTRAKCIJU FAZNO-ZADRŽAVAJUĆIH FOURIEROVIH DESKRIPTORA

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Abstract: The magnitude-based Fourier descriptors (FD) are frequently used in shape-based image retrieval, due to their efficiency and effectiveness. Unlike the phase-preserving Fourier descriptors, the magnitude-based Fourier descriptors are inherently invariant under rotation and starting point change, but they discard all valuable information contained in the phase of the Fourier coefficients (FCs). In order to preserve the coefficients' phase, the orientation and starting point of the shape must be determined. In this paper, we conducted a comprehensive evaluation of different state-of-the-art methods for determining nominal shape orientation, which can be used to extract phase-preserving Fourier descriptors: the point of maximal radius, the axis of least inertia (moments), the phase of the first harmonic, the cross-correlation, the Procrustes distance and the pseudomirror points. The methods were compared in terms of sensitivity to non-rigid transformations, retrieval performance, computational complexity and computational time. The experimental results give insight into the pros and cons of all analyzed methods.

Keywords: Fourier descriptors, image retrieval, shape description, nominal shape orientation, reference points

Sažetak: Amplitudno-bazirani Fourierovi deskriptori često se koriste u pretraživanju i dohvat slika na osnovu oblika, zahvaljujući njihovoj efikasnosti i efektivnosti. Za razliku od fazno-zadržavajućih Fourierovih deskriptora (FD), amplitudno-bazirani Fourierovi deskriptori su inherentno invarijantni na rotaciju i izbor početne tačke, ali odbacuju sve vrijedne informacije sadržane u fazi Fourierovih koeficijenata (FCs). Kako bi se zadržale informacije sadržane u fazi Fourierovih koeficijenata, nominalna orijentacija i početna tačka konture oblika moraju biti jedinstveno određene. U ovome radu obavljena je sveobuhvatna analiza različitih state-of-the-art metoda za određivanje nominalne orijentacije oblika, koje mogu biti iskorištene za ekstrakciju fazno-zadržavajućih Fourierovih deskriptora: tačka maksimalnog radijusa, osa najmanje inercije (momenti), faza osnovnog harmonika, kros-korelacija, Procrustes rastojanje i pseudo-ogledalne tačke. Metode su uspoređivane u smislu osjetljivosti na ne-rigidne transformacije, performanse dohvata, računске složenosti i vremena izračunavanja. Eksperimentalni rezultati daju uvid u prednosti i nedostatke svih analiziranih metoda.

Ključne riječi: Fourierovi deskriptori, pretraživanje i dohvat slika, opisivanje oblika, nominalna orijentacija oblika, referentne tačke

INTRODUCTION

One of the challenging tasks in the field of image processing is to effectively and efficiently describe a shape, as the most important discriminative element in content-based image retrieval. In contrast to colour and even texture, shape is very sensitive to many transformations such as scaling, translation, rotation, projective transformation,

deformation, indentation, occlusion etc., thus it is often more difficult for description and analysis.

Shape description methods may be classified into two groups: contour based, and region based [1], [2]. The contour based techniques analyse only the contour of the shape, while the region based methods use the interior of the shape as well. The contour based techniques are more compact and often more efficient and effective, but fail in tasks where the shape interior is important for classification (e.g. logo recognition, traffic signs etc.). The region based methods perform well in tasks where the contour of the shape is not continuous, or where protrusion, indentation or occlusion of the shape are present.

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However, the region based descriptors are rarely compact, and often use more computational resources.

The FDs are established as compact shape descriptors, with low computational complexity and high retrieval performance. Although FDs may be used as region based [3], they are quite more often used as a global contour-based shape description technique. The derivation of the contour based FDs consists of two main steps. The first step is to compute the Discrete Fourier transform (DFT) over the shape signature function such as: Complex coordinates, Centroid/Radial distance, Perimeter area function, just to name a few [4]-[10]. The second step is to apply different normalization procedures over the FCs to obtain FDs that are invariant under translation, rotation, change of scale and starting point. Invariance of the FDs under translation is usually obtained by discarding the DC component (zeroth coefficient), since this is the only coefficient affected by translation. The scale invariance is achieved by dividing the FCs with the magnitude of the first harmonic [7], [8], [11], [12], or the sum of magnitudes of all harmonics [13], [14]. In order to obtain an invariance under rotation and starting point change, many authors discard the phase of the FCs and use only the magnitude of the FCs [7]-[9], [11], [13]. Due to this simplistic approach, valuable information about the shape is inevitably lost. Thus, two completely different shapes can have the same magnitude of FCs [14]. The phase of the FCs can be preserved and used for shape description only if the resulting descriptors are invariant to rotation and starting point change. This is possible if the nominal orientation of the shape is known. According to the best of the authors' knowledge, there are only few papers that addressed different methods for determining nominal orientation(s), suitable for usage with the phase-preserving FDs.

