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Same Mathematical Structure, Different Design: How Does Task Format Affect Creative Problem-Posing Performance?

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Abstract:

Introduction: The purpose of the study was to investigate the effect of task format on pre-service mathematics teachers' creative problem-posing performance.

Methods: In this quantitative study, a figural and a written pattern related to daily life with the same mathematical structure were presented to participants and they were asked to write as many problems as they could. The problems were analyzed based on whether they were viable and then on the components of fluency, flexibility, and originality of creativity.

Results: The results of the study indicated that, although the flexibility and originality scores in the figural pattern were higher, the only statistical difference was observed in the originality component. Moreover, it has been found that some of the participants wrote similar problems in both tasks; however, the problems in the figural pattern were inclined to be more difficult.

Discussion: Developing students' creative-thinking skills is a main purpose of mathematics education research (Mann, 2006). However, the results of the research have indicated that the teachers' knowledge of how to develop students' creative-thinking skills is still lacking (Shriki & Lavy, 2012) and, consequently, students are provided with few opportunities to experience creative thinking and learning in class (Silver, 1997; Sriraman, 2005). Problem posing has a close association with creativity as well as with problem solving (Haylock, 1997; Silver, 1997). Therefore, to develop creative-thinking skills, learning environments should be enriched with problem-posing tasks. The results of this research do not assert that only one of the figural or written pattern types must be preferred to stimulate creative-thinking skills in the context of problem posing.

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Instead, the results emphasize that each of the written and figural patterns has its own strengths.
Limitations: The fluency, flexibility, and originality scores are affected by the sample size. Conducting similar studies on larger samples may provide more valid conclusions about possible differences. This study has taken two different task formats into account as follow; being in written form related to daily life or including figural patterns. Figural patterns can also be structured based on whether they explicitly provide the pattern rule (Barbosa & Vale, 2016). The effect of these types of problem-posing tasks on creativity components is another case that can be investigated.
Conclusions: Considering these results together, while including problem-posing tasks for both written and figural patterns to improve the flexibility and originality components of creativity is supported, the use of problem-posing tasks for figural patterns may be further recommended.

Key words: problem posing, creativity, task presentation, pattern, pre-service mathematics teachers.

Introduction

In several countries, recommendations for the reform of school mathematics have suggested an important role for problem posing (Silver, 2013). For example, the National Council of Teachers of Mathematics (NCTM, 2000) suggested that teachers should "ask students to formulate interesting problems based on a wide variety of situations, both within and outside mathematics" (p. 257). Kilpatrick (1987) emphasized the importance of problem posing by stating that "problem formulating should be viewed not only as a goal of instruction but also as a means of instruction" (p. 123). One of the main reasons for the importance attributed to problem posing is its relationship with creative thinking skills (Harpen & Sriraman, 2013; Haylock, 1997; Leung, 1997). Creativity that is seen as synonymous with divergent thinking (Cropley, 2006) is the ability to generate information or ideas from given information or ideas, where the emphasis is on the quantity and quality of the outputs (Balka, 1974). Problem posing is an open-ended activity (Silver, 1995). Therefore, participants will be able to write a wide variety of problems based on their knowledge and experience. Since these problems will provide insight into their divergent thinking skills, problem posing can be seen as an assessment tool in determining creative thinking skills (Leung, 1997).

Cai, Hwang, Jiang, and Silber (2015) have suggested a series of research problems to be answered through problem posing. One of these problems is the following: "How do different characteristics of problem situations affect

subjects' problem posing?" (pp. 9-10). The limited number of studies conducted in this context indicated that the presentation of the problem-posing tasks in visual and written (Işık & Kar, 2011; Chapman, 2012), free and structured (Silber & Cai, 2017), or with and without quantitative data (Leung & Silver, 1997) formats affects the participants' problem-posing performance (writing mathematically valid or arithmetically and semantically complex problems). This research aims to take the contributions of these studies one step further and thus examines the effect of the task format on creative problem-posing performance. More specifically, pre-service mathematics teachers were asked to write problems on patterns with the same mathematical structure (i.e. a growing pattern that grows by a fixed number) but presented in different formats, and their problems were compared on the fluency, flexibility, and originality dimensions of creativity.

The results of this study may offer some contribution to mathematics education research. First, we still have an inadequate understanding of what tasks develop creativity thinking, how they contribute to creative thinking ability, how they can be integrated into learning environments, and how the outcomes of these tasks can be judged (Novotná, Verbovanec, & Török, 2013; Shriki, 2010; Silver 1994; Singer & Voica, 2017). Thus, the results of this study can shed light on the choice of tasks to be used in research aiming at developing creativity thinking skills. Second, differences can be seen in the participants' fluency, flexibility, or originality scores due to the problem-posing task format. Thus, determining how the problem-posing task format affects these components may contribute to a more careful evaluation of the participants' creative problem-posing performance. Last, since problem posing generates more diverse and flexible thinking and encourages children to take greater responsibility for their learning, students will gain various benefits from problem posing (English, 1997). In addition, as many researchers (Crespo & Sinclair, 2008; Lee, Capraro, & Capraro, 2018; Stickles, 2011) emphasize, teachers' competency and knowledge in using and teaching problem posing are affective on students' problem-posing ability. Therefore, in-/pre-service mathematics teachers should be able to demonstrate these skills to develop students' creative problem-posing skills. Given this, the results of this study will also be able to give an idea about the ability of their creative problem-posing performance.

1 Theoretical framework

1.1 Creativity and problem posing

Creativity is highlighted as a basic ability in mathematics documents (e.g. NCTM, 2000) that needs to be developed in students. Creativity is associated with mathematical thinking (Singer, Pelczer, & Voica, 2011) and problem posing and solving (Leung, 1997; Silver, 1997) and offers opportunities to develop

students' mathematical understanding (Mann, 2006; Shriki, 2010). Due to its complex nature, creativity has been defined, characterized, and interpreted in various ways based on researchers' points of view (Ayllon, Gomez, & Balleste-Claver, 2016; Treffinger, Young, Selby, & Shepardson, 2002). For instance, creativity is the ability to engage with given material and produce new ideas, where the quality and quantity of the outputs are emphasized (Balka, 1974). As Vale, Pimentel, and Barbosa (2018) found, creativity involves abilities such as challenging assumptions, breaking boundaries, recognizing patterns, seeing things in new ways, imagining, communicating, and solving and inventing problems. In these and other definitions and explanations of creativity, the most common feature that researchers point out is producing something new.

