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A Detailed Examination of 8th Grade Students' Mathematical Reasoning Process[1]

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Abstract:

Introduction: This study aims to reveal the mathematical reasoning process of 8th-grade students and the reasoning indicators they use in this process.

Methods: The study was carried out in line with the data obtained from the Mathematical Reasoning Test (MRT) administered to 292 8th-grade students.

Results: The mathematical reasoning performances of the participants were generally at low and medium levels. Evidence was found that students used reasoning indicators such as "… for/because of…, /therefore/so/thus…", "if … then…", "because…", "should be/should be..." and "equal..." when reasoning.

Discussion: It is important that when evaluating the solutions to questions that require mathematical reasoning, students must be interviewed and confirmed how they reached those solutions.

Limitations: The limitations of the research are, firstly, that it is limited to 8th graders, and secondly, that students will get bored if the number of questions in the MRT is high.

Conclusions: Students must be asked to justify their reasoning while solving problems that require mathematical reasoning.

Key words: mathematical reasoning, reasoning indicators, 8th grade students.

[1] This study is a part of the doctoral dissertation prepared by the first author under the supervision of the second author.

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Introduction

As science and technology develop, the skills expected from individuals have also differentiated and changed. The skills referred to as 21st-century skills are learning and innovation skills (creativity, innovation, critical thinking, problem-solving, communication, collaboration), knowledge, media and technology skills (information literacy, media literacy, and technology literacy), life and career skills (flexibility, adaptability, initiative, self-management, social and intercultural skills, productivity, responsibility, leadership) (P21, 2019). In this century, individuals need to be able to think logically and express their thoughts correctly. Mathematical reasoning skill is needed to cope with the problems encountered in daily life and to solve problems with logical solution methods based on existing data. It should be ensured that learning is linked to daily life and other disciplines at school. It is underlined that children's knowledge of basic logic positively affects their cognitive, affective and social development (Can & Can, 2020). Kramarski et al. (2001) found that there is a relationship between learning mathematics and mathematical reasoning and that those who reason better can produce more effective solutions to problems and make better associations. On the other hand, it is pointed out that reasoning is necessary in the problem-solving process and that reasoning is indispensable in learning mathematics (Lestari, 2019). It is expected that schools will take necessary precautions in this context so that individuals can acquire such skills and use them in their daily lives. Looking at the 21st century, schools should prepare their students as individuals who can think critically, reason, and organize their ideas (Sumarsih et al., 2018).

Reasoning is used in the sense of the line of thought, the way of thinking, adopted to produce claims and reach conclusions (Lithner, 2004). Reasoning is very important in the development of comprehension and association in mathematics (Stylanides, 2007; Yankelewitz et al., 2010). Relating mathematics both within itself and to daily life and other disciplines requires an emphasis on mathematical reasoning and also forms a basis for further mathematical reasoning (Brodie, 2010). Developing students' mathematical reasoning will enable them to acquire many higher-level thinking processes. According to Holyoak and Morrison (2005), to solve a mathematical problem, an individual must think at a higher level, reason about possible outcomes, and decide between alternative outcomes.

1 Mathematical reasoning in middle school mathematics curriculum in Turkey

In the Middle School Mathematics (5th, 6th, 7th, 8th grades) Curriculum, the importance of making logical judgments in problem-solving competence and other processes is mentioned, and the basic competencies foreseen for effective learning and correct use of the reasoning skills used in mathematics are given (MNE, 2018).

1. Problem-solving
2. Mathematical process skills:
    a. Communication
    b. Reasoning
    c. Mathematical modeling
    d. Association
3. Sensory abilities
4. Psychomotor skills
5. It is stated that information and communication technologies are among the basic skills that are expected to be taught in the curriculum and that learning environments should be prepared to develop reasoning. In this curriculum, it is particularly emphasized that the use of information and communication technologies that make it possible to see different forms of representation of concepts and the relationships between them and enable students to discover mathematical relationships is emphasized. With the help of these technologies, environments should be prepared to develop students' skills such as problem-solving, communication, and reasoning by modeling (MNE, 2018, p. 6).

In the curriculum (MNE, 2018), reasoning is defined as the process of reaching new knowledge by using mathematics-specific tools (symbols, definitions, relationships, tables, graphs, etc.) and thinking techniques (induction, deduction, comparison, creativity, proof, etc.), based on existing knowledge. Since reasoning skill makes individuals' lives easier, it is emphasized that reasoning skill should be developed during the mathematics curriculum. In this regard, it is very important to use appropriate mathematics teaching techniques when necessary, to include information and communication technologies and problem-solving activities, and to include studies aimed at improving students' problem-solving competencies, communication, association, and reasoning skills.