In this paper, we will compare and discuss the following state-of-the-art methods: the cross-correlation [15], the point of maximal radius [16], the axis of least inertia (moments) [17], the phase of the first harmonic [18], the Procrustes distance [19], [20] and the pseudomirror points [14], [21]. All mentioned methods are based either on aligning shapes during the matching, or determining a nominal shape orientation prior to the matching. We have conducted a detailed comparison of the aforementioned methods in terms of their sensitivity to non-rigid shape transformations (spurious peaks, indentation and missing parts) and their influence on the shape retrieval performance, computational time and complexity. The pros and cons of all methods are identified and discussed using suitable experimental setups.

The paper is organized as follows. A brief introduction on the FDs is presented in Section 2. Different methods for shape alignment are described in Section 3. The experimental results are discussed in Section 4. The concluding remarks are presented in the last section.

1. FOURIER DESCRIPTORS

For the subsequent analysis, it will be assumed that a shape contour is given with N boundary points $P_n=(x_n, y_n)$, represented in the form of complex numbers

$$Z_n = x_n + jy_n \quad (1)$$

where $n=0,1,\dots, N-1$. The Discrete Fourier Transform is computed over (1) by:

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} Z_n e^{-j2\pi nk/N} \quad (2)$$

where $k=0,1,\dots, N-1$. The FCs a_k are used to derive the FDs. Translation of the shape affects only the coefficient a_0 (depending on shape signature), thus *translation invariance* is achieved by disregarding the DC component a_0 [13]. In order to introduce *scale invariance*, many FD implementations [4], [7]-[9], [18] use $S_c=|a_1|$ as the normalization coefficient, or $S_c=\sum_{i=1}^{N-1}|a_i|$ in [13] which yields a better retrieval performance. If a shape is rotated for an angle ϕ round its centroid and the point P_{mis} used as a shape contour starting point instead of P_0 , the FCs are expressed as:

$$a_k^{(new)} = e^{j\phi} e^{j2\pi km/N} a_k^{(old)} \quad (3)$$

where $k=0,1,\dots, N-1$. Hence, the rotation affects the phase of all harmonics equally with a constant value, while the change of the starting point manifests as de-rotating the k -th harmonic for an angle of $2\pi km/N$. The most simplistic workaround to obtain the *rotation and starting point invariance* is to simply disregard the phase and obtain the magnitude-based FDs $|a_k|$, hence discarding important information contained in phase description.

However, invariance under rotation may be achieved by de-rotating the shape for angle e.g. $-\arg\{Z_m\}$ if starting point P_m is known. The starting point may be determined using two approaches: based on nominal orientation of the shape, or determining a common starting point of two shapes during their matching. Once the shapes are perfectly aligned, sharing the same starting point and having a zero rotation, an invariant FD may be formulated as

$$F_{TS} = \left\{ \frac{a_{-M/2}}{S_c}, \dots, \frac{a_{-1}}{S_c}, \frac{a_1}{S_c}, \frac{a_2}{S_c}, \dots, \frac{a_{M/2}}{S_c} \right\} \quad (4)$$

where a_i are computed using (2), and M_i is the number of FCs needed for representation, which denotes a small empirically chosen number ($M \leq 30 < N/2$) independent of the number of points N . Note that the DFT is a periodic sequence with period N , since $a_{-m} = a_{N-m}$ holds.

2. SHAPE ALIGNMENT AND METHODS FOR DETERMINING NOMINAL SHAPE ORIENTATIONS

Point with the maximal radius. This is the simplest and the most intuitive method for shape alignment. It is proposed by Kuhl et al. [16]. The point with the maximal distance (P_s) from the centroid (Z_c) is used as shape orientation reference. The point with the maximal radius is computationally efficient because it is simple to compute. Moreover, the point which determines the nominal orientation is derived during descriptor extraction, therefore shape matching is very fast.

