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Signless Laplacian energy of a first KCD matrix

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Abstract. The concept of first KCD signless Laplacian energy is initiated in this article. Moreover, we determine first KCD signless Laplacian spectrum and first KCD signless Laplacian energy for some class of graphs and their complement.

1 Introduction

In this article, a connected, simple and undirected graph G having n vertices and m edges is considered. The degree of vertex v_i is represented as d_i. The complement $\overline{G}$ of G has two vertices adjacent if they are not adjacent in G [12]. The regular graph G has degree r. For undefined terminologies refer [12].

The idea of energy of a graph G was outlined by Gutman [11]. It defines energy of G as the sum of absolute eigenvalues of G. Tremendous work on this concept is available in the literature [11, 16]. Recently various graph-energy-like quantities: Laplacian [10], signless Laplacian [1, 8], distance [13] etc. are studied. Further, two non-isomorphic graphs having same spectra are cospectral otherwise noncospectral. Various energies are available in the literature

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defined using the degree of a vertex. The vertex degree and edge degree together makes the concept more fascinating and opens new areas of research. With this urge a new matrix using vertex and edge degree together was defined by Mirajkar et al. in [14].

The first Karnatak College Dharwad matrix i.e., first KCD matrix [14] $KCD_1(G)$ is defined as

$$kcd_{1_{ij}} = \begin{cases} (d_i + d_j) + d_{ij} & \text{if } v_i \text{ is adjacent to } v_j \ , \\ 0 & \text{otherwise,} \end{cases}$$

with d_i and d_j representing vertex degrees v_i and v_j respectively, the edge degree is $d_{ij} = d_i + d_j - 2$.

$\beta_1 \geqslant \beta_2 \geqslant ... \geqslant \beta_n$ represents the first KCD eigenvalues of G. The first KCD energy $E_{KCD_1}(G)$ [14] is

$$E_{KCD_1}(G) = \sum_{i=1}^{n} |\beta_i|. \tag{1}$$

If $D(G)$ and $KCD_1(G)$ are the diagonal matrix and first KCD matrix respectively, then for a (n, m) graph G, the first Karnatak College Dharwad Laplacian matrix [15] $L_{KCD_1}(G)$ is

$$L_{KCD_1}(G) = D(G) - KCD_1(G).$$

It has first KCD Laplacian eigenvalues $\eta_1 \geqslant \eta_2 \geqslant ... \geqslant \eta_n$ in the non-increasing order. The corresponding first Karnatak College Dharwad Laplacian energy [15] $LE_{KCD_1}(G)$ is

$$LE_{KCD_1}(G) = \sum_{i=1}^{n} \left| \eta_i - \frac{2m}{n} \right|, \tag{2}$$

where $\eta_i, i = 1, 2, ..., n$ represents the first KCD Laplacian eigenvalues of G.

If $D(G)$ and $A(G)$ are the diagonal matrix and adjacency matrix respectively, then the signless Laplacian matrix [1] of G is

$$SL(G) = D(G) + A(G).$$

It has $\mu_1^+ \geqslant \mu_2^+ \geqslant ... \geqslant \mu_n^+$ as the signless Laplacian eigenvalues of G.
The signless Lapalcian energy [1] of G is

$$SLE(G) = \sum_{i=1}^{n} \left| \mu_i^+ - \frac{2m}{n} \right|.$$

Analogous to energy of a graph G the concept of $SLE(G)$ also has numerous chemical applications. For some of the spectral properties of $SL(G)$ refer [1, 3, 5, 6, 7]. Further more, the concept of $SL(G)$and $SLE(G)$ has motivated us to develop the idea of the first KCD signless Laplacian matrix and first KCD signless Laplacian energy of G.

With this motivation, in the second section, we introduce the first KCD signless Laplacian matrix and first KCD signless Laplacian energy of G, followed by general basic results on them. Further, in third section we determine first KCD signless Laplacian spectrum and first KCD signless Laplacian energy for some class of graphs and their complements.

