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Identifying Mathematical Literacy Demands in Turkish, Singaporean and Australian Textbooks

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Abstract:

Introduction: Textbook tasks are considered as tools for implementing, endorsing mathematical thinking and thereby creating chances for mathematics learning. Therefore, textbook tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged. Among the essential sources of textbook tasks include worked examples and exercises. Because these worked examples and exercises in the textbooks are mostly used by students either in the classroom or at home, they definitely affect students' conceptual understanding of mathematics and may inspire students to work individually or collaboratively with their peers. Thus, given the importance of mathematical literacy for learning and understanding of math, one should investigate the chances students can have through it. This study aims to reveal the inclusion of the mathematical literacy demands in the fifth-grade mathematics textbooks from Turkey, Singapore, and Australia.

Methods: Being qualitative in nature, the current study employed a document analysis method to examine the textbooks. The cycle of mathematical literacy processes, defined in PISA framework, was used as a framework during the analysis to investigate mathematical literacy demands.

Results: Findings of the analysis of mathematical literacy demand in real life problems indicated that textbooks from all three countries had provided more opportunities for the competencies of two mathematical literacy processes, formulating and employing, while a small portion of these problems requires higher level cognitive skills to interpret/evaluate their mathematical solutions and make decisions for real life, which is the third process in the cycle; therefore, most of the real-life problems in the textbooks could not provide the chances for

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completing the whole mathematical literacy cycle. Moreover, textbooks from all three countries provided more chances for experiencing mathematical literacy processes in to-be-solved questions rather than worked examples.

Discussion: In general, textbooks from all three countries have included a small portion of the problems relating real life. Related literature also proves evidence for lack of real-life opportunities in the mathematics textbook tasks. Moreover, textbooks from all countries had provided more opportunities for the competencies of formulating and employing while a small portion of these problems requires higher level cognitive skills to interpret/evaluate their mathematical solutions and make decisions for real life. These results are not in accordance with the mathematics education calls voiced in national and international standards of mathematics education: Raising individuals with both mathematical thinking and reasoning skills and a useful foundation of mathematical knowledge and skills needed in all areas of life. Most of the real-life questions in the textbooks could not provide the chances for completing the whole mathematical literacy cycle. Textbooks' weaknesses in their inclusion of MLP may also cause impediments in the development of students' skills of handling the problems that they confronted in daily life.

Limitations: The present study only included one textbook from each country while these textbooks were representing the authenticity of the other textbooks in these countries. Moreover, this study examined the opportunities of mathematical literacy only provided in the textbooks while the actual implications of these opportunities may differ across classrooms in these countries.

Conclusions: Students' inappropriate practices with the real-life problems may cause them to not successfully solving these kinds of problems. Instead, employing more real-life problems in the classroom activities may result in higher student understanding. Moreover, examples and tasks from daily life are helpful to provide students with meaningful contexts and enable students to relate to their familiar experiences. The absence of providing necessary problem-solving opportunities in a range of different types in the textbooks may cause students to not solve specific types of problems. Moreover, textbooks should also include these problem-solving opportunities to construct students' conceptual appreciations of problem structures. Thus, one implication this study can make is that Singaporean and Australian textbook creators should include more sufficient practices of the whole MLP cycle in their problems to make sure students acquire the principal latent components of the problems.

Key words: mathematical literacy competencies, mathematics textbooks, comparative education.

Introduction

Over the past 50 years, there has been an increase in researchers' interest in students' mathematics performances in the international large-scale assessments such as the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) (e.g., Andrews, Ryve, Hemmi, & Sayers, 2014; Kjærnsli & Lie, 2004). The results of these assessments are also carefully followed by countries around the world and have had an impact on their education systems (e.g., Breakspear, 2012; Hopkins, Pennock, Ritzen, Ahtaridou, & Zimmer, 2008). Having higher performances on PISA appeared to emphasize existing policies and thus there was no motivation for extensive change (Dobbins, 2010) while decreases in performance forced countries to take actions for change in their educational policies, assessments, curriculum standards and performance goals (e.g., Martens, Nagel, Windzio, & Weymann, 2010; Takayama, 2008). Upon PISA's influence on thought and action in mathematics education, Stacey et al. (2015) indicated that PISA results have reinforced to take actions and inspired projects that propose enhancing achievement. These projects have also aimed at improving content and design of the textbooks due to the general acceptance that enhancement in textbooks can stimulate the way mathematics is taught and learned (e.g., Ball & Cohen, 1996; Hirsch, 2007; Weiss, Knapp, Hollweg, & Burrill, 2002).

Textbooks, considered a 'de-facto national curriculum' (Mayer, Sims, & Tajika, 1995), constitute the major structures of teaching programs and regarded as artifacts that evolves educational policy into the teaching practices; therefore, carries intended curriculum toward implemented curriculum (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). As a result, research on textbook offers a broader reflection on curriculum standards and teaching practices in the classroom environment (Mayer et al., 1995). In order to stand out in international competition, to evaluate the performance of their students and to direct education policies, countries have given increasing importance to the skills evaluated in international large-scale assessments, such as PISA; and this has affected the design of the textbooks, including the competenciesto be targeted (Hopkins et al., 2008). When we analysed the key competencies of the PISA, Mathematical Literacy Processes (MLP) is encountered as a central concept (de Lange, 2003). PISA, through MLP cycle, examines how student are well prepared for their future lives in terms of their learning of mathematics (OECD, 2007). MLP cycle starts with a problem situated in the real-life context, continues with formulating the problem in mathematical terms, employing the mathematical computations and finalized with interpretation and evaluation of mathematical solution to check for its adequacy in answering the original question. If it is not adequate, the cycle should start from the formulating again to enhance the model. Considered at the heart of PISA mathematical assessment, MLP adaptation of

the textbooks also be a significant concern when giving importance of textbooks that are the primary resource for mathematics learning as they provide students with the tasks delivering fundamental bases for fostering meaningful learning of the concept, practicing basic mathematical skills, and engaging students in doing "important" mathematics (Valverde et al., 2002; Van Zanten & Van den Heuvel-Panhuizen, 2018).