Axis of least inertia (moments). Moments are widely used for shape description and analysis [17], [22]–[25]. Moments are used to determine the Axis of least inertia (ALI), which is defined as the line for which the integral of the square of the distances to points on the shape boundary is minimal [26], [27]. It is important to note that the ALI passes through the centroid of the shape.

Orientation using the phase of the first harmonic.

Many authors tried to normalize phase preserving descriptors using the phase of the first or higher harmonics [16], [18], [28]–[30]. These approaches use the phase of known harmonics to extract (or eliminate) the effects of rotation and starting point. Bartolini et al. [18] proposed to use the following FCs normalization:

$$\begin{aligned}\hat{R}_k &= \frac{R_k}{R_1}, \hat{R}_k = 0 \\ \hat{\theta}_k &= \theta_k - \frac{\theta_{-1} + \theta_1}{2} + k \frac{\theta_{-1} - \theta_1}{2}\end{aligned}\quad (5)$$

where $a_k = R_k e^{j\theta_k}$ and $\hat{a}_k = \hat{R}_k e^{j\hat{\theta}_k}$ are the original and translation, rotation, scale and starting point change invariant descriptor, respectively.

Cross-correlation method. Larsson et al. [15] proposed to use cross-correlation between shape signatures to determine the best match. Instead of using cross-correlation in spatial domain, they used multiplication of scaled FCs in frequency domain. Unlike the previously described methods, cross-correlation does not explicitly search for the nominal orientation of the shape, but rather finds the optimal matching between two shapes during matching stage. This approach reduces the descriptor extraction time, but increases the computational complexity of the descriptor matching procedure.

Procrustes distance. The Procrustes distance is used to find the optimal similarity between shapes by minimizing the Procrustes distance [19], [20]. The procedure is based on using one of the shapes as a reference. The reference shape is fixed, and the other is rotated around the origin, until an angle of rotation ϕ is found such that the sum of the squared distances between the corresponding points $Z_n^{(1)}$ and $Z_n^{(2)}$ is minimized. If the starting point is important (for FDs, Wavelet descriptors etc.) then every point must be used considered as starting point, the shape needs to be de-rotated, and the Procrustes distance has to be

computed. The minimal value of Procrustes distance determines the starting point and rotation.

Pseudomirror points. This method is based on finding specific points on the shape contour that are used as the orientation reference [14], [21]. These points represent the points for which the phase-preserving FDs of the shape and the mirrored version of the shape are the most similar. Since many pseudomirror points may be found, usually a smaller number ($Q \in [8, 16]$, where $Q \in \mathbb{N}$) of points are chosen to be the starting points for the FDs. In matching stage, the FDs computed in these points are compared point-wise using City-block distance, and the minimal value is chosen as similarity value. This method combines the best of all above - it uses the phase of all harmonics and a subset of contour points for cross-correlation, preserves a lower computational complexity, and improves shape discriminability. In contrast to other proposed methods, the descriptor size is increased from M to $Q \times M$.

3. EXPERIMENTAL RESULTS

Three important parameters are analyzed for all methods described in the previous section: 1) sensitivity to non-rigid transformations (spurious peaks, indentation, missing parts), 2) retrieval performance (Bulls-eye Score, Precision and recall diagrams), and 3) computational cost (computational time and complexity).

3.1. Sensitivity to non-rigid transformations

In order to investigate and properly illustrate the sensitivity of different methods to non-rigid transformations, the following experiments were conducted. Two different shapes (representing a bone and a beetle) were compared respectively to five corresponding shapes: shape with spurious peaks, shape with indentation, shape with a large missing part, a very similar shape and a non-similar shape. The shapes used in the experiments are given in Figure 1.a). The resulting shape orientations and similarity measure scores (in terms of phase-preserving FDs) are depicted in Table I and Table II. For illustration purposes, the centroids of the shapes are marked with a square symbol, starting points of the contour are marked with the circle symbol, and one corresponding point on each shape is marked with a filled square symbol. It is important to note that non-rigid transformations dislocate the centroid of the shape. Thus, same points on the shapes may form different angles with the centroid due to scale change, and serve only as illustrations. As it can be observed in Table I and Table II, the method that uses the point with maximal radius fails in determining nominal orientation in case of spurious peaks (illustrated in Table I-1a and Table II-1a), especially when these peaks affect the position of the point with the maximal radius. In contrast, moderate indentations which do not alter the location of the point with maximal radius also do not significantly affect the nominal orientation (Table II-2a). Missing parts or large indentation may change the nominal orientation for a significant amount (Table I-2a, Table I-3a and Table II-3a).

