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GAME BOARD OR ABACUS? GREEK COUNTER CULTURE REVISITED[1]

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Abstract. A late 5th century BC funerary altar from the necropolis of Krannon (Central Greece) depicts a bearded man and a boy on either side of a board with five lines carved on a block. The fact that the man is seated and the horizontal position of the board reveal important information about Greek education and the history of Greek numeracy. This paper analyses the iconography of the relief, the link between the *Five Lines* game (*Pente grammai*) and abaci, examines the possible identification of the man as a "pebble arithmetician", of the boy as a student, and suggests a new reconstruction of the reckoning system operated on an abacus composed of five horizontal lines. A special practical function is proposed for the half-circle at one end of the

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[1]The expression "counter culture" is a verbal pun first used by R. Netz in his seminal paper (2002) on the role of counters in the cognitive history of ancient Greece.

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abacus. This five lines pattern and the related material, especially counters, are considered from a wider perspective, a system of cultural practices associated with boards and counters throughout the Greek world.

Abacus, arithmetics, boardgame, tombstone, numeracy, *Pente grammai,* Achilles and Ajax

Περίληψη. Ένας επιτύμβιος βωμός που χρονολογείται στα τέλη του 5ου αιώνα π.Χ. από την νεκρόπολη της Κραννώνας (Κεντρική Ελλάδα) φέρει ανάγλυφη παράσταση ενός γενειοφόρου άνδρα και ενός αγοριού εκατέρωθεν ενός πίνακα με πέντε γραμμές σκαλισμένου πάνω σε έναν κυβόλιθο. Η καθιστή στάση του άνδρα καθώς και η οριζόντια θέση του πίνακα προσφέρουν σημαντικές πληροφορίες για την εκπαίδευση και την ιστορία της αρίθμησης στον ελληνικό κόσμο. Στο παρόν άρθρο επιχειρείται η εικονογραφική ανάλυση του αναγλύφου, εξετάζονται η σχέση μεταξύ του παιχνιδιού των πέντε γραμμών και των αβάκων, η πιθανή ταύτιση του άνδρα ως «ειδικού στην αρίθμηση με ψήφους», του αγοριού ως μαθητή, και προτείνεται μια νέα ανασύνθεση του υπολογιστικού συστήματος που πραγματοποιούταν πάνω σε άβακα αποτελούμενο από πέντε οριζόντιες γραμμές. Μια ειδική πρακτική λειτουργία αποδίδεται στο ημικύκλιο στο ένα άκρο του άβακα. Το μοτίβο των πέντε γραμμών και το σχετιζόμενο με αυτό υλικό, ιδίως οι ψήφοι, αναλύονται σε μια ευρύτερη προοπτική, στο πλαίσιο ενός συστήματος πολιτισμικών πρακτικών που συνδέονται με πίνακες και ψήφους σε ολόκληρο τον ελληνικό κόσμο.

Άβακας, αριθμητική, επιτραπέζιο παιχνίδι, επιτύμβια στήλη, αρίθμηση, πέντε γράμμαι, Αχιλλέας και Αίαντας

Riassunto. Un rilievo funerario databile alla fine del V sec a.C. proveniente dalla necropoli di Krannon (Grecia Centrale) raffigura un uomo barbato e un ragazzo disposti ai lati di una tavola con cinque linee incisa su un blocco. Il fatto che l'uomo sia seduto e la posizione orizzontale della tavola rivelano importanti informazioni riguardanti l'istruzione e le tecniche di calcolo diffuse nel mondo greco. Il contributo esamina l'iconografia del rilievo, il rapporto tra il gioco delle *Cinque linee* (*Pente grammai*) e gli abachi ed analizza la possibile identificazione dell'uomo come un esperto di calcoli aritmetici basato sul conteggio dei sassolini e del ragazzo come un allievo. Inoltre, viene proposta in questa sede una nuova ricostruzione del sistema di calcolo praticato mediante un abaco composto da cinque linee orizzontali; una speciale funzione pratica è proposta per il semicerchio posto ad una delle estremità dell'abaco. Il motivo a cinque linee e il materiale associato del rilievo in

questione, in particolare i gettoni per il calcolo, sono analizzati in una più ampia prospettiva, nel quadro di un sistema di pratiche culturali che implicano l'uso di tavole e pedine diffuso in tutto il mondo greco.

Abaco, Achille e Aiace, aritmetica, calcolo, cinque linee, gioco da tavola, rilievo funerario

Introduction

A late 5th century BC funerary altar from Krannon in Thessaly (Central Greece) (Fig. 1)[3] depicts an unusual scene: on the right, a bearded man, wearing a tunic and a mantle, sits on an elegant *klismos* chair with curved legs and a high backrest. He extends his right hand above a rectangular block, the top of which is carved with five horizontal lines, and he points to a half-circle on the far end of the board. On the left, a young person, most likely a boy with cropped hair, wearing a *chitoniskos*, stands behind the stone, raising his left hand and looking down at a small dog trying to catch something in his hand.[4] The name of the deceased, partly erased, ΦΑΝΑΙΟΣ ("bringing light"),[5] is carved above his head on the panel's frame.

At first glance, the board seems to represent the structure of the so-called *Five Lines* game (πέντε γραμμαί), a race game where each player moves counters according to the throw of dice as well as to strategic skill.[6] The aim is to be the first to gather his/her counters on the middle or "sacred line", where the tokens are safe, especially in a knock-off variant.[7] The relief could show a gaming scene between a male adult and a boy. No counter is visible

[3] Diachronic Museum of Larisssa; BATZIOU-EFSTATHIOU – KARAGOUNIS 2013, 13 (with fig.). We thank Stella Katakouta, Ephoria and Museum of Larissa and Eftychia Alevizou for their precious help.

[4] On this species of small dog, called *melitê*, see VESPA 2022, in press. See a youth wearing a similar folded chiton on a funerary relief from central Greece in BOSNAKIS 2012, cat. B1, pl. 13. Cf. the similar groups of a boy or girl with a dog on Attic steles: CAT 0.785, 0.787, 0.855, 0.864, 0.865, 0.874, 0.915. Dog jumping against a boy or girl: CAT 1.317, 1.321.

[5] The online *Lexicon of Greek Personal Names* provides six entries ranging from 375-2nd century BC (Northern Greece and Asia Minor), http://clas-lgpn2.classics.ox.ac.uk.

[6] For the collection of ancient written sources and the reconstruction of the rules, see SCHÄDLER 2009.

[7] On the knock-off variant, see KIDD 2017a. The reconstructed game in two variants can be played online, see https://locusludi.ch/play-ancient-online-games.

on the board, but such detail may have been painted and have eroded with time.

Several anomalies in the scene, however, invite us to question the methodological criteria that allow us to identify a game-board. On the Krannon relief, the identification of a *Pente grammai* board can be argued on the basis of iconographic, archaeological, and literary evidence. Yet, it could also depict a counting board with an orientation that suggests new experimentations in Greek reckoning methods.

Figure 1: Marble funerary altar, from Krannon (H. 78.6 cm, W. 54/58 cm, D. 41/55 cm). Diachronic Museum of Larissa. Photo courtesy Larissa Museum.

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1. Depicting *Five Lines* games

The Krannon relief does not conform to the expected iconography of board games. These scenes are mainly found on archaic Attic vase painting produced between ca. 540 and 480 BC.[8] About 170 vases show the same heroic context: two warriors, often identified as the best homeric fighters, Achilles and Ajax,[9] equipped with armour and weapons, sit opposite each other on block-like seats (*thakoi*), denoting an outdoor setting, and they play on a rectangular stone (Fig. 2; 510 BC)[10] which may look like an altar[11] or may be topped by a portable board (Fig. 3; 500 BC).[12] More rarely the game takes place on a slab with transverse supports (Fig. 4; ca. 520 BC),[13] as on miniature archaic clay models (Fig. 7; ca. 580 BC).[14] Only in a few daily life scenes do the players sit at a three-legged table (*trapeza*) outdoors, as indicated by the presence of a tree (Fig. 5; 520-510 BC).[15] On no vase-painting is the board placed on the knees of the players as is common in Etruscan (Fig. 8; 4th century BC) and Roman iconography.

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[8] For catalogues see BUCHHOLZ 1987; KOSSATZ-DEISSMAN 1981; WOODFORD 1982.

[9] Ten examples are recorded. The earliest vase of the series is the amphorasigned by the painter and potter Exekias, Vatican, Museo Gregoriano Etrusco Vaticano 344; *BAPD* 310395; KOSSATZ-DEISSMAN 1981, no. 397; MACKAY 2010, 327-351; SANNIBALE 2018.

[10] Amphora, Essen, Museum Folkwang A 176/K1049; *BAPD* 351214; KOSSATZ-DEISSMAN 1981, no. 409; BUCHHOLZ1987, no. 117, fig. 57a; BUNDRICK 2017, 61, fig. 6. See also the Etruscan mirror (4th century BC), Rome, Villa Giulia 6425; BUCHOLZ 1987, no. 14, 142-143, fig. 53.

[11] With volutes decoration, see e.g. Boston, MFA 01.8037; *BAPD* 200007;KOSSATZ-DEISSMAN 1981, no. 392, pl. 96 and no. 421, pl. 100; BUCHHOLZ 1987, no. 30 and no. 394, pl. 96. See also: cup, Museo nazionale etrusco di Marzabotto 489; BALDONI 2017, 420, fig. 2.

[12] Lekythos, Paris, Louvre L 34 (MNB 911). See also: column krater frgt, Atlanta, Michael Carlos Museum 2004.33.2; *BAPD* 9017853; BUNDRICK 2017, 60, fig. 5.

[13] Amphora, once in Florence; BUCHHOLZ1987, no. 106, fig. 56a. See also the amphora, Cambridge, Fitzwilliam Museum G50; *BAPD* 320078; WOODFORD 1982, D2, pl. IVb. Lekythos, London, British Museum B 638; WOODFORD 1982, F2, pl. IVc. A variant depicts two winged players: cup, Copenhagen, National museum 13521; *BAPD* 504; BUCHHOLZ 1987, 132, fig. 49e-f.

[14] Copenhagen, National museum 1950; USSING 1884, 3 pl. 1; PRITCHETT 1968, 197, pl. 7, 1; SCHÄDLER 2009, 175, fig. 1 and 180, fig. 5. For parallels, SCHÄDLER 2019; CHIDIROGLOU – SCHÄDLER – SCHIERUP, 2022, in press.

[15] Pelike, attributed to the Plousios Painter, private collection; R. GUY in LEIPEN 1984, 11, no. 7. See also the pelike, New York, MMA 68.27; WOODFORD 1982, pl. VI.

Figure 2: Attic black-figure amphora (H. 49.5 cm). Essen, Museum Folkwang A176/K1049. © Museum Folkwang Essen/Artothek.

Figure 3: Attic black-figure lekythos (H. 14 cm). Paris, Louvre MNB 911. Photo Marie-Lan Nguyen. Creative Commons license.

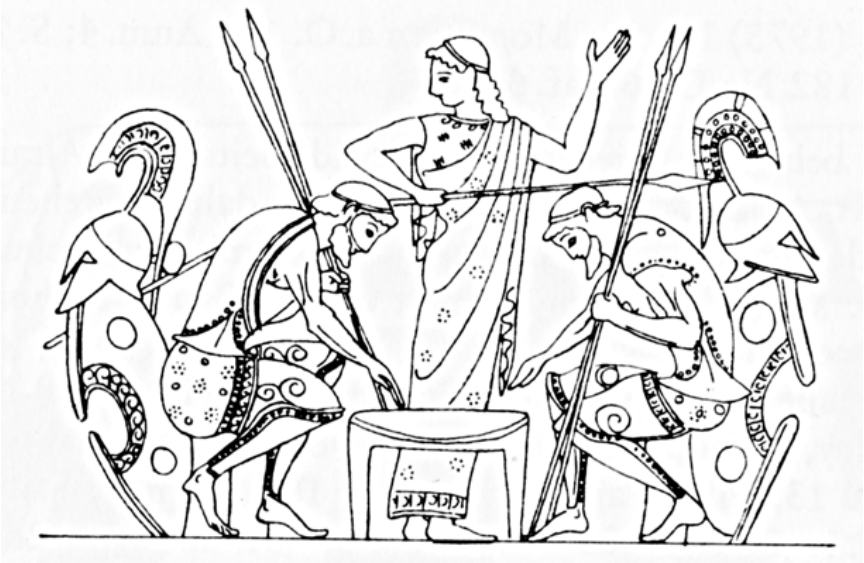

Figure 4: Attic black-figure amphora, once in Florence. After BUCHHOLZ 1987, no. 106, fig. 56a.

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Figure 5: Attic black-figure pelike (H. 36 cm), private collection. After LEIPEN 1984, 11.

Figure 6: Attic black-figure kyathos (H. 14.7 cm). Brussels, Musées royaux d'Art et d'Histoire R2512. Photo Museum.