Textbook tasks are considered as tools for implementing activities (Mason & Johnston-Wilder, 2004), endorsing mathematical thinking (Lithner, 2003), and thereby creating chances for mathematics learning (Sullivan, Clarke, & Clarke, 2012). Therefore, textbook tasks "can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged" (Henningsen & Stein 1997, p. 525). The most essential sources of textbook tasks include worked examples and exercises (Sullivan et al., 2012). Because these worked examples and exercises in the textbooks are mostly used by students either in the classroom or at home, they definitely affect students' conceptual understanding of mathematics and may inspire students to work individually or collaboratively with their peers (Hopf, 1980; Sullivan et al., 2012). Thus, given the importance of mathematical literacy for learning and understanding of math, one should investigate the chances students can have through it. Since mathematics educators involves MLP through international assessments, specifically PISA, it necessitates to investigate the extent to which these mathematical literacy demands exist in the textbook tasks. Thus, this study proposes a reflection on how MLP exist in the textbook tasks while mathematical literacy demands may also exist in the textbook explanations, illustrations, as well as in-class activities, etc. The relevancy of the possible existence of mathematical literacy demands beyond the textbook tasks cannot be ignored but is not considered in the current study. This study aims to reveal the inclusion of the MLP (as defined in PISA study) in the fifth-grade mathematics textbooks from Turkey, Singapore, and Australia. The specific research question was: To what extent do mathematical literacy demands take place in the sets of questions in the Australian, Turkish and Singaporean textbooks?

Selection of countries

In the selection of the countries from which we chose mathematics textbooks, we examined mathematics mean score of all countries in PISA 2018 and decided to select three countries with an average mathematics test score in PISA 2018, which was a) statistically below the OECD average, b) statistically above the OECD average, and c) had statistically the same average as the OECD average, respectively (see Table 1). The reasoning behind our selection was to track the

similarities/differences of the MLP characteristics of mathematics textbooks from the countries at a different achievement rate.

Table 1

PISA 2018 average test score

Country	*Rank*	*Score*
Turkey	43	454
Singapore	2	569
Australia	29	491

Note: Rank represents the arrangement of each country among 79 countries held PISA 2018. The highest score in the 2018 PISA Math Test was 591 and the lowest score was 325 while the OECD average score was 489.

The results of international assessments, such as PISA, affect education systems around the world (e.g., Breakspear, 2012; Hopkins et al., 2008). High performance on PISA has been shown to strengthen the presence of systems (Dobbins, 2010); on the other hand, declines in performance urge countries to take steps to change their education practices and policies (e.g., Martens et al., 2010; Takayama, 2008). Following the publication of the PISA 2015 results, Turkey has taken important decisions concerning the education practices including mathematics teaching, which also continued after PISA 2018 results. These decisions include the revision of mathematics education programs at all levels and the concept of "mathematics literacy" for the first time, which is included in the basic principles of mathematics education program (MoNE, 2017). The Ministry of National Education has organized workshops for school principals and teachers to adopt the MLP theory and apply it in the classroom. The content of the transition to high school exam also changed significantly and included MLP questions. In the last step, secondary school textbooks are gradually being changed starting from the fifth grade in 2018. For this reason, this study aimed to compare the first sample of the redesigned Turkish textbooks with their counterparts in Australia and Singapore.

Mathematical literacy processes

PISA 2018 Assessment and Analytical Framework (OECD, 2019) defines mathematical literacy as

> "…an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded

> judgments and decisions needed by constructive, engaged and reflective citizens." (p. 75)

Mathematical Literacy Processes (MLP) refer to an individual's capacity to formulate, employ, and interpret mathematics. MLP describe how students carry the mathematical context of the problem into the interpretation, application and evaluation of the mathematical outcomes through formulating the problem mathematically and employing necessary mathematical concepts and procedures (OECD, 2019). According to PISA 2018 analytical mathematics framework, the formulating process is defined as "how effectively students are able to recognise and identify opportunities to use mathematics in problem situations and then provide the necessary mathematical structure needed to formulate that contextualised problem into a mathematical form" while the employing process is defined "how well students can perform computations and manipulations and apply the concepts and facts that they know to arrive at a mathematical solution to a problem formulated mathematically". Finally, the interpreting/evaluating process is defined "how effectively students can reflect upon mathematical solutions or conclusions, interpret them in the context of a real-world problem, and determine whether the results or conclusions are reasonable" (OECD, 2019, p.77). Students' ability to solve mathematics questions is dependent upon their abilities in these three processes. Either educational policy or classroom level discussions can also be informed by students' performances in these processes (OECD, 2019).

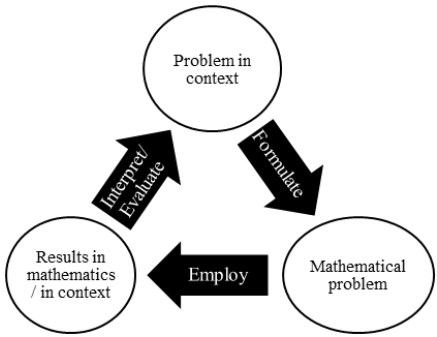


Figure 1. A model of mathematical literacy processes (MLP) in practice (adapted from OECD, 2019, p. 77).

There is a demand in the world to prepare students with the mathematical knowledge applicable to the real-life situations. Wu (2009) indicated that some countries make necessary efforts to ensure their students are prepared well toward real-life situations by aligning their curriculum and assessment practices accordingly. She also highlighted that mathematical literacy should be embodied

in the curriculum and textbooks if the main goal is to acquire it as an educational outcome, which explains our effort to work on the textbooks. MLP takes students into consideration as problem solvers in PISA; however, in an assessment context, it is not usually expected them to involve in all three processes as a cycle (Niss, Blum, & Galbraith, 2007). Students, as problem solvers, usually complete a partial cycle or completed the whole processes couple of times to revise their prior decisions. In our case, the demand for mathematics literacy cycle in textbooks is handled in two dimensions. First, an investigation was made for the inclusion of each process of the MLP in the textbook tasks. Then, the progress in the entire MLP cycle in textbook tasks was examined.