At the beginning of the twentieth century, creativity was considered, with a narrow perception, as a common characteristic feature of geniuses who revolutionized their fields. Over the course of time, such thinking started to shift. Contemporary approaches define creativity as the tendency of creative thinking and behaving, a perspective that provides stronger bases for educational applications (Silver, 1997). On this basis, attention is drawn to the difference between professional-level and school-level creativity. Sriraman (2005) stated that students are not expected to bring innovations in the field of mathematics as mathematicians did, but they are able to offer a new perspective of interpretation or solution to a math problem. Similarly, Leikin (2011) associated creative thinking at the school level with finding different solutions and ideas, establishing various mathematical connections, applying different techniques, and original thinking.

When the explanations of creativity are examined, students are asked to produce various ideas and solutions. Therefore, activities that support creative thinking skills should enable different solutions or explanations. Accordingly, open-ended mathematical tasks are widely used in the determination and development of creative thinking skills, as this allows the production of many solutions or answers (Chamberlin & Moon, 2005; Know, Park, & Park, 2006; Silver, 1994; Vale et al., 2018). Problem posing is seen as an open-ended task (Haylock, 1997; Pehkonen, 1995) and is accepted as an important didactic tool in measuring and fostering mathematical creativity (Cai et al., 2015; Moore-Russo & Demler, 2018). As Leung (1997) pointed out, "given the 'creating a problem' characteristic of problem posing and the 'bring into something new' nature of creativity one might see problem posing as a kind of creativity" (p. 81). In this study, the problem-posing definition made by Stoyanova and Ellerton (1996) was adopted because pre-service mathematics teachers were asked to write problems based on their knowledge and experience. This definition is as follows: "Problem posing will be defined as the process by which, on the basis of

mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems." (p. 518)
The three primary criteria used in judging creative productions in problem-solving and -posing tasks are fluency, flexibility, and originality (Guilford, 1950; Torrance, 1988). First, fluency represents the number of solutions in a problem-solving context while representing the number of problems in a problem-posing context. Second, flexibility refers to the total number of different productions (different solution strategies or problems with different mathematical structures) produced in problem-solving or problem-posing tasks. Vale et al. (2018) indicated that fluency skill is very important because the first step to problem solving or generating anything creative is having as many ideas as possible to choose from. Balka (1974) found that a high fluency score does not obviously indicate a high creative ability; however, a high fluency score accompanied by a high flexibility score may give a better indication of creative ability in mathematics. Last, the recognition of originality is decisive in determining creativity (Carreira & Amaral, 2018). For example, Vale, Pimentel, Cabrita, Barbosa, and Fonseca (2012) have identified originality as the ability to think in an unusual way and as associated with the uniqueness of the ideas. Thus, originality is the ability to create fresh, unique, unusual, totally new, or extremely different ideas or products. In this respect, originality is seen as the pinnacle of creativity (Vale et al., 2018). Conversely, some other researchers have considered originality as a feature of the sample and identified it as the rareness of the responses given in the sample. This definition also provides an opportunity for statistical analysis. For example, Leikin (2009) accepted problems posed by less than 15% of participants as original. However, many other researchers accepted problems posed by less than 10% of participants as original (Kar, Özdemir, Öçal, Güler, & İpek, 2019; Harpen & Sriraman, 2013; Yuan & Sriraman, 2010). In such circumstances, the posed problems considered original do not have to be mathematically complex. In this study, in order to make statistical comparisons, the originality component was used in the sense of the rareness of the responses, and the lower 10% level was embraced as a criterion.

1.2 Task formats and problem-posing performance

Researchers have used many different types of problem situations to investigate problem posing, ranging from simply deleting a question statement from a textbook problem to more open-ended problem situations (Cai et al., 2015). Research using different problem-posing tasks indicates that there are various factors that affect problem-posing performance. One such factor is task format. Different meanings are attributed to the task format. For example, some research examines the impact of the task directive on problem-posing performance (Chen,

Van Dooren, Chen, & Verschaffel, 2007; Ellerton, 1986; Lowrie & Whitland, 2000). In such studies, the participants are asked to pose problems for their friends, generate simple, medium, and difficult problems, and pose problems for students from different grade levels. For example, Ellerton (1986) explained that the directive "pose a difficult problem for your friends" was intended to help the students project their thinking beyond themselves and, thus, affect their problem-posing performance. Lowrie and Whitland (2000) asked third-grade students to pose problems for second- and fourth-grade students. As expected, students posed arithmetically more complex problems for fourth-grade students.

One of the basic elements of problem posing is the understanding of the task's mathematical structure (Brown & Walter, 2005). Freudenthal (1991) defined that "structure is a form abstracted from its linguistic expression" (p. 20). Participants who pose problems should be able to understand the mathematical structure of the tasks and use these structures for the problems they will pose. The complexity of the mathematical structure is a factor affecting problem-posing performance (Leung & Silver, 1997; Stickles, 2011). In her study aiming to investigate pre- and in-service mathematics teachers' problem-posing performances based on a given problem, Stickles (2011) indicated that the complexity of the data used in the task has an effect on problem-posing performance.

Stoyanova and Ellerton (1996) classified problem-posing tasks as structured, semi-structured, and free situations. Some research studies point out that these types of situations are another factor affecting participants' performance in writing mathematically valid problems (Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005; Silber & Cai, 2017). For example, Silber and Cai (2017) found that pre-service mathematics teachers posed more semantically complex problems in structured problem-posing tasks (e.g. "in the Road-Trip Open-Ended Task, pose a problem that has '1130 miles' as the answer") compared to those posed in free tasks (e.g. "pose a problem based on the Road-Trip Open-Ended Task") aimed at measuring the same ability. The researchers conjectured that the main reason for this difference was the mathematical relations were considered more thoroughly in the structured task.