2 First *KCD* signless Laplacian energy

Definition 1 *If* $D(G)$ *and* $KCD_1(G)$ *are the diagonal matrix and first* KCD *matrix respectively, then for a* (n, m) *graph* G*, the first Karnatak College Dharwad signless Laplacian matrix* $SL_{KCD_1}(G)$ *is defined as*

$$SL_{KCD_1}(G) = D(G) + KCD_1(G).$$

It has first KCD *signless Laplacian eigenvalues* $\eta_1^+ \geqslant \eta_2^+ \geqslant ... \geqslant \eta_n^+$ *in the non-increasing order, where* η_1^+ *and* η_n^+ *are the highest and lowest first* KCD *signless Laplacian eigenvalues of* G *respectively.*

The corresponding first Karnatak College Dharwad signless Laplacian energy $SLE_{KCD_1}(G)$ *is defined as*

$$SLE_{KCD_1}(G) = \sum_{i=1}^{n} \left| \eta_i^+ - \frac{2m}{n} \right|, \tag{3}$$

where $\eta_i^+, i = 1, 2, ..., n$ *are the first* KCD *signless Laplacian eigenvalues of* G*.*

Lemma 2 *If* G *is a regular graph, then*

$$SLE_{KCD_1}(G) = E_{KCD_1}(G). \tag{4}$$

Proof. The regular graph G satisfies

$$\eta_i^+ - \frac{2m}{n} = \beta_i, \quad i = 1, 2, ..., n \tag{5}$$

Making an appeal to Eq. (5) in (3), we get

$$SLE_{KCD_1}(G) = \sum_{i=1}^{n} |\beta_i| = E_{KCD_1}(G).$$

□

Corollary 3 *If* G *is a regular graph, then*

$$SLE_{KCD_1}(G) = LE_{KCD_1}(G) = E_{KCD_1}(G).$$

Proof. From Eq. (4), we have

$$SLE_{KCD_1}(G) = E_{KCD_1}(G). \tag{6}$$

Further, from [15], we consider

$$LE_{KCD_1}(G) = E_{KCD_1}(G) \tag{7}$$

Finally, from Eqs. (6) and (7), we obtain the required result.

□

3 Estimation of first *KCD* signless Laplacian energy for some class of graphs and their complements

This section considers first $SLE_{KCD_1}(G)$ for few class of graphs and respective complements.
If $\eta_1^+ \geqslant \eta_2^+ \geqslant ... \geqslant \eta_k^+$ are the distinct first KCD signless Laplacian eigenvalues of G having the multiplicities $t_1, t_2, ..., t_k$ then, the first KCD signless Laplacian spectrum of G is denoted as

$$SL_{Spec_{KCD_1}}(G) = \begin{pmatrix} \eta_1^+ & \eta_2^+ & ... & \eta_k^+ \\ t_1 & t_2 & ... & t_k \end{pmatrix},$$

where $t_1 + t_2 + ... + t_k = n$.

Further calculations are based on the following

Lemma 4 [13] If

$$M = \begin{pmatrix} M_0 & M_1 \\ M_1 & M_0 \end{pmatrix}$$

represents a symmetric matrix with block partition, then the eigenvalues of M are the eigenvalues of the matrices $M_0 + M_1$ and $M_0 - M_1$.

Theorem 5 *If* $K_{\frac{n}{2},\frac{n}{2}}$ *has* $|V(K_{\frac{n}{2},\frac{n}{2}})| = n = \{2l; l = 3, 4, ...\}$, *then*

$$SL_{Spec_{KCD_1}}\left(K_{\frac{n}{2},\frac{n}{2}}\right) = \begin{pmatrix} \frac{n}{2} & \left(\frac{3-2n}{2}\right)n & \left(\frac{2n-1}{2}\right)n \\ n-2 & 1 & 1 \end{pmatrix}$$

and

$$SLE_{KCD_1}\left(K_{\frac{n}{2},\frac{n}{2}}\right) = 2n^2 - 2n.$$

Proof. For $K_{\frac{n}{2},\frac{n}{2}}$ of order n, we have

$$SL_{KCD_1}\left(K_{\frac{n}{2},\frac{n}{2}}\right) = \begin{pmatrix} \frac{n}{2}I_{\frac{n}{2}} & (2n-2)J_{\frac{n}{2}} \\ (2n-2)J_{\frac{n}{2}} & \frac{n}{2}I_{\frac{n}{2}} \end{pmatrix}.$$