Studies of textbooks

It has been popular to compare textbooks and their problem specifications especially after the first results of the TIMSS study was issued. Several studies were conducted to compare textbooks of diverse educational systems to reveal indigenous objectives and practices (Gatabi et al., 2012). Specifically, textbook tasks (including to-be-solved problems and worked examples) have been receiving admired attention among studies focusing on textbook analysis as the most prevalent non-mathematical topic (e.g., Fan, 1998; Fan & Zhu, 2000; Li, 2000; Sun, 2011). Analysis of the textbook tasks generally consists of answer types, concepts, and complexities (e.g., Huntley & Terrell, 2014; Zhu & Fan, 2006). While some studies were conducted on specific mathematical topics (e.g., Vincent & Stacey, 2008), some others focused on the entire textbook of a specific grade level (e.g., Brändström, 2005; Dole & Shield, 2008).

This summary only refers to a few studies that focused on the analysis of textbook tasks in terms of the competencies included in the international assessments. Bao (2004) worked on the old and new mathematics textbooks used in China to investigate the difficulty level of the items of the international assessments. As a result of his study, he expressed how important the alliance between the problems used in the textbooks and the anticipated student learning. Gatabi et al. (2012) compared Australian and Iranian mathematics textbooks in terms of the inclusion of mathematical literacy in the mathematical tasks as indicated in the relevant literature, and concluded that Australian mathematics textbooks contain different types of mathematical tasks ranging between simple formulation to the real-life contextualization. On the other hand, Iranian textbook provides less diversity in the types of mathematical tasks providing students very minimal chance in the real-life contextualization, which is the main process of the mathematical literacy. Vincent and Stacey (2008) investigated different textbooks used in Australia at the eighth-grade level to reveal how competent the problems in the textbooks with the classification of the TIMSS video study. As a

result of their study, the problems included in the textbooks were well aligned with the ones used in TIMSS video study.

To sum up, the studies mentioned above illustrated that analysis of the problems included in the textbooks can provide opportunities to understand student learning; however, the analysis should be based upon a framework that may reveal the usefulness of the textbooks in terms of the curricular objectives. On the other hand, these studies did not incorporate an essential element of the textbook tasks: Their concern with using real-life context utilizing mathematical literacy framework. Considering the value of mathematical literacy in learning mathematics, mathematical literacy demands are also worth including in the analysis of textbook tasks.

1 Methodology

Being qualitative in nature, the current study employed a document analysis method to examine Mathematical Literacy Processes (MLP) in mathematics textbooks. Document analysis includes recording the existing records and documents related to the subject to be investigated and then coding these documents according to a certain norm or system (Cohen, Manion, & Morrison, 1994).

1.1 Selection of textbooks

In this study, the textbooks were selected based on purposive sampling strategy. Table 2 gives information about the selected textbooks. Turkey and Singapore utilize standardized textbooks in their classrooms while mainstream teaching materials are used in Australia. In Turkey, textbooks are compulsory in primary and secondary education. The adoption of a textbook for instruction depends on the approval of the Ministry of National Education (MoNE). Turkish textbooks are evaluated based on four basic dimensions: 1) the conformity to the instructions of MoNE, 2) scientific competence, 3) the level of achievement of instructional objectives, and 4) quality of visual and content design. Similarly, Singaporean textbooks need to be approved by Singapore's MoNE before they can be adopted. As a representative of mathematics textbooks in Singapore, the textbook series, New Syllabus D Mathematics, was selected for the study. In Australia, different curriculum standards are developed in the different states. These standards can be employed by authors who would like to write a textbook to be used in the curriculum.

Table 2

List of textbooks used in the study

Country	*Textbook Series*	*Publisher*
Turkey	Middle School Mathematics Course Book 5	MEB Publishing
Singapore	New Syllabus Primary Mathematics 5A	Shinglee
Australia	Mathematics for Australia 5	Haese Mathematics

1.2 Units of analysis

This study employed the model of Mathematical Literacy Processes (MLP), given in Figure 1, as the unit of data analysis. PISA 2018 Assessment and Analytical Framework defines mathematical literacy in three processes: formulating situations mathematically, employing mathematical concepts, facts, procedures and reasoning, and interpreting and evaluating mathematical outcomes (see Table 3 for a detailed explanation of mathematical literacy processes) (OECD, 2019). The current study utilized these processes and utilized the following criteria to better describe textbooks' attempts to apply mathematical processes.

Mathematical Literacy Framework also acknowledges that mathematical literacy helps individuals to recognize the role that mathematics plays in the world; hence a central aim is to provide situations encountered in daily life (OECD, 2019). Therefore, we only focused on the questions relating to real-life or taken directly from a real-life phenomenon. Moreover, textbooks tasks, in general, have two basic opportunities for students to foster their understandings of the concepts. They include exercises for students to practice their math learning (we named them “to-be-solved questions – TBSQ”) and the questions with an answer that aims to extend or to example a concept or definition for robust students’ understanding (we named them “worked examples – WE”). If a TBSQ/WE includes mathematical literacy processes, we evaluated TBSQ and WE separately in terms of mathematical literacy inclusion. Within this regard, Table 4 shows the distribution f) of the contents to be analyzed in textbooks. According to Table 4, a total of 948 content items were investigated to determine the MLP’s inclusion in the textbooks. Australian textbooks provided more real-life content than Turkish textbooks while Singaporean textbooks had the least amount of content regarding the real-life context to be examined in the current study.

