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A PHYSICALLY INSPIRED MODEL FOR ESTIMATING CLEAR-SKY SOLAR IRRADIANCE

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Article Info	Abstract
Received: 18.06.2024 *Accepted: 22.07.2024* **Keywords:** Solar irradiance, Empirical modeling, Clear sky	The estimation of solar irradiance under clear sky conditions is still a research topic. Many times, accessible empirical models are applied in practice, even if more uncertainty is expected than in the case of complex models. This paper reports a class of innovative empirical models for estimating clear sky global solar irradiance. Although the models are of an empirical nature, the mathematical equations are inspired by the physics behind the radiative transfer in the atmosphere. The models are developed with high-quality data recorded on the Solar Platform of the West University of Timisoara, Romania. Considering the local character of the empirical models, the development process is described in detail so that it can be replicated in other locations. Having only deterministic quantities as input, the proposed models harmoniously combine the two key requirements for practical applications: accessibility and accuracy.

1. Introduction

The estimation of solar resource is carried out in two stages. Firstly, the solar irradiation under clear sky conditions is computed by integrating a solar irradiance model [1]. Secondly, the result is adjusted to the actual sky conditions by applying an Ångström-Prescott type equation [2]. As a result, the estimation of clear-sky solar irradiance becomes a focal prerequisite in the evaluation of solar resources for PV projects.

The simple models for estimating solar irradiance under clear sky can be divided into two distinct classes. The first class includes empirical models, usually built by fitting radiometric data collected in a single location. These models are the simplest and most accessible. Empirical models have a limited applicability due to the encapsulation of the local

atmosphere particularities into the coefficients. The second class includes parametric models, which admit one or more atmospheric parameters at the input. Consequently, the area of applicability is extended. There are many clear-sky solar irradiance models developed over the time around the globe. We direct the reader to two review articles [3-4]. These papers discuss from various perspectives models for estimating solar irradiance under clear sky conditions belonging to both classes.

This paper reports innovative empirical models for estimating global solar irradiance under clear sky conditions. Despite their empirical nature, the mathematical equations are inspired by the physics behind the atmospheric transmittance parameterization. Considering only deterministic physical quantities as input, the models aim for balance between accessibility and accuracy, two key requirements in practice. The prosed models form a new class (or group), which is further denoted G-EM, i.e. global-empirical models.

The paper is structured as follows: Section 2 briefly presents the parametric equations for the estimation of solar irradiance under clear sky, discusses the relevant measured data and underlines some ab-initio limitations in the empirical modeling of solar irradiance. The proposed empirical models for clear sky global solar irradiance are introduced and assessed in Sec. 3. The main conclusions are gathered in Sec. 4.

2. Data and Methods

2.1. Atmospheric transmittance

Considering the processes of absorption and scattering of solar radiation in the atmosphere as being independent, simple equations for the atmospheric transmittance can be deduced in a self-consistent and mathematically elegant framework [5]. Each atmospheric constituent is associated with a subunit attenuation factor, which depends on the atmospheric optical mass and the amount of the constituent in the atmosphere. In one of the simplest mathematical expressions (e.g. the PS model [6]), the direct-normal G_{dn}, diffuse G_d and global G components of solar irradiance are expressed as follows:

$$G_{dn}(m,j,z) = G_{SC}\varepsilon_j \overline{\tau}_{O3}\overline{\tau}_w\overline{\tau}_g\overline{\tau}_R\overline{\tau}_a \tag{1}$$

$$G_d(m,j,z) = \frac{1}{2}G_{SC}\varepsilon_j \overline{\tau}_{O3}\overline{\tau}_w\overline{\tau}_g\left(1-\overline{\tau}_R\overline{\tau}_a\right)\cos\left(\theta_z\right) \tag{2}$$

$$G(m,j,z) = G_{dn}\cos\left(\theta_z\right) + G_d \tag{3}$$

where G_{sc} = 1369 W/m^2 defines the solar constant in the PS model [6], ε_j denotes the daily correction of the solar constant according to the Sun-Earth distance, j denotes the Julian day and θ_z denotes the zenithal angle. The subunit factors $\overline{\tau}_i$ represent the specific atmospheric transmittances associated to the following processes: ozone absorption (i = O3), water vapor absorption ($i = w$), trace gases absorption ($i = g$), Rayleigh scattering (i = R) and aerosol scattering ($i = a$). Equations for $\overline{\tau}_i$ are derived and discussed in Ref. [6].