Each type of representation gives an opportunity to look at the relationship between the data from a different perspective (Ainsworth, 2006; Knuth, 2000). Problem-posing tasks can also be designed to include different representation types (Cai, Jiang, Hwang, Nie, & Hu, 2016), and this has been found to affect participants' problem-posing performance (e.g. Işık & Kar, 2011; Cai et al., 2013; Chapman, 2012). In this regard, problem-posing tasks can be presented in a way that they are related to daily life or involve a figure, graphic, or table. There are different views regarding the effect of such task formats, which are also taken into consideration in this study, on problem-posing performance.

Knuth (2000) indicated that graphics provide direct information by presenting infinitely many points while symbolic representations do this indirectly. Accordingly, problem-posing performance would be expected to be higher in tasks involving these types of representations. For example, Cai et al. (2013) found that eleventh-grade students had more difficulty with problem posing for an equation system with two unknowns compared to a linear graph with one unknown. In another study, Işık and Kar (2011) reported that pre-service mathematics teachers showed a lower performance in problem posing through visual representations compared to open-ended written stories having the same mathematical structure. They hypothesized as the main reason for this situation that the interpretation of visual representations would require higher-level cognitive skills. Chapman (2012), conversely, presented the picture task, one of several different problem-posing tasks, to a similar sample (a picture reflecting the comparison meaning of the subtraction operation). However, in the posed problems, the focus of the participants' interpretations was more on the meaning of pairing. Chapman (2012) indicated that they focused more on the picture's context instead of on its purpose when posing problems. Similarly, while emphasizing the importance of context in problem posing, English (1997) also indicated that it is necessary to focus primarily on the mathematical structure of the tasks. In this respect, focusing only on the problem's context without analyzing its mathematical structure would also negatively affect problem-posing performance.

When the aforementioned studies are investigated, it is understood that the presentation of the tasks has an effect on problem-posing performance. These studies commonly attribute the meaning of writing task-related, mathematically solvable, or arithmetically and semantically more complex problems to problem-posing performance. Bonotto and Santo (2015) further indicate that "problem posing is a form of mathematical creation: the creation of mathematical problems in a specific context" (p. 105). However, these studies do not provide information about whether task format has an influence on creative problem-posing performance. Additionally, there are various studies examining the relationship between problem-posing performance and creative thinking ability (e.g. Ayllon, et al., 2016; Harpen & Sriraman, 2013; Singer, et al., 2011; Yuan & Sriraman, 2010). Although these studies do not focus on the effect of the task format on creative problem-posing performance, it is still possible to draw some conclusions from them. First, it was emphasized that creativity skill has a stronger relationship with the flexibility and originality components compared to the fluency component. Silver and Cai (2005) indicated that fluency scores can be seen as an insignificant way to evaluate students' creativity. Contrastingly, originality is the dimension that receives the most emphasis among all the components of creativity (Kontorovich, Koichu, Leikin, & Berman, 2011). Kar

et al. (2019) analyzed the problems posed by pre-service mathematics teachers for a constantly growing visual pattern task that requires the arrangement of blocks according to the fluency, flexibility, and originality components of creativity. It was found in this study that the originality component predicts problem-solving performance more strongly. In this visually presented pattern, various problems could be written about different situations such as the surface area or volume of the blocks. Therefore, it was highly possible for a participant to pose many problems with different mathematical structures. This situation may result in higher flexibility and originality scores instead of fluency scores.

Second, studies particularly conducted with students in younger age groups emphasize that they pose a limited number of problems with a similar mathematical structure for visually presented patterns. For example, Cai (1998) asked sixth-grade Chinese and American students to pose problems for a figural pattern based on an array of dots (having a mathematical structure such as 6, 9, 12,…). The research results found that 80% of the problems devised in both groups were aimed at the first four given figures in the task. In addition, more than 68% of the students wrote problems comparing the number of dots in the figures. These findings indicate a lower performance in all creativity components. Besides the students' lack of pattern knowledge, their lack of experience in problem posing might be one of the reasons for these results. Another possible reason for this situation is that students might be affected by the pattern problems solved in the mathematics lessons. Heavily preferring the problems solved in the lessons might lead to lower scores, especially in the flexibility and originality components of creativity. For example, Rosli et al. (2015) analyzed the effect of problem-posing and -solving order on pre-service middle school teachers' problem-posing and -solving performances on a figural pattern (a ladder pattern with the mathematical structure of 1, 3, 6, 10,…). The research findings showed that most problems posed by the group that previously took the problem-solving test were similar to the problem structures in the problem-solving task (drawing any step in the pattern, asking the number of blocks corresponding to any step, asking the general pattern rule).

Last, in some research (Cai & Hwang, 2003; Xu, Cai, Liu, & Hwang, 2020), figural and daily life-related patterns are used together, and participants' problem-posing performance is investigated. However, these studies have not been directly structured on the differences in the fluency, flexibility, and originality aspects of creative thinking skill, based on the problems posed for the figural and written patterns. Moreover, the patterns used in these studies have different mathematical structures. Despite these limitations, these studies provide different results about the structure of the posed problems for figural and daily life-related patterns. For example, Cai and Hwang (2003) asked students in the sixth and seventh grades to pose problems for figural (a pattern including an

array of dots in a square and growing in a quadratic form) and written patterns related to daily life (the doorbell situation growing by a fixed number in the order of 1, 3, 5, 7…). As the research found, more problem categories were identified in the written than in the figural pattern. Consequently, it can be hypothesized that the higher number of problem categories in the written pattern indicated that the flexibility score would be much higher compared to that in the figural pattern. However, Xu et al.'s (2020) analysis of the problems that middle school students posed for figural (the pattern growing in a quadratic form and including an array of dots) and written patterns related to daily life (the doorbell situation with the mathematical structure of 1, 3, 5, 7,…) indicated that the higher number of problem types in the figural pattern demonstrated that the students' flexibility scores would be higher in the figural pattern compared to those in the verbal pattern. Furthermore, the rates of the problems that were written by adding new situations were 2% and 4%, respectively, in the figural and written patterns, which indicate a more difficult aspect compared to other problem types. These results assume that the written pattern provided an opportunity to produce more original problems.