2 Research on mathematical reasoning

In the studies on mathematical reasoning in the literature, it is seen that research is mostly done on efforts to develop this skill by applying thinking methods to students. For example, Kramarski et al. (2001) concluded that there is a relationship between reasoning and both mathematical learning and mathematics achievement. It has been stated that those who make better judgments make better connections by producing effective solutions to problems. Erdem and Gürbüz (2015) determined that the application of high-level open-ended problems that require reasoning can be effective in improving students' mathematical reasoning. Poçan et al. (2017) investigated the effect of demographic characteristics on middle school students' mathematical reasoning levels. They found that there was a significant difference according to the educational status of the parents, but there was no significant difference according to gender, grade level, and number of siblings. Yöndeş and Taş (2018) found that the reasoning skills of middle school students were positively affected by playing intelligence games and suggested that they should be encouraged to deal with problems that require reasoning (intelligence games, etc.) rather than the usual formulaic problems. Çoban (2019) found that there was a significant difference in the mathematical reasoning of 6th-grade students with differentiated teaching methods on the subject of integers. In addition, it was observed that positive attitudes toward mathematical reasoning were developed in the interviews conducted with the students. Nurjanah et al. (2021) divided the students into two groups and examined the contribution of the subject of plane geometry to the students' reasoning success through applied and computer-based teaching. Both the participants' mathematical concepts and reasoning skills were positively affected. Albaqawi (2023) concluded that female students studying in middle school have low inductive and deductive reasoning skills. It has been suggested to prepare mathematics textbooks and curricula that will enable the development of these skill types.

Mathematical reasoning is indispensable for students to discover, understand, and learn the basic subjects of mathematics (Umay, 2003). As a matter of fact, in the analysis of the answers given by students at different grade levels at the middle school level, studies on reasoning show that students have difficulty in making predictions, reasoning, and understanding and that they are prejudiced against mathematics and their success is low (Koay, 1998; Lamprianou & Lamprianou, 2003; Duatepe et al., 2005; Kramarski & Zoldan, 2008; Angraini et al., 2023). Especially in recent times, the mathematical reasoning thinking level, which is aimed at the competencies expected from individuals, has come to the fore, and in this regard, many studies investigating mathematical reasoning levels have been included in the literature (Erdem & Gürbüz, 2015; Lithner, 2017; Bal-İncebacak & Ersoy, 2018; Benli & Gökkurt Özdemir, 2018). With the

changing world and renewed technology, how students think mathematically has been researched, and the difficulties experienced by students in mathematics have been discussed. Yankelewitz et al. (2010) showed similar success in the same mathematical reasoning tasks, although the grade levels of the students were different. In studies conducted by Erdem (2015) and Kutluca and Tum (2021) at the 7th-grade level, it was determined that students' mathematical reasoning skill levels improved in learning environments enriched with different teaching methods. Albaqawi et al. (2023) concluded that the mathematical, inductive, and deductive reasoning levels of 8th-grade female students in middle school were low. It has been emphasized in many studies that developing mathematical reasoning and teaching this skill to students is important (Mason, 2001; Jonsson et al., 2022; Angraini et al., 2023). Different ways of thinking, providing different learning environments, and investigating skill levels are important points in gaining mathematical reasoning skills. Koçyiğit and Yenilmez (2022) found that STEM-focused teaching applications to 10th-grade high school students increased the students' mathematical reasoning skill levels. It was also found that mathematics teachers encouraging students to make mathematical reasoning by giving them tasks during their lessons had a positive effect on generalization and justification (Mastuti et al., 2022). In a study by McJames et al. (2023), it was determined that teachers' frequent use of creative reasoning questions in their lessons had a significant effect on students' course success. Bai et al. (2023) administered a test consisting of 15 questions to 8th-grade students in the mathematical abilities dimension (abstraction and generalization, reasoning and proof, problem solving) and the Bloom taxonomy cognitive level dimension (analysis, evaluation, and creation). It was concluded that the students had the lowest scores in these dimensions and that the gender variable did not have any effect on higher-order thinking skills. In line with the literature review, it can be said that there are no studies aimed at determining what reasoning indicators and strategies 8th-grade students may use in the mathematical reasoning process.

3 Purpose and importance of the study

Mathematical reasoning enables students to think about mathematics in meaningful and non-memorized ways. Reasoning is a mental process, and the student need to be able to solve the problems s/he encounters correctly and meaningfully while reasoning. In the 21st century, individuals are expected to have the ability to reason, make connections, and express their ideas in different areas of thought (critical, creative, etc.) in the face of problems. As it is known, the prerequisite for solving problems correctly is to understand and reach conclusions by reasoning. As the student's level of mathematical reasoning increases, likely, that his/her ability to analyze the situation using some

indicators will likely increase. How students think in the mathematical reasoning process, what reasoning indicators they use to provide solutions, in other words, their mathematical reasoning pictures, is an important issue that needs to be investigated. In this context, the study aims to determine the mathematical reasoning process of 8th-grade students and the reasoning indicators they use in this process. It is thought that the present study will contribute to the literature by determining the relationship between 8th-grade students' mathematical reasoning skill levels and reasoning indicators. This study sought answers to the following questions:

- What kind of reasoning indicators do 8th-grade students use when reasoning?
- What are the mathematical reasoning levels of 8th-grade students?
- What is the relationship between 8th grade students' mathematical reasoning skill levels and mathematical reasoning indicators?
- What are the strategies used by 8th-grade students while solving questions in MRT?

4 Methodology

4.1 Research design

In the study, explanatory design, one of the mixed methods types, was used. Mixed research is defined as a type of research in which the researcher reaches findings by collecting and analyzing data to find answers to research questions and makes inferences using quantitative and qualitative methods or approaches in a single study (Tashakkori & Creswell, 2007). To explain the general situation in more detail, qualitative data are collected in the context of quantitative data using an explanatory design (Creswell & Piano Clark, 2011). It is quantitative research as the mathematical reasoning levels of the participants are revealed through descriptive statistics. In the qualitative dimension of the research, mathematical reasoning indicators were created through content analysis of the explanations given by the participants for the questions in the MRT. In addition, it was associated with qualitative data obtained from semi-structured interviews to reveal the students' reasoning in depth. By examining the quantitative and qualitative data together in the study, a more detailed picture was obtained about how the participants reasoned.