The maximal radius point method is extremely sensitive to centroid dislocation, and consequently very sensitive to non-rigid transformations. Thus, similar shapes may be oriented differently when using the point with the maximal radius, as shown in Table II-4a.

Moments (or ALI) performs moderately as shape orientation reference. The displacement between the shapes is relatively small as shown in Table I-1b, Table I-3b, and Table II-2b, Table II-3b. It is important to note that moment based orientation determines the reference axis and not a single reference point, there could be two possible starting points - a fact that the authors in [17] have not taken into consideration. This is illustrated in Table II-1b. Another weakness of moment based orientation is that in case of the shapes with holes and protrusions, they may determine wrong orientation, which is pointed out in Figure 1.b).

The phase of the first, or the most dominant harmonic, is frequently used as a shape orientation reference. It exhibits average performance, and it is relatively simple to derive. However, since the phase of the first harmonic is used to determine nominal orientation, in cases of circular harmonic locus, or first harmonic with negligible magnitude, this method may fail. This is additionally illustrated with the example given in Figure 1.c), where this method fails to align shapes properly.

The cross-correlation and Procrustes methods cope well with non-rigid transformations. This is illustrated in Tables I and II (1d-3d, 1e-3e). It is interesting that the cross-correlation based method may create arbitrary orientations for complex shapes, as shown in Table II-5d. To additionally evaluate this, the cross-correlation function for shape matching given in Table II-5d is presented in Figure 1.d).

The point for which the cross-correlation achieves the maximum (marked with $\circ$), does not correspond to the intuitive orientation (which is marked with $\bullet$). Thus, FDs are not computed from the corresponding starting points, and the similarity is reduced. In addition, the cross-correlation method may find better matching (score) for non-similar shapes, which does not correspond to the intuitive shape alignment. Hence, this method may reduce the discriminability among shapes.

Unlike other methods, the Procrustes distance based method manages to align shapes almost perfectly. This does not come without a price, since the Procrustes distance based method requires significant computational effort. Moreover, the process of finding the starting point is more complicated. If the starting points are not chosen correctly, the accuracy of the similarity measure using the FDs may suffer (Table I-2e and Table 3e, and Table II-2e).

The pseudomirror point based orientation method uses a wider set of starting points as orientation references. They also suffer from similar weaknesses as the cross-correlation method, since for a larger number of pseudomirror points (Q) shapes may be aligned to their non-nominal orientation. However, if less than 8-16 points are chosen as orientation reference, the miss-alignment of the shapes is less probable than for cross-correlation based method (which uses all contour points).

If values of shape similarity are analyzed (Table I and Table II), values in columns denoted as spurious peak (Sp), indentation (Ind), missing part (MP) and similar shape (Sim) should have similarity score values as low as possible, ideally zero, while the non-similar shape (Non-Sim) column should have values as large as possible, which indicate that shapes are not similar. It is clear that