2 Methods

2.1 Participants

In this research, 45 junior pre-service mathematics teachers who were studying middle school mathematics teaching program participated. Their curriculum involves Mathematics Teaching I and II courses, which focus on how to teach middle school mathematics topics to students and are given in the fall and spring semesters of the third grade. The purpose of this study was explained to 65 participants, in two sections, who were taking a Mathematics Teaching II course. Of them, 49 voluntarily agreed to participate in the study. However, four did not participate in either one of the two meetings where the problem-posing tests were held. Thus, these students' responses were not included in the analyses.

In the teacher training program, the participants are provided with a four-year education. Those who complete the program are certified to teach mathematics to middle school students (11-14 age groups). The program focuses on how to teach mathematics and includes courses on general culture and pedagogical and mathematical content knowledge. The participants take two courses in instructional methods for mathematics during their third year in the program. Furthermore, within the framework of these courses, training on problem posing and solving is provided, including training on the definition and importance of problem posing and its association with problem solving, and problem-posing types. Within the scope of these courses, Polya's (1957) problem-solving steps are introduced to the participants via sample problems. Problem-posing activities were also conducted to expand the problems in the looking-back step, the last

step of the problem-solving process. In addition, in the looking-back step, participants were encouraged to solve problems with different strategies. There were also some discussions held on the problems posed or solved by participants. As it is well known, such activities support creative thinking skills, although the participants did not take a course focusing only on creativity. In this paper, pseudonyms have been used for each participant.

2.2 Data collection tool and analysis

In order to determine the effect of the task format on pre-service mathematics teachers' creative problem-posing performances, a Problem-Posing Test (PPT) consisting of two parallel pattern tasks was created (see Figure 1). There are two main reasons for choosing the patterns subject. First, pattern tasks can be presented in different formats by relating them to daily life or visual shapes. In this respect, the patterns topic has been chosen since it allows preparing various tasks in different formats. Second, patterns tasks were widely used in the studies investigating the relationship between problem posing and creativity or between problem posing and problem solving (e.g. Cai, 1998, 2003; Kar et al., 2019; Rosli et al., 2015; Silber & Cai, 2017), because patterns tasks provide opportunities to include different mathematical concepts in the problems to be posed. In this respect, they can provide in-depth information about students' flexible thinking skills. The pattern rule in both tasks is in the format of $a_n=3n+1$, where the n-term refers to term order. Additionally, the curricula of the fifth to eight grade levels (11 to 14 age groups), where pre-service mathematics teachers would teach, mainly emphasize the teaching of a constantly growing pattern that grows by a fixed number (Ministry of National Education [MNE], 2018). In this context, the data collection tool was limited to such pattern types in which the participants had more experiences and would teach in the future.

The patterns used in the studies focusing on problem posing were generally presented in two different formats; daily life related or figural (Cai & Hwang, 2003; Harpen & Sriraman, 2013; Kar et al., 2019; Rosli et al., 2015; Silber & Cai, 2017). In these studies, these two variables were taken as the basic criteria that determine the task formats. In this respect, two patterns tasks were prepared. First pattern task was related to daily life but did not involve figural shape. On the other hand, the second task involved figural shape, but did not involve a daily life story. First, a pattern related to daily life was prepared and then a figural pattern suitable for mathematical structure of the first pattern was prepared. In the first PPT task, an open-ended story associated with daily life, which discussed the number of guests coming to a party every time a doorbell rings, was included. Cai and Hwang (2003) used this task to investigate sixth grade Chinese and American students' problem-posing skills. This task has also widely been used in other problem-posing studies related to patterns (e.g. Cai, 2003;

Silber & Cai, 2017; Xu et al., 2020; Yuan & Sriraman, 2010). Similarly, the figural pattern task in the PPT, which discusses an array of matchsticks in a square, has been widely used in problem-solving tasks (e.g. Rivera & Becker, 2007, 2008).

Party Task (PT)

Kübra had a birthday party and guests enter when the doorbell rings.
The first time the doorbell rang, 4 guests entered.
The second time the doorbell rang, 7 guests entered.
The third time the doorbell rang, 10 guests entered.

The pattern continued in the same way. At every following doorbell ringing, three more people came in than those coming at the previous doorbell ringing.

Write as many problems as you can for the open-ended story above.

Matchstick Task (MT)

The first three terms of a figural pattern, which has been structured with matchsticks, are given below. Write as many problems as you can for the following pattern.



1. Term 2. Term 3. Term

Figure 1. Problem-posing tasks.

The tasks in the PPT were given to pre-service mathematics teachers at different times, with the PT given first and the MT one week later. They were allowed 40 minutes for each task. The problems posed by the participants for both tasks were analyzed through inductive and deductive content analyses (Strauss & Corbin, 1998). In the problem posing tasks, firstly, the participants were asked to pose mathematically valid problems which should be related to given tasks. In this context, the problems were coded as viable and nonviable (see Figure 2) in line with the analysis schemas found in the literature (e.g. Harpen & Sriraman, 2013; Silver & Cai, 2005). The authors of this study, two coders, independently analyzed whether the posed problems were viable or nonviable. Then, the coders compared these analyses and the coding process completed. In light of the total completed analysis, it was determined that some responses were not pattern questions, the solution could not be found by the current data's use, the quantitative data in the problem were not consistent, and the problem was not clear enough. These responses were coded as nonviable and sample responses of these situations are given in the findings section.

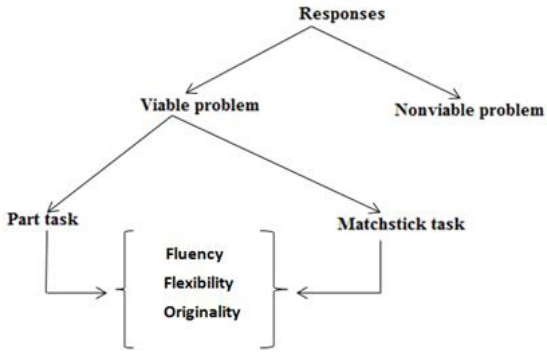


Figure 2. Schema related to the analysis of the problems.

In the second phase, viable problems were analyzed based on the categories of fluency, flexibility, and originality (see Figure 2). The fluency score was determined by calculating the total number of every pre-service mathematics teacher's viable problems. For example, if a participant was unable to write a viable problem in the PT, and another participant managed to write 5 viable problems, their fluency scores were coded as 0 and 5 points, respectively.