4.2 Participants

The research was conducted with the participation of 292 eighth-grade students studying in five state middle schools in the center of a province in Turkey. Maximum diversity sampling was used to ensure that the research results were as representative as possible of students in this age group. In this context, students taking the high school entrance exam provide an advantage in terms of the

implementation of the study. Since students are responsible for all the achievements of the mathematics course in the high school entrance exam, they must learn the mathematics achievements and solve questions that require mathematical reasoning. In determining the schools participating in the study, care was taken to select schools with different levels of success (2 low, 2 medium, 1 high) in consultation with the provincial directorate of national education. 149 of the participants are female students and 143 are male students. To keep the identities of the participants confidential, they were given codes as S1, S2, S3…

4.3 Data collection

As a data collection tool, the Mathematical Reasoning Test (MRT), which consists of 22 questions developed using the Ministry of National Education (2020) and middle school mathematics textbooks and can reveal students' reasoning, was prepared. It is assumed that all 8th-grade students have achieved the achievements targeted by the Ministry of National Education at the end of the semester. The average score that can be obtained from the test is a minimum 0 and a maximum 5. To determine whether the questions in the test were questions related to mathematical reasoning, the opinions of 2 field educators, 1 curriculum expert, 1 measurement-evaluation expert, and 4 mathematics teachers (9-10-12-15 work years) were consulted. To check the clarity of the questions, MRT was evaluated by Turkish language experts. As a result of the preliminary application with 50 students with different achievements, it was decided that a 55-minute period for the test was appropriate and the questions were understood by the students. Item analysis was conducted to obtain the final version of the questions in the test. It is stated that items with item-total correlations of .30 and higher distinguish individuals well, items between .20 and .30 can be included in the test or corrected if deemed necessary, and items with lower than .20 should be removed from the test (Büyüköztürk, 2011). Therefore, it was decided to remove 5 questions from the test because their coefficients were below .30. Additionally, the Cronbach Alpha coefficient of MRT was calculated as .88. These analysis results show that the test can be applied reliably for students at this level. Thus, the test consisting of 17 questions has taken its final form. In addition, it was aimed to support the quantitative data with qualitative data by conducting interviews regarding the students' solutions to both create indicators of their mathematical reasoning in MRT and whether they made correct evaluations or inferences.

4.4 Data analysis

The answers given to the questions in MRT were analyzed using statistical package programs. In the analysis of the answers, the scoring scale developed by Erdem (2011) was used. On the scoring scale, the scoring of each question varies between 0 and 5 points (fully correct, partially correct-A, partially correct-B, partially correct-C, incorrect and blank answers). Students' mathematical reasoning skill levels were determined according to their answers to the questions in MRT. The levels of the students regarding this test were determined according to the skill level ranges given in Table 1.

In the analysis of qualitative data, a content analysis technique was used and judgment indicators were created. The reliability of the coding done by two researchers while creating the indicators was determined as 85%. To ensure scoring reliability, two experienced mathematics educator experts independently scored the answers to 17 questions in MRT separately. The consistency between the ratings made independently by two experts was determined as .90. On the other hand, direct quotes from the interviews with the students were also included to increase the validity of the research.

Table 1

Mathematical reasoning levels according to MRT score average

Level	***Score Interval ($\bar{X}$)***
Quite Low	0.00-0.99
Low	1.00-1.99
Medium	2.00-2.99
High	3.00-3.99
Quite High	4.00-5.00

5 Results

In this section, the level of mathematical reasoning of 8th grade students is given. In addition, the reasoning indicators that emerge when students make mathematical reasoning are presented. The strategies they use to solve MRT questions when reasoning are also given. The mathematical reasoning levels of the participants are given in Table 2.

Table 2

Statistics on mathematical reasoning levels

Level	*Frequency (f)*	*Percentage (%)*
Quite Low	56	19
Low	88	30
Medium	64	22
High	54	18
Quite High	30	10

As seen in Table 2, it was determined that 19% of 8th grade students had quite low mathematical reasoning, 30% had low, 22% had medium, 18% had high and 10% had quite high mathematical reasoning. It was observed that the mathematical reasoning levels of the 8th-grade students participating in the study were mostly at low and medium levels. In this case, it can be said that 8th-grade students are not at a sufficient level of reasoning, and thinking while solving mathematical problems and reaching the desired goal based on the knowledge given.