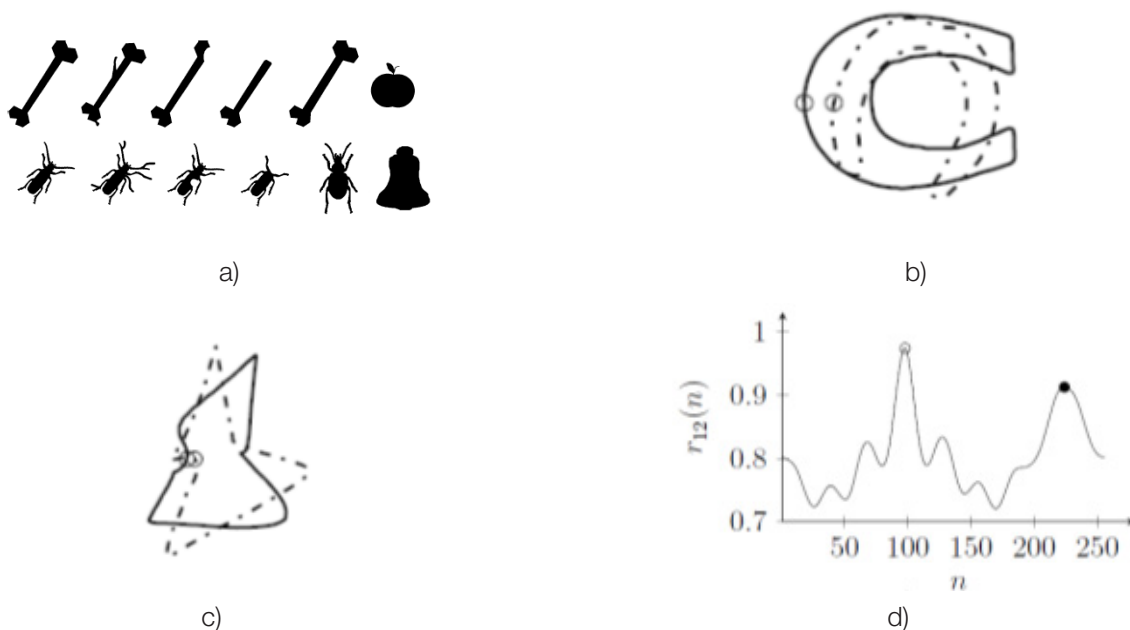


Figure 1: a) Representative shapes used for testing sensitivity under non-rigid transformations, b)-d) Illustration of drawbacks of different methods: b) alignment of shapes using moments, c) aligning shapes using first-harmonic in case of circular harmonic locus, d) cross-correlation similarity measure for different starting point.

the Cross-correlation and the Pseudomirror points perform best when dealing with non-rigid transformations, but they decrease the discrimination ability between non-similar shapes at the same time.

The method based on maximal radius point is clearly the most sensitive under non-rigid transformations. The rest of the methods exhibited average performance under non-rigid transformations.

Table I: Nominal orientations and shape similarity measures for different shape orientation methods, for “Bone” shape. The centroids of the shapes are marked with a $\square$ symbol, starting points of the contour are marked with the $\circ$ symbol, and one corresponding point on each shape is marked with a $\blacksquare$ symbol.

	Spurious peaks $\Delta = 0.6429$ 1a	Indentation $\Delta = 0.4238$ 2a	Missing part $\Delta = 0.4643$ 3a	Similar $\Delta = 0.0604$ 4a	Non-similar $\Delta = 0.9807$ 5a
max radius					
	$\Delta = 0.5168$ 1b	$\Delta = 0.2845$ 2b	$\Delta = 0.4193$ 3b	$\Delta = 0.0586$ 4b	$\Delta = 1.2437$ 5b
moments					
	$\Delta = 0.5217$ 1c	$\Delta = 0.2925$ 2c	$\Delta = 0.3318$ 3c	$\Delta = 0.0607$ 4c	$\Delta = 0.8779$ 5c
phase 1st harm.					
	$\Delta = 0.5104$ 1d	$\Delta = 0.2999$ 2d	$\Delta = 0.3387$ 3d	$\Delta = 0.0850$ 4d	$\Delta = 0.8522$ 5d
cross-correlation					
	$\Delta = 0.5166$ 1e	$\Delta = 0.5542$ 2e	$\Delta = 0.5251$ 3e	$\Delta = 0.0654$ 4e	$\Delta = 1.3472$ 5e
Procrustes					
	$\Delta = 0.5279$ 1f	$\Delta = 0.2870$ 2f	$\Delta = 0.3276$ 3f	$\Delta = 0.0586$ 4f	$\Delta = 0.8243$ 5f
pseudomirror					

Table II: Nominal orientations and shape similarity measures for different shape orientation methods, for “Beetle” shape. The centroids of the shapes are marked with a $\square$ symbol, starting points of the contour are marked with the $\circ$ symbol, and one corresponding point on each shape is marked with a $\blacksquare$ symbol.