The flexibility score was calculated by considering the number of different problem types. According to Silver (1997), flexibility refers to "shifts in approaches taken when generating responses to a prompt" (p. 76). This definition is considered as a reference point and problem categories were determined accordingly. At the same time, in the analysis process, researchers benefitted from categories determined in various research about problems posing with respect to patterns (Cai, 2003; Cai & Hwang, 2003; Xu et al., 2020). For instance, Cai (2003) analyzed problems posed by 4th - 6th grade students about figural patterns and he determined dots in one figure, dots in more than one, comparing number of dots, and draw figure categories. Xu et al. (2020), on the other hand, determined number of guests, adding conditions, and general pattern about ring number or guests for PT. Considering these studies, each coder determined problem types. Problems asking different situations, problems including different mathematical concept, rule or operation were evaluated in different categories. Some pre-service mathematics teachers just only changed the numerical data and posed the same type of problems (sample problems are presented under findings section). This type of problems had the same structure and thus, they were joined under the same problem type. Coders, then, compared the types of problems they determined and finalized the classification. The pre-service mathematics teachers' flexibility scores were calculated by considering

how many problem types were included. For example, if a participant presented three out of 23 problem types in the PT, his or her flexibility score will be three.

In PT and MT, problem types were categorized with respect to the structure of problems under six categories: i) asking the number of guests/matchsticks, ii) asking the pattern rule, iii) asking the ring number/figure number, iv) adding extra conditions specified in the task, v) adding extra conditions or information not specified in the task, and vi) maintenance of the pattern. Some problems posed concentrated on the calculation of the number of guests (or the number of matchsticks). In this context, the order of ringing a bell (or a figure) with respect to the number of guests (or matchsticks) or the comparison of a ringing a bell (or figures) with respect to guests (or matchsticks) were the topics of the problems posed. These problems were collected under asking the number of guests/ matchsticks category. Considering the asking the pattern rule category, problems were posed to determine the general rule of the patterns. Some pre-service mathematics teachers asked the new pattern rule by changing the number of incoming guests when the bell rings at the first time in the PT. In MT, in addition to the number of matchsticks, some participants asked the general rule of the pattern over different situations such as the number of squares in each shape. This category was expanded in a way that it covers such responses. Under asking the ring number/figure number category, the order of ringing a bell (or the order of shapes) with respect to the number of guests (or the number of matchsticks) was asked. Some pre-service mathematics teachers considered the conditions related to mathematical properties in numerical values while posing problems. For instance, the number of matchsticks (or guests) or the number of the order of ringing a bell (or a shape) is the focus issues while posing problems. These problems are categorized under adding extra conditions specified in the task category. In adding extra conditions or information not specified in the task category, the addition of a condition or information which is not seen in a pattern is the matter of this category. Lastly, some pre-service mathematics teachers posed problems which necessitated continuing a few steps of a pattern (e.g. as in MT, asking to draw a few steps of a shape). These types of problems were collected under maintenance of the pattern category. Sample answers for the problem types under these six categories are presented in findings section.

Finally, the originality of each of the responses was determined according to their rareness. To determine the originality score, the problem types observed in less than 10% of the pre-service mathematics teachers were defined. As 45 participants in total participated in the study, the problems written by five or more were not accepted as original. Thirteen and 18 problem types were judged as original in the PT and MT, respectively. Each original problem type was given 1 point to determine the participants' originality scores. For example, if a

participant presented two original problems in the MT, his or her originality score will be two.

To determine whether there was a difference between the pre-service mathematics teachers' fluency, flexibility, and originality scores in the PT and MT, a Wilcoxon test was applied, as the data were not distributed normally. Furthermore, the effect size was also calculated in the conditions where statistical differences were a matter of concern. According to Cohen's (1998) interpretation of the effect size, anything greater than .5 is large, .5-.3 is moderate, .3-.1 is small, and anything smaller than .1 is trivial. The responses to and detailed descriptions of every stage of the analysis process are presented in the findings section below.

3 Results

3.1 Distribution of the conditions where the problems posed are viable and nonviable

The pre-service mathematics teachers gave 332 responses in the PT, and the response mean per participant was 7.4. As for the MT, there were 330 responses in total, and the response mean per participant was 7.3. In the PT and MT, 9.9% (33) and 12.7% (42) of the responses were nonviable problems, respectively. Sample problems for the nonviable category are given in Table 1.

Table 1

Examples of nonviable problems

No	Problem	Task type	Nonviable
1.	It is known that the number of girls and boys in the first coming group is not equal. After how many doorbell rings will the number of girls and boys among the guests come to be equal? (Gülseren)	PT	Insufficient data
2.	The total number of guests coming to the birthday party is 4 times the number of guests arriving at the second ring. Accordingly, how many times did the doorbell ring in total? (Durmuş)	PT	Inconsistent data
3.	At which doorbell ring do 71 people enter? (Dilda)	PT	Inconsistent data
4.	Which term shape has a perimeter of 240 cm? (Aslı)	MT	Insufficient data
5.	Find the general pattern term (Fatma)	MT	Not clear enough
6.	How many matchsticks were used in the term where the number of matchsticks was prime for the first time? (Girgin)	MT	Not a pattern problem

According to Table 1, as the number of girls and boys entering at every doorbell ring in the first problem was not defined, the problem could not be solved. In this respect, the problem had insufficient data. According to the second problem in Table 1, 7 guests would enter at the second doorbell ring. In this case, 4x7=28 total of guests would come to the party. However, the total of guests at the party would never be 28 in any case. Similarly, in the third problem, at which doorbell ring do 71 people enter is asked. With reference to the pattern rule of an=3n+1, n∈N would not be provided such that 3n+1=71. Therefore, the data in the problems written by both pre-service mathematics teachers was inconsistent.
In the fourth problem in Table 1, it was asked which term was the figure with a circumference of 240 cm. The term number in the task being n, at the figure's circumference 2n+2 matchsticks would be obtained. However, this problem was impossible to solve since the length of a matchstick was not provided. Thus, the problem had insufficient data. In the table's fifth problem, the general pattern term was asked. Various pattern rules would be structured depending on the number of matchsticks, the circumference of the figure, and the number of squares in this figural pattern. Therefore, statements given in explanation of the problem were not clear enough. In the last problem, the number of matchsticks was a prime number for the first time in the second figure. The answer to this problem would be directly achieved by referencing the first three figures given in the pattern. Therefore, the problem does not require the use of the pattern in the given task.