Table 3

Levels and indicators of mathematical reasoning

Mathematical Reasoning Indicators	*Mathematical Reasoning Levels*											
	Quite High		*High*		*Medium*		*Low*		*Quite Low*		*Total*	
	C.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.
for… /because of…/therefore/so/thus…	14;46	4;5	18;40	3;3	13;23	6;7	11;16	7;9	4;5	4;4	60;130	24;28
	18;51		21;43		19;30		18;25		8;9		84;158	
if … then…	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.
	11;24	1;1	11;21	2;2	4;5	4;4	5;5	3;3	0;0	1;1	31;55	11;11
	12;25		13;23		8;9		8;8		1;1		42;66	
because…	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.
	6;6	2;2	7;12	2;2	2;2	1;1	4;5	2;3	0;0	0;0	19;25	7;8
	8;8		9;14		3;3		6;8		0;0		26;33	
should be/should be...	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.
	10;18	0;0	11;14	2;2	12;13	3;3	6;7	1;1	1;1	2;2	41;53	8;8
	10;18		13;16		15;16		7;8		3;3		49;61	
Equal	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.	T.J.	F.J.
	13;20	0;0	5;10	0;0	7;9	5;6	2;2	1;1	0;0	0;0	27;41	6;7
	13;20		5;10		12;15		3;3		0;0		33;48	
Total	61;122		61;106		57;73		42;52		12;13		234;366	

X; Y: Number of Students; Number of times the indicator was used
T.J.: True Justification, F.J.: False Justification

As seen in Table 3, five indicator groups emerged depending on the explanations given by 292 8th-grade students in their solutions to the questions in the MRT, according to their mathematical reasoning levels. Indicators with similar meanings were evaluated in the same group. These indicators, from the most

used to the least used, emerged such as [for… /because of… /therefore/therefore/so/thus…], [should be/should be…], [if… then…], [equal] and [because…]. In addition, 234 students made explanations for the questions in the mathematical reasoning test and used reasoning indicators a total of 366 times. Since reasoning is the process of concluding by basing an individual's thoughts on a logical framework, the indicators used by 8th-grade students while making explanations are also classified as True Justification (T.J.) and False Justification (F.J.). Depending on the explanations they made, the students reasoned 201 times with T.J. and 318 times with F.J. Based on Table 3, it can be said that as students' mathematical reasoning levels increase, they use reasoning indicators with T.J. more. It can also be said that as the level of mathematical reasoning decreases, the number of indicators used also decreases. It can be pointed out that students with a quite high level of reasoning use indicators in the right place for the right reasons. In addition, it can be conceived that students with quite low levels of reasoning often use indicators for the wrong reasons.

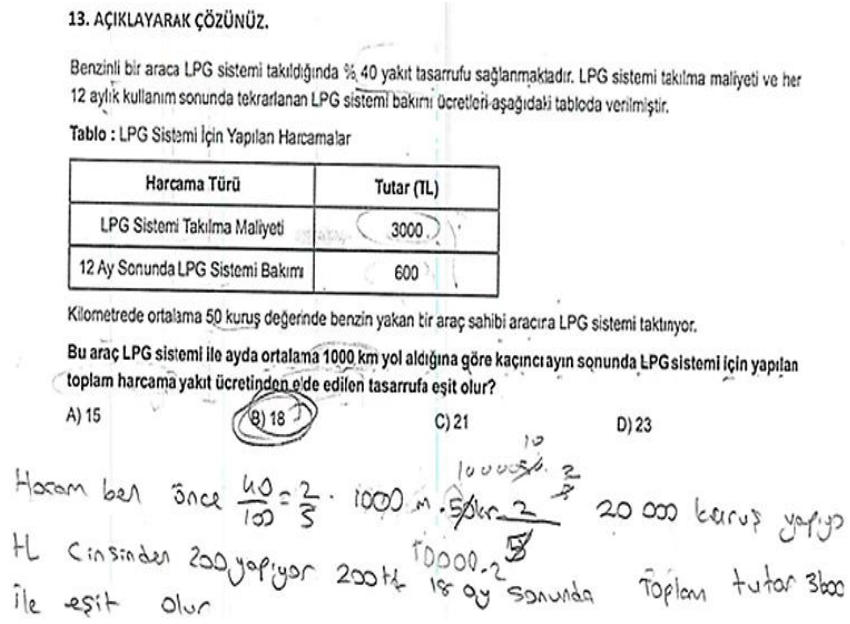
Table 4

Strategies expected to be used in questions on MRT

Question	*Sub-Learning Area*	*Strategies Expected to be Used*	*N*
1.	Factors and Multiples	Finding the Minimum Quantity Based on GCD	137
2.	Factors and Multiples	Finding the Minimum Quantity Based on LCM	156
3.,4.	Square Root Expressions	Using Approximate Perfect Squares	301
5.	Square Root Expressions	Making Relationships Between Numbers Given a Certain Total	138
6.	Data analysis	Simplifying Numbers	151
7.	Algebraic Expressions and Identity	Using Identities	112
8.	Algebraic Expressions and Identity	Finding the Minimum Quantity by Factoring	115
9.	Linear Equations	Finding a Solution Using Percentages	75
10.	Linear Equations	Finding a Solution Using Slope	60
11.-12.	Inequality-Linear Equations	Finding a Solution Using Slope	146
13.-14.-17.	Triangles-Geometric Objects	Benefiting from the Information Provided	217
15.-16.	Transformation Geometry-Geometric Objects	Using Close Factors to Get the Largest Multiplication Result	26

Table 4 lists the strategies that 8th-grade students are expected to use when reasoning on each question of the MRT. In the 15th and 16th questions of the test, it was observed that the strategy of "Using Close Factors to Get the Largest Multiplication Result", which was expected to be used to reach correct results, was used the least. In this regard, it can be said that this strategy is rarely used by students in reasoning and solving these two problems. It can be conceived that students benefited from the strategy they were expected to use to reach correct conclusions by reasoning about other questions. It can also be said that in the 2nd

question in the MRT, the students made great use of the strategy of "Finding the Minimum Quantity Based on LCM" and reached their conclusion by reasoning. To better illustrate the above results, the reasoning indicators and justifications used by some students in some questions are directly given below.