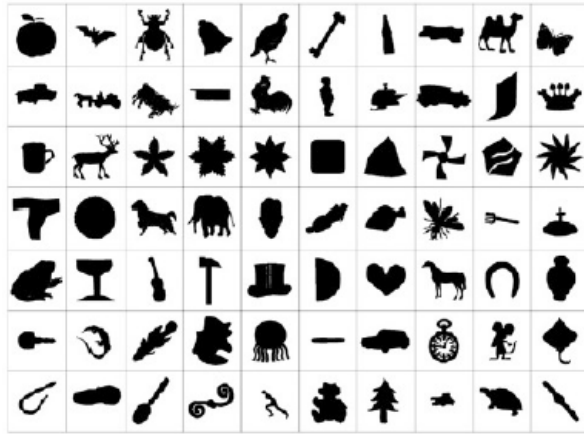
	Spurious peaks $\Delta = 1.0165$ 1a	Indentation $\Delta = 0.1424$ 2a	Missing parts $\Delta = 0.6775$ 3a	Similar $\Delta = 0.7258$ 4a	Non-similar $\Delta = 1.0403$ 5a
max.radius					
	$\Delta = 0.8433$ 1b	$\Delta = 0.1672$ 2b	$\Delta = 0.6499$ 3b	$\Delta = 0.3901$ 4b	$\Delta = 0.9338$ 5b
moments					
	$\Delta = 0.8600$ 1c	$\Delta = 0.1566$ 2c	$\Delta = 0.6535$ 3c	$\Delta = 0.3685$ 4c	$\Delta = 0.9405$ 5c
phase 1st. harm.					
	$\Delta = 0.8001$ 1d	$\Delta = 0.2018$ 2d	$\Delta = 0.8654$ 3d	$\Delta = 1.0148$ 4d	$\Delta = 1.0008$ 5d
cross-correlation					
	$\Delta = 0.6688$ 1e	$\Delta = 0.7599$ 2e	$\Delta = 0.6664$ 3e	$\Delta = 0.4098$ 4e	$\Delta = 1.0451$ 5e
Procrustes					
	$\Delta = 0.6649$ 1f	$\Delta = 0.1433$ 2f	$\Delta = 0.6537$ 3f	$\Delta = 0.3458$ 4f	$\Delta = 0.9195$ 5f
pseudomirror					

3.2. Retrieval performance

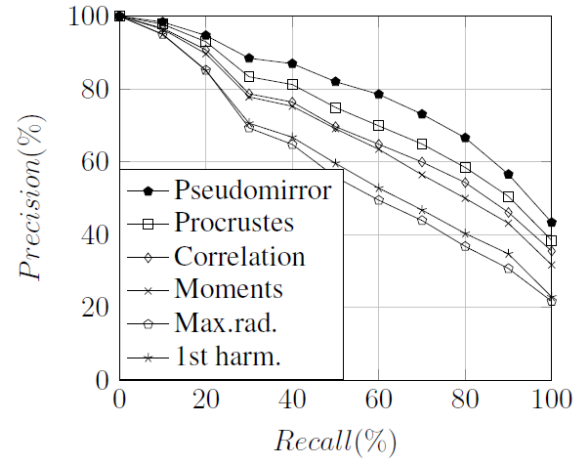
The retrieval performance of phase preserving FD that use different shape orientation (alignment) methods is analyzed on the prominent MPEG7 CE-1 Set B [31] which consists of 1400 shapes representing real life objects, classified into 70 classes with 20 similar shapes for each class (Figure 2.a)). This database is convenient for shape-based image retrieval testing since it includes rotation, scaling, skew, stretching, defection, indentation and articulation of shapes.

Two commonly adopted measures of retrieval performance are computed: precision and recall (PR) diagrams, and the Bulls-Eye score [15]. The PR diagrams obtained on MPEG-7 CE-1 Set B are given in Figure 2.b), while the achieved Bulls-eye scores are given in Table III.

As it may be seen from the experimental results, the pseudomirror point based orientation method achieves the best performance. Unlike the cross-correlation and the Procrustes distance, this method uses only few points for shape orientation reference, hence it reduces the cross-class similarity and improves retrieval performance.



a)



b)

Figure 2: a) MPEG7 CE-1 Set B dataset representative shapes (70 classes with 20 variations per class), b) Precision and recall diagrams of phase-preserving Fourier descriptors based on different alignment methods, on MPEG-7 dataset.

The maximal radius method is very sensitive to non-rigid transformations, which justifies the lower performance. Mostly due to inadequate scale normalization, which is elaborated in [13], the first-order harmonic based method exhibited the lowest performance.

3.3. Computational Cost

We have investigated two computational cost measures: the computational complexity and the computational time.