3.2 Creativity results of the party and matchstick tasks

Fluency

In an effort to determine the fluency scores, the number of viable problems of each pre-service mathematics teacher was calculated, showing that 90.1% (299 responses) and 87.3% (288 responses) of the responses given in the PT and MT, respectively, were viable problems. The highest fluency score of a participant in the PT was 13, while the lowest was 1, resulting in an average fluency score of 6.6. The highest fluency score of a participant in the MT was 12, while the lowest was 0, resulting in an average fluency score of 6.4. According to the Wilcoxon test's results, there was no statistically significant difference between the participants' fluency scores in the tasks ($z=-.669$, $p=.503>.05$).

Flexibility

To calculate the flexibility scores, the problems created with the same mathematical structure in every task were counted as one. For instance, the two following problems structured by Dilda in the PT were counted as one problem type.

Problem 1: What is the difference between the number of guests entering at the 11th and 7th doorbell rings?
Problem 2: What is the difference between the number of guests entering at the 13th and 9th doorbell rings?

In the PT, 23 problem types were identified in the pre-service mathematics teachers' responses and were then sorted into six categories. The names of these categories and sample problems are given in Table 2. Among these six categories, most of the problem types were seen in adding extra conditions or information not specified in the task category (Table 2, for PE17-22 problem types), while the least were seen in maintenance of the pattern category (1 problem type). When considering the distribution of the number of problems, 45.2% were oriented toward asking the number of guests. Asking the ring number (20.7%) and adding extra conditions or information not specified in the task (20.4%) were the other categories for which a relatively greater number of problems were written. On the contrary, maintenance of the pattern (2%) or adding extra conditions specified in the task (3.3%) were the categories for which the least number of problems were written.

In the MT, 30 problem types were determined and were then sorted into six categories (see Table 3). According to Table 3, most of the problem types were seen in the adding extra conditions or information not specified in the task category (ME17-29), while the least were seen in maintenance of the pattern category (MF30). When considering the distribution of the number of problems in these categories, in approximately half of the 288 viable problems (46.5%), was oriented toward asking the number of matchsticks. In addition, the other two categories in which the greatest numbers of problems were posed were under asking the figure number (19.5%) and adding extra conditions or information not specified in the task categories (18.7%). Conversely, the least number of problems were written in adding extra conditions specified in the task (1.7%) and maintenance of the pattern (1.4%) categories.

In the PT, the pre-service mathematics teachers were able to write 10 problem types at most and 1 problem type at least. The average flexibility score was 4.7. In the MT, they were able to write 9 problem types at most and 0 problem types at least. The average flexibility score was 5.2. According to the results of the Wilcoxon test, no statistically significant difference between the participants' flexibility scores was determined ($z=-1.549$, $p=.121>.05$).

Originality

In the PT, 13 original problems were determined: one original problem (PA3) in the category of asking the number of guests; two original problems (PB6-7) in the category of asking the pattern rule; two original problems (PC10-11) in the category of asking the ring number; five original problems (PD12-16) in the

category of adding extra conditions specified in the task; and lastly, three original problems (PE20-22) in the category of adding extra conditions or information not specified in the task.

Table 2

Problem categories determined in the party-task.

Categories		*F(%)***	*Exemplary problem*
$PA_{1\text{-}4}$*:	Asking the number of guests	135(45,2)	How many people came to the party between the 70^{th} and 75^{th} doorbell rings? (Buket, PA_3)
$PB_{5\text{-}7}$:	Asking the pattern rule	25(8,4)	If 5 people entered at the first doorbell ring instead of 4, and other guests continued entering according to the same rule, how many people would come in at the 5^{th} doorbell ring? (Şahika, PB_6)
$PC_{8\text{-}11}$:	Asking the ring number	62(20,7)	At which doorbell ring did the 200^{th} person enter? (Karabayır, PC_{11}) If 22 people enter at a doorbell ring, what number is this doorbell? (Alçiçek, PC_9)
$PD_{12\text{-}16}$:	Adding extra conditions specified in the task	10(3,3)	If the guests already present at the party following the 17^{th} doorbell ring were sitting in threes at the tables, how many people would be standing? (Çelebi, PD_{12}) How many times does the doorbell need to be rung to make the total number of guests present at the party 10 times the order of the last doorbell ring? (Demir, PD_{16})
$PE_{17\text{-}22}$:	Adding extra conditions or information not specified in the task	61(20,4)	The first guests entered at 10:00 p.m., ringing the first doorbell. Afterward, every half-hour, a doorbell was rung. Accordingly, how many guests in total would have entered the party by 12:40 a.m.? (Mazlum, PE_{19}) How many handshakes in total would be among the guests entering following the first four doorbell rings? (Kibar, PE_{21})
PF_{23}:	Maintenance of the pattern	6(2)	Please write the first 10 terms of this pattern. (Fatma, PF_{23})