Equations (1-3) were written in such a way that only the dependence on deterministic parameters (atmospheric optical mass m, Julian day j and site altitude z) is highlighted. The equations (1-3), with $\overline{\tau}_i$ from the PS clear-sky solar irradiance model, represent the source of inspiration for the G-EM class of models.

2.2. Database

The empirical G-EM models were developed with high-quality radiometric data recorded on the Solar Platform of the West University of Timisoara, Romania [7]. The atmospheric parameters, such as the Ångström turbidity coefficient β, Ångström exponent α and water vapor column content w, were retrieved from AERONET station Timisoara [8]. The complete post-processed dataset (388 entries), with quality control checked and the perfectly clear sky condition manually verified, is available online as Supplementary Material attached to Ref. [1]. The dataset was randomly split into two subsets of the same dimension. The first subset, further denoted F-dataset, was used to fit the equations while the second subset, further denoted T-dataset, was used to test the resulted models. This is a current practice in solar radiation modeling, e.g. [9].

To evaluate the effect of the site altitude in the empirical modeling of solar irradiance, supplementary radiometric data were collected from two mountain stations: Izana, Spain and Tamanrasset, Algeria. The two stations are registered in the Baseline Surface Radiation Network (BSRN) [10] with the indicatives IZA and TAM. The IZA station is located on the top of the mountain, at an altitude of 2372.9 m. The climate is temperate with dry and warm summers. The TAM station is located in the desert at an altitude of 1385 m. The climate is arid and hot. Each of the two stations is collocated with an AERONET station, from where atmospheric parameters were collected. Only data verified in clear sky conditions were retrieved. The test dataset from IZA contains 837 entries, while the test dataset from IZA contains 414 entries.

2.3. Ab-initio limitations

Solar irradiance components at the ground level are primarily conditioned by deterministic parameters (m, j and z) and atmospheric parameters (e.g. β, α, w). Figure 1 displays the direct-normal $G_{dn,m}$, diffuse $G_{d,m}$ and global G_m components of solar irradiance with respect to the cosine of zenithal angle. The additional subscript index m indicates a measured value. Visual inspection shows that global solar irradiance G_m can be well expressed in terms of $\cos(\theta_z)$. The geometric locus of $G_{dn,m}[\cos(\theta_z)]$ is substantially dispersed by the atmospheric parameters variability, especially by the fluctuation in the atmospheric aerosol loading (indicated by β). Thus, deterministic modeling of $G_{dn,m}$ is expected to be accompanied by large errors. When estimating G, the impact of the variability in atmospheric aerosol loading is reduced by the compensatory summation in Eq. (3). For example, when β increases, G_{dn} decreases due to the intensification of the scattering process. But the same intensification of the scattering process will simultaneously lead to an increase in G_d.

