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# Different Types of Subitizing Activity: A Teaching Experiment with Preconservers

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**Abstract:**
***Introduction:*** Subitizing, a quick apprehension of the numerosity of a small set of items, is consistently utilized to support early number understanding. Perceptual subitizing is the innate ability to recognize less than five items without consciously using other mental or mathematical processes. Conceptual subitizing, which requires higher-level abilities, means perceiving the quantities as groups and performing a mental process on them. Research on conceptual and perceptual subitizing indicates some limitations about the activities regarding the children's early number development. So, MacDonald and Wilkins (2016) developed a framework that explained the types of activities that young children used while subitizing. In this framework, five sets of perceptual subitizing activity were described to explain how young children's perceptual subitizing activity changed. Besides, two types of conceptual subitizing activities were defined to explain how children's limited or flexible number understanding related to their subitizing activity. These seven different types of activities characterize the changes in children's subitizing actions. The study aims to investigate the relationship between children's number understanding and subitizing activity.
***Methods:*** A teaching experiment was conducted with two preschool-aged children to analyze what perceptual and conceptual processes children relied upon when subitizing. The teaching experiment consisted of twenty-six sessions. The interviews were conducted to determine whether children are able to conserve numbers or not, and whether they rely on a variety of different types of subitizing activity or not. After the interviews, 26 teaching sessions were carried out with two preconserver children.
***Results:*** In the experimental process, it was observed that the children rely on the color of items, the gap between items, and symmetrical aspects of items when perceptually subitizing. However, they could not manage to transition their subitizing activity from perceptual to conceptual subitizing. The study indicates that children's subitizing skills were closely related to their number conservation development.

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***Discussion:*** Based on the findings from this study, for Eren and Beren, subitizing activities were found to be perceptually limited. Specifically, it was found that four types of perceptual subitizing emerged to explain how symmetry, the gap between items, color of items, and canonical patterns promoted strategies that children relied on when constructing number understanding. During the teaching experiment, although these children carried out the activities that required the separating and combining numbers and seeing the relationship between the subgroups and the composite groups, they used perceptual units in this process. The relationship between the number conservation activity and the conceptual subitizing activity requires the coordination of thinking structures related to both ordering and classification. However, it was found that the children could not move from perceptual to the conceptual subitizing.
***Limitations:*** As all studies have some limitations, this study has, too. One of the limitations of the study is the sample size/number of participants. But teaching experiments aim to get a deep understanding, studying with a small sample is an obligation. Secondly, this study focused on some compounds of subitizing such as perceptual and conceptual ones.
***Conclusion:*** In order to make the transition from perceptual subitizing to conceptual subitizing children should have more experiences with subitizing activities.When designing mathematical games and assessments for young children, being aware of different types of subitizing categories may provide better support children's number understanding and subitizing.

**Key words:** subitizing, number conservation, early math, teaching experiment.

## Introduction

Subitizing is instantly perceptual apprehension and identification of the numerosity of a small group of items. In the first half of the twentieth century, researchers considered counting was an inadequate indicator of a true understanding of number, and they emphasized the importance of subitizing (e.g., Douglass, 1925). Carper (1942) stated that subitizing was more accurate than counting and influential for abstracting. In the second half of the twentieth century, educators developed different models for counting and subitizing. For some researchers, subitizing is a much more basic skill than counting and a necessary precursor to counting (Fitzhugh, 1978; Klahr & Wallece, 1976). The others stated that children mostly employ counting rather than subitizing which is defined as a shortcut to counting (Brownell, 1928; Gelman & Gallistel, 1978). Gelman and Gallistel (1978) expressed that subitizing is simply a form of rapid counting. However, questions regarding the relationship between number development and subitizing activity subsist (MacDonald, 2013). Subitizing investigations are situated primarily in the field of neuroscience and psychology

and focused mainly on the perceptual mechanisms that support subitizing activity. These studies stated that item orientation (e.g., patterned versus unpatterned items) influences activity relying upon perceptual mechanisms (Arp & Fagard, 2005; Logan & Zbrodoff, 2003). In the fields of mathematics education, it was found that subitizing activity changed as children were able to subitize sets with a larger number of items and a wider variety of orientations (Clements & Sarama, 2007; Sarama & Clements, 2009). This change was explained as children's ability to link spatial imaginary to number development. Some studies concluded that factors such as the spatial arrangement, physical size of the dots, and color of the dots affect subitizing ability (Leibovich, Kadhim, & Ansari, 2017; MacDonald, 2013, 2015; Whelley, 2002). For example, children can easily subitize in rectangular arrangements and it is followed by linear, circular, and scrambled arrangements (Beckwith & Restle, 1966). In addition, certain arrangements are easier for specific numbers. For example, children make fewer errors for ten dots than for eight with the "domino five" arrangement, but fewer errors for eight dots for the "domino four" arrangement. (Leibovich, Kadhim, & Ansari, 2017). Whelley (2002) argued that if there is a parallelism between the colors and the groups (for instance, three dots clustered are all red and two dots clustered are all black), children's subitizing skills become much more efficient. When there is no such a matching (for instance, four dots clustered have two red and two black items and three dots clustered have two red and one black item), their subitizing accuracy decreased. In addition, the performance in subitizing varies based on the gaps between the items and patterned arrangements (Logan & Zbrodoff, 2003; Mandler & Shebo, 1982).