Table 3

Mathematical literacy processes and the underlying mathematical capabilities (OECD, 2019)

Competency	*It refers to*	*List of activities*
Formulate	Individuals being able to recognize and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualized form.	- identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables - recognizing mathematical structure in problems or situations - simplifying a situation or problem to make it amenable to mathematical analysis - identifying constraints and assumptions behind any mathematical modeling and simplifications gleaned from the context - representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models - representing a problem differently, including organizing it according to mathematical concepts and making appropriate assumptions - understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically - translating a problem into mathematical language or a representation - recognizing aspects of a problem that correspond with known problems or mathematical concepts, facts, or procedures. - using technology to portray a mathematical relationship inherent in a contextualized problem. - selecting an appropriate model from a list.
Employ	Individuals being able to apply mathematical concepts, facts, procedures, and reasoning to solve mathematically formulated problems to obtain	- devising and implementing strategies for finding mathematical solutions - using mathematical tools, including technology, to help find exact or approximate solutions - applying mathematical facts, rules, algorithms, and structures when finding solutions - manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations - making mathematical diagrams, graphs, and constructions and extracting mathematical information from them

	mathematical conclusions.	- using and switching between different representations in the process of finding solutions - making generalizations based on the results of applying mathematical procedures to find solutions - reflecting on mathematical arguments and explaining and justifying mathematical results.
Interpreting /evaluating		- performing a simple calculation - drawing a simple conclusion - selecting an appropriate strategy from a list - interpreting a mathematical result back into the real-world context - evaluating the reasonableness of a mathematical solution in the context of a real-world problem
	reflect upon mathematical solutions results or conclusions and interpret them in the context of real-life problems.	- understanding how the real-world impacts the outcomes and calculations of a mathematical procedure or model in - order to make contextual judgments about how the results should be adjusted or applied - explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem - understanding the extent and limits of mathematical concepts and mathematical solutions - critiquing and identifying the limits of the model used to solve a problem - evaluating a mathematical outcome in terms of the context

1.3 Coding procedures

To respond to the research question, the questions relating to the real-life were analyzed in terms of their alignment with the MLP. To check inter-rater reliability, three coders who are proficient in both English and Turkish languages and had experiences with textbooks participated in the coding process of the textbooks. Due to the multiplicity of data to be encoded, in the beginning, 215 content items in the Singaporean textbook were independently coded by three coders. According to the Fleiss' Kappa formula, the inter-rater reliability of the first codes was calculated to be 81.5%. Then, coders met and discussed the items that caused disagreement until agreement upon each disputed item is reached. After coders, an expert in the field of mathematics education also coded the same content. According to the Cohen's Kappa formula, the interrater agreement rate was found to be 88.7% between the coders and the expert. Discussions on the causes of disagreement resulted in a consensus. Then, the remaining content items were shared among coders, and coders worked independently. Finally,

coded data were examined both qualitatively and quantitatively to identify the similarities and differences among the mathematics textbooks of three countries in terms of their adaptation of MLP. In the presentation of the findings, the percentages of each of the MLP in the textbook questions were determined. Afterward, progress in the MLP cycle was analyzed to reveal how much of the daily life questions in the textbooks have gone through the entire MLP. The findings were also descriptively compared in terms of the distribution of TBSQ and WE.

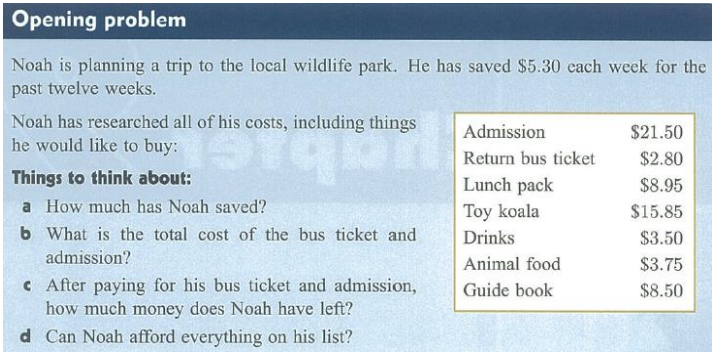
Table 4

Distribution of the contents to be analyzed (f)

	TBSQs	*WE*	*Total Real-Life Questions*	*Total Questions*
Turkey	160	86	246	643
Singapore	138	77	215	537
Australia	472	15	487	1091
Overall	770	178	948	2271

1.4 Sample codes

To provide a clear understanding of the coding procedure, the below is provided some samples of codes regarding the units of analysis. Figure 2 presents a TBSQ from Australia fifth grade textbook. The question is about the cost of a trip to a local wildlife park and includes the prices of possible preferences. The very first step MLP cycle is having a problem in the context and this question has a real-life context of planning a trip. Then we choose it for evaluation and start checking its inclusion of MLP. Parts a, b, and c of the question requires students to formulate the situation and employ the value based on their formula. For example, part a of the question requires students to come up with a formula by writing a mathematical sentence of the problem (12 x 5.30) and then computing the total savings in dollars. Building upon the solutions of prior parts of the question, part d requires evaluating the results in the context (interpret/evaluate) whether Noah can effort everything in his list. This problem covered all processes in MLP; thus, completed the whole MLP cycle.



Opening problem

Noah is planning a trip to the local wildlife park. He has saved $5.30 each week for the past twelve weeks.

Noah has researched all of his costs, including things he would like to buy:

Admission	$21.50
Return bus ticket	$2.80
Lunch pack	$8.95
Toy koala	$15.85
Drinks	$3.50
Animal food	$3.75
Guide book	$8.50

Things to think about:

a How much has Noah saved?

b What is the total cost of the bus ticket and admission?

c After paying for his bus ticket and admission, how much money does Noah have left?

d Can Noah afford everything on his list?

Figure 2. TBSQ from Australian textbook. Processes: Formulate, Employ, Interpret/Evaluate; Progress in MLP cycle: Whole process.