Computational complexity. The computational complexity for each of the proposed methods is given in Table III. All methods, except the Procrustes distance, require at least $O(n \log n)$ operations in descriptor extraction stage, due to the computation of the Fast Fourier transform. The alignment using the Procrustes distance requires a computation of the FDs in the matching stage. The cross-correlation and the first-harmonic based alignment require the same complexity for inverse Fourier transform. Moreover, the Procrustes distance is not a compact descriptor, since all contour points of the shape need to be stored. Usually, the methods that provide simpler matching are more desirable (e.g. point with maximal radius, moments, pseudomirror points), because they may be applied in the real-time. This means that increased descriptor extraction time and complexity may sometimes be overlooked, for the sake of improving retrieval performance, since descriptor extraction may be conducted offline.

Computational time. Two different computational times were analyzed: the average time needed to extract the descriptor from N points (descriptor extraction), and the average time needed to match two descriptors using N points, or M descriptor coefficients - depending on the method of descriptor matching. For each experiment, time was computed as the average of 1.000.000 iterations. The experiments were conducted on a PC with Intel Core i3 processor, 4MB of RAM and Ubuntu 16.04 operating system.

First, the computational time for the extraction of the descriptors using different number of contour points N was analyzed. The results are presented in Table IV. It is clear that the Procrustes distance based method is the fastest one, since it only requires scale normalization. The pseudomirror points method is the most computationally expensive, since it involves searching and sorting pseudomirror points using the Fourier transform coefficients. The rest of the methods perform similarly, with acceptable descriptor extraction times.

The descriptor matching times for different shape orientation methods are given in Table V. The methods based on these points with the maximal radius and moments only require computation of the City-Block distance, therefore they are extremely fast. The pseudomirror points based descriptor requires computation of Q^2 City-

Table III: Computational complexity and Bulls-Eye scores achieved by Fourier descriptors based on different shape alignment methods on MPEG-7 dataset

Method	Comput. complexity (descriptor extract.)	Computat. complexity (matching)	Bulls Eye score
Max. rad.	$O(n \log n)$	$O(n)$	60.74
Moments	$O(n \log n)$	$O(n)$	71.50
1st harm.	$O(n \log n)$	$O(n \log n)$	58.50
Cross-correlation	$O(n \log n)$	$O(n \log n)$	72.57
Procrustes	$O(n)$	$O(n^2)$	76.38
Pseudomirror	$O(n \log n)$	$O(n)$	82.03

Table IV: Average shape descriptor extraction time in microseconds

Nr. pts. (N)	Max. rad.	Moments	1st harm.	Cross-corr.	Procrustes	Psm. (Q=8)
64	62	93	107	14	3.5	2061
128	73	109	136	21	5.6	4349
256	94	134	191	34	10.1	9429
512	138	189	302	54	18.6	20666

Table V: Average descriptor matching time in microseconds

Nr. coeff. M (Nr. pts. N)	Max. rad.	Moments	1st harm.	Cross-corr.	Procrustes	Psm. (Q=8)
20 (64)	2.4	2.4	2576	77	4402	339
40 (128)	3.1	3.1	4550	91	9735	549
60 (256)	3.7	3.7	8906	110	22511	727
80 (512)	4.4	4.4	15893	182	58067	943

block distances and finding their minimum, thus it needs more computational time. Nevertheless, methods such as the Procrustes distance or the first harmonic method based on the Dynamic Time Warping distance require significantly more computational time than the pseudomirror points method, which considerably increases the computational time proportionally to the number of shape contour points.

4. CONCLUSION

In this paper we presented an evaluation of the most prominent methods for determining a nominal shape orientation and a shape alignment used to extract phase-preserving FDs. The results confirmed that it is difficult to propose a single method which is versatile, simple to implement, fast, and achieves promising shape retrieval performance. The experimental results showed that the method based on the point with the maximal radius is extremely fast, relatively simple, but may perform adequately only on a limited set of problems. The pseudomirror point based method achieved the best retrieval performance, at the cost of a longer descriptor extraction time and larger descriptor size. The moment based and cross-correlation based methods may be described as fast and compact, but they have slightly reduced retrieval performance. The Procrustes method is the best choice when it comes to pure shape alignment. However, its conjunction with phase-preserving FDs is questionable, especially since other more computationally expensive methods, such as the structural Hierarchical Procrustes method [20], exploit the Procrustes distance in a more efficient manner and obtain considerably better retrieval performance. In the end, the choice of the method is dependable on the shape dataset, demands for descriptor compactness, the application demands and the real-time requirements. Methods based on the moments or the cross-correlation may be suitable when the computational time is important, while the pseudomirror point based shape orientation may be convenient if a higher retrieval performance or mirror-invariance is needed.

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