Here $I_{\frac{n}{2}}$ is the identity matrix having order $\frac{n}{2}$ and $J_{\frac{n}{2}}$ is the $\frac{n}{2} \times \frac{n}{2}$ matrix with all entries as 1.
Further, above matrix can be expressed as

$$SL_{KCD_1}\left(K_{\frac{n}{2},\frac{n}{2}}\right) = \begin{pmatrix} M_0 & M_1 \\ M_1 & M_0 \end{pmatrix},$$

where $M_0 = \frac{n}{2}I_{\frac{n}{2}}$ and $M_1 = (2n-2)J_{\frac{n}{2}}$.
Now,

$$M_0 + M_1 = \frac{n}{2}I_{\frac{n}{2}} + (2n-2)J_{\frac{n}{2}}$$

and

$$M_0 - M_1 = \frac{n}{2}I_{\frac{n}{2}} - (2n-2)J_{\frac{n}{2}}.$$

By using ***Lemma 4***, the spectrum of $SL_{KCD_1}\left(K_{\frac{n}{2},\frac{n}{2}}\right)$ is

$$SL_{Spec_{KCD_1}}\left(K_{\frac{n}{2},\frac{n}{2}}\right) = \begin{pmatrix} \frac{n}{2} & \left(\frac{3-2n}{2}\right)n & \left(\frac{2n-1}{2}\right)n \\ n-2 & 1 & 1 \end{pmatrix}.$$

Further, employing the Eq. (3), we derive

$$\begin{aligned} SLE_{KCD_1}\left(K_{\frac{n}{2},\frac{n}{2}}\right) &= \sum_{i=1}^{n}\left|\eta_i^+ - \frac{2m}{n}\right| \\ &= \sum_{i=1}^{n}\left|\eta_i^+ - \frac{n}{2}\right| \\ &= (n-2)\left|\frac{n}{2}-\frac{n}{2}\right| + \left|\left(\frac{3-2n}{2}\right)n - \frac{n}{2}\right| \\ &\quad + \left|\left(\frac{2n-1}{2}\right)n - \frac{n}{2}\right| \\ &= 2n^2 - 2n. \end{aligned}$$

□

Definition 6 [9] The crown graph $CR(c)$ for an integer $c \geqslant 3$ represents the graph with vertex set $\{u_1, u_2, ..., u_c, v_1, v_2, ..., v_c\}$ and edge set $\{u_i v_j, 1 \leqslant i, j \leqslant c, i \neq j\}$.

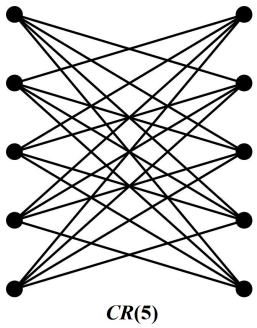


Figure 1: Crown graph

Theorem 7 *For* $CR(c)$ *with an integer* $c \geqslant 3$,

$$SL_{Spec_{KCD_1}}(CR(c)) = \begin{pmatrix} 5-3c & 5c-7 & 11c-7-4c^2 & 4c^2-9c+5 \\ c-1 & c-1 & 1 & 1 \end{pmatrix}$$

and

$$SLE_{KCD_1}(CR(c)) = 8(2c^2-5c+3).$$

Proof. For $CR(c)$ of order $n = 2c$, we have

$$SL_{KCD_1}(CR(c)) = \begin{pmatrix} (c-1)I_c & (4c-6)(J_c - I_c) \\ (4c-6)(J_c - I_c) & (c-1)I_c \end{pmatrix}.$$

Here I_c is the identity matrix having order c and J_c is the square matrix having order c with all entries equal to 1.
Further, above matrix can be expressed as

$$SL_{KCD_1}(CR(c)) = \begin{pmatrix} M_0 & M_1 \\ M_1 & M_0 \end{pmatrix},$$

where $M_0 = (c-1)I_c$ and $M_1 = (4c-6)(J_c - I_c)$.
Now,

$$M_0 + M_1 = (5-3c)I_c + (4c-6)J_c$$

and

$$M_0 - M_1 = (5c-7)I_c - (4c-6)J_c.$$

By using ***Lemma* 4**, the spectrum of $SL_{KCD_1}(CR(c))$ is

$$SL_{Spec_{KCD_1}}(CR(c)) = \begin{pmatrix} 5-3c & 5c-7 & 11c-7-4c^2 & 4c^2-9c+5 \\ c-1 & c-1 & 1 & 1 \end{pmatrix}.$$

Further, employing the Eq. (3), we prove

$$\begin{aligned} SLE_{KCD_1}(CR(c)) &= \sum_{i=1}^{n=2c} \left|\eta_i^+ - (c-1)\right| \\ &= 8(2c^2 - 5c + 3). \end{aligned}$$

□

Definition 8 [2, 4] The cocktail party graph $CP(a)$ (for $a \geqslant 3$) is the graph consisting of two rows of paired vertices in which all vertices but the paired ones are connected with an edge.