On the other hand, some of the real-life problems in the textbooks were not found to be compatible with some mathematical literacy competencies and could not complete the MLP cycle. The following WE from the Singapore textbook is a sample for this situation (see Figure 3). When the solution process is examined, firstly, the expression in daily life was translated into a mathematical sentence (formulate) and then calculations were made based on the mathematical expression written (employ). Hence, this WE is coded having formulation and employ competencies, and stopped at the employing step of the MLP cycle.

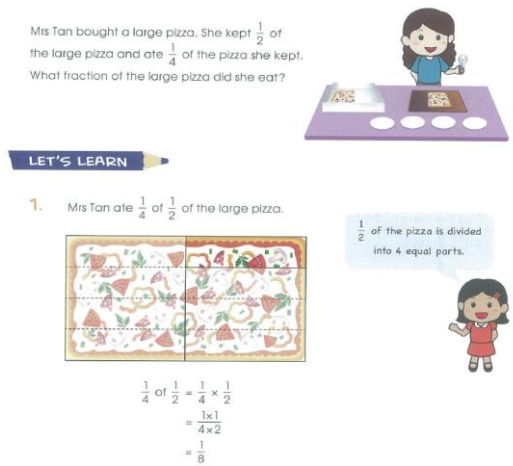


Figure 3. WE from Singaporean textbook. Processes: Formulate, Employ; Progress in MLP: Formulate & Employ.

2 Findings

The Mathematical Literacy Processes (MLP) cycle begins with a problem from the real-life context. Figure 4 shows the distribution of real-life problems and their compatibility with the MLP defined in the framework of PISA mathematical literacy. The bottom line of the figure shows the proportion of real-life questions in textbooks across countries. It seems Australian textbook offer students more opportunities to deal with real-life problems than the others, but textbooks have close distributions in the range of 38% to 45%. When we consider analyzing each MLP in all textbooks, we can also track similar distributions. Across the textbooks from all countries, real-life problems most of the time requires students to formulate a given situation and/or to employ their formula. However, a small portion of these problems (especially in Singaporean (5.6%) and Australian (5.5%) textbooks) require students to interpret/evaluate their mathematical solutions and make decisions in the context. Moreover, Turkish fifth-grade textbooks provide more opportunities for their students to practice each MLP in their real-life problems.

As a result of the investigation illustrated in Figure 4, Figure 5 is created to present the progress of textbook problems in the MLP cycle, as described in the PISA framework (Figure 1). Almost all real-life problems (96.7%) in Turkish textbooks require students to formulate the situation given in the context. Then more than two-third of these problems also requires employing the formula to have a solution in the world of mathematics. At the next stage, there is a dramatic drop at the percentage coverage of the real-life problems that require interpreting mathematical results and evaluating these results in the real-life context.

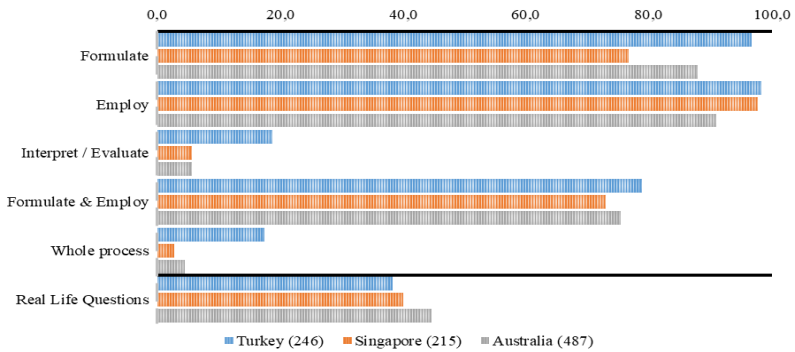


Figure 4. MLP Distributions across Real-life problems from textbooks (%). Note: The total number of real-life problems is provided within parenthesis.

Similar results are also evident in textbooks from other countries. Let's talk about the points where Turkish textbooks differ from other textbooks in terms of adaptation to the mathematics literacy cycle. First of all, more daily life problems enter the cycle of mathematical literacy from the first stage (formulation) of the process. Even though there is a decrease in the percentage distribution, more problems in the employing phase progress in the MLP cycle. Despite significant declines in the proportion of textbook problems completing the final stage of the MLP, completion rate of the whole MLP cycle in Turkish textbooks are higher by several times than others.

As a result of the investigation illustrated in Figure 4, Figure 5 is created to present the progress of textbook problems in the MLP cycle, as described in the PISA framework (Figure 1). Almost all real-life problems (96.7%) in Turkish textbooks require students to formulate the situation given in the context. Then more than two-third of these problems also requires employing the formula to have a solution in the world of mathematics. At the next stage, there is a dramatic drop at the percentage coverage of the real-life problems that require interpreting mathematical results and evaluating these results in the real-life context. Similar results are also evident in textbooks from other countries. Let's talk about the points where Turkish textbooks differ from other textbooks in terms of adaptation to the mathematics literacy cycle. First of all, more daily life problems enter the cycle of mathematical literacy from the first stage (formulation) of the process. Even though there is a decrease in the percentage distribution, more problems in the employing phase progress in the MLP cycle. Despite significant declines in the proportion of textbook problems completing the final stage of the MLP, completion rate of the whole MLP cycle in Turkish textbooks are higher by several times than others.

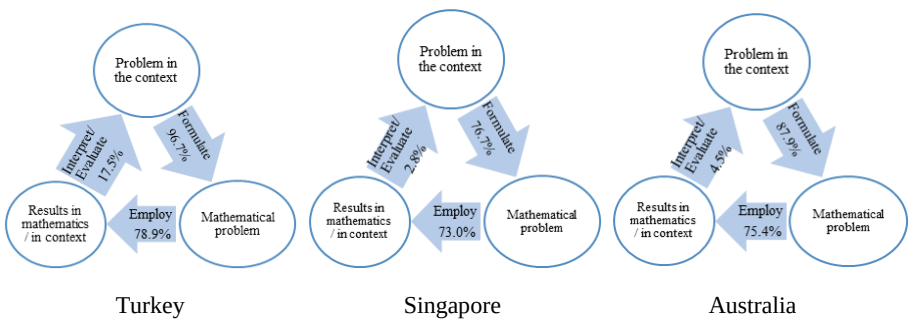


Figure 1. The alignment of textbooks with MLP (OECD, 2019).