13. AÇIKLAYARAK ÇÖZÜNÜZ.

Benzinli bir araca LPG sistemi takıldığında % 40 yakıt tasarrufu sağlanmaktadır. LPG sistemi takılma maliyeti ve her 12 aylık kullanım sonunda tekrarlanan LPG sistemi bakımı ücretleri aşağıdaki tabloda verilmiştir.

Tablo : LPG Sistemi İçin Yapılan Harcamalar

Harcama Türü	Tutar (TL)
LPG Sistemi Takılma Maliyeti	3000
12 Ay Sonunda LPG Sistemi Bakımı	600

Kilometrede ortalama 50 kuruş değerinde benzin yakan bir araç sahibi aracına LPG sistemi taktırıyor.

Bu araç LPG sistemi ile ayda ortalama 1000 km yol aldığına göre kaçıncı ayın sonunda LPG sistemi için yapılan toplam harcama yakıt ücretinden elde edilen tasarrufa eşit olur?

A) 15 B) 18 C) 21 D) 23

Hasan ben önce $\frac{40}{100} = \frac{2}{5} \cdot 1000$ m. 50 kr 20 000 kuruş yapıyo TL cinsinden 200 yapiyor 200 tl 18 ay sonunda Toplam tutar 3600 ile eşit olur

Figure 1. S47's answer to the Q13 in the MRT.

In the 13th question in the MRT, S47 did not mark the choice by just making operations when reasoning. In his statement, he converted how much fuel will be saved when LPG is installed into the TL unit. It can be said that the student, who thinks that the result obtained and the total cost should be equal, makes a judgment by correctly justifying the data in the table by using the "equal…" indicator. The student calculated the fuel fee depending on the month after the LPG system. The student's MRT average is 4.29 and he has a quite high level of mathematical reasoning.

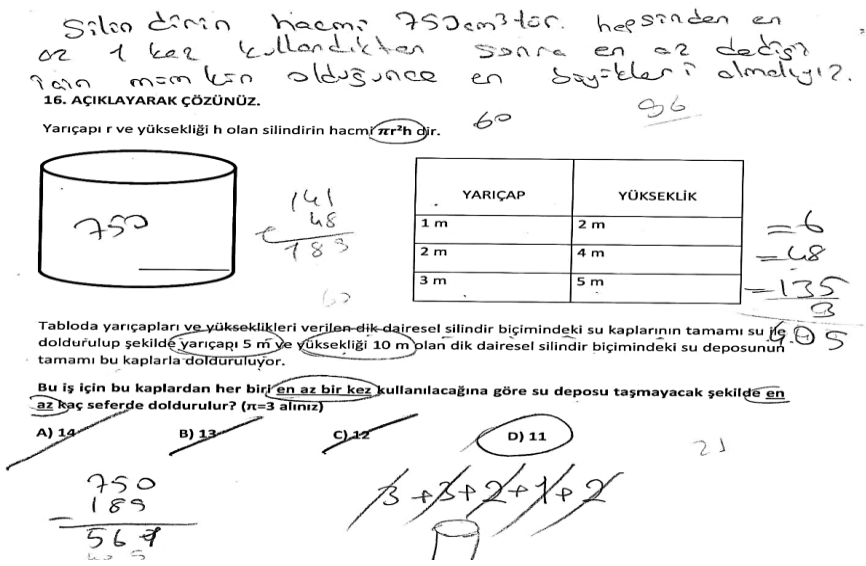


Figure 2. S135's answer to Q16 in the MRT.

It can be said that in the 16th question of the MRT, S135 used the indicator "should be..." when reasoning, and after finding the volume of the cylinder, she made a judgment with true justification by explaining that the containers to be used should have the largest possible volume since it is desired to use the least number of containers. The student's MRT average is 4.76 and she has a quite high level of mathematical reasoning.

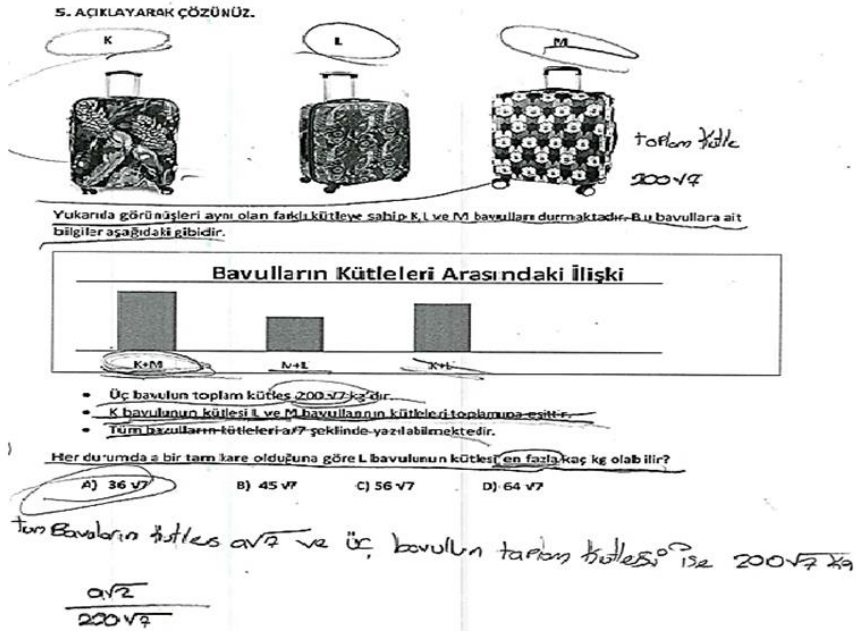


Figure 3. S160's answer to Q5 in the MRT.