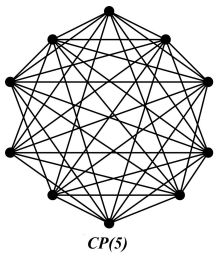


Figure 2: Cocktail party graph

Theorem 9 *For* $CP(a)$ *with an integer* $a \geqslant 3$,

$$SL_{Spec_{KCD_1}}(CP(a)) = \begin{pmatrix} 2a-2 & 18-14a & 16a^2-34a+18 \\ a & a-1 & 1 \end{pmatrix}$$

and

$$SLE_{KCD_1}(CP(a)) = 8(a-1)(4a-5).$$

Proof. For $CP(a)$ of order $n = 2a$, we have

$$SL_{KCD_1}(CP(a)) = \begin{pmatrix} (8a-10)(J_a-I_a)+(2a-2)I_a & (8a-10)(J_a-U_a) \\ (8a-10)(J_a-U_a) & (8a-10)(J_a-I_a)+(2a-2)I_a \end{pmatrix}.$$

Here I_a is the identity matrix having order a, J_a is the square matrix having order a with all entries equal to 1 and U_a is a square matrix with all entries as zeros, except the entries on the anti-diagonal being 1.
Further, above matrix can be expressed as

$$SL_{KCD_1}(CP(a)) = \begin{pmatrix} M_0 & M_1 \\ M_1 & M_0 \end{pmatrix},$$

where $M_0 = (8a-10)(J_a-I_a)+(2a-2)I_a$ and $M_1 = (8a-10)(J_a-U_a)$.
Now,

$$M_0 + M_1 = 2(8a-10)J_a - (6a-8)I_a - (8a-10)U_a$$

and

$$M_0 - M_1 = (8a-10)U_a - (6a-8)I_a.$$

By using ***Lemma 4***, the spectrum of $SL_{KCD_1}(CP(a))$ is

$$SL_{Spec_{KCD_1}}(CP(a)) = \begin{pmatrix} 2a-2 & 18-14a & 16a^2-34a+18 \\ a & a-1 & 1 \end{pmatrix}.$$

Further, employing the Eq. (3), we prove

$$\begin{aligned} SLE_{KCD_1}(CP(a)) &= \sum_{i=1}^{n=2a} \left|\eta_i^+ - 2(a-1)\right| \\ &= 8(a-1)(4a-5). \end{aligned}$$

□

Theorem 10 *If* $\overline{K_{\frac{n}{2},\frac{n}{2}}}$ *has* $|V(\overline{K_{\frac{n}{2},\frac{n}{2}}})| = n = \{2l; l = 3, 4, ...\}$, *then*

$$SL_{Spec_{KCD_1}}\left(\overline{K_{\frac{n}{2},\frac{n}{2}}}\right) = \begin{pmatrix} 5-\frac{3n}{2} & \frac{2n^2-9n+10}{2} \\ n-2 & 2 \end{pmatrix}$$

and

$$SLE_{KCD_1}\left(\overline{K_{\frac{n}{2},\frac{n}{2}}}\right) = 4(n-2)(n-3).$$

Proof. For $\overline{K_{\frac{n}{2},\frac{n}{2}}}$ of order n, we have

$$SL_{KCD_1}\left(\overline{K_{\frac{n}{2},\frac{n}{2}}}\right) = \begin{pmatrix} (2n-6)J_{\frac{n}{2}} - \left(\frac{3n}{2}-5\right)I_{\frac{n}{2}} & O_{\frac{n}{2}} \\ O_{\frac{n}{2}} & (2n-6)J_{\frac{n}{2}} - \left(\frac{3n}{2}-5\right)I_{\frac{n}{2}} \end{pmatrix}.$$