2.1 Analysis of the situation in TBSQ and WE

We were also concerned about textbooks' providing children similar learning opportunities in conducting MLP. To this end, Figure 6 has shown the distribution of MLP in terms of to-be-solved questions (TBSQ) and worked examples (WE). Generally speaking, textbooks from all countries have all provided more chances for experiencing MLP in TBSQ. Textbooks from Singapore and Australia included no WE going through the entire MLP cycle, while the Turkish fifth-grade mathematics textbook provides some cases for exemplifying uses of MLP in WE. When we analyze the situations in the MLP, the Australia textbook has provided a few chances for the students to engage in MLP in WE (4.50%). Singaporean textbook has provided equal opportunities for enacting formulating and employing processes in WE (34%) while none of WE completed the whole cycle, which means did not include the process of evaluating and interpreting results. The case of Turkish fifth-grade math textbooks also favored TBSQ for MLP while they have provided students with the chance for getting acquainted with each MLP in the WE.

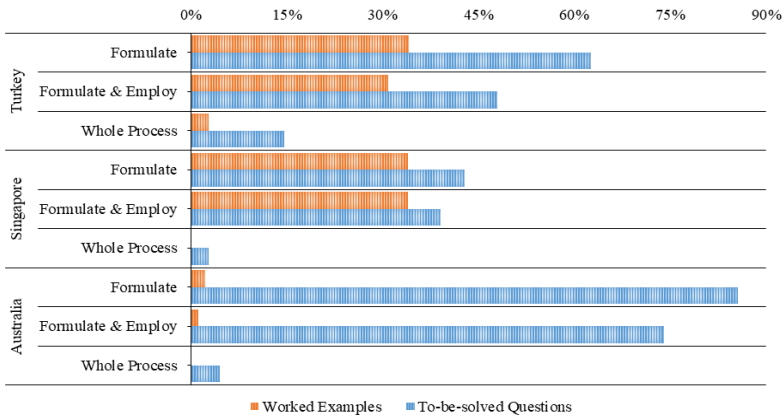


Figure 6. Opportunities for mastering MLP.

3 Discussion and conclusions

The current study aimed to reveal the inclusion of the PISA mathematical literacy process in the fifth-grade mathematics textbooks from Turkey, Singapore and Australia. Garner (1992, p. 53) points out that "textbooks serve as critical vehicles for knowledge acquisition in school" and may "replace teacher talk as the primary source of information". In general, textbooks from all three countries have included a small portion of the problems relating real life. Related

literature also prove evidence for lack of real-life opportunities in the mathematics textbook tasks (e.g., Pepin & Haggarty, 2007; Wijaya, van den Heuvel-Panhuizen, & Doorman, 2015). Students' inappropriate practices with the real-life problems may cause them to not successfully solving these kind of problems (Wijaya et al., 2015). Instead, employing more real-life problems in the classroom activities may result in higher student understanding (Gu et al., 2004). Moreover, examples and tasks from daily life are helpful to provide students with meaningful contexts and enable students to relate to their familiar experiences (Pepin & Haggarty, 2007). Within this regard, pure mathematically contextualized problems should not be included in the textbooks rather than the problems underlining real-life contexts (Alajmi, 2012).

An analysis of mathematical literacy demands in real life problems indicated that textbooks from all countries had provided more opportunities for the competencies of formulating and employing while a small portion of these problems requires higher level cognitive skills to interpret/evaluate their mathematical solutions and make decisions for real life. Therefore, cognitively lower anticipations expressed in the present study overlapped with the findings of the previous researches (e.g., Huntley & Terrel, 2014; O'Keeffe & O'Donoghue, 2015; Zhu & Fan, 2006). On the other hand, these results are not in accordance with the mathematics education calls voiced in national and international standards of mathematics education: Raising individuals with both mathematical thinking and reasoning skills and a useful foundation of mathematical knowledge and skills needed in all areas of life (MoNE, 2017; NCTM, 2000).

Most of the real-life questions in the textbooks could not provide the chances for completing the whole mathematical literacy cycle. Among school subjects, mathematics has a long history of being driven by textbooks and curriculum materials that teachers use to teach mathematics topics (Remillard, 2005). Today, the mathematics textbook remains a major classroom resource for teaching and learning mathematics (Nicol & Crespo, 2006). Furthermore, students spend considerable time using mathematics textbooks at school or at home to do the assigned homework, and teachers from primary to make use of mathematics textbooks to understand the curriculum objectives and to shape their pedagogical practices from primary to tertiary level (Zakka, Oluyemi, & Twaki, 2015). Thus, mathematics textbooks play a significant role in conveying mathematical knowledge and competencies including mathematical literacy (Tall, Smith, & Piez, 2008). Hence, textbooks weaknesses in their inclusion of MLP may also cause impediments in the development of students' skills of handling the problems that they confronted in daily life.

Textbooks from all countries have all provided more chances for experiencing MLP in TBSQs. Particularly, Singaporean and Australian fifth-grade math

textbooks are provided no worked examples progressing whole MLP. Worked examples help to understand the nature of mathematics with their effective use in demonstrating different methods in complex tasks, developing concepts, and showing relationships in explanation and proof (Liz et al., 2006). From this point of view, the contributions of worked examples to mathematical understanding cannot be denied. It is stated that well-structured examples support students' problem-solving skills by providing meaningful, visually rich solution steps that will enable students to achieve this goal with a specific goal to be achieved (Atkinson, Derry, Renkl, & Wortham, 2000; Shen & Tsai, 2009; Van Loon-Hillen, Van Gog, & Brand-Gruwel, 2012).