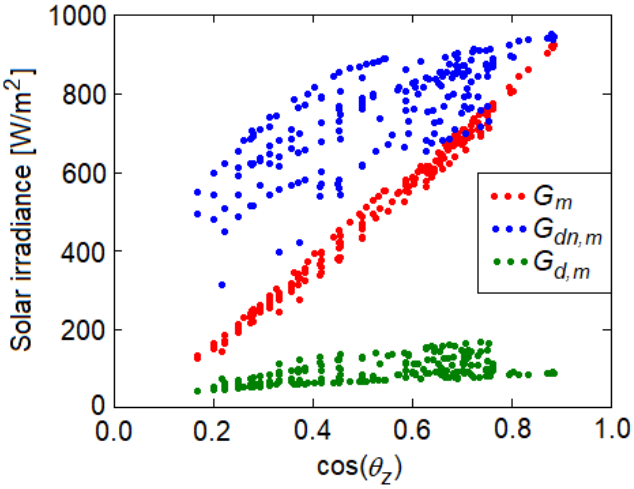


Figure 1. Direct-normal $G_{dn,m}$, diffuse $G_{d,m}$ and global G_m solar irradiances in the F-dataset

3. Results and discussions

This section introduces the empirical models proposed for estimating the global horizontal solar irradiance under clear sky and explains their relationship with the source of inspiration, namely the PS parametric model. The quality of equation fit on data is measured by the determination coefficient R^2. In the second part, the results of the performance evaluation of the models are presented and analyzed. The models' performance is evaluated in terms of normalized mean bias error (*nMBE*) and normalized root mean square error (*nRMSE*). The three statistical indicators (R^2, *nMBE* and *nRMSE*) are defined in Appendix A.

3.1. G-EM: an innovative class of empirical models

The following empirical models for the global solar irradiance have been fitted on the F-dataset:

$$G_1(m) = a_1 \left(\cos\theta_z\right)^{b_1} \tag{4}$$

$$G_2(m, j) = G_{SC}\varepsilon_j a_2 \left(\cos\theta_z\right)^{b_2} \tag{5}$$

$$G_3(m, j, z) = \frac{1}{2} a_3 G_{SC}\varepsilon_j \overline{\tau}_{O3}(m \cdot l_{O3,p})\overline{\tau}_g(m)\left[1 + \overline{\tau}_a(m \cdot \beta_p)\overline{\tau}_R(m, z)\right]\left(\cos\theta_z\right)^{b_3} \tag{6}$$

with

$$\overline{\tau}_g\left(m\right) = \exp\left[-m \cdot \left(-5.4 \cdot 10^{-5} - 3.8 \cdot 10^{-6} m + 0.0099 \cdot m^{-0.62}\right)\right] \tag{7a}$$

$$\overline{\tau}_R(m, z) = \exp\left[-m(z) \cdot \left(0.709 + 0.0013 \cdot m(z) - 0.5856 \cdot m(z)^{0.058}\right)\right] \tag{7b}$$

$$\overline{\tau}_{O3}(m \cdot l_{O3}) = \exp\left[-m \cdot l_{O3} \cdot \left(0.0184\text{-}0.0004 m \cdot l_{O3} + 0.022\left(m \cdot l_{O3}\right)^{0.66}\right)\right] \tag{7c}$$

$$\overline{\tau}_a(m \cdot \beta) = \exp\left[-m \cdot \beta \cdot \left(1.053\text{-}0.083 m \cdot \beta + 0.3345\left(m \cdot \beta\right)^{0.668}\right)\right] \tag{7d}$$

$\overline{\tau}_g\left(m\right)$, $\overline{\tau}_{O3}(m \cdot l_{O3})$, $\overline{\tau}_a(m \cdot \beta)$ and $\overline{\tau}_R(m, z)$ express the specific atmospheric transmittances in the PS model [6], associated to trace gases absorption, ozone absorption, aerosols scattering and Rayleigh scattering, respectively. The fitted coefficients and the determination coefficient R^2 are: a_1 = 1054.4, b_1 = 18545, R^2 = 0.991 for G_1, a_2 = 0.78986, b_2 =1.19472, R^2 = 0.993 for G_2 and a_3 = 1.62362, b_3 = 1.16804, R^2 = 0.993 for G3. The equations (4) and (5) were also fitted on the direct-normal solar irradiance from F-dataset, achieving very modest determination coefficients R^2 = 0.595 and R^2 = 0.625, respectively.