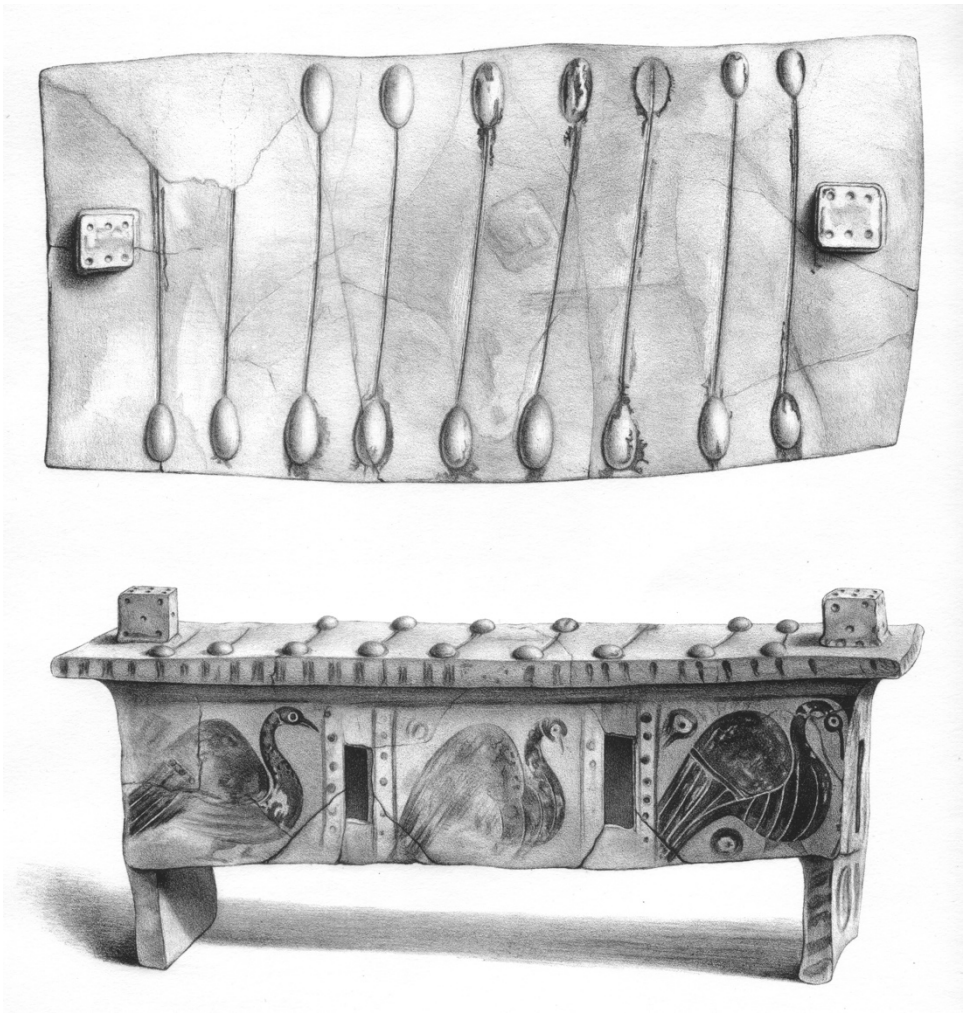

Figure 7: Attic terracotta model of gaming table (12 x 37 cm), from Attica? Copenhagen, National Museum 1950. After USSING 1884, pl. 1.

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Figure 8: Bronze Etruscan mirror (H. 35 cm). Collezione Giulio Sambon, inv. Sambon 439, ST. 1753. In deposito presso Soprintendenza Archeologia, Belle Arti e Paesaggio della città metropolitana di Milano. After G. KÖRTE, *Etruskische Spiegel*, V, Berlin, 1897, pl. 109.

The warriors are depicted manipulating small pieces often painted with contrasted black and white colours (Fig. 2, 3). The meaning of their action, however, is still hotly debated. Since the 19th century, some scholars have assumed that the warriors were casting lots to determine their fate.[16] The block-like shape of the table is also reminiscent of other contexts, such as voting[17]and reckoning. There may be a visual reference to an abacus on the Attic black figure amphora (Fig. 2) where the warriors are moving counters on a monumental block placed on a base before the goddess Athena. Athena's name is painted on the block in the genitive case (*Athenaias*, "this belongs to Athena"), which calls to mind public abaci with state property inscriptions also in the genitive case.[18] Are the warriors depicted playing, casting votes, drawing lots, consulting an oracle, or counting?

Play is the usual interpretation of these scenes. In 1869, Louis Becq de Fouquières suggested a form of *petteia* game, with *pettoi/pessoi*, "counters", based on Pollux's description of a game composed of five lines with a middle "sacred" line, *hiera grammê*.[19] In 1963, John D. Beazley compared the scene depicted on the bilingual amphora attributed to the Andokides and Lysippides painters with the archaic clay model of a gaming table with nine lines and two dice in Copenhagen (Fig. 7) and a 4th century Etruscan mirror where the warriors play on a board seen from above with seven lines and two dice (Fig. 8),[20] and he suggested to identify the game with the *Five Lines*

[16] On drawing lots on an altar, see GERHARD 1843, 28-29, pl. XIX ("Im Lager on Troja, Brett und Würfelspieler, Loosorakel"); BUNDRICK 2017; BALDONI 2017. On the iconic value of the *bomos* as communication with the divine, FRONTISI-DUCROUX 2013.

[17] On the voting-table for Achilles' weapons, see BUCHHOLZ 1987, nos. 188-195, figs 58-59.

[18] See the marble counting tables in Corinth (second half of the 5th century BC), marked as state properties: DONATI 2010, 10-12, 20-23, cat. no. 12, fig. 15 (inscr.: *damosia [trapeza] Korinthion /Dios bouleôs,* with traces of painted columns), cat. no. 13, fig. 16 (inscr.: *strata[gion Korinthion]*). The earliest abacus in the temple of Aphaia in Aegina dates to the same period (510-500 BC); IMMERWAHR 1986. Yet, other interpretations are possible: see GERLEIGNER 2020 on the inscription *Athenaia/Aias* as a palindrome in a scene featuring Aias, Kassandra, and Athena; in Essen, the painter may have played too with pairing Athena with Aias. In other contexts, the inscription *Athenaias* on an altar in vase painting can also indicate the name of the goddess, see FRONTISI-DUCROUX 2013, fig. 5.

[19] Poll., *Onom.*, 9.87; BECQ DE FOUQUIÈRES 1869, 394-397. On the literary transmission of the expression "sacred line" and of the name of the game, see ZUCKER 2021

[20] Bronze mirror, ex. Sambon coll., ex. Museo del Teatro della Scala, Soprintendenza per l'Archeologia, le Belle Arti ed il Paesaggio della Lombardia de Milan; KÖRTE 1897,

game (*Pente grammai*).[21] Ulrich Schädler confirmed this hypothesis thanks to a black-figure kyathos in Brussels (Fig. 6; 490 BC), where the structure of the board is seen from above with five lines ending with counters on both sides.[22] On a few vases, the dice are not visible, but the numbers produced by the dice thrown by the players are inscribed in full letters.[23]

The agonistic and emotional components of the activity are highlighted by the characters' animated gestures. The game has not yet come to an end, and the issue is still uncertain: a counter is seized to be moved (Fig. 2, 3) or it is presented to the partner;[24] an open hand is raised in surprise,[25] stretched out to invite the partner to play (Fig. 2).[26] Two extended fingers (Fig. 5) belong to polysemic gestures also used in agonistic contexts (scoring, declaring forfeit...), which link the figures visually and increase the liveliness of the action. The scene disappears from the repertory of vase painting after ca. 480 BC (or a little later),[27] and it does not seem to be used in other media like tomb reliefs.[28]

The scene on the Krannon altar presents several characteristics that suggest that its meaning is different from the vase-painting series. No interactive play is performed: the bearded man is sitting, the boy is standing, and they do not face each other: the man looks at the boy, the boy looks down at the dog. The man points his finger towards the semi-circle, the boy

144-146, pl. 109; MIRABELLA ROBERTI 1976, 33, no. 176; SCHÄDLER 2009, 177-178, figs 3a et 3b.

[21] BEAZLEY 1963, no. 114, 1-7 about the amphora in Boston MFA 01.8037; *BAPD* 200007.

[22] Kyathos, Brussels, Musées royaux d'Art et d'Histoire R2512; *BAPD* 306097; SCHÄDLER 2009, 176-177, fig. 2; MASSAR 2019.

[23] E.g. lekythos, Boston, Museum of Fine Arts 95.15: 2 and 4 (*diofero- tetarafero)*; *BAPD* 303417; KOSSATZ-DEISSMAN 1981, no. 403, pl. 98. Knucklebones produce other numbers: 1-3-4-6. However, the inscriptions could also designate the number of pieces already placed on the sacred line, not the result of the throw.

[24] Hydria, New York, MMA 65.11.12; *BAPD* 275093; KOSSATZ-DEISSMAN 1981, no. 425.

[25] Amphora, Munich, Antikensammlung 1417; *BAPD* 302081; KOSSATZ-DEISSMAN 1981, no. 411, pl. 99.

[26] Amphora, Basel, Antikenmuseum und Sammlung Ludwig BS 21/328; *BAPD*7052; KOSSATZ-DEISSMAN 1981, no. 391, pl. 96.

[27] One red-figure exception: Berlin, Antikensammlung 319; *BAPD* 214735; KOSSATZ-DEISSMAN 1981, no. 420.

[28] With the possible exception of the Eretria relief, see BUCHHOLZ 1987, 133, fig. 50b; DASEN 2018, 33 (fig.).

seems to be entertained by his pet. To define the action, the relief has to be analysed in a wider cultural context, focusing on the posture of the man and of the boy, on the identification of the type of board, and the orientation of the five lines board associated with a semi-circle, also present in archaeological evidence.

2. The representation of a *technê*?

The man's posture is also found in the small series of archaic and classical Greek gravestones commemorating a category of professionals proud of mastering a *technê* mixing intellectual and practical skills, like medicine or music.[29] The activity can be characterized by an emblematic attribute alluding to a master/disciple relation with a younger person. Thus, on a gravestone from Boeotia (440 BC),[30] a standing bearded man is portrayed as a music teacher, wearing a *himation* and presenting a lyre to a naked young man standing before him. Similarly, medical practitioners are depicted with the instruments of their practice. On an early fifth century gravestone from Asia Minor,[31] a bearded medical doctor is sitting on an elegant folding stool, *diphros*, wearing fine clothes and boots, holding a long walking stick, the attribute of citizens, with two cupping devices carved between his head and that of a naked young man bringing medical instruments that allude to his training. On a late Hellenistic funerary relief from Asia Minor (Fig. 9; 1st century BC),[32] another medical doctor is sitting on a chair with a backrest, facing an altar where a priest and two individuals seem to be honouring the deceased. Different attributes and gestures display his intellectual and practical achievements: the transmission of written knowledge is suggested by the closed scroll held in his left hand and by the other scroll brought to him, while his two outstretched fingers symbolically address an audience; moreover, a set of six surgical instruments is engraved in the field before his head, alluding to his manual skills.

[29] BERGER 1970, 145-149, 155–158. Commemorated individuals can have hired professions and not belong to the social elite. Cf. KOSMOPOULOU 2001.

[30] Basel, Antikenmuseum Basel und Sammlung Ludwig BS 206; BERGER 1970, 149, fig. 160.

[31] Basel, Antikenmuseum Basel und Sammlung Ludwig BS 236; BERGER 1970, esp. 11-23.

[32] Staatliche Museen zu Berlin, Antikensammlung, Sk 804; SCHWARZMAIER 2014.

Figure 9: Marble relief (H. 67 cm, L. 83 cm, Th. 7.5 cm). Antikensammlung, Staatliche Museen zu Berlin- Preussischer Kulturbesitz Sk 804. Photo Johannes Laurentius.

On a third century tombstone, probably from northern Greece, a similar composition fixes the memory of the deceased as a scholar transmitting another type of skill, mathematics (Fig. 10).[33] On the left, the man is sitting comfortably on an elegant seat, *diphros,* with a foot stool, pointing with his index finger at a large board next to a cithara. The inscription gives his name and his profession, *Ptolemaios geometros [geometres]*, "Ptolemaios, mathematician". Below the *pinax* stands a young boy, his hands on his belly, in a pose suggesting obedience and concentration. As Alain Schärlig demonstrated, the letters on the *pinax* conform to Greek alphabetic numeracy: it represents the most ancient visual testimony of a multiplication table or so-called "Table of Pythagoras", possibly recited by the boy, as was

[33] Geneva, MAH Musée d'art et d'histoire, Ville de Genève 27937; CHAMAY–SCHÄRLIG 1998; SCHÄRLIG 2001a, 99-102, fig. 4; SCHÄRLIG 2001b.

usual in elementary education.[34] The cithara on the wall displays that music and numeracy were "sisters" *technai* to train harmony and a structured reasoning;[35] its presence may indicate that both teachings could be provided by the same person.[36]

Figure 10: Marble stela (H. 77 cm, L. 48.4 cm, Th. 10.5 cm). Geneva, MAH Musée d'art et d'histoire, Ville de Genève 27937. © Musée d'art et d'histoire, Ville de Genève, photographer Flora Bevilacqua.

[34] Cf. Arist., *Top.*, 8.14, 163b on learning by heart the multiplication table of the first 10 numbers, *kephalismoi*. On tables of additions, multiplication, including exercises of metrological conversions written for or by schoolchildren on papyri, SIDOLI 2015, 389-390.