Turkish fifth-grade textbooks provide more opportunities for their students to engage in each MLP in their real-life problems. Turkish textbooks also have more problems demanding to go through of MLP cycle. However, the PISA test score indicates Turkish students' comparatively low performance in mathematical literacy items than those from Australia and Singapore. In a study comparing contents and problem-solving requirements in American and Korean textbooks, Hong and Choi (2014) indicated that neither Korean nor American students' performances on the international assessments might be caused by the textbook itself. An explanation for this outcome can be the "teacher effect." Love and Pimm (1996, p. 398) suggest that "the teacher normally acts as a mediator between the student and the text" whose interpretations are "based not only on her constructions of the intention of the author but on her accumulated experience of teaching". Previous studies pointed out that higher student learning occurs when mathematical tasks requiring higher level cognitive demands are presented on a regular basis (Stigler & Hiebert, 2004; Tarr et al., 2008). The key element to enhance student achievement in the international assessments may be to use textbooks that include more problems with higher level cognitive demands along with appropriate implementation of the problems in the classroom environment by teachers (Hong & Choi, 2014). Thus, it would be suggested to investigate the appropriate practices of the mathematical literacy by focusing on teachers' implementations, which is again a key role in implementing the intended curriculum. Are teachers including Turkish able to incorporate whole MLP cycle? In-depth analysis focusing on the implementation of MLP practices in the classroom environment will give us better insights toward international assessment results.

The absence of providing necessary problem-solving opportunities in a range of different types in the textbooks may cause students to not solve specific types of problems (Riley, Greeno, & Heller, 1983). Moreover, Sweller, Chandler, Tierney and Cooper (1990) also indicated that mathematics curriculum, such as textbooks, should also include these problem-solving opportunities to construct students' conceptual appreciations of problem structures. American National

Council of Teachers of Mathematics (NCTM, 2000) also highlighted the necessity of students' exposure with diverse mathematical problems in the mathematics learning environment. Thus, one implication this study can make is that Singaporean and Australian textbook creators should include more sufficient practices of the whole MLP cycle in their mathematical problems to make sure students acquire the principal latent components of the problems.

3.1 Limitations

There existed some limitations in this study. First, the present study only included one textbook from each country while these textbooks were representing the authenticity of the other textbooks in these countries. Second, this study examined the opportunities of mathematical literacy only provided in the textbooks while the actual implications of these opportunities may differ across classrooms in these countries.

3.2 Implications

For future studies, we suggest that not only problems but also the content of the textbooks should be included in the analysis of the textbooks to provide a wider view of the mathematics literacy demands in them. Although Turkish textbooks provide more opportunities for improving mathematics literacy competencies, students from Turkey lack behind of the students from Australia and Singapore according to the international assessment results, such as TIMSS and PISA (Mullis, Martin, Foy, & Hooper, 2016; OECD, 2019). Studies utilizing different methodologies to investigate teachers' utilization of textbooks in mathematics teaching would contribute on validating the results obtained in the current study.

References

Alajmi, A. H. (2012). How do elementary textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait. *Educational Studies in Mathematics*, *79*(2), 239-261.

Andrews, P., Ryve, A., Hemmi, K., & Sayers, J. (2014). PISA, TIMSS and Finnish mathematics teaching: An enigma in search of an explanation. *Educational Studies in Mathematics*, *87*, 7-26.

Atkinson, R. K., Derry, S. J., Renkl, A., & Wortham, D. (2000). Learning from examples: Instructional principles from the worked examples research. *Review of Educational Research*, *70*(2), 181-214.

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is or might be the role of curriculum materials in teacher learning and instructional reform? *Educational researcher*, *25*(9), 6-14.

Bao, J. (2004). A comparative study on composite difficulty between new and old Chinese mathematics textbooks. In L. Fan, N.-Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics* (pp. 208-227). Singapore: World Scientific.

Brändström, A. (2005). *Differentiated tasks in mathematics textbooks: An analysis of the levels of difficulty* [Doctoral dissertation]. Luleå Tekniska Universitet.

Breakspear, S. (2012). The policy impact of PISA: An exploration of the normative effects of international benchmarking in school system performance. *OECD Education Working Papers, No. 71.* OECD Publishing.

Cohen, L., Manion, L., & Morrison, K. (1994). *Educational research methodology*. Athens: Metaixmio.

de Lange, J. (2003). Mathematics for literacy. In B. Madison & L. Steen (Eds.), *Quantitative literacy: Why numeracy matters for schools and colleges* (pp. 75-89). New Jersey: The National Council on Education and the Disciplines.

Dobbins, M. (2010). Education policy in New Zealand - Successfully navigating the international market for education. In K. Martens, N. Alexander Kenneth, M. Windzio, & A. Weymann (Eds.), *Transformation of education policy*. Basingstoke: Palgrave.

Dole, S., & Shield, M. (2008). The capacity of two Australian eighth-grade textbooks for promoting proportional reasoning. *Research in Mathematics Education*, *10*(1), 19-35.

Fan, L. (1998). Applications of arithmetic in the United States and Chinese textbooks: A comparative study. In G. Kaiser, E. Luna, & I. Huntly (Eds.), *International comparison in mathematics education* (pp. 151-162). London: Falmer Press.

Fan, L., & Zhu, Y. (2000). Problem solving in Singaporean secondary mathematics textbooks. *The Mathematics Educator*, *5*(1/2), 117-141.

Gatabi, A. R., Stacey, K., & Gooya, Z. (2012). Investigating grade nine textbook problems for characteristics related to mathematical literacy. *Mathematics Education Research Journal*, *24*(4), 403-421.

Garner, R. (1992). Learning from school texts. *Educational Psychologist*, *27*(1), 53-63.