## 1 Categories of subitizing

As a result of the Building Block program, two types of subitizing were described by Sarama and Clements (2009). Perceptual subitizing is the innate ability to recognize less than five items without consciously using another mental or mathematical process. Conceptual subitizing, which requires higher-level abilities, means perceiving the quantities as groups and performing a mental process on them (Clements, 1999). Sarama and Clements (2009) describe it as a child's ability to rely less on the items' orientations and more on their own number understanding. Research on the conceptual and perceptual subitizing (Clements, 1999; Sarama & Clements, 2009; Starkey & McCandliss, 2014) indicates some limitations about the activities regarding the children's early number development (MacDonald & Wilkins, 2019). MacDonald and Wilkins (2016) conducted a 22-session teaching experiment and found that different types of perceptual subitizing skills explain shifts in preschool children's abstraction process. They developed a framework that explained the types of activities that young children used while subitizing (MacDonald & Wilkins, 2016). In this framework, five sets of perceptual subitizing activity were

described to explain how young children's perceptual subitizing activity changed (Table 1). In addition, two types of conceptual subitizing activities were defined to explain how children's limited or flexible number understanding related to their subitizing activity (Table 2). These seven different types of activities characterize the changes in children's subitizing actions (Clements, Sarama, & MacDonald, 2019; MacDonald, 2013, 2015; MacDonald & Wilkins, 2016, 2019).

### *1.1 Five types of perceptual subitizing*

MacDonald and Wilkins (2016) state that perceptual activity starts from a child's reliance on fixed and spatial patterns, and moves toward a process where units rely on their flexible and dynamic representations.

Table 1

*Five types of perceptual subitizing.*

| *Types of perceptual subitizing* | *Descriptions* | *Examples* |
|---|---|---|
| Initial Perceptual Subitizing (IPS) | Children define the visual movement or the shape of the dots. | Children may describe seeing five dots and explain how they see it stating "because it looks like a flower." |
| Perceptual Subgroup Subitizing (PSS) | Children can numerically express small subgroups of two or three items, but they cannot subitize the composite group. | Children may say "I saw two and three dots." or "I saw two plus three.", but do not use this information to describe the composite group. |
| Perceptual Ascending Subitizing (PAS) | Children can first define clustered items as subgroups and then the composite group. | Children may first state "I saw 2 and 3 dots." and then, correctly define the total composite group. |
| Perceptual Descending Subitizing (PDS) | Children may first define the composite group and then, define the perceived cluster of dots as subgroups. | Children may say "I saw five because I saw two and three." |
| Perceptual Counting Subitizing (PCS) | Children first describe seeing one more or one less than the composite group, then, count down or up to the composite group. | Children may express what they saw as "4…..5" or "6….5". |

Note: These types of perceptual subitizing are described and categorized by MacDonald and Wilkins (2016) based on the answers of the children who participated in the study (Clements, Sarama, & MacDonald, 2019).

In the Initial Perceptual Subitizing (IPS), children began by relying on their eye scans or the shape of the item orientations when subitizing (MacDonald & Wilkins, 2016). For instance, the IPS predicts that when "domino four" dots are shown to the children they may say "I saw four because there was a square." The Perceptual Subgroup Subitizing (PSS) refers to the child's ability to subitize the subgroups, but not compose these subgroups to describe the composite group (for instance, they may say "I saw 3 and 3."). Similar to the IPS in the PSS activities children rely on the clustered items but not construct a number scheme to describe the entire group. The Perceptual Ascending Subitizing (PAS) and Perceptual Descending Subitizing (PDS) are much more multidimensional than the IPS and PSS (2016). During the PAS and PDS children again use the spatial patterns, and in addition, they can (a) subitize the subgroups and then, identify the composite group (for instance, they may say "I saw 3 and 3, and there are 6 dots.") or (b) subitize the composite group, and then, identify the subgroups (for instance, they may say "I saw 6 because there were 3 and 3.") (2016). Particularly, in the (b) condition the difference between the PDS and conceptual subitizing is that children still consider the physical characteristics such as spatial patterns, symmetry, color, etc. in defining the subgroups. During the Perceptual Counting Subitizing (PCS) children subitize the subgroup and then count forward one and back one (for instance, they may say "I saw 5…..6.) (MacDonald & Wilkins, 2016). Researcher stated that when children start performing actions related to number comprehension and stop relying visually on subgroups, they begin developing conceptual processes in relation to conceptual subitizing.