[35] Cf. Archytas fr. B1 on four "sister" sciences, astronomy, geometry, "logistic" (arithmetic) and music; HUFFMANN 2020.

[36] The relief could evidence a change in education in the Hellenistic period, see J. Chamay in CHAMAY – SCHÄRLIG 1998.

On the Krannon gravestone, the man stretches out similarly his hand towards the attribute of his *techné*, this time a board with five horizontal lines. The board could be a game-board or a counting-board alluding to the skill of a "pebble arithmetician", as Markus Asper coined it,[37] mastering different types of calculation on the surface marked with lines, a category of teacher hardly mentioned by Greek authors, though arithmetic was an important part of elementary education.[38]

3. Game-tables ... or counting-tables?

The Krannon board depicts five lines associated with a half-circle at one end. This structure is found on a series of archaeological monuments of varying size and craftmanship, ranging from large manufactured blocks to terracotta tiles, including carved pavement or natural stones. Beside the basic five lines pattern, extended variants are composed of seven or eleven lines. They were first listed in 1965 by William K. Pritchett who identified these as boardgames for a *Five Lines* game.[39] In 2001, A. Schärlig provided a catalogue of 29 examples, all identified as abaci. The evidence is constantly expanding thanks to new finds,[40] but the distinction between game-tables or abaci is still an open debate between scholars.

The first criterion to distinguish a game-table from a counting board is the presence of numerals and monetary symbols in or near the lines or along the table showing that these were used to calculate, as on the large board found in the sanctuary of Amphiaraos in Oropos (Fig. 11).[41] As earlier studies have established, ancient Greeks had two main ways to write a number in an abbreviated way before the Indo-Arabic numeracy:[42] acrophonic, when the number is written with the first letter of its name, such

[37] ASPER 2009, 108-109.

[38] In Athens, accountants were elected by lot to keep the state accounts which shows that numeracy education was widely shared among citizens; cf. Arist., *Ath. Const.*, 48.3.

[39] PRITCHETT 1965 and 1968 (13 examples). See also BEAZLEY 1963 who identifies them as boardgames.

[40] SCHÄRLIG 2001a, 61-104, with earlier bibliography. See also KNOEPFLER 2000, 78-81. For new material, see e.g. the catalogue of IGNATIADOU 2019 (all boards are identified as *Pente grammai*).

[41] PETRAKOS 1997, no. 762, fig. 84; SCHÄRLIG 2001a, 67-68 fig. 3.3. See also Corinth: DONATI 2010, cat. no. 12-13; Delphi: MATHÉ 2009. Hosios Loukas: ROUSSET 2013. For the photograph of the abacus, see Claire Tuan: http://lespierresquiparlent.free.fr/ecrire-les-nombres.html.

[42] SCHÄRLIG 2001a, 43-60.

as Δ for *dekatê*, "ten". In this system, the same sign always represents the same value whatever its position. Thus, ΔΔΔ means 30. According to A. Schärlig, this system appeared in the late 5th century BC.

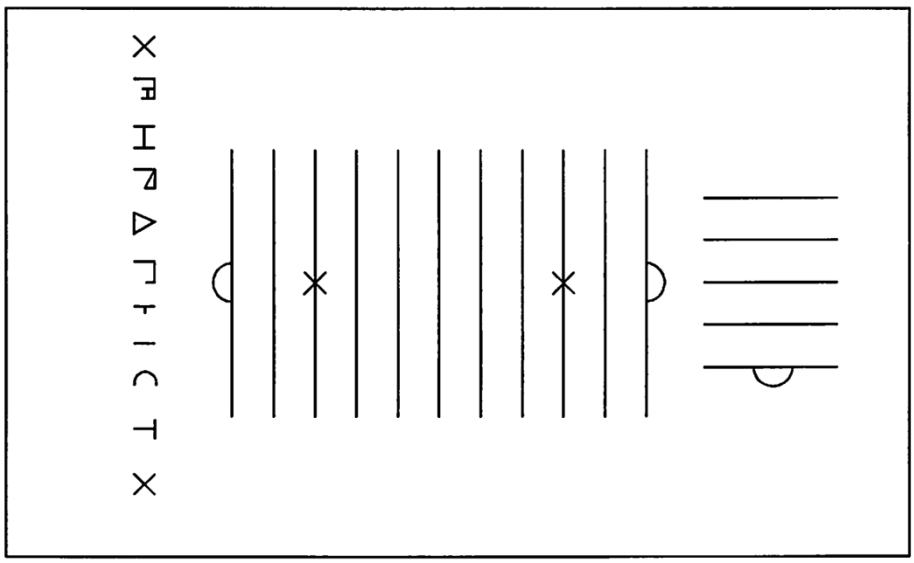


Figure 11: Marble abacus (L. 127 cm, W. 78 cm, Th. 8.5 cm, 215 kg), from Oropos, Amphiaraion. Line drawing Alain Schärlig.

The alphabetical numeracy also appears in the 5th century BC, but only becomes prevalent by the 3rd century BC. The numbers are written in capitals or minuscules using the 24 letters of the Greek alphabet, completed with three archaic letters (with varying forms), *digamma* Ϝ for 6, *koppa* Ϙ for 90, and *sampi* Ϡ for 900. The first nine letters correspond to the units, from 1 to 9 (Α = 1, Β = 2, Γ = 3, aso), the following letters to tens, from 10 to 90 (Ι = 10, Κ = 20, Λ= 30, aso) and the final ones to hundreds, from 100 to 900 (Ρ = 100, Σ = 200, Τ = 300, aso). Again, the same sign always represents the same value, whatever its position in the written number. Thus 333 can be written as ΤΛΓ or ΛΓΤ[43]

The Indo-Arabic system allows two different operations to be performed: calculating and writing a number. The two Greek numeracy systems are characterized by a hiatus: they were efficient for writing a

[43] SCHÄRLIG 2001a, 53.

number but reckoning could not be performed in a satisfactory way. Neither of these two systems allowed the four operations to be carried out efficiently.

Thus, until the Indo-Arabic system became prevalent in the modern period, writing a number and reckoning (addition, subtraction, multiplication, division) were two distinct processes. Calculation was a material operation implying the manipulation of physical objects.[44] Any small device could be used as counting unit, pebbles or little stones, shells, glass counters... a well-documented technique in Greece. The earliest testimony is found in Herodotus who states that "in writing and in reckoning with pebbles (*psephoi*) the Greeks move the hand from left to right, but the Egyptians from right to left."[45]

Finger calculation could be associated with reckoning with counters. In a fictive letter by Alciphron (2nd century AD), the farmer Agelarchides complains about loans and high mortgages: "They're an awful nuisance, these people that roam about with their counters (*psephoi*) and their bent fingers."[46]

On an Etruscan gem (Fig. 12; 350-325 BC),[47] these two operations are performed and clearly dissociated: a beardless man is sitting on a *diphros* and counting by moving with his left hand three counters on a three-legged table (*trapeza*). In his right hand he is holding a tablet on which a number is engraved, possibly a total that he is checking. From top to bottom, alphabetic numbers are carved in four rows.[48]

The monetary values carved on large abaci range from a fraction to thousands. These numbers are too high for gambling gains or players' losses. These values, the monumental size of large blocks, such as those of Oropos (Fig. 11), Salamis and Epidauros, their careful carving and their location in public trading spaces argue in favour of their reckoning function. Most of them come from sanctuaries, which played a well-documented banking role

[44] Cf. below Pl., *Leg.,* 819b-c on children training calculation with different types of objects.

[45] Hdt 2.36.4 (trans. A.D. Godley, Loeb). Pebble arithmetic is also mentioned in Aesch., *Ag.,* 570. For the collection of sources, SCHÄRLIG 2001a, 15-33.

[46] Alciphron 2.5 (Agelarchides to Pytholaos) (trans. A.R. Benner, F.H. Fobes, Loeb, modified).

[47] Paris, Cabinet des Médailles 58.1898; ZAZOFF 1968, 193, no. 1199; MENNINGER 1969, 868-869, fig. 134; AMBROSINI 2011, 73-74, no 2.2, fig. 112 a-c.

[48] Some unusual details in that image could lead to question its authenticity, especially the frame of the tablet showing possible miter cut, as Carina Weiss informs us (personal communication).

at different levels (collecting fees, taxes, money deposits, currency changes, treasury record...).[49]

Figure 12: Etruscan carnelian gem (H. 1 cm, l. 0.9 cm). Paris, Cabinet des Médailles 58.1898. Photo Serge Oboukhoff © BnF-CNRS-MSH Mondes.

[49] IMMERWAHR 1986, 199: "The find spots also, which are often sanctuaries, favour the identification as abacus". See also KNOEPFLER 2000, 78, n. 330; MATHÉ 2009. On the banking and money-changing role of sanctuaries, see e.g. BOGAERT 1968, esp. 279-304; CHANKOWSKI-SABLÉ 2014; DOYEN 2014.

When no numerical mark is visible, the size and the material more modest, the design reduced to the basic five lines, were the boards used for counting or to play *Pente grammai*?[50] A second argument proposed by scholars is the orientation of the board. Karl Menninger and others have reconstructed a standard computation system based on vertical columns as on the Salamis board, with small crosses for separating the myriads on the left and the units on the right.[51]

This vertical pattern may be suggested on an Apulian volute-krater representing Darius III and a Persian embassy (Fig. 13; 330-320 BC).[52] In the lower register, a bearded man is sitting, wrapped in a *himation* in the Greek manner, and performs a calculation on a small *trapeza*. No column is marked by lines but the pebbles are placed below a row of acrophonic

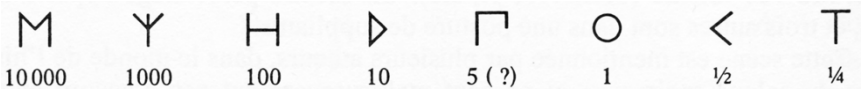
Figure 13: Red-figure italiote crater (H. 130 cm). Naples, National Museum 3253 (81947). Photo courtesy Naples National Museum.

[50] Cf. KNOEPFLER 2000, 78, about the Eretria board: "on connait des tables à 5 lignes seulement – un exemple inédit se trouve à Érétrie même – et il pourrait s'agir alors de tables de jeu".

[51] NAGL 1914; MENNINGER 1969, 857; SCHÄRLIG 2001.

[52] On this scene which is a *hapax*, see VILLANUEVA-PUIG 1998; SCHÄRLIG 2001a, 97-99.

symbols, based on a decimal system, for 10'000, 1000, 100, 10, 5 drachmae, 1 obole and specific signs for fractions of the obole.



However, the position of the counters may also reflect pictorial needs, not reckoning procedures. As on the Etruscan gem (Fig. 12), the man is stretching out his right hand to move counters while he holds in the left a tablet inscribed with TALNTA H (*sic*), 100 talents. Marie-Christine Villanueva-Puig identifies him as the treasurer of King Darius III, and the two figures in Oriental dress as Persian tribute-bearers, bringing a bag of money or gold on the right, precious plates on the left.

Henry R. Immerwahr concluded that beside the indication of numerals, "the principal difference between board game and abacus is that in the former the lines must run parallel to the side where the player is sitting, while the abacus requires vertical lines, which enable the user to calculate either on the lines or in the columns between them."[53]

A third criterion for differentiating board games from abaci is based on three puzzling particularities.[54] The first element is the small cross marking the middle lines on extended eleven lines variants (line 3, line 6, line 9) (Fig. 11). The meaning of these crosses is difficult to determine in a game context. The crosses could indicate the middle or "sacred" lines of a large *Five Lines* board,[55] but no cross is visible on identified *Five Lines* boards with dice, like miniature clay game boards (5-11 lines) (nine lines on Fig. 7) or Etruscan mirrors (seven lines on Fig. 8). Their marking could thus be a distinctive visual cue for an abacus with a possible numerical function suggested below.

The second element is the presence of small painted circles or circular depressions at the end of each line, like the marble slab from Eretria (Fig. 14).[56] These circles have no clear use for counting but they may have served to place racing counters, as depicted on an Attic skyphos (Fig. 6), on Attic miniature clay game boards (Fig. 7), and on an Etruscan mirror (Fig. 8), and they could thus characterize game-boards.

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[53] IMMERWAHR 1986, 199.

[54] Cf. IMMERWAHR 1986, 200, n. 15 for a list of "obscure features".

[55] PRITCHETT 1968, 187-215, esp. 193 pl. 4.1 (Salamis), 195 pl. 5.1 (Delos); SCHÄDLER 2009, 184.

[56] Marble (reemployed stele), Eretria Museum M 785; KNOEPFLER 2000, 78, fig. 13.

A third element is the semi-circle which is often associated with small five lines boards, as on the Eretria slab (Fig. 14).[57] It has no ludic function, as it is too small for keeping aside captured or reserve pieces. No explanation has yet been provided on its role for calculation, but the Krannon board allows us to make a hypothesis.[58]

Figure 14: Marble slab (reused stela?) (30 x 47.5 x 8 cm), from Eretria, western gate. Eretria Museum M 785. Photo Courtesy of the Swiss School of Archaeology in Greece, Andreas Skiadaressis.