In the following, some explanations regarding the G_1, G_2 and G_3 models are presented. Replacing Eq. (1) and Eq. (2) in Eq. (3), we get:

$$G(m, j, z) = G_{SC}\varepsilon_j \cdot \frac{1}{2}\overline{\tau}_{O3}\overline{\tau}_w\overline{\tau}_g\left(1 + \overline{\tau}_R\overline{\tau}_a\right) \cdot \cos\left(\theta_z\right) \tag{9}$$

Equation (9) emphasizes the three different parts of a parametric model for global horizontal irradiance: the adjusted solar constant $G_{SC}\varepsilon_j$, the atmospheric transmittance

$\frac{1}{2}\overline{\tau}_{O3}\overline{\tau}_{w}\overline{\tau}_{g}\left(1+\overline{\tau}_{R}\overline{\tau}_{a}\right)$ and the cosine law. Comparing G_1 with G (Eq. 9) it can be seen that the fit coefficients encapsulate an average value of the solar irradiance at ground and the average characteristics of the atmospheric transmittance. Such models were developed by Long and Ackerman [11]. G_2 includes explicitly the adjusted solar constant, the fit coefficients encapsulating only the average characteristic of the atmospheric transmittance. G_3 has a higher degree of complexity, explicitly including the specific atmospheric transmittances associated to the trace gases absorption and Rayleigh scattering, which depend only on the atmospheric optical mass. For the Rayleigh scattering, altitude dependence is taken into account by means of the atmospheric optical mass. The specific atmospheric transmittances associated to ozone absorption and aerosol scattering are considered with the prescriptions for the ozone content in the atmosphere $l_{O3,p}$ = 0.35 cm·atm and atmospheric turbidity β_p = 0.05. $l_{O3,p}$ and β_p represent local average values.

Equations (4 - 6) define the three proposed models for estimating the clear-sky solar irradiance . Equations of similar form constitute the class G-EM.

3.2. Performance assessment

The performance of the G-EM models was assessed against data from T-dataset. The main results of testing the models are collected in Figure 2. This figure shows the solar irradiance components G_{dn} and G estimated with the G_1 (Eq. 4), G_2 (Eq. 5) and PS models. G_3 is not represented graphically in Fig.2, because its performance is similar to that of G_2. What should differentiate it is the site altitude, a fact that cannot be tested in the parental location. Visual inspection shows that G_1 and G_2 estimate with a high accuracy the global solar irradiance in the parental location. Remarkably, the accuracy is substantially better than in the case of the PS model. However, this failure of PS must be understood in the context that PS was developed based on a spectral code and the blackbody radiation law, without any connection with the testing site.

If we look again at Figure 2, we notice that both models, G_1 and G_2, fail to estimate the direct-normal component G_{dn}. Instead, the PS model, based on the input information (ozone column content, water vapor column content and the Ångström turbidity coefficient), estimates G_{dn} much better.

Without any input information regarding the content of the atmosphere, the empirical models will give increasing errors as the atmosphere moves away from that “average” captured by the empirical coefficients. Figure 3 shows an analysis of the errors depending on the

Ångström turbidity coefficient β, the Ångström exponent α and the water vapor column content w. The estimation error increases with the increase in the atmospheric aerosol loading and aerosol absorptivity. The minimum errors occur in moments when the Ångström turbidity coefficient is close to the average values. The increase in air humidity contributes to the increase in the estimation errors.