Gu, L., Huang, R., & Marton, F. (2004). Teaching with variation: An effective way of mathematics teaching in China. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 309-345). Singapore: World Scientific.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 524-549.

Hirsch, C. R. (2007). *Perspectives on the design and development of school mathematics curricula*. Reston: National Council of Teachers of Mathematics.

Hopkins, D., Pennock, D., Ritzen, J., Ahtaridou, E., & Zimmer, K. (2008). *External Evaluation of the Policy Impact of PISA*. OECD doc. EDU/PISA/GB(2008)35/REV1. Paris: OECD.

Hong, D. S., & Choi, K. M. (2014). A comparison of Korean and American secondary school textbooks: The case of quadratic equations. *Educational Studies in Mathematics*, *85*(2), 241-263.

Huntley, M. A., & Terrell, M. S. (2014). One-step and multi-step linear equations: A content analysis of five textbook series. *ZDM, 46*(5), 751-766.

Kjærnsli, M., & Lie, S. (2004). PISA and scientific literacy: Similarities and differences between the Nordic countries. *Scandinavian Journal of Educational Research, 48*, 271-286.

Li, Y. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education, 31*(2), 234-241.

Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics, 52*(1), 29-55.

Liz, B., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in mathematics education. In *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 126-154).

Love, E., & Pimm, D. (1996). This is so: A text on texts. *International Handbook of Mathematics Education, 1*, 371-409.

Martens, D. K., Nagel, A. K., Windzio, M., & Weymann, A. (2010). *Transformation of Education Policy*. Basingstoke: Palgrave.

Mason, J., & Johnston-Wilder, S. (Eds.). (2004). *Fundamental constructs in mathematics education*. Psychology Press.

Mayer, R. E., Sims, V., & Tajika, H. (1995). Brief note: A comparison of how textbooks teach mathematical problem solving in Japan and the United States. *American Educational Research Journal, 32*(2), 443-460.

MoNE. (2017). *Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. Sınıflar).* Ankara: National Education Publishing.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Mathematics*. Retrieved from: Boston College, TIMSS & PIRLS International Study Center.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Nicol, C. C., & Crespo, S. M. (2006). Learning to teach with mathematics textbooks: How preservice teachers interpret and use curriculum materials. *Educational Studies in Mathematics, 62*(3), 331-355.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. Galbraith, H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education, the 14th ICMI study* (pp. 3-32). New York: Springer.

OECD (2007). *Programme for international student assessment (PISA) 2006: Science competencies for tomorrow's world*. OECD Publishing.

OECD (2019). *PISA 2018 Assessment and Analytical Framework.* Paris: PISA, OECD Publishing. https://doi.org/10.1787/b25efab8-en.

O'Keeffe, L., & O'Donoghue, J. (2015). A role for language analysis in mathematics textbook analysis. *International Journal of Science and Mathematics Education, 13*(3), 605-630.

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *Zentralblatt for the Didactics of Mathematics, 33*(5), 158-175.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of educational research*, *75*(2), 211-246.

Riley, M. S., Greeno, J. G., & Heller, J. I. (1983). Development of children's problem solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153-196). New York: Academic Press.

Shen, C. Y., & Tsai, H. C. (2009). Design principles of worked examples: A review of the empirical studies. *Journal of Instructional Psychology*, *36*(3), 238-245.

Sullivan, P., Clarke, D., & Clarke, B. (2012). *Teaching with tasks for effective mathematics learning* (Vol. 9). Springer Science & Business Media.

Sun, X. (2011). ''Variation problems'' and their roles in the topic of fraction division in Chinese mathematics textbook examples. *Educational Studies in Mathematics, 76*(1), 65-85.

Stacey, K., Almuna, F., Caraballo, R. M., Chesné, J. F., Garfunkel, S., Gooya, Z., ... & Perl, H. (2015). PISA's influence on thought and action in mathematics education. In *Assessing Mathematical Literacy* (pp. 275-306). Springer, Cham.

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership, 65*(1), 12-16.

Sweller, J., Chandler, P., Tierney, P., & Cooper, M. (1990). Cognitive load as a factor in the structuring of technical material. *Journal of Experimental Psychology: General, 119*, 176-192.

Takayama, K. (2008). The politics of international league tables: PISA in Japan's achievement crisis debate. *Comparative Education*, *44*(4), 387-407.

Tall, D., Smith, D., & Piez, C. (2008). Technology and calculus. *Research on technology and the teaching and learning of mathematics*, *1*, 207-258.

Tarr, J. E., Reys, R. E., Reys, B. J., Chavez, O., Shih, J., & Osterlind, S. (2008). The impact of middles grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education, 39*, 247-280.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks.* Dordrecht, Netherlands: Kluwer Academic Publisher.

Van Loon-Hillen, N., Van Gog, T., & Brand-Gruwel, S. (2012). Effects of worked examples in a primary school mathematics curriculum. *Interactive Learning Environments, 20*(1), 89-99.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM Mathematics Education, 50*(5), 827-838.

Vincent, J., & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal*, *20*(1), 82-107.

Weiss, I. R., Knapp, M. S., Hollweg, K. S., & Burrill, G. (Eds.) (2002). *Investigating the influences of standards: a framework for research in mathematics, science, and technology education*. Washington: National Academy Press.

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics*, *89*(1), 41-65.

Wu, M. (2009). A comparison of PISA and TIMSS 2003 achievement results in mathematics. *Prospects*, *39*(1), 33.

Zakka, Z. M., Oluyemi, S., & Twaki, G. (2015). Analysing gender representation in primary 5 and 6 mathematics textbooks in Nigeria. *International Journal of Educational Studies*, *2*(2), 109-114.

Zhu, Y., & Fan, L. (2006). Focus on the representation of problem types in intended curriculum: A comparison of selected mathematics textbooks from Mainland China and the United States. *International Journal of Science and Mathematics Education*, *4*(4), 609-626.