[57] Salamis: SCHÄRLIG 2001a, 66, fig. 3.2. Oropos: SCHÄRLIG 2001a, 67, fig. 3.3. Goritsa/Demetrias: SCHÄRLIG 2001a, 72-73, fig. 3.7 a-b. Dhekelia: SCHÄRLIG 2001a, 75, fig. 3.11; Eretria: SCHÄRLIG 2001a, 80-81, fig. 3.18; Delphi: MATHÉ 2009.

[58] E.g. SCHÄRLIG 2001, 190: "Ces éléments constituent pour nous un mystère complet"; MATHÉ 2009, 175: "on ne s'explique pas leur utilité".

4. Reckoning with five lines and a semi-circle

Ancient authors allude to the changing value of a pebble according to its place on the counting board. According to Diogenes (3rd century AD), Solon (6th century BC) compared the sycophants to "the pebbles (*psephoi*) used in calculations; for just as each pebble stood now for more, now for less, so the tyrants would treat each of their courtiers now as great and famous, now as of no account."[59] Similarly, Polybius (2nd century BC) compares the value of Philip's courtiers to counters:

> These men are really like the pebbles (*psephoi*) on reckoning-board (*abakion*). For the pebbles, according to the will of the reckoner, have the value now of a *chalkous* [an eighth of an obol], and the next moment of a talent; while courtiers, at the nod of the king, are now happy, and the next moment lying piteously at his feet.[60]

But how was the board oriented? No text describes this, nor does archaeological evidence provide a clear indication in this regard. The Krannon relief may provide a significant new piece of information to reconstruct the Greek reckoning system: the board is placed before the man sitting with the five horizontal lines and the semi-circle on the far end. This represents a major change from the current assumption that the calculator was facing vertical columns and the evidence has a significant impact on the computation possibilities. In the system based on vertical lines, only limited operations could be performed as we explained above. Yet, using horizontal lines, the four basic reckoning operations (addition, subtraction, multiplication, division) are far easier to perform. The procedure is not explained by Greek, nor Latin authors, but diachronic comparisons provide models. In the Renaissance, abaci were similarly oriented horizontally, and the annexe demonstrates how the small Greek abacus could similarly allow addition, subtraction, multiplication and division.

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[59] Diog. Laert. 2.1.59 (trans. R.D. Hicks, Loeb).

[60] Polyb. 5.26.13 (trans. W.R. Paton, Loeb): Ὄντως γάρ εἰσιν οὗτοι παραπλήσιοι ταῖς ἐπὶ τῶν ἀβακίων ψήφοις· ἐκεῖναί τε γὰρ κατὰ τὴν τοῦ ψηφίζοντος βούλησιν ἄρτι χαλκοῦν καὶ παραυτίκα τάλαντον ἰσχύουσιν, οἵ τε περὶ τὰς αὐλὰς κατὰ τὸ τοῦ βασιλέως νεῦμα μακάριοι καὶ παρὰ πόδας ἐλεεινοὶ γίνονται.

In the 1503 textbook *Margarita Philosophica* (*Pearl of Wisdom*), Gregor Reisch illustrates the section devoted to arithmetic, music, geometry, and astronomy with a scene of contest between two calculation methods performed by historical figures (Fig. 15): on the left the philosopher Boethius, born in Pavia (5th century AD) is writing a calculation with Indo-Arabic numerals, whereas on the right Pythagoras (6th century BC) calculates with counters on an abacus with four horizontal lines as on the Krannon relief, the last one marked with a cross indicating the line of thousands in German Renaissance calculation textbooks.[61] In the background, the female personification of Arithmetic watches Boethius. Numbers are depicted on her dress, including an Indo-Arabic 4 still in the form of a loop. Similar scenes of reckoning on a board with horizontal lines are illustrated in Jakob Köbel, *Eynn Newe geordent Reche büchlein vf den linien mit Rechepfenigen* (Fig. 16; 1514) and Adam Ries, *Rechenung nach der lenge/ auff den Linihen vnd Feder/.../Mit grüntlichem vnterricht des visierens* (Fig. 17; 1550).

The main formal difference with the Krannon relief is that Renaissance boards have four lines while the Greek ones had five. This strengthens our hypothesis: in the Roman numeration and in our current oral numeration, we use four denominations based on a decimal system: the units, the tens, the hundreds and the thousands. Thus, when we go beyond nine units, we go to the tens. When we go beyond nine tens, we go to the hundreds, and similarly when we go beyond nine hundreds, we go to the thousands. But when one passes the nine thousands, there is no proper superior entity. The system is built with the previous denominations. Thus, one passes to tens of thousands and then to hundreds of thousands, and so on. This can explain the presence of four lines on the simplest line surfaces found in the Renaissance: there is a line for each denomination (units, tens, hundreds, and thousands). Ancient Greeks, however, used five denominations of reference, namely the units, the tens, the hundreds, the thousands, and the myriads, which are equivalent to ten thousand. And it is only beyond the myriads that they built superior numbers with smaller denominations.

The enigmatic function of the semi-circle can be addressed by a possible reckoning explanation, especially as it is found associated with reckoning eleven lines tables on large monumental abaci in Salamis and Oropos (Fig. 11), among others.

[61] SCHÄRLIG 2001a, 190.

Figure 15: Frontispiece of Libri IIII Arithmetice Speculative, Gregor Reisch, *Margarita Philosophica*, Freiburg, 1503.

Figure 16: Frontispiece of Jakob Köbel, *Ain new geordnet Rechenbiechlin auf den linien mit Rechen pfeningen*, Augsburg, 1514.

Figure 17: Frontispiece of Adam Ries, *Rechenung auff der linihen und federn*, Frankfurt, 1560.

We hypothesize that the half-circle was associated with a specificity of the Greek monetary system. The talent, worth 6000 drachmae, was an important reference value, but it could hamper the calculator as it interfered with a regular base-10 system. This duality is present on the abacus from Salamis where a numeracy series is engraved with T for one talent, i.e. 6000 drachmae, the other with ΠΧ for 5000 drachmae according to the decimal system. Several anecdotes show that this dual system was sometimes abused for fraud.[62] Managing stable exchange operations was crucial for money-

[62] Lys. (5th century BC), *Against Diogeiton* 32.6 reports that at the death of his brother Diodotus, Diogiton sends a dowry of five thousand drachmae, instead of one talent of 6000

changers and bankers. Bankers could thus convert the results into a common decimal system. This leads us to formulate the following hypothesis: the zone delimited by the semi-circle is an intermediate exchange zone. This closed area was used to place pebbles representing talents in a transitory manner. The Krannon relief suggests that such money-change operations were part of teaching exercises, as evidenced by later period arithmetic papyri for students.[63]

5. Which board?

5.1. *Abax, trapeza...*

Archaeological evidence is often ambiguous. The function of boards carved with five, seven or eleven lines is uncertain when no numeracy is indicated. Similarly, the use of small pieces, stones, pebbles, shells, beads is indetermined when the archaeological context is unclear. The same type of objects could be morphologically used for both counting and play. Gaming pieces from archaeological contexts were produced in various materials, like shells, as the ones on an archaic board in Copenhagen (Fig. 7), or glass as in Hellenistic Greece.

The ambiguity of the material evidence is paralleled by the polysemy of the vocabulary. Similar terms designate a flat surface used for games, money-changing, and reckoning. The commonest one is *abax* and its diminutive *abakion* (ἄβαξ, ἀβάκιον; lat. *abacus*), which is well-attested for counting as well as for gambling. In a lost Middle Comedy play by Alexis (4th century BC), a character asks a slave for an *abax* and counters, *psephoi,* for calculating costs, a request that suggests that such boards could be carried around and most likely made of wood.[64]The term *abax* also defined a type of board among the objects used by gamblers in the list of Pollux's *Onomasticon*:[65]

drachmae, which means that he used a decimal system to cheat the woman. We thank S. Dayan for this reference.

[63] See BURKHALTER 2014, 565-567, on exercising converting money into drachmae in a period of change variations (e.g. 43 talents 2000 drachmae must be converted in base 10 drachmae).

[64] Alexis, *The Man who had a Cataract* (*Apeglaucomenos)*, fr. 15 K.-A (= Ath. 117 c-e): "Bring an abacus and somecounting pebbles!".

[65] Poll., *Onom.*, 10.150. See a similar list in 7.203.

> Utensils for the gamblers: board, little board, sieve, dice, dice to shake, knucklebones, dice-bags, dice-boxes, rimmed table, pebbles, counters.
>
> Κυβευτοῦ σκεύη· ἄβαξ, ἀβάκιον, κόσκινον, κύβοι, διάσειστοι κύβοι, ἀστράγαλοι, φιμοί, κημοί, τηλία, κήθια κηθίδια, ψῆφοι, πεττοί.

Other sources mention such game boards and allude to their weight. In the *Historical Commentaries*, Carystius of Pergamum (2nd century BC) condemns the lack of self-control of King Philip II, addicted to drinking as well as to gambling. His *abax* was so light that it could be thrown under a bed:

> Carystius says in his *Historical Commentaries*: When Philip decided to get drunk, he used to say the following: "We need to start drinking; because if Antipater is sober, that's enough." On one occasion, when he was shooting dice and someone announced that Antipater had arrived, he had no idea what to do and shoved the board (*abax*) he was using to keep score under his couch.[66]

Gambling implies betting money,[67] and two distinct but related operations, game and counting, could take place successively on the same surface. As Photius records it: "Small abacus: on which they played and bet and did the calculations".[68]

The term *trapeza* (τράπεζα) designates a three-legged table also used for reckoning in money-changing operations and for a board game. Bankers, τραπεζίται, are thus described as operating on tables, τράπεζαι, at the agora,[69] and in a Lysias' plea, the trading activity of a merchant is associated

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[66] Carystius, fr. 3, FHG 4.357 = Ath. 10.435D (trans. Ch. Burton Gulic, Loeb).

[67] Poll., *Onom.*, 9.96: "Whoever won by number of units had to pick up the amount of money, bet on the table." (Ὁ δ' ὑπερβαλλόμενος τῷ πλήθει τῶν μονάδων ἔμελλεν ἀναιρήσεσθαι τὸ ἐπιδιακείμενον ἀργύριον).

[68] Phot., α 25: ἀβάκιον· ἐφ' οὗ ἐκύβευον καὶ ἐφ' οὗ τοὺς λογισμοὺς ἐποιοῦντο.

[69] Pl., *Ap.*, 17c: "in the marketplace at the bankers' tables" (ἐν ἀγορᾷ ἐπὶ τῶν τραπεζῶν). Cf. the modern etymology of banker, after "bank", "the bench".

with both a *trapeza* and an *abakion,* like the man on the Etruscan gem (Fig. 11).[70]

In Late Antiquity, τράπεζα is synonymous with τάβλα,[71] derived from the Latin *tabula,* which refers to a reckoning table as well as game-board.[72] The materiality of the object is mentioned in two epigrams. In the poem attributed to Agathias scholasticus (6th century AD), gambling is performed on a stone (marble?) board (τάβλα/τράπεζα):

> On a board. Seated by this table made of pretty stone, you will start the pleasant game of dice-rattling.
>
> εἰς τάβλαν. Ἑζόμενος μὲν τῇδε παρ' εὐλάιγγι τραπέζῃ/παίγνια κινήσεις τερπνὰ βολοκτυπίης.[73]

In another epigram, the τάβλα is made of wood and used for a game with bone pieces, perhaps a *Five Lines* game as suggested by the reference to Palamedes:

> On a board. Your bones, O Palamedes, should have been sawn up and made into instruments of the art that is derived from war. For being in the wars you did invent another war, the war of friends on a wooden field.
>
> Εἰς τὴν τάβλαν. Ὀστέα σου, Παλάμηδες, ἔδει πρισθέντα γενέσθαι/ ὄργανα τῆς τέχνης τῆς ἀπὸ τοῦ πολέμου·/ ἐν πολέμοις/ γὰρ ἐὼν ἕτερον πόλεμον κατέδειξας,/ ἐν ξυλίνῳ σταδίῳ τὸν φιλικὸν πόλεμον[74]

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[70] Lys., *For Kallaischros,* fr. 50: "with the small abacus and the small table by putting himself on sale" ('μετ' ἀβακίου δὲ καὶ τραπεζίου πωλῶν ἑαυτόν).

[71] Cf. Suda τ 7 ed. Adler: Τάβλα. ὄνομα παιδιᾶς. ταύτην ἐφεῦρε Παλαμήδης εἰς διαγωγὴν τοῦ Ἑλληνικοῦ στρατοῦ σὺν φιλοσοφίᾳ πολλῇ·

[72] *Tabula* for play, see e.g. Varro, *Ling.*, 10.22: "A board on which they play *ludus latrunculorum*"(*ut in tabula solet in qua latrunculis ludunt*). *Tabula lusoria*: Mart. 14.12 (a two-sided board for *Alea* and *Ludus Latrunculorum).*

[73] *Anth. Pal.*, 9.767 (trans. W.R. Paton, Loeb, modified).

[74] *Anth. Pal.*, 15.18 (trans. W.R. Paton, Loeb, modified).