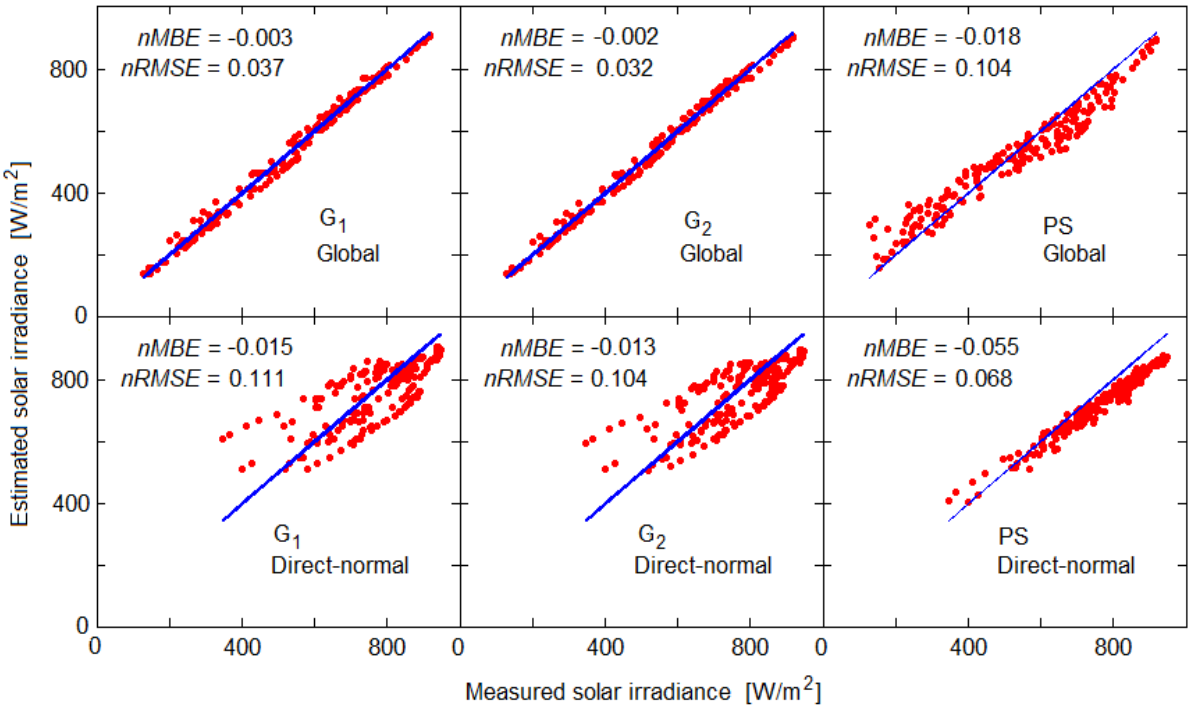


Figure 2. Estimated vs. measured clear sky solar irradiance components in Timisoara

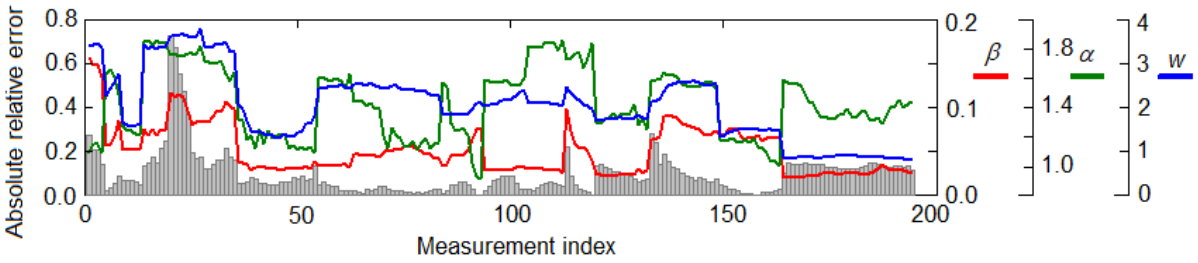


Figure 3. Absolute relative error at the estimation of direct-normal solar irradiance with G_2 for each entry in the test dataset. The Ångström turbidity coefficient β, Ångström exponent α and the water vapor column content w, are superimposed

Table 1. Performance of models M2 (Eq, 5), M3 (Eq. 6) and PS at the stations Izana (IZA) and Tamanrasset (TAM). The models' performance is evaluated in terms of normalized mean bias error (nMBE) and normalized root mean square error (nRMSE).