### *1.2 Two types of conceptual subitizing*

MacDonald and Wilkins (2016) defined a framework for two types of conceptual subitizing that describe how children's limited or flexible number understanding related to their subitizing activity (Table 2).

Table 2

*Two types of conceptual subitizing*

| *Types of conceptual subitizing* | *Descriptions* | *Examples* |
|---|---|---|
| Rigid Conceptual Subitizing (RCS) | Children define seeing the composite group, regardless of the orientation and color of the items, and constantly describe a fixed subgroup when explaining how they see it. | For instance, children may tell "I saw four, and I saw 2 and 2." for a set containing four dots. Children cannot describe a set of four objects as subgroups of 1 and 3 at the same time. |

| | | |
|---|---|---|
| Flexible Conceptual Subitizing (FCS) | Children define the composite group, regardless of the orientation or color of the items, and can define two or more subgroups in different tasks. | For instance, children may tell they saw five and explain it saying "I saw 2 and 3". For the same sequence, they may also say "I saw 2, 2, and 1." |

Note: These types of perceptual subitizing are described and categorized by MacDonald and Wilkins (2016) based on the answers of the children who participated in the study (Clements, Sarama, & MacDonald, 2019).

Rigid Conceptual Subitizing (RCS) refers to the fact that children cannot describe any more than one subgroup when they are subitizing. Flexible Conceptual Subitizing (FCS), on the other hand, refers to the fact that children can describe two or more subgroups while subitizing (for instance, the subgroups of 2-2 and 3-1) (MacDonald & Wilkins, 2016). The RCS is much more basic than the FCS because in the latter type children may describe more subgroups.

In terms of the learning trajectories developed by Sarama and Clements (2009), it is assumed that subitizing skills improve over time in a process from the perceptual subitizing to the conceptual subitizing. The study aims to describe the development of the children's schemas in relation to the subitizing activity. Specifically, this present study addresses the following questions:

1. Which types of perceptual and conceptual subitizing do the preschool children rely upon?
2. How do preschool children's perceptual and conceptual subitizing develop over time?

## 2 Method

The constructivist teaching experiment method was used to analyze the changes that occurred in the subitizing activity of preschool children. Cobb and Steffe (2011) state that the teaching experiment offers researchers many opportunities to repeatedly test students' learning trajectories to better understand the cognitive changes in their thinking patterns. For this reason, in this study, the constructivist teaching experiment method, which enables the use of flexible activities that will respond to the change in students' mathematical thoughts, was used. There are two reasons for using the teaching experiment method in a study: developing a hypothesis (exploratory teaching) and developing a model of student thinking based on an existing hypothesis (experimental teaching) (Steffe & Thompson, 2000). The hypotheses of this study are based on the framework put forward by MacDonald and Wilkins (2016). The purpose of using the teaching experiment in the current study is the patterning of thinking models that explain the conceptual process that each child employs when subitizing. One of the important features of the teaching experiment method is that the researcher is

in the role of a teacher. The aim is to establish an interactive communication with the children through this role (Steffe, 1991). Thus, the researcher's role in a constructivist teaching experiment is described as a teacher-researcher.

### *2.1 Participants*

In this study, participants were recruited from a preschool in the south of Turkey. Twenty children initially participated in this study. Screening interviews conducted with them to determine their ability to count, conserve numbers, and subitize item orientations composed of from two to five items. Following the interview process, four children were selected to participate in the teaching experiment based on their ability to conserve number, count, and subitize. Two of them preconserver and two of them conserver. Preconserver children, Eren and Beren, are the focus of this article to provide more insight into the relationship between subitizing activity and conservation of number. Because they were six years old and relatively older children who were lack of ability to conserve numbers and conceptually subitizing.

### *2.2 Screening interviews*

The interviews were conducted to determine whether children are able to conserve number or not and whether they rely on a variety of different types of subitizing activity. The first clinical interviews assessed children's number conservation. In the second interviews, the children's subitizing activity and counting ability were assessed. To assess children's ability or inability to conserve number one-to-one correspondence tasks were used. In the second clinical activities, they were asked to subitize dots on the cards (Figure 1), draw what they remember and then, count five or more dots in linear and random orientations. The reason for asking the students to draw what they remember was to identify their schemas when subitizing. The reason for asking them to count the dots given in linear and random orientations was to reveal their ability to keep track of items while counting.