Some terms seem to be more specific for gambling, such as τηλία. It designates a rimmed table that Pollux compares to a baker's tray, characterized by a high border, probably for keeping the dice from falling off.[75] Archaeological board games, especially for *Alea*, played with two or three dice, often have such a rim. In Roman Ephesus, the table and the board are fused on a marble table reproducing on top an *Alea* tray with handles.[76]

Apart from the Late Antiquity/Byzantine τάβλα, there is no classical Greek equivalent of the generic Latin *tabula lusoria* is not found. What seems to matter is not the material playing surface, but the structural system, because the design could be drawn on any flat surface, made of wood, terracotta, stone, as well as on the natural ground. A possible generic term is διαγραμμισμός or διαγραμμίζειν, "to play on lines", after γραμμή, "the line", and γραμμίζειν "to draw lines".[77] Specific terms refer to the grid of lines, like *Polis*, "the City-State", a structure of squares that may have referred to a town with an orthogonal plan,[78] or *hiera grammē*, "the sacred line", for the *Five Lines* game.

The commonest family of terms, however, is based on the name of the pieces, *pessoi/pettoi*, which are a type of *psephoi*.[79] *Petteia/pesseia*, "a game with pieces" designates in a generic way all games where pieces are moved. The terminology thus underscores that play is primarily an activity: for Pollux, playing boardgames means "to move counters", *pessonomein*, and the player

[75] It is also used for quails' fight, Poll., *Onom.*, 9.108: "And having drawn a circle on a board similar to a baker's tray, they set the quails in battles with one another." (Καὶ τηλίᾳ μὲν ὁμοίᾳ τῇ ἀρτοπώλιδι κύκλον ἐμπεριγράψαντες ἐνίστασαν τοὺς ὄρτυγας ἐπὶ ταῖς μάχαις ταῖς πρὸς ἀλλήλους).

[76] SCHÄDLER 2016.

[77] Phot., *Lex.*, δ 301: "*Diagrammizein*: to play on lines" (Διαγραμμίζειν· ἐπὶ γραμμαῖς παίζειν). For the sources, see COSTANZA 2019, s.v. διαγραμμισμός (Poll., *Onom.*, 9.99), and VESPA 2020, 88-89. In Philemon (4th century BC) fr. 175 K.-A, it designates a form of *Polis*: "a game of 60 black and white squares. It is also called lines" (Διαγραμμισμός· παιδιά τις ἑξήκοντα ψήφων λευκῶν καὶ μελαινῶν ἐν χώραις ἑλκομένων. Τὸ δὲ αὐτὸ καὶ γραμμὰς ἐκάλει); on the identification of *diagrammismos*, a Cross-Lines game, see NELSON 2020.

[78] On the possible reference to the Hippodamian town plan and the different types of grids (6 x 6, 7 x 8, 8 x 8, 8 x 9, 9 x 9), see SCHÄDLER 2002, 97-98. For a 6 x 7 squares *ludus latrunculorum* board, see M. Chidiroglou in this volume.

[79] On *psephos* as hyperonym including *pessoi* and other terms, see Poll., *Onom.*, 9.97, who explain that *psephoi*, pebbles, could be used as *pessoi* to play the *Five Lines* game (ἐπεὶ δὲ ψῆφοι μέν εἰσιν οἱ πεττοί, πέντε δ'ἑκάτερος τῶν παιζόντων εἶχεν ἐπὶ πέντε γραμμῶν). The contrary is not attested.

is "the one who moves counters", *petteutès*.[80] The *Alea* game similarly is designated by the throw of the die, *alea*, and *XII scripta* possibly by the highest throw, two sixes, as U. Schädler suggested quoting Nonius.[81] Similarly, to reckon is "to move pebbles", *psephizein*, an active operation with materialised numbers, *psephoi.*[82] In Latin, the equivalent is the generic *calculorum lusus* for all kinds of games played with pieces,[83] based on *calculus*, "little stone", used as a counter for calculation (it produced our modern "calculation") as well as a piece for boardgames, which could be made of different material.[84]

5.2. Psephoi *and divination*

However, moving pieces, *psephoi or pessoi*, according to rules was not limited to boardgames and abaci. This action also characterized other regulated practices performed on similar flat, structured surfaces. *Psephoi* used for voting were thus counted on an *abax*.[85] The overlap between games and divination is well-documented. In mantic procedures, specialists could perform "pebble divination", *psephôn mantikê*, a skill that Apollo gave to Hermes.[86] The name of the board, however, differs for divination, which is not practiced on an *abax*, but on a *pinax* (πίναξ), though it also involves a structure and calculation, as in casting nativities.[87] In an epigram by

[80] Poll., *Onom.*, 9.97: τὸ δὲ πεττεύειν καὶ ἡ πεττεία καὶ τὸ πεσσονομεῖν καὶ ὁ πεττευτή. Repeated *ibid.* 7.203.

[81] Non., 170.22: XII *puncta tesserarum*.

[82] According to CASEVITZ 2021, these words have no etymologies because they are borrowed, which underscores their presence among circulating practices and knowledge.

[83] E.g. Plin., *Ep.*, 7.24,5, about Ummidia Quadratilla "amusing herself with boardgames" *(laxare animum lusu calculorum).*

[84] *Calculi* are used for *ludus latrunculorum* in Ov., *Am.,* 3.358. See also Ov., Tr., 478 (made of stone); Panegyric on Piso, 193 and 196 (made of glass).

[85] On voting pebbles counted on an *abax*, see e.g. Arist., *Ath. Pol.,* 69.1.

[86] Apollod., *Bibl.*, 3.10.2: "But Hermes wished both to get the wand for the pipe and to acquire the art of divination. So he gave the pipe and learned the art of divining by pebbles" (trans. Sir J.G. Frazer, Loeb) (ὁ δὲ καὶ ταύτην λαβεῖν ἀντὶ τῆς σύριγγος ἤθελε καὶ τὴν μαντικὴν ἐπελθεῖν: καὶ δοὺς διδάσκεται τὴν διὰ τῶν ψήφων μαντικήν). Philochorus, FGrHist 328 F 195: "There may be many who cast pebbles, but few prophets" (Πολλοὶ θριοβόλοι, παῦροι δέ τε μάντιες ἄνδρες).

[87] On a mathematician casting nativities on a *pinax,* Plut., *Rom.*, 12.3: "Tarutius […], who, besides being a philosopher and a mathematician, had applied himself to the art of casting nativities on a board, in order to indulge a speculative turn of mind, and was thought to excel in it." (trans. B. Perrin, modified, Loeb) (ἦν Ταρούτιος […], φιλόσοφος μὲν ἄλλως καὶ μαθηματικός, ἁπτόμενος δὲ τῆς περὶ τὸν πίνακα μεθόδου θεωρίας ἕνεκα καὶ δοκῶν ἐν αὐτῇ περιττὸς εἶναι).

Agathias scholasticus, the farmer Calligenes consults an astrologer who throws pebbles, *psephoi*, on a *pinax* to cast a horoscope, and then counts with his fingers.[88] More contiguities are to be found between the indeterminacy that characterizes play, especially with randomizing items like dice, and divination techniques as a way of acting on chance, as Hamayon demonstrated.[89]

On a red-figure lucanian bell-krater, two satyrs wearing cloaks seem to practice divination with pebbles (Fig. 18; ca. 380–370 BC). On the left, a satyr, entirely wrapped in a *himation* is standing on a two-stepped base, holding a *thyrsus* with his left hand. He is facing another satyr, wearing a cloak drawn up to veil his head, sitting on a stool on a platform before a *trapeza*, pointing his finger at 6 aligned pieces. As on the Krannon relief, there is no interaction between the figures and a scene of play can be excluded. Arthur D. Trendall suggested that the scene may derive from a lost satyric theatre play and could parody an oracular consultation.[90] Pausanias and Pseudo Lucian allude to such divinatory processes based on the observation of the *schemata*, positions, of the knucklebones thrown directly on a table, *trapeza*.[91]

Evidence from other cultural contexts confirms that game-boards could also be used for divination. A cuneiform tablet (177-176 BC) thus records on one side the rules of the *Twenty-Squares* game, while on the other side a grid is inscribed with the twelve zodiac signs and predictions.[92]

Byzantine authors elaborate on the correspondences between the structure of the board, especially of the *Alea* boards, with three series of

[88] *Anth. Pal.*, 11.365: "Taking his counters and spreading them on a tray, and bending his fingers, he said to Calligenes [...]" (trans. W.R. Paton) (ὃς δὲ λαβὼν ψηφῖδας, ὑπὲρ πίνακός τε πυκάζων, δάκτυλά τε γνάμπτων, φθέγξατο Καλλιγένει [...]). On finger calculation, see GAVIN – SCHÄRLIG 2019.

[89] HAMAYON 2021 with earlier bibliography.

[90] TRENDALL 1987. He suggests that it is practiced on a board with a structured surface indicated by the line of pieces.

[91] Lucian, *Amores*, 16. In Bura, the oracles produced by the different combinations are inscribed on a *pinax*; Paus. 7.25.10. On this procedure well evidenced in Asia Minor epigraphy, NOLLÉ 2007.

[92] FINKEL 1995. See also MEYER 1982 and BECKER 2007 for a comparison between the Egyptian game of twenty squares and liver models. In other cultures, divinatory technique can have a mathematical dimension, as in the *sikidy* system of Madagascar performed on a structured surface; see CHEMILLIER *et alii* 2007.

squares symbolising the course of life, past, present and future.[93] In the 7th century AD, John of Antioch described the cosmic symbolism of the *Alea* game or *XII scripta* that he attributes to Palamedes: the twelve sections of the board represent the universe, with the twelve signs of the zodiac, the seven dice in the dice box correspond to the seven planets, the tower with the sky.[94]

Figure 18: Lucanian bell-krater (H. 26.3 cm). Princeton, Princeton University Art Museum y1986-33. © Photo Museum.

[93] Cf. Isid., *Etym.*, 18.64. Eustath. in Hom. *Od.* 1397, 46–47 reports that Diodorus Cronus compared the move of the *pessoi* with the revolution of stars; see KIDD 2017a, 91. On the cosmic symbolism of the game-board, SCHÄDLER 2000; on the funerary symbolism of *Alea* boards in catacomb, see U. Schädler in this volume.

[94] Fr. 41 Roberto *Excerptum Salmasianum* p. 390, 2 (= K. Müller, *FHG*, IV p. 550); repeated by the Suda τ 7 Adler: s.v. τάβλα, ὄνομα παιδιᾶς. Ταύτην ἐφεῦρε Παλαμήδης εἰς διαγωγὴν τοῦ ἑλληνικοῦ στρατοῦ σὺν φιλοσοφίᾳ πολλῇ· τάβλα γάρ ἐστιν ὁ γήινος κόσμος, ιβ' δὲ κάσοι ὁ ζωδιακὸς ἀριθμός, τὸ δὲ ψηφοβόλον καὶ τὰ ἐν αὐτῷ ζ' κοκκία τὰ ζ' ἄστρα τῶν πλανήτων, ὁ δὲ πύργος τὸ ὕψος τοῦ οὐρανοῦ· ἐξ οὗ ἀνταποδίδοται πᾶσι πολλὰ καὶ κακά. A similar comparison is made by Malalas (6th century AD), *Chron.*, 5.9 Thurn-Meier. The *Polis* game is also called *Plinthion* after πλίνθος, the brick, which may allude to the rim of the mould used for pressing bricks; it designates also the square of troops, Joseph., *AJ*, 13.4.4 and the plural πλινθία the *regiones caeli* marked by the Augurs (Plut., *Rom.* 22.1).

5.2. Skirapheia*: game-boards or abaci?*

The find spots of large blocks in sanctuaries with five or eleven lines led some scholars hypothesize that they were board-games used by pilgrims while waiting for their turn to take an oracle, and to dismiss the elements associated with reckoning practices.[95] A similar reinterpretation of monumental abaci already took place in the Roman and Byzantine period.

The function of sanctuaries as large financial centres vanished in the Roman period, but the stone public abaci remained visible, though no more in use for reckoning purposes. Lexicographs seem to have been informed that these were boardgames and associated the term *skirapheia*, "gambling house", with the temple of Athena Skiras. Suetonius thus explains that the Athenians meet in the sanctuaries (ἱεροῖς) to play, and Pollux specifies that "gambling houses are called *skirapheia*, because it was mainly in Athens that dice were thrown in the temple of Athena Skiras and in the area around this temple".[96] Erudite reconstructions also fused with another tradition, that of boards for oracular practices, like Photius: "Skiron: this is a place in Athens where fortune tellers reside; there is also the shrine of Athena Skiras; and 'Skira' the festivals."[97]

Conclusion. Greek counter culture, where is the game?

In a seminal paper published in 2002, Reviel Netz coined the notion of "Greek counter culture" advocating the development of a cognitive history of Greek numeracy, which is characterized by the manipulation of counters and part of a larger semiotic system involving several contiguous activities, such as mathematics, games, and divination. The Krannon gravestone contributes to this complex cultural history at several levels.

First, the monument is exceptional because it lifts the anonymity of a mathematics teacher belonging to little known professionals who transmitted their knowledge mainly orally. Literary sources provide very few information about practical mathematics: how was it taught, by whom, what was its place in the Greek education system? Greek authors focus on

[95] PRITCHETT, 1968, 199-200. See also e.g. WIDURA 2015, 71-81.