In the 5th question of the MRT, while making a judgment, S160 used the indicator "if…then…" for wrong reasons by writing the same data as the existing data in the question. The student's MRT average is 0.58 and he has a quite low level of reasoning. In general, it is possible to say that students at quite low and low levels of mathematical reasoning give wrong answers for wrong reasons.

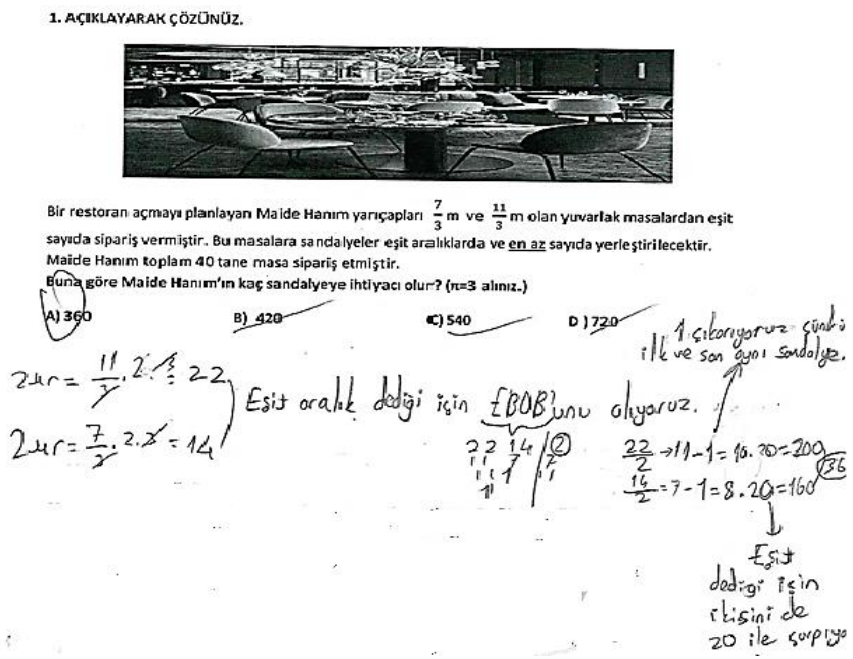
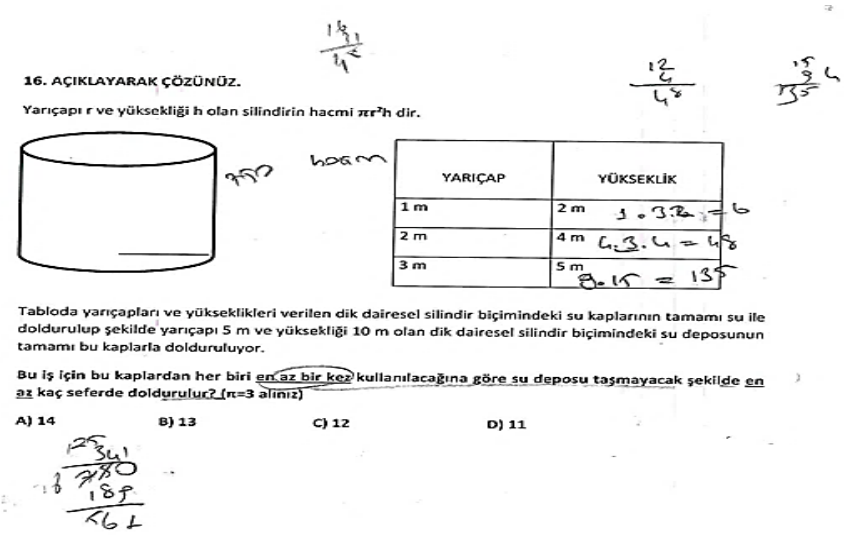


Figure 4. S144's answer to Q1 in the MRT.

It can be stated that in MRT, while making a judgment in Q1, S144 benefited from the strategy of "Finding the Minimum Quantity Based on GCD", which is expected to be used to reach the correct conclusion. The student's MRT average is 4.17 and he has a quite high level of mathematical reasoning.

Various interviews were held with the students, whose explanations for their solutions to the questions in the MRT were not well understood, to reveal in more detail how they thought of their solutions. Some of these interviews are given below.



16. AÇIKLAYARAK ÇÖZÜNÜZ.

Yarıçapı r ve yüksekliği h olan silindirin hacmi $\pi r^2 h$ dir.