Here $I_{\frac{n}{2}}$ is the identity matrix having order $\frac{n}{2}$, $J_{\frac{n}{2}}$ is the $\frac{n}{2} \times \frac{n}{2}$ matrix with all entries as 1 and $O_{\frac{n}{2}}$ is a zero matrix of order $n/2$.
Further, above matrix can be expressed as

$$SL_{KCD_1}\left(\overline{K_{\frac{n}{2},\frac{n}{2}}}\right) = \begin{pmatrix} M_0 & M_1 \\ M_1 & M_0 \end{pmatrix},$$

where $M_0 = (2n-6)J_{\frac{n}{2}} - \left(\frac{3n}{2}-5\right)I_{\frac{n}{2}}$ and $M_1 = O_{\frac{n}{2}}$.
Now,

$$M_0 + M_1 = (2n-6)J_{\frac{n}{2}} - \left(\frac{3n}{2}-5\right)I_{\frac{n}{2}} = M_0 - M_1.$$

By using ***Lemma*** 4, the spectrum of $SL_{KCD_1}\left(\overline{K_{\frac{n}{2},\frac{n}{2}}}\right)$ is

$$SL_{Spec_{KCD_1}}\left(\overline{K_{\frac{n}{2},\frac{n}{2}}}\right) = \begin{pmatrix} 5-\frac{3n}{2} & \frac{2n^2-9n+10}{2} \\ n-2 & 2 \end{pmatrix}.$$

Further, employing the Eq. (3), we derive

$$\begin{aligned} SLE_{KCD_1}\left(\overline{K_{\frac{n}{2},\frac{n}{2}}}\right) &= \sum_{i=1}^{n}\left|\eta_i^+ - \left(\frac{n}{2}-1\right)\right| \\ &= 4(n-2)(n-3). \end{aligned}$$

□

Theorem 11 *For* $\overline{CR(c)}$ *with an integer* $c \geqslant 3$,

$$SL_{Spec_{KCD_1}}\left(\overline{CR(c)}\right) = \begin{pmatrix} c & 4-7c & 4c^2-9c+4 & 4c^2-c \\ c-1 & c-1 & 1 & 1 \end{pmatrix}$$

and

$$SLE_{KCD_1}\left(\overline{CR(c)}\right) = 8(c-1)(2c-1).$$

Proof. For $\overline{CR(c)}$ of order $n = 2c$, we have

$$SL_{KCD_1}\left(\overline{CR(c)}\right) = \begin{pmatrix} (4c-2)J_c-(3c-2)I_c & (4c-2)I_c \\ (4c-2)I_c & (4c-2)J_c-(3c-2)I_c \end{pmatrix}.$$

Here I_c is the identity matrix having order c and J_c is the square matrix having order c with all entries equal to 1.
Further, above matrix can be expressed as

$$SL_{KCD_1}\left(\overline{CR(c)}\right) = \begin{pmatrix} M_0 & M_1 \\ M_1 & M_0 \end{pmatrix},$$

where $M_0 = (4c-2)J_c-(3c-2)I_c$ and $M_1 = (4c-2)I_c$.
Now,

$$M_0 + M_1 = (4c-2)J_c + cI_c$$

and

$$M_0 - M_1 = (4c-2)J_c - (7c-4)I_c.$$

By using ***Lemma 4***, the spectrum of $SL_{KCD_1}\left(\overline{CR(c)}\right)$ is

$$SL_{Spec_{KCD_1}}\left(\overline{CR(c)}\right) = \begin{pmatrix} c & 4-7c & 4c^2-9c+4 & 4c^2-c \\ c-1 & c-1 & 1 & 1 \end{pmatrix}.$$

Further, employing the Eq. (3), we prove

$$\begin{aligned} SLE_{KCD_1}\left(\overline{CR(c)}\right) &= \sum_{i=1}^{n=2c} |\eta_i^+ - c| \\ &= 8(c-1)(2c-1). \end{aligned}$$

□

Theorem 12 *For* $\overline{CP(a)}$ *with an integer* $a \geqslant 3$,

$$SL_{Spec_{KCD_1}}\left(\overline{CP(a)}\right) = \begin{pmatrix} -1 & 3 \\ a & a \end{pmatrix}$$

and

$$SLE_{KCD_1}\left(\overline{CP(a)}\right) = 4a.$$

Proof. For $\overline{Cp(a)}$ of order $n = 2a$, we have

$$SL_{KCD_1}\left(\overline{Cp(a)}\right) = \begin{pmatrix} I_a & 2U_a \\ 2U_a & I_a \end{pmatrix}.$$