[96] Poll., *Onom.*, 9.96 (personal trans.): διότι μάλιστα Ἀθήνησιν ἐκύβευον ἐπὶ σκίρῳ ἐν τῷ τῆς Σκιράδος Ἀθηνᾶς νεῷ. Cf. Suet. Περὶ τῶν παρ'˝Ελλησι παιδιῶν, 1.41-48 Taillardat: "they called *skirapheia* the places to play dice" (σκιράφεια ἐκάλουν τὰ κυβευτήρια).

[97] Phot. σ 521. Σκίρον: τόπος Ἀθήνησιν, ἐφ' οὗ οἱ μάντεις ἐκαθέζοντο · καὶ Σκιράδος Ἀθηνᾶς ἱερόν · καὶ ἡ ἑορτὴ Σκιρά · οὕτω Φερεκράτης. See also Hsch. σ 891 Σκειρόμαντις. On this memorial reconstruction, see VESPA, in preparation.

"elite theoretical mathematics",[98] though pebbles were a means to acquire the abstract properties of numbers. Despite the important role of mathematics, no other representation of a teacher is known,[99] apart from the Hellenistic relief of Ptolemaios *geometres* with the table of Pythagoras (Fig. 10). The Krannon funerary altar is also significant as it shows a teacher in action, performing elementary calculations on an abacus as well as more complex exercises with the semi-circle above the five lines.

Second, the monument reminds us that arithmetic could be ludic.[100] In Plato's *Laws*, the Athenian underscores that mathematics should be taught since infancy in a pleasant way adapted to the age of the children. The training is not based on counters but on other objects providing a material dimension to calculation:

> First, as regards counting, lessons have been invented for the merest infants to learn, by way of play and fun, modes of dividing up apples and chaplets, so that the same totals are adjusted to larger and smaller groups, and modes of sorting out boxers and wrestlers, in byes and pairs, taking them alternately or consecutively, in their natural order. Moreover, by way of play, the teachers mix together bowls made of gold, bronze, silver and the like, and others distribute them, as I said, by groups of a single kind, adapting the rules of elementary arithmetic to play.[101]

[98] ASPER 2009, 107: "Recently, however, a consensus has emerged that Greek mathematics was heterogeneous and that the famous mathematicians are only the tip of an iceberg that must have consisted of several coexisting and partly overlapping." On the two forms of mathematics, practical and theoretical, see e.g. Pl., *Phlb*., 56 D 3–57 A 3. See also the overview of GAVIN – GENEQUAND 2021.

[99] NETZ 2002b; SIDOLI 2020, 190.

[100] According to NETZ 2002a, one should also consider theoretical mathematics as some form of game practiced in the upper circles of Athenian society. The 45 arithmetic riddles preserved in the *Anth. Pal.* 14, evidence the porosity between mathematical knowledge and play in the form of enigma; see KWAPISZ 2000 and GRANDOLINI 2006. More broadly on education, BETA 2020 and 2021, and in particular mathematical games and military education, MACÉ 2022. On this *longue durée* ludic tradition of problems to solve, see the early medieval period *propositiones ad acuendos juvenes*, GAVIN – GENEQUAND 2021.

[101] Pl., *Leg.*, 819b-c (trans. R.G. Bury, Loeb). See also Pl., *Resp*., 536a-537a.

Boardgames could have been part of elementary education. As R. Netz and others noted, dice throws and moving the pieces also taught numerical skills.[102] In the *Republic*, the Athenian contends that boardgames make for skilful children (*ikanoipaides*):

> no one playing with pieces (*petteutikos*), or dice (*kubeutikos*), would become sufficiently competent, if he had treated it as a mere sideline and not practiced it since childhood?[103]

The relationship between boardgames and mathematics is present in several platonic dialogues. In *Gorgias,* Socrates thus lists boardgames in the category of *technai* that structure the mind:

> Socrates: But there is another class of arts (*technai*) which achieve their whole purpose through speech and—to put it roughly—require either no action to aid them, or very little; for example, numeration, calculation, geometry, playing with pieces (*petteia*), and many other arts (*technai*).[104]

Like a new Palamedes, the child moving counters on the board invents a civilised order, made of lines, *grammai*, forming the structure of the board, letters as well as numbers.[105]

When no numerical mark is visible, the size and the material modest, the design reduced to the basic five lines, were the boards used for counting or to play *Pente grammai*? We argue that both games and reckoning could be performed on this surface with a similar horizontal orientation, both for

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[102] NETZ 2002a, 342: "hence the relative ease with which they are analysed by computer programs. At a certain abstract level, the computer, the modern player and the ancient player, all display the same set of numerical skills." See also BOUVIER 2020, 178-179.

[103] Pl., *Resp.,* 2.374c (trans. Chr. Emlyn-Jones, W. Preddy, Loeb).

[104] Pl., *Grg.,* 450d (trans. W.R.M. Lamb, Loeb): Ἕτεραι δέ γέ εἰσι τῶν τεχνῶν αἳ διὰ λόγου πᾶν περαίνουσι, καὶ ἔργου, ὡς ἔπος εἰπεῖν, ἢ οὐδενὸς προσδέονται ἢ βραχέος πάνυ, οἷον ἡ ἀριθμητικὴ καὶ λογιστικὴ καὶ γεωμετρικὴ καὶ πεττευτική γε καὶ ἄλλαι πολλαὶ τέχναι.

[105] See the demonstration of BOUVIER 2020.

training as well as leisure. It could explain Heraclitus' alliterations in the first part of his enigmatic sentence: "Time is a child who moves counters to play" (αἰὼν παῖς ἐστι παίζων, πεσσεύων). The aim of the alliteration may not be rhetorical but to clarify that the child is moving pieces on a structured board to play, *paizôn,* not for calculating.[106]

Finally, the scene displays the common fundamental cultural role of arithmetics and games, often neglected because of the oral transmission of both *technai*. The abacus and *Pente grammai* share a similar metaphoric value referring to civic and cosmic order. In the *Defense of Palamedes*, Gorgias (5th century BC) lists the inventions credited to Palamedes, the Greek civilising hero who used his mantic power to interpret divine signs and provided instruments to organise human society:

> […] I am a great benefactor for you, both for the Greeks and for all humans, not for those now alive but for those yet to come. For who would have transformed human life from resourceless to resourceful, and from disordered to ordered, by inventing military formations, fundamental for victories, written laws, the guardians of justice, writing, an instrument of memory, measures and weights, resourceful means of exchange for commerce, number, the guardian of wealth, signal fires, the strongest and swiftest messengers, and *pesseia,* a painless way of passing leisure time?
>
> τίς γὰρ ἂν ἐποίησε τὸν ἀνθρώπειον βίον πόριμον ἐξ ἀπόρου καὶ κεκοσμημένον ἐξ ἀκόσμου, τάξεις τε πολεμικὰς εὑρὼν μέγιστον εἰς πλεονεκτήματα, νόμους τε γραπτοὺς φύλακας [τε] τοῦ δικαίου, γράμματά τε μνήμης ὄργανον, μέτρα τε καὶ σταθμὰ συναλλαγῶν εὐπόρους διαλλαγάς, ἀριθμόν τε χρημάτων φύλακα, πυρσούς τε κρατίστους καὶ ταχίστους ἀγγέλους, πεσσούς τε σχολῆς ἄλυπον διατριβήν;[107]

Marco Vespa demonstrated how these discoveries form a coherent semiotic system based on σχήματα, lines forming letters or numbers, and hence producing alphabet, mathematics, measures and weights, structuring

[106] On this polysemy, DASEN 2020.
[107] Gorg. fr. B 11a, 30 DK (trans. A. Laks, G.W. Most, Loeb, modified).

the battle field, abaci as well as game boards.[108] The abacus on the Krannon relief embodies this enculturation system: the training of numerical competences is associated with learning the alphabet as a numeration system, while playing the *petteia* game introduces young players to social rules and to self-control, such as *agôn* and fair play.[109]

Petteia games and abaci are of course cognitively distinct. The operation performed on the abacus only depends on the skill of the reckoner, while the results of games played with dice, like *Pente grammai,* are determined by aleatory throws beside the strategy of the players.[110]

Another difference between *petteia* and abaci is that a positive value has never been denied to mathematics, while games could be denounced as a moral and civic threat when turned into gambling for money. Alcidamas' speech *Odysseus Against the Treachery of Palamedes*, thus accuses Palamedes of having invented money as well as *kubeia,* gambling, synonymous of excessive behaviour and waste:

> And *kuboi*, in turn, he introduced as the greatest vice: pains and penalties for the losers, mockery and reproach for the winners. For the gains from *kuboi* are useless gains, and most are wasted immediately.
>
> καὶ κύβους αὖ μέγιστον κακὸν κατέδειξε, τοῖς μὲν ἡττηθεῖσι λύπας καὶ ζημίας, τοῖς δὲ νενικηκόσι καταγέλωτα καὶ ὄνειδος. τὰ γὰρ ἀπὸ τῶν κύβων προσγιγνόμενα ἀνόνητα γίγνεται, τὰ δὲ πλεῖστα καταναλίσκεται παραχρῆμα.[111]

The lively attitude of the boy beside the teacher is also significant: it is a visual code qualifying the expected young age of the teacher's student, and

[108] For a detailed analysis of the literary sources, from Gorgias to Philostratus, see VESPA 2020a.

[109] On these civic values, cf. HAMAYON 2015, 2021; DASEN 2015 and 2021.

[110] On possible reckoning and board game on the same block, see the slab with 6 lines and a numerical series beside two (*Polis*?) grids in Rhamnous, cf. SCHÄRLIG 2001a, 70-71, fig. 3.6.

[111] *Odysseus, Against the Treachery of Palamedes*, 27 (trans. KIDD 2017a, 129). On the social disorder generated by *kubeia*, gambling in a generic sense, see KIDD 2017b and VESPA 2020b. On the pre-monetary value of games, WENDLING 2015.

suggesting an enjoyable context. The meaning of the *melitê* dog on the relief completes an educational program. The animal participates actively to the visual construction of an ideal instruction. In ancient Greece, the joyful and affectionate relationship between this animal companion and young individuals was synonymous with introducing youths to another important range of social rules, such as those associated with taming as well as hunting.[112] On the five lines board, calculation training may have ended playfully with *Pente grammai*.[113]

Appendix: How to use the five-lines abacus from Krannon *by Jérôme Gavin*

This appendix aims to demonstrate how the four basic arithmetical operations could be performed on the Krannon abacus. If there is no Greek or Roman text explaining how to use an abacus, there is far more evidence in the Renaissance. In his monumental work, Barnard[114] elaborates a classification of different counting methods with counters based on the analysis of 120 treatises (including different editions). This classification shows that if the mathematical background underwent little variation, the abacus' shape could vary greatly. Focusing only on the methods using a surface with lines, we note that in some methods the counters must only be placed on the lines. In others they must only be placed between the lines. But in the most common processes, the counters can be placed on or between the lines. To finish this quick overview in an exhaustive way, Barnard kept aside a fourth category for more complex processes.

By comparing these different techniques with the specificities of our five-line calculation surface and by using clues gathered throughout the ancient Greek world, we have developed a user's guide, according to the principle of calculation simplicity. Indeed, between two hypothetical calculation methods, one must, in the absence of concrete decisive elements, choose the simplest.

[112] VESPA 2019 and VESPA 2022, in press. See also SEIGLE 2018. In Rome, cf. BRADLEY 1998.

[113] Cf. the similar conclusion of G. Bakewell in this volume on the alternance of games and logic, dialectic, and mathematic exercises in Plato's Academy.

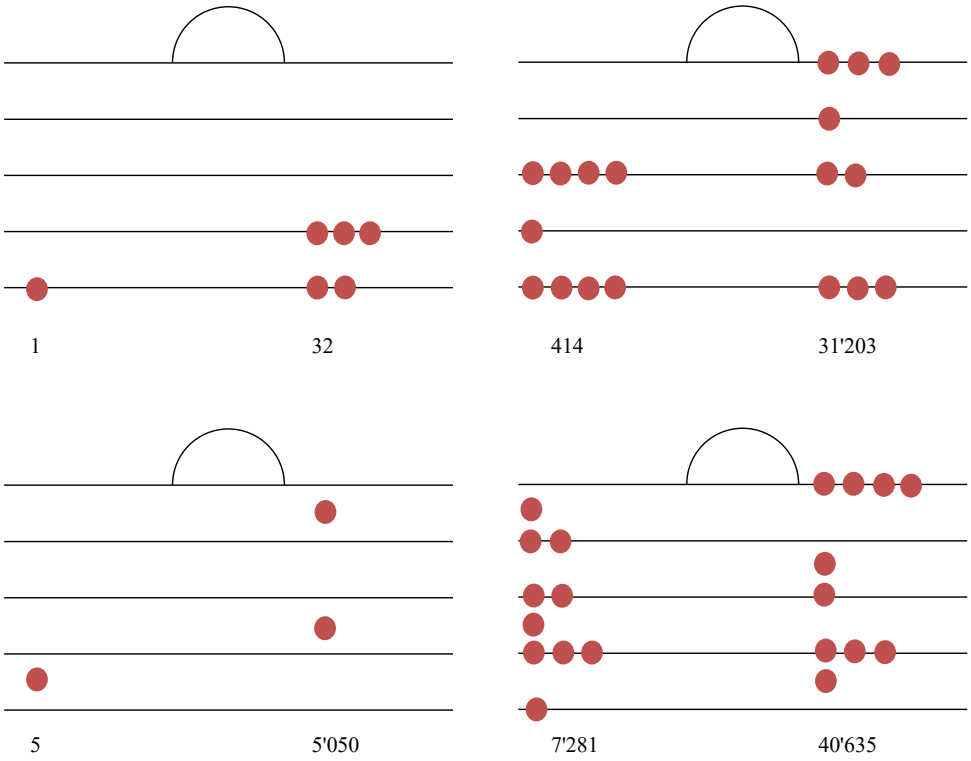
[114] BARNARD 1916, 254.