Table 3

Problem categories and types determined in the matchstick-task

Categories	*F(%)***	*Exemplary problem*
$MA_{1\text{-}6}$*: Asking the number of matchsticks	134(46,5)	What is the sum of the number of matchsticks in the 107th and 83rd figure? (Dilda, MA_4) How many matchsticks are needed to construct 10 squares? (Mazlum, MA_6)
$MB_{7\text{-}10}$: Asking the pattern rule	35(12,2)	Please determine the general rule giving the number of matchsticks at the circumference of the figure in the pattern. (Mazlum, MB_{10})
$MC_{11\text{-}14}$: Asking the figure number	56(19,5)	In the figures, diagonals are only drawn inside the squares. Accordingly, which figure contains 26 diagonals therein? (Ayşenur, MC_{14})
$MD_{15,16}$: Adding extra conditions specified in the task	5(1,7)	If the total number of matchsticks in a figure structured according to this pattern is a three-digit number, what is the smallest number of matchsticks used? (İrem, MD_{16})
ME: Adding extra conditions or information not specified in the task		
Length ($ME_{17\text{-}21}$)	23(7,9)	The length of a matchstick is 3 cm. Accordingly, how many matchsticks are used in a figure with a circumference of 450 cm? (Aslı, ME_{17}) If a matchstick is 2 cm in length, what is the diagonal length of the largest rectangle in the 11th figure? (Büşra, ME_{21})
Area ($ME_{22\text{-}24}$)	12(4,2)	If the length of a matchstick is 4 cm, how many matchsticks are used to construct a figure with an area of 320 cm^2? (Çiğdem, ME_{22})
Time ($ME_{25,26}$)	5(1,7)	In Figure 1, it takes 5 seconds for a spider to move from an edge of the figure and then return to that same edge. If the spider moves at the same speed, how many seconds does it take the spider to rotate the circumference of the 98th figure? (Aslı, ME_{25})
Figure type (ME_{27})	1(0,4)	What is the total number of rectangles in the 8th figure? (Elvan, ME_{27})
Connection number (ME_{28})	5(1,7)	Inasmuch as a flower pattern is drawn at each of the matchsticks' connection points, how many flowers are drawn in the 10th figure? (Mazlum, ME_{28})
Price (ME_{29})	8(2,8)	Inasmuch as the price of 4 matchsticks is 25 cents, how much does the 12th figure cost? (Nur, ME_{29})

MF_{30}: Maintenance of the pattern	4(1,4)	Accordingly, please draw the sixth figure (Karakurt, MF_{30})

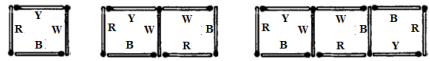
* MA1-6: It means that there are six problem types in category A.
** The data was calculated in Frequency (Percentage) over the number of viable problems.

In Table 2, PC11 is an original problem written by Karabayır. The problem asks at which doorbell ring the 200th guest comes in. Additionally, some of the original problems are as follows:
Kübra gives blue hats to the first group coming in, yellow hats to the second group, and red hats to the next group. She distributes colored hats respectively in this manner. Accordingly, if the doorbell is rung 17 times, how many hats of which color are there in this group at most? (PB7, Emine)
In the event that everyone at the party sits in fours around the tables, one person stands while they sit in threes, and no one stands, inasmuch as it is known that the total number of people is less than 80, how many people are there inside at most? (PE20, Halil)
Inasmuch as it is known that Kübra's best friend enters at either the fifth or sixth doorbell ring, what it the possibility that s/he is the 67th person to enter the party? (PE22, Seval)
In the MT, 18 problem types were considered as original: one original problem (MA4) in the category of asking the number of matchsticks; three original problems (MB8-10) in the category of asking the pattern rule; two original problems (MC13-14) in the category of asking the figure number; two original problems (MD15-16) in the category of adding extra conditions specified in the task; nine original problems (ME17, ME20-22, and ME24-28) in the category of adding extra conditions or information not specified in the task; and lastly, one original problem (MF30) in the category of maintenance of the pattern (see Table 3). Examples of original problem types which are not representatively given in Table 3 are presented in Table 4.

Table 4

Sample problems for the original category in the matchstick-task

No	*Categories*	*Exemplary problems*
1	MB_8	The squares in the figures are numbered with consecutive natural numbers as follows: the odd-numbered squares are colored in red; the even-numbered squares are colored in blue. How many red squares are there in the 22nd figure? (Mazlum) 
2	MB_9	Selin colored the matchsticks in red, blue, white, and yellow. Every figure is structured counterclockwise in the order of R, B, W, Y,

		starting from the vertical matchstick on the left. Hence, in which figure are all the colors used in equal numbers? (Emine) 
3	MD_{15}	There are 20 matchsticks in every box. How many boxes should be used to construct the 21st figure? (Seher)
4	ME_{24}	The length of a matchstick is 3 cm. If Ayşe has 93 matchsticks, how many cm^2 does the area of the last figure to be structured have? (Seval)

The greater number of problem types in the MT resulted in an increase in the number of problems in the original category. However, the fact that a problem observed in less than 10% of the pre-service mathematics teachers does not guarantee that it is complex. For instance, Seher posed a problem oriented to the division of the total number of matchsticks by 20 and discussed the interpretation of the remainder in the first problem in Table 4. The problem takes the calculation of the number of matchsticks in the 21st figure a step forward. Even though the problem is included in the original category, when considering the pre-service mathematics teachers' level, it is difficult to say that the problem is a complex one. Similarly, the fourth problem in Table 4 is concentrated on the arithmetical operations oriented to the area concept, while the first and the second problems discuss the formation of different patterns over a pattern. In Table 3, the problem written in the ME27 category asks the total number of rectangles in any one of the figures. These problem types require more advanced reasoning skills beyond the arithmetical operations.

The pre-service mathematics teachers' average originality scores in the PT and MT are 0.51 and 0.89, respectively. According to the Wilcoxon test's results, a significant difference in favor of the MT among the originality scores is determined ($z=-2.174$, $p=.03<.05$, $r=.23$). The effect level of the statistical difference among the originality scores is in a small range.

4 Discussion and conclusions

This study aimed to contribute to the problem-posing literature by investigating the effect of task format on pre-service mathematics teachers' creative problem-posing performance. Similarities and differences are determined among the problems that the participants posed in both tasks. The rates of the number of responses and the viable problems are high and close to each other for both tasks. That the average number of problems posed in the tasks is above seven shows that the participants have a high problem-posing performance. In addition, the rate of nonviable problems in each task is low and close to each other (9.9% and 12.7% in the PT and MT, respectively). Considered together, these quantitative

results show that the participants' problem-posing performances do not vary depending on the task's representation type.

Conversely, when considering the quality of the problems posed for both tasks, it is evident that some differences and similarities are remarkable. When the distributions for the fluency component of creativity are compared, the similarities become more prominent. The maximum numbers (13 and 12, respectively) and means (6.6 and 6.4, respectively) of the problems written in the PT and MT are very close to each other. The statistical results also show that there is no significant difference. Similarly, some of the studies in the literature indicate that compared to the other components, fluency is the least effective in interpreting participants' performances and does not provide adequate data by itself. As an example, in a study on the effect of the components of creativity, as reflected in problem-posing, on pre-service mathematics teachers' problem-solving abilities, it was determined that the fluency score was the least predictive component (Kar et al., 2019).