Here I_a is the identity matrix having order a and U_a is a square matrix with all entries as zeros, except the entries on the anti-diagonal being 1.
Further, above matrix can be expressed as

$$SL_{KCD_1}\left(\overline{Cp(a)}\right) = \begin{pmatrix} M_0 & M_1 \\ M_1 & M_0 \end{pmatrix},$$

where $M_0 = I_a$ and $M_1 = 2U_a$.
Now,

$$M_0 + M_1 = I_a + 2U_a$$

and

$$M_0 - M_1 = I_a - 2U_a.$$

By using ***Lemma 4***, the spectrum of $SL_{KCD_1}\left(\overline{CP(a)}\right)$ is

$$SL_{Spec_{KCD_1}}\left(\overline{CP(a)}\right) = \begin{pmatrix} -1 & 3 \\ a & a \end{pmatrix}.$$

Further, employing the Eq. (3), we establish

$$\begin{aligned} SLE_{KCD_1}\left(\overline{CP(a)}\right) &= \sum_{i=1}^{n=2a} \left|\eta_i^+ - 1\right| \\ &= 4a. \end{aligned}$$

□

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References

[1] N. Abreu, D. M.Cardoso, I. Gutman, E. A. Martins, M. Robbiano, Bounds for the signless Laplacian energy, *Linear Algebra Appl.*, **435** (2011) 2365–2374. ⇒ 22, 23, 24

[2] N. Biggs, *Algebraic Graph Theory*, Cambridge University Press, New York, 1993. ⇒28

[3] D. Cvetković, Signless Laplacians and line graphs, *Bull. Acad. Serbe Sci. Arts Cl. Sci. Math. Natur.*, **131,** 30 (2005) 85–92. ⇒24

[4] D. Cvetković, M. Doob, H. Sachs, *Spectra of Graphs: Theory and Applications*, Academic Press, New York, USA, 1980. ⇒28

[5] D. Cvetković, P. Rowlinson, S. K. Simić, Signless Laplacians of finite graphs, *Linear Algebra Appl.*, **423,** 1 (2007) 155–171. ⇒24

[6] D. Cvetković, P. Rowlinson, S. K. Simić, Eigenvalue bounds for the signless Laplacian, *Publ. Inst. Math.*, **81,** 95 (2007) 11–27. ⇒24

[7] D. Cvetković, P. Rowlinson, S. K. Simić, *An Introduction to the Theory of Graph Spectra*, Cambridge University Press, Cambridge, UK, 2010. ⇒24

[8] D. Cvetković, S. K. Simić, Towards a spectral theory of graphs based on the signless Laplacian I, *Publ. Inst. Math.*(Beograd), **85** (2009) 19–33. ⇒22
[9] J. A. Gallian, A dynamic survey of graph labeling, *Electron. J. Comb.*, **17** (2018), #DS6. ⇒27
[10] I. Gutman, B. Zhou, Laplacian energy of a graph, *Linear Algebra Appl.*, **414** (2006) 29–37. ⇒22
[11] I. Gutman, The energy of a graph, *Ber. Math. Stat. Sekt. Forschungsz. Graz.*, **103** (1978) 1–22. ⇒22
[12] F. Harary, *Graph Theory*, Addison-Wesely, Reading, Mass, 1969. ⇒22
[13] G. Indulal, I. Gutman, A. Vijaykumar, On distance energy of graphs, *MATCH Commun. Math. Comput. Chem.*, **60** (2008) 461–472. ⇒22, 25
[14] K. G. Mirajkar, A. Morajkar, First KCD matrix and first KCD energy of a graph, Accepted for publication in *Mathematical Forum*, **28** (2020). ⇒23
[15] K. G. Mirajkar and A. Morajkar, Laplacian energy of a first KCD matrix, Communicated for publication in *Commun. Combin., Cryptogr. & Computer Sci.* ⇒23, 25
[16] B. Zhou, More on energy and Laplacian energy, *MATCH Commun. Math. Comput. Chem.*, **64** (2010) 75–84. ⇒22

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