Representing a number

To represent a number on the abacus, we base ourselves on the five lines. Each line represents the five main denominations of the Greek numeration, namely 1, 10, 100, 1000 and 10'000. Each pebble placed on one of these lines represents one time the value of the line.

To avoid cluttering the abacus with too many pebbles on each row, we set as a rule that a pebble placed between two lines indicates five times the value of the line below. Thanks to this, we can represent any number on the abacus in a simplified or reduced way.

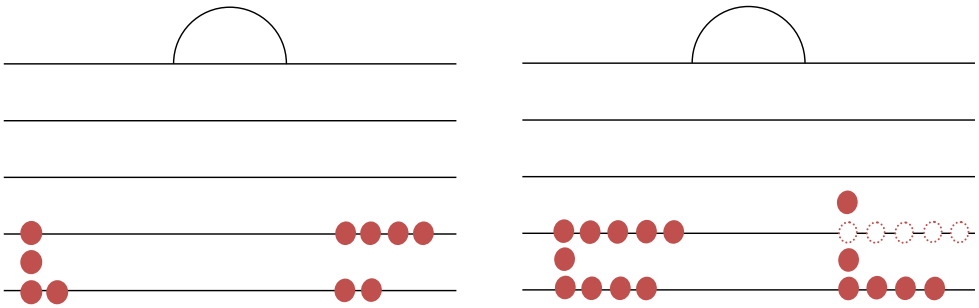
Showing a number on the abacus is thus intuitive and only requires a basic training.



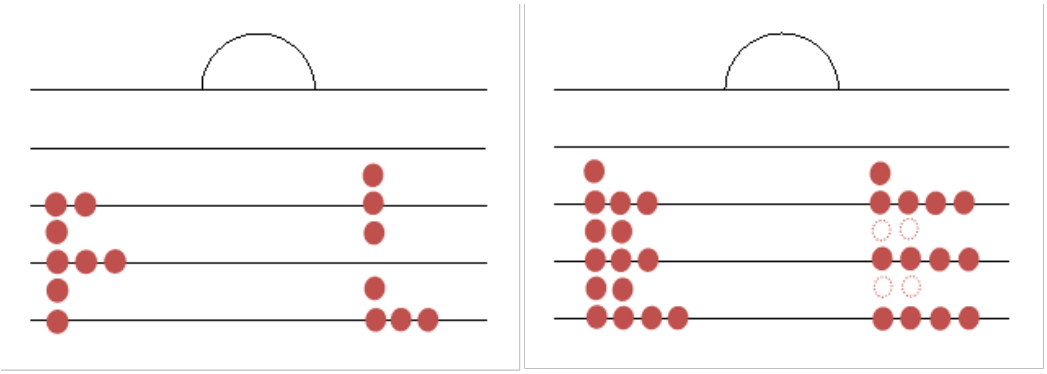
Addition

To add 17 and 42, we start by plotting the two numbers on either side of the abacus. To find the sum, simply slide the pebbles together. If necessary, reduce the numbers to get a smaller result.

On the left of the first abacus, we have the number 17 and on the right 42. On the left of the second abacus, we have the result of the addition (59) in non-reduced form and on the right in reduced form.



To add 286 and 658, we proceed in the same way.



On the left side of the first abacus, we have the number 286 and on the right 658.

On the left of the second chart, we have the result of the addition (944) in unreduced form and on the right in the simplified form.

The simplicity and efficiency of the process explains why it took centuries for written calculation to become the standard in the West. One also becomes aware of the physical aspect of the calculation. As R. Netz summed it up: "We imagine numbers as an entity seen on the page; the

Greeks imagined them as an entity grasped between the thumb and the finger".[115]

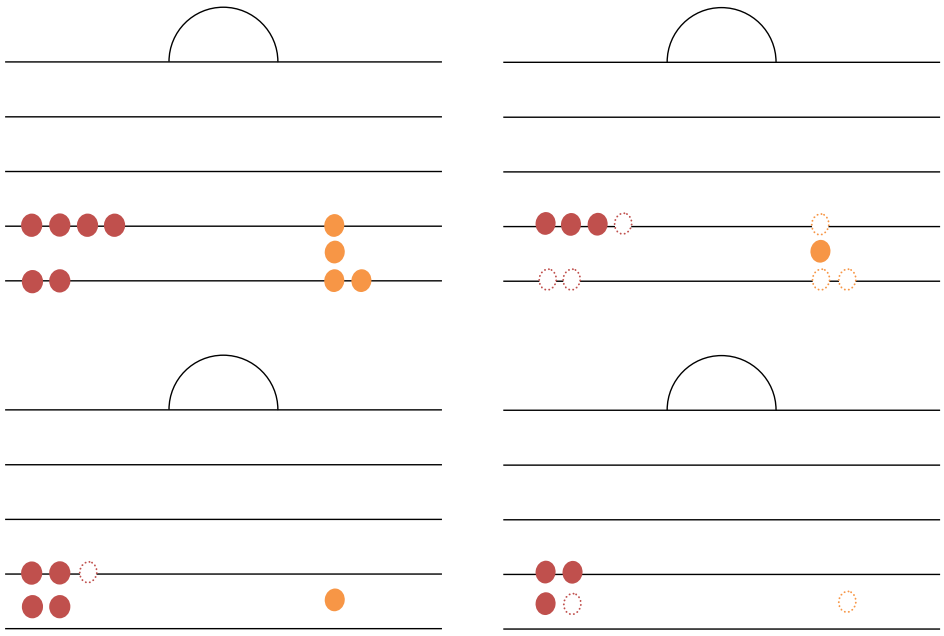
In order to understand ancient calculation, one must put aside our own reality which consists in using the same system to record numbers and perform calculations. Numerical calculation was done with pebbles, while recording the result was done differently.

Subtraction

To subtract 17 from 42, we start by plotting the two numbers on either side of the abacus. To find the difference, we must remove the smaller number from the larger one. In the abacus system, we must remove from the larger number, the pebbles corresponding to the smaller number. To do this, we start by removing simultaneously the pebbles that have the same values.

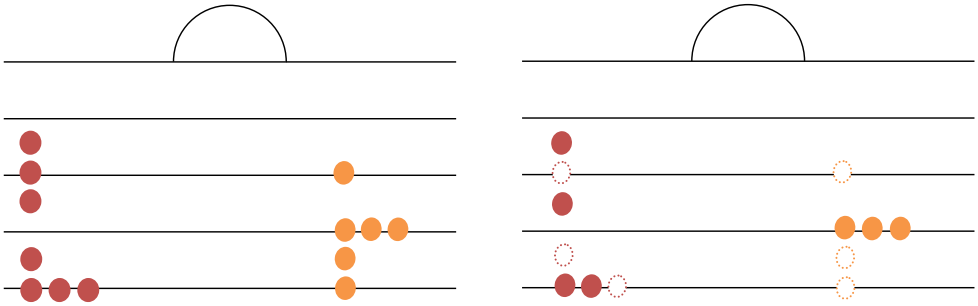
All that is left to do is to remove the last pebble that has no equivalent for the moment. To do this, we transform a pebble of a higher value into pebbles of a lower value. Here, we transform 10 in two steps.

The result is 25.

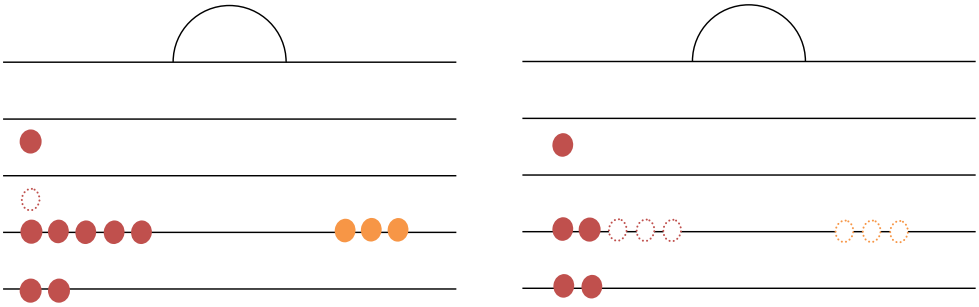


[115] NETZ 2002a, 329.

To subtract 136 from 658, we proceed in a similar way by first removing simultaneously the pebbles which have the same value.



We then remove the three pebbles that have no equivalent for the moment. In order to do this, we transform a pebble of a higher value into pebbles of a lower value. Here, we transform 50 into five times 10.



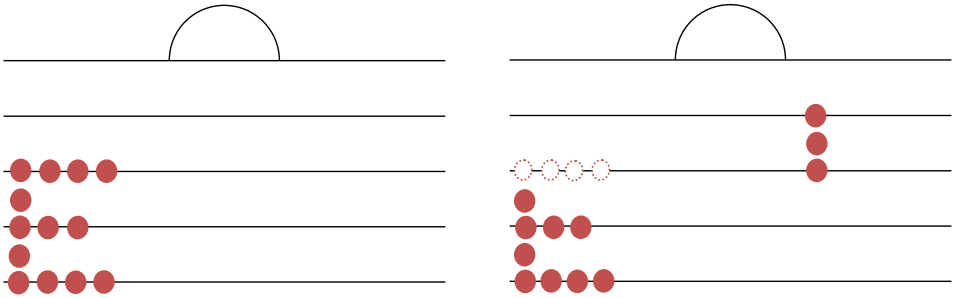
The result is 522.

Multiplication

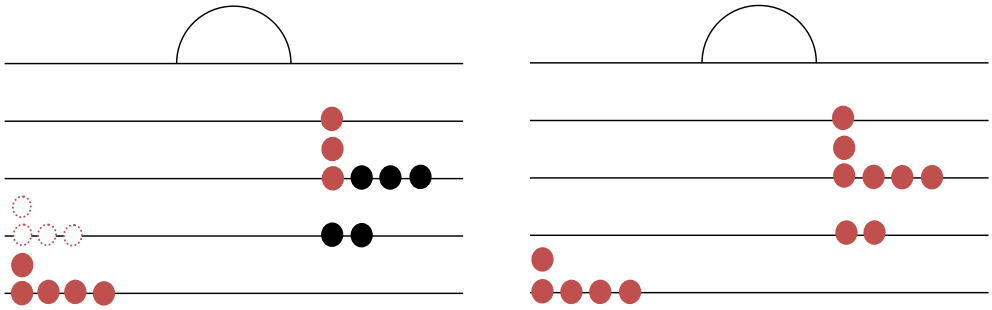
To multiply 489 by 4, first place 489 on the abacus.

Then we multiply the 4 hundreds by 4, which equals 16 hundreds that we place on the right side of the abacus.

Once the hundreds have been multiplied, they can be removed from the left side of the abacus.

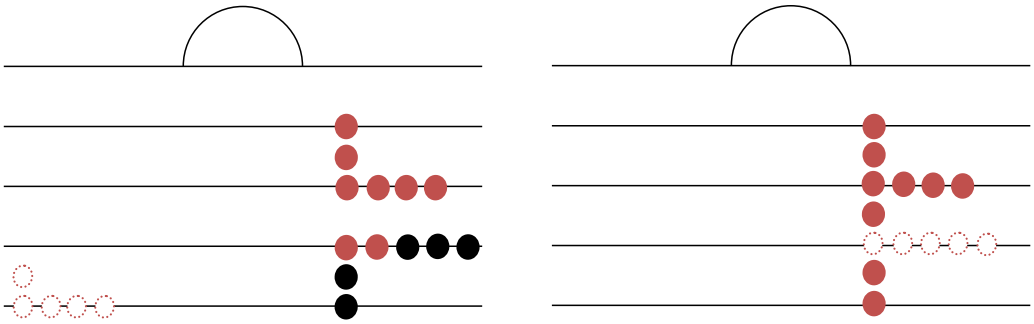


Then we multiply the 8 tens by 4, which equals 32 tens (represented in black) that we add to the right. Now that the tens have been multiplied, they can be removed from the left part of the abacus. Reductions are made if necessary.



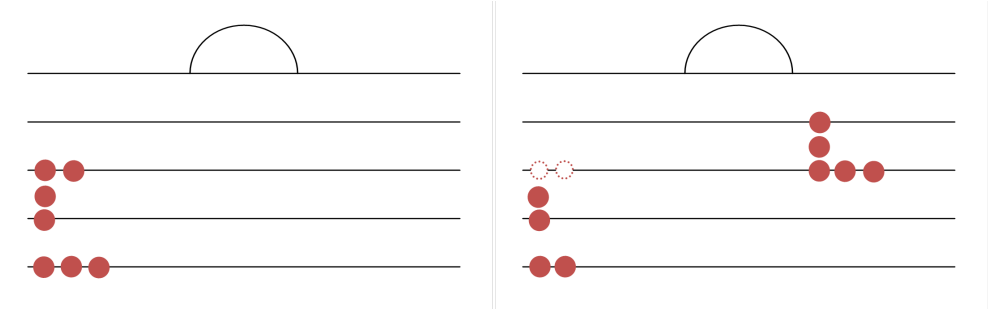
The 9 units are then multiplied by 4, giving 36 units (shown in black) which are added to the right. Once the units are multiplied, they can be removed from the left side of the chart.