The first step in producing creative things is to generate as many different ideas as possible. However, producing many ideas does not always guarantee creative results and, therefore, fluency should be judged in conjunction with flexibility and originality (Balka, 1974; Carreira & Amaral, 2018; Vale et al., 2018). Despite the fact that no statistically significant difference was determined between the participants' flexibility scores, when the problem types are comparatively analyzed, differences are found between the tasks regarding the number of problem types. While 23 problem types were determined in the PT, 30 problem types were determined in the MT. It is understood that the MT, in this respect, has more effectively stimulated the posing of a variety of problems than the PT has because the figural patterns provided an opportunity to associate with various mathematical concepts, including area, circumference, length, angle, and probability, as well as stimulated an analysis of the relationships among the information pieces.

Asking the number of the matchsticks/guests or figure/ring were the most preferred problem types in both tasks. These types of problems have been widely emphasized in educational tasks or textbooks for patterns (Zaskis & Liljedahl, 2002). Hence, the obtained results can be associated with the pre-service mathematics teachers' learning experiences. Unfortunately, given this research's data, no detailed explanation can be given regarding the reasons for this situation. Therefore, more examinations with qualitative approaches are needed.

Differences in the types of problems have emerged, especially in the category of adding extra conditions or information not specified in the task. While this category covered six problem types in the PT, this number increased to 13 in the MT. In the PT, associations with situations such as probability, combination, time, and gift prices gained prominence, while in the MT, associations with

situations such as length, area, circumference, figural types, and number of matchstick connections were more likely. Additionally, more problem types were in favor of the MT in the categories of asking the number of matchsticks and asking the pattern rule. Consequently, the MT has provided more opportunities in creating a variety of problem types, in association with the pattern of various mathematical concepts and in thinking of different pattern types.

When looking at the results of the originality component, a statistically significant difference in favor of the MT was determined, and its effect size was small. In the MT, nine problems in the category of adding extra conditions or information not specified in the task were judged as original, while in the PT, this number was only three. In parallel to other research (Harpen & Sriraman, 2013; Yuan & Sriraman, 2010), the problem types observed in less than 10% of the participants' responses were accepted as original, although the less-observed problems are not meant to be difficult or very interesting. As such, when the qualities of the problems posed in the originality category are compared, mathematically more difficult and interesting problems were observed in the PT. However, some of the participants who posed original problems in the PT wrote original problems (for example, the problems posed by Aslı and Emine in the findings section) adopting the same approach in the MT task as well, which is another remarkable situation. In these cases, it has been observed that the problems in the MT were mathematically more difficult. Thus, it is understood that the figural pattern contributed to the participants posing mathematically more difficult and interesting problems. However, how this occurred and the way of thinking of the participants who were unable to write original problems in one task but who wrote original problems in another task still remain questions to be answered. Hence, researching such questions with qualitative approaches may provide more enlightening results, especially regarding the originality component of creativity.

Developing students' creative thinking skills is a main purpose of mathematics education research (Mann, 2006). However, the results of the research have indicated that the teachers' knowledge of how to develop students' creative thinking skills is still lacking (Shriki & Lavy, 2012) and, consequently, students are provided with few opportunities to experience creative thinking and learning in class (Silver, 1997; Sriraman, 2005). Problem posing has a close association with creativity as well as with problem solving (Haylock, 1997; Silver, 1997). Therefore, to develop creative thinking skills, learning environments should be enriched with problem-posing tasks. Since the patterns are open-ended, allow establishing an in-depth connection with a great variety of mathematical concepts, and contribute to the development of problem-solving and -posing skills as well as communication abilities in students, they also stimulate students'

creative development (Vale et. al., 2012). The results of this research do not assert that only one of the figural or written pattern types must be preferred to stimulate creative thinking skills related to problem posing. Instead, the results emphasize that each of the written and figural patterns has its own strengths. For example, the problems related to the use of the combination and probability concepts were structured in the PT, whereas these problem types were not found in the MT. However, the MT more effectively stimulated the association with a variety of mathematical concepts and, in this context, the writing of mathematically more difficult problems. Considering these results together while including problem-posing tasks for both written and figural patterns to improve the flexibility and originality components of creativity is supported, and the use of problem-posing tasks for figural patterns may be further recommended.

Since this research's data were obtained from only a small group of pre-service mathematics teachers at a university, the results cannot be generalized. However, we still believe that the outcomes we have obtained provide educational value and will make a potential contribution to studies connecting creativity and problem posing. The results of this study indicated that participants' problem-posing performances in the components of creativity differ according to the task format. Although the participants scored higher in the flexibility and originality components for the MT, a statistical difference was observed only in the originality component. Given this, the task format should be considered as an important variable when evaluating participants' problem-posing performance through the originality component. Thus, clearer evaluations can be made about participants' creativity.

Considering the original problem types, pre-service mathematics teachers posed problems that were cognitively more difficult to solve in the MT. Due to its quantitative nature, this study does not provide data about the complexity of the problems. Therefore, this situation must be questioned in further studies. In this study, the originality was evaluated by considering the rareness of the posed problems. For this reason, it does not necessarily mean that each original problem type implies a difficult problem. Unlike the problems found in the textbooks, originality is also defined as posing problems with surprising relationships between concepts and components (Singer et al., 2011). Accordingly, the analyses to be conducted can also highlight the complexity of the problems. This perspective of originality can be adapted in further studies. Thus, the effect of the task format on the originality component can be revealed in detail. Therefore, it is crucial to carry out in-depth research in this field.

In addition, the fluency, flexibility, and originality scores are affected by the sample size. Conducting similar studies on larger samples may provide more valid conclusions about possible differences. This study has considered task formats in written form related to daily life and including figural patterns.

Figural patterns can also be structured based on whether they explicitly provide the pattern rule (Barbosa & Vale, 2016). The effect of these types of problem-posing tasks on creativity components is another case that can be investigated. Finally, another criterion is the task's mathematical structure, which affects participants' performance in pattern tasks (Lannin, Barker, & Townsend, 2006). In this research, a growing pattern that grows by a fixed number is used. Therefore, it should be possible to carry out similar research studies over the repeating or quadratic patterns. Such research can further deepen our understanding of the impact of different variables, such as a task's representation and structure, on the ability of creative problem-posing performance.

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