YARIÇAP	YÜKSEKLİK
1 m	2 m
2 m	4 m
3 m	5 m

Tabloda yarıçapları ve yükseklikleri verilen dik dairesel silindir biçimindeki su kaplarının tamamı su ile doldurulup şekilde yarıçapı 5 m ve yüksekliği 10 m olan dik dairesel silindir biçimindeki su deposunun tamamı bu kaplarla dolduruluyor.

Bu iş için bu kaplardan her biri en az bir kez kullanılacağına göre su deposu taşmayacak şekilde en az kaç seferde doldurulur? (π=3 alınız)

A) 14 B) 13 C) 12 D) 11

Figure 5. S286's answer to Q16 in the MRT.

When S286's solution to Q16 in MRT was examined, it was seen that he calculated the volumes of the containers to be used to fill the water tank and the volumes of the tank to be filled, but then did it without explaining the operations. In the interview with the student regarding this solution, he said, "I found the volume of the warehouse and the volume of the others, then I thought about subtracting the volumes of the containers from the volume of the warehouse, but then I stopped doing it because I thought I would make a mistake." The student's MRT average is 2.47 and he has a medium mathematical reasoning level.

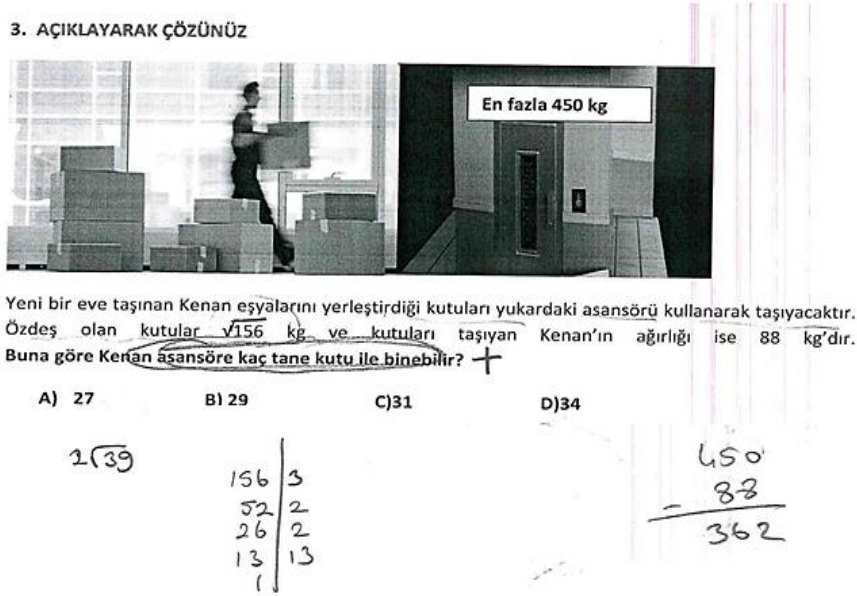


Figure 6. S285's answer to Q3 in the MRT.

S285 subtracted Kenan's mass from the maximum mass that the elevator could carry in the 3rd question on MRT. She wrote the mass of each box as a$\sqrt{b}$ using square root expressions. During the interview with her, she said, "*I found the maximum amount the elevator can carry is 362. I changed* $\sqrt{156}$ *to* $2\sqrt{39}$. *"I couldn't interpret what I should do next*". Based on this statement, it can be said that the student could not answer the question correctly because she could not use the expected strategy of "using approximate perfect squares" in square root expressions. The student's MRT average is 2.53 and she has a medium mathematical reasoning level.

6 Discussion and conclusions

This study was conducted to determine 8th-grade students' mathematical reasoning process and the reasoning indicators they use in this process. As a result of the analyses carried out for this purpose, various results were obtained.

First, it was determined that the mathematical reasoning levels of the participants were generally low and medium. In this case, it can be stated that 8th grade

students do not have a sufficient level of reasoning, and thinking while solving mathematical questions and reaching the desired goal based on the knowledge given. In the research conducted by Erdem and Gürbüz (2015), it was found that the mathematical reasoning of middle school students was at low and medium levels. A similar conclusion was reached by Poçan et al. (2017). Studies have shown that reasoning is a difficult process (Erdem, 2015; Yenilmez & Ata, 2019) and that both middle school students and teachers make a lot of effort mentally (Öz & Işık, 2018; Benli & Gökkurt Özdemir, 2021; Özaydın, 2022). It was concluded that when learning environments that are enriched using different and entertaining teaching methods are designed, students' mathematical reasoning improves and they begin to love mathematics (Erdem, 2015; Kutluca & Tum, 2021).

Second, depending on the explanations given by the 8th-grade students in their solutions to each question in the MRT, the reasoning indicators such as "… for/because of…/, therefore,/so/thus…", "if … then…", "because…", "should be/should be..." and "equal..." emerged. Mason (2001) highlighted that using indicators similar to these when students reason improves mathematical reasoning. A similar conclusion was reached by Mason et al. (2012). When this result was detailed, it was seen that the students mostly used the indicators "for… /because of…/therefore/so/thus…" and "should be/should be…" in their explanations while reasoning. On the other hand, it was revealed that students used the indicators "equal…" and "because…" the least. These results are similar to the results of studies in the literature. For example, it was mentioned that when doing mathematical reasoning, it was necessary to focus on students' explanations and how the rules worked, rather than directly applying the rules (Norqvist, 2018). McJames et al. (2023) pointed out the importance of students' reasoning by deciding on their strategies for solving problems in lessons. This result shows that it is necessary to encourage students to reason by considering the questions "Why?", and "How?". In this way, students who are encouraged to use reasoned indicators in their solutions are prevented from learning by rote and using stereotypical methods.