Station	Model	*nMBE*	*nRMSE*
IZA	M2	-0.148	0.155
	M3	0.004	0.071
	PS	-0.069	0.075
TAM	M2	-0.091	0.108
	M3	0.082	0.120
	PS	-0.090	0.121

To evaluate the influence of altitude on the estimates of the empirical models, M2, M3 and PS were run at the two mountain stations, IZA and TAM, described in Sec. 2.2. The results are presented in Table 1. At the IZA station, at over 2300 meters altitude, the M3 estimates register double accuracy compared to M2. In addition to M2, the M3 equation captures more of the physics of the radiative transfer in the atmosphere and Rayleigh scattering explicitly depends on altitude. Thus, the improvement of M3 performance over M2 can be partially attributed to the consideration of altitude as an input. Surprisingly, M3 and PS have similar performances at the IZA station. At the TAM station, at roughly 1300 meters altitude, M2, M3 and PS register similar performances, significantly lower than at IZA. This behavior has two causes. The TAM station is 1 km lower than the IZA station. As a result, the altitude effect in the M3 estimates is minor and no longer differentiates clearly M3 from M2. In another perspective, TAM is a desert station. In the dataset against which the models were tested, the Ångström turbidity coefficient varies from $\beta = 0.003$ to $\beta = 0.4$ (an atmosphere strongly loaded with desert dust). The instances when the atmosphere is strongly loaded with desert dust induce significant errors in the estimation (see Fig. 3 and the comments around). These errors are much smaller at the IZA station, where the atmosphere is permanently clean, lightly loaded with aerosols ($0.002 < \beta < 0.048$).

4. Conclusions

The study highlights how a parametric model for solar irradiance can be reduced to an empirical class. The study is developed based on the very simple PS model, that is used for illustration. The use of more advanced models could lead to the development of better global solar irradiance estimators with only deterministic physical quantities as input.

New simple and accurate empirical equations for estimating the global solar irradiance under clear sky conditions in the Banat Plain have been proposed. Tests against measured data

show that an empirical model with deterministic inputs (Julian day, atmospheric optical mass and site altitude) can estimate global solar irradiance with the same accuracy as a parametric model.

At a general level, global horizontal is the sole solar irradiance component that can be accurately estimated under clear sky based on empirical models. The direct-normal and diffuse components strongly depend on the atmospheric parameters (especially the aerosol loading of the atmosphere), which induces a significant uncertainty in the estimates. Existing models for the empirical estimation of the diffuse solar irradiance must be applied with great caution.

Appendix

A. Statistical indicators of accuracy

A measure of how well an empirical model fits a set of measured data is the coefficient of determination R^2:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(c_i - m_i)^2}{\sum_{i=1}^{N}(m_i - \mu)^2} \tag{A1}$$

c and m refer to the computed and measured values, respectively, while N denotes the number of elements in the dataset. $\mu = \frac{1}{N}\sum_{i=1}^{N} m_i$ denotes the arithmetic mean of the measured data. The coefficient of determination R^2 represents the fraction of variability in a dataset that is accounted for by the empirical model.

Mean bias error (*MBE*) and root mean square error (*RMSE*) are two statistical indicators frequently used for measuring the performance in solar energy modeling:

$$MBE = \frac{1}{N}\sum_{i=1}^{N}(c_i - m_i) \tag{A2}$$

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(c_i - m_i)^2\right]^{1/2} \tag{A3}$$

MBE is a measure of systematic errors (or bias) while *RMSE* is a measure of random errors.

In order to compare the performance of a model applied to samples of different magnitudes of the elements, the indicators are used normalized to the mean of the measured data:

$$nMBE = MBE/\mu \quad \text{(A4)}$$

$$nRMSE = RMSE/\mu \quad \text{(A5)}$$

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