Reductions are made if necessary and the result is read off the right side.



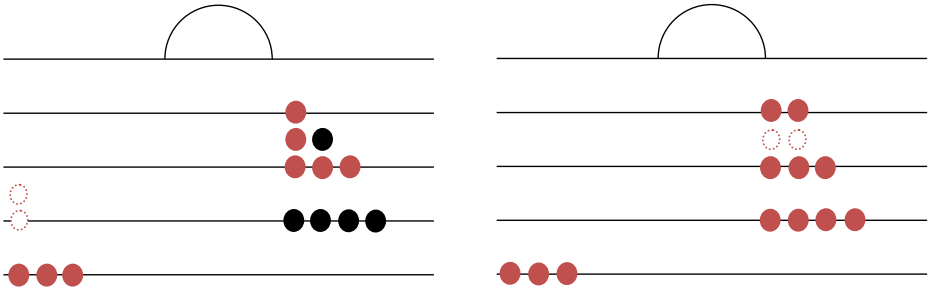
The result is 1956.

To multiply 263 by 9, we place 263 on the abacus. Then we multiply the 2 hundreds by 9, which equals 18 hundreds that we place on the right side of the abacus. Once the hundreds are multiplied, they can be removed from the left side of the abacus.



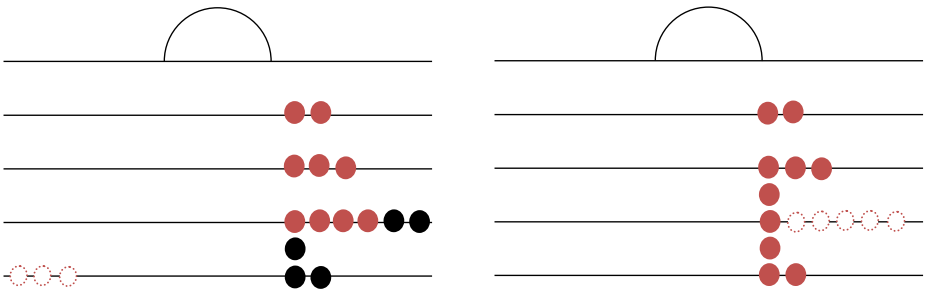
We then multiply the 6 tens by 9, which equals 54 tens (represented in black) that we add to the right. Once the tens are multiplied, we can remove them from the left part of the abacus.

Reductions are made if necessary.



The 3 units are then multiplied by 9, for a total of 27 units (shown in black) that are added to the right. Once the units are multiplied, they can be removed from the left side of the chart.

Reductions are made if necessary and the result is read off the right side.



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The result is 2367.

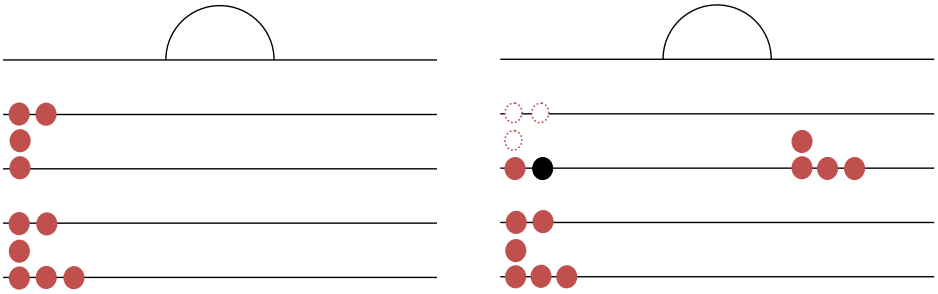
This procedure is based on the knowledge gathered from the multiplication table attested on the Geneva relief (fig. 10) and in the works of Aristotle. It also implements a kind of long-lasting distributivity since it is also found in Eutocius of Ascalon (5th century AD).[116]

In the case of a multiplication by 70, one multiplies by 7 first, which is easy, then by 10, which is even easier. By extension in the case of a multiplication by 72, one multiplies by 7, by 10 and then by 2.

Division

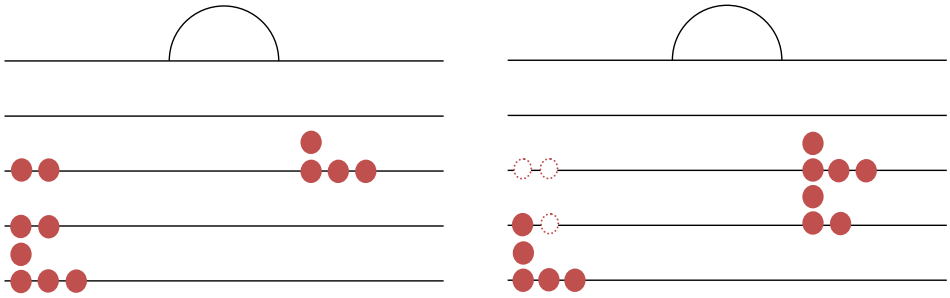
To divide 2628 by 3, we start by placing 2628 on the abacus. The division of 2 thousands by 3 is not possible, so we go to the 26 hundreds.

26 hundreds divided by 3 equals 8 hundreds with a remainder of 2 hundreds.



We then move to the tens.

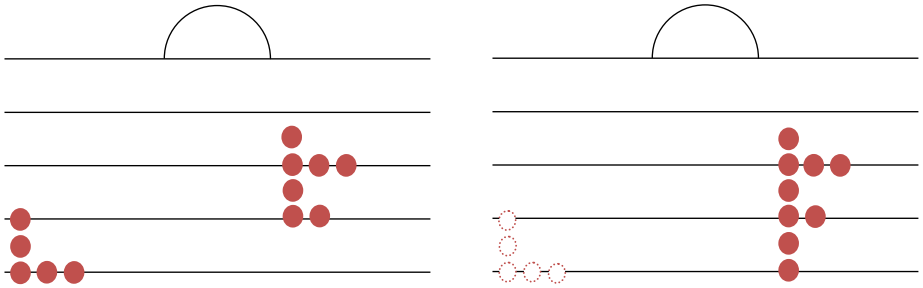
22 tens divided by 3 makes 7 tens with a remainder of 1 ten.



[116] More about this system in GAVIN – SCHÄRLIG 2018.

Lastly, we go to the units (18).

18 units divided by 3 makes 6 units without remainder.

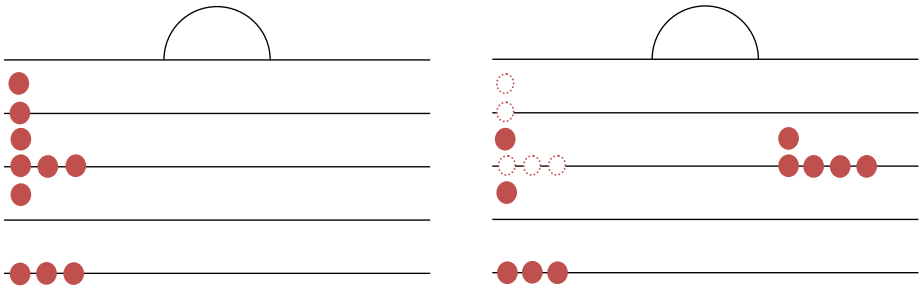


The result is 876.

To divide 6853 by 7, we start by putting 6853 on the abacus.

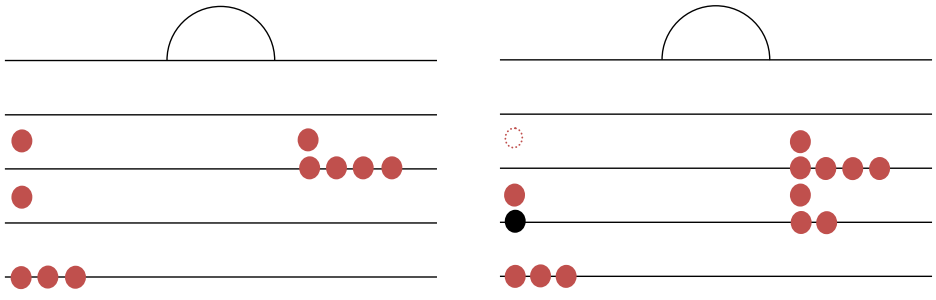
The division of 6 thousands by 7 is not possible, so we move to the hundreds (68).

68 hundreds divided by 7 equals 9 hundreds with a remainder of 5 hundreds.



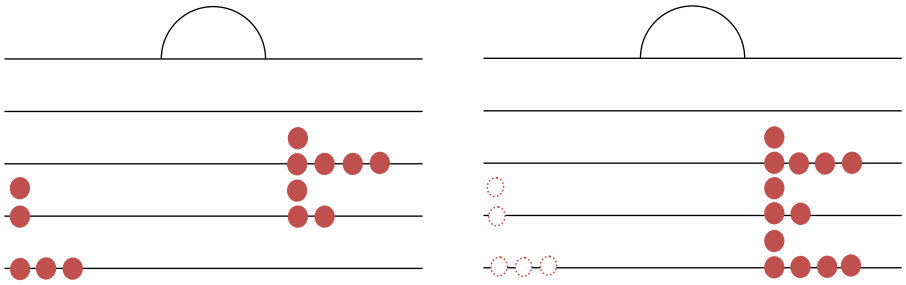
Then we go to the tens (55).

55 tens divided by 7 equals 7 tens with a remainder of 6 tens.



Then we go to the units (63).

63 units divided by 7 equals 9 units without a remainder.



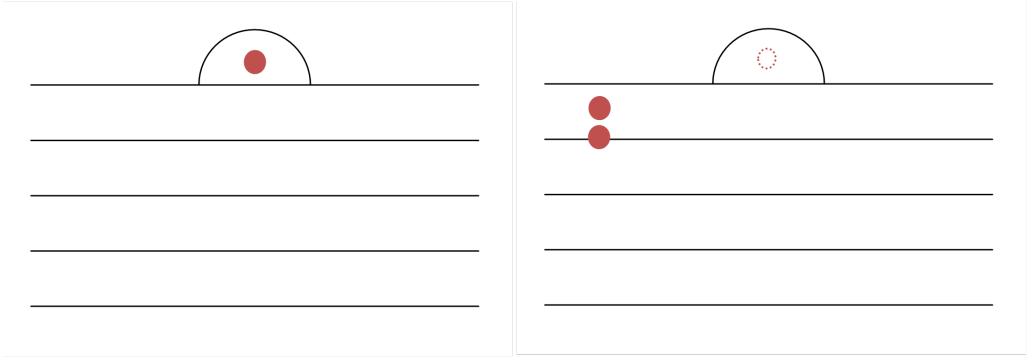
The result is 979.

This process is very efficient with a smaller divisor than 10, but it is much more laborious when the divisor is bigger.

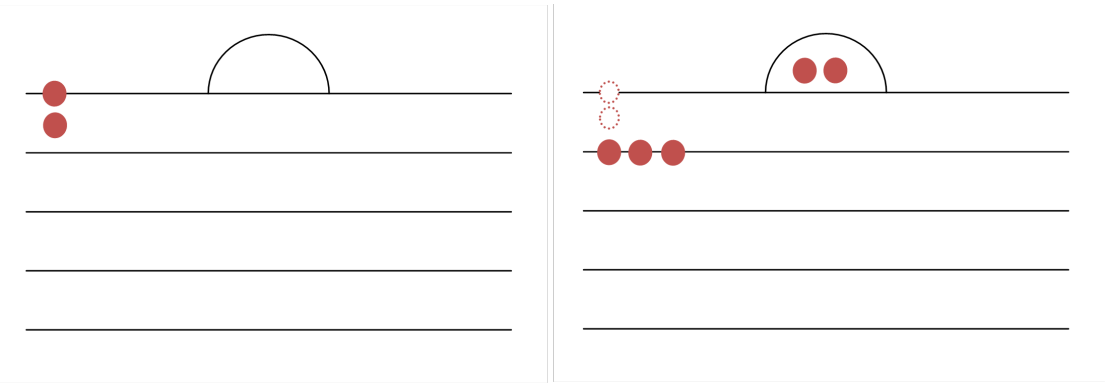
Using the semi-circle

As explained in the article, our hypothesis concerning the semi-circle is that it constituted an intermediate exchange zone in order to manage in a simple way the singularity represented by the talent.

Thus, a counter placed inside the semi-circle represented a talent before being transformed into drachmae.



We see that the talent placed in the intermediate zone is easily transformed visually into 6000 drachmae.



Here we see how 15'000 drachmae become 2 talents and 3000 drachmae.

Thus, with five simple lines and a few counters, one can perform all the elementary operations of arithmetic in an efficient way. Such simplicity to support human resourcefulness: what a great lesson for mankind.

Abbreviations

BAPD Beazley Archive Pottery Database https://www.beazley.ox.ac.uk/carc/pottery

CAT Clairmont Ch. W. 1993. *Classical Attic Tombstones*, Kilchberg.

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