Third, it was observed that among the strategies that students were expected to use in solving MRT questions, the strategy of "Using Close Factors to Get the Largest Multiplication Result" was used the least in solving the 15th and 16th questions. When this result was detailed, it was revealed that the students rarely used this strategy in their solutions by reasoning in two questions, and they made correct justifications by reasoning about the contents of the other questions. On the other hand, it was observed that the students reached their conclusion by making good use of the "Finding the Minimum Quantity Based on LCM" strategy expected in the 2nd question. These results are similar to the results of studies in the literature. For example, Jonsson et al. (2022) pointed out that the

methods used for correct reasoning and proof had a positive effect on increasing the reasoning levels of high school students, the student's self-expression, and cooperation in the classroom.

Finally, when the students' papers were examined, it was seen that some students reached only operation-oriented solutions when reasoning and could not make an explanation. What is expected from students is to follow a path that is aware of the relationship between the knowledge given and how they can reach correct conclusions with justifications while pursuing a solution. When in-depth interviews were conducted with the students about the way they followed in solving some questions, they said that they marked the "smallest" or "largest" results in the options based on the "minimum" or "maximum" expressions in the question stem. This shows that they make solutions by heart, without understanding what they read and without reasoning. It has been observed that those who solved the questions incorrectly mostly made such explanations, while those who solved the questions correctly made explanations using correct reasoning indicators. It is stated that teaching students rules rather than concepts in teaching mathematics makes them dependent on operations (Dahlan & Wibisono, 2021; Abidin et al., 2020). It can be said that this is due to students' insufficient conceptual knowledge of mathematics subjects. Geteregechi (2020) suggested that it is necessary to encourage students to understand the mathematical concepts necessary for reasoning. Additionally, Jäder et al. (2020), when middle school students were asked to solve problems that required reasoning, it was determined that the students had insufficient conceptual knowledge and generally solved problems with classical rules and procedures. To solve this problem, care should be taken to ensure that the mathematical reasoning questions asked to students are high-level open-ended questions without answer choices. Similarly, it is possible to come across studies suggesting that open-ended questions should be used to reveal mathematical reasoning correctly (Cifarelli & Cai, 2005; Erdem, 2015; Erdem & Soylu, 2020; Özdemir et al., 2022; Ulinnuha et al., 2021; Rizos & Gkrekas, 2023).

On the other hand, the test used in the current study was found to be effective in terms of revealing the reasoning indicators in detail by making explanations while students were reasoning. Therefore, if multiple-choice questions are to be used, they must be questions in which students can express themselves and whose solutions are requested with justification. It is underlined in the literature (Erdem, 2015; Yenilmez & Ata, 2013; Kutluca & Tum, 2021; Albaqawi, 2023) that questions that require reasoning should be of a type that can reveal whether students understand the relevant subject, detect where they have difficulties, and reveal which strategies they choose and why. In this context, comparatively revealing the indicators that students at different grade levels can use when making mathematical reasoning may be the subject of future research.

7 Limitations and implications

In summary, based on the results of the research, it is strongly recommended that justified indicators must be taken into consideration in revealing and evaluating students' real mathematical reasoning. The most important thing to note here is that when evaluating the solutions to questions that require mathematical reasoning, students must be interviewed and confirmed how they reached those solutions. Otherwise, it should not be forgotten that there may be students who reasoned with true justification in the questions but could not write them down with correct explanations or expressions. To carry out this process correctly, it is necessary to evaluate the answers as soon as the questions are applied to the students and, when necessary, to immediately interview the students without forgetting their solutions. One of the limitations of the research is that it is limited to the mathematical reasoning of 8th-grade students. Comparisons can be made by conducting similar studies at other grade levels to reveal, evaluate, and improve mathematical reasoning processes. In this context, it can be seen what kind of mathematical reasoning indicators emerge at which grade level. Another thing to consider is whether the questions are related to the topics students have studied so far. During the application, students were not notified that any topic was not covered.

Another limitation is that students' performance may be negatively affected due to fatigue or boredom with the last questions in the MRT. This process emerged in the pilot application and was also mentioned by some students. Thus, the number of questions in the MRT was reduced. However, during real application, it was observed that some students got bored after a while. Current research can be conducted on large samples to determine how many questions should be composed of questions on average in tests measuring mathematical reasoning for students and how much time should be given for each question. In connection with this, it is also important to plan the time to be given for reasoning tests, taking into account the number of questions, so that it does not exceed the duration of the lesson in each country. Otherwise, if students cannot use the break they are accustomed to, they may be reluctant to reveal their true potential, and this may affect the result. In this context, it can be stated as a suggestion that the application of mathematical reasoning tests, which require high-level thinking, not only for 8th-grade students but also for all grade levels in middle school, should be planned in a way that does not exceed the duration of the lesson and should consist of fewer questions. Thus, considering the age factor, one can gain more insight into how students can use indicators and true or false justifications in their solutions to mathematical reasoning tests consisting of a small number of questions in middle school. In this context, students must be asked to justify their reasoning while solving problems that require mathematical reasoning.

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