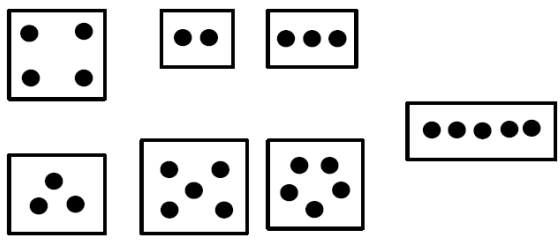


*Figure 1.* Subitizing cards used in the clinical interviews.

### *2.3 Procedure*

As stated, earlier interviews were carried out with twenty pre-school children to determine: (i) whether the children conserved the numbers (ii) they relied on different types of subitizing (iii) they could keep track of items when counting or not. Interview protocols developed by MacDonald (2013) were employed.

*Teaching experiment activities*

The teaching experiment continued for 13 weeks, 2 hours per week. A total of 26 teaching sessions were carried out. The most basic activity was "draw what you saw". It was adapted from MacDonald and Wilkins (2019). In this activity children are asked to subitize the dots, to express verbally how many dots they see and then, to draw or use counters to show the teacher-researcher what they saw or remember. The concentration game was adapted from Clements and Sarama's (2009) activities. In this game, 6-8 cards were faced down on the table. Childrenwere asked to match the cards with the dots and the cards with the numbers taking into account the number of dots. This game was also used to match the dot sets to support children in identifying the subgroups of numbers. For instance, four dots were given in a domino four orientation, while the other card with four dots clustered two-two. Ice-cream game was developed by MacDonald and Wilkins (2016). In the game, there was an ice cream cone with six ice cream scoops. Each ice cream scoop had a different orientation of dots. The goal was to roll a dice and match dots or numerals on the dice and the dots in the ice cream scoop. When matching was justified, the ice cream scoop could be colored. For instance, in regard to six dots, there are subgroups of 2-2-2, 4-2, 5-1, and 3-3 on each ice cream scoop. Children roll the dice and match the number or dot set on the dice with the dot set on the ice cream scoop and paint the matching ice cream scoop. The child who first paints all the ice cream scoops wins the game. Digital games are also used in teaching activities. The digital game entitled Monster Mansion-Number Match on the ABCya game website (https://www.abcya.com/) was employed in the study. In addition, the digital games related to subitizing which developed by Clements and Sarama (2019) were also used. In these games, when the children correctly subitize the sets of objects they see for a short time, they approach the planet with a spacecraft.

### *2.4 Data analysis*

Teaching experiments are based on a process that includes student actions and expressions to inform the researcher about how students make sense of specified mathematical concepts (Steffe & Thompson, 2000). In addition, the teaching experiment process offers the opportunity to explain the actions and expressions of the students in response to the researcher-teacher action and expressions. The conceptual analysis described by Glasersfeld (1995) was used to analyze the interaction between the teacher-researcher and the students. The conceptual analysis makes it possible to observe what students say and do as a result of

what teacher-researcher says and to recognize how each student's own mathematical reality changes with activities performed in the teaching experiment sessions (Steffe & Thompson, 2000). For example, when analyzing a child's response to a task the teacher-researcher attends to the actions, words, and materials rely on when justifying his/her thinking. This data provides the teacher-researcher with more information about each child's development of a number scheme. It is described as ongoing which includes critical events that enable children to define their mathematical schemes that they use when subitizing and is repeated following each teaching experiment (Powell, Francisco, & Maher, 2003). During the ongoing analysis, the focus was on which type of subitizing, namely perceptual and conceptual, the children relied on. The second step of the analysis involved a retrospective analysis. To carry out this analysis, all teaching sessions were completed and recorded on video and transcribed. In addition, the written work by the children and the notes taken by the teacher-researcher were examined and combined with the data transcribed producing the data sets. The data sets include evidence showing the changes in the children's schemas about subitizing. At the final step, the fine-grained analysis was carried out to see at which level the existing schemas were used by students when they dealt with subitizing and how this process worked (Siegler, 2007). First, the existing schemas used by children were analyzed. To see how the conceptual change in the child's thinking process occurred in each session, the actions of each child, including both verbal and gestures and facial expressions, were examined from transcribed data and video recordings. Continuous and systematic analysis were carried out to strengthen the implications regarding the changes in children's understanding, and the evidence was obtained to demonstrate the accuracy or invalidity of the inferences.

## 3 Findings

The findings of the study are obtained from the data collected from the interviews and the teaching experiment with two children. As stated earlier the clinical interviews targeted the children's comprehension and views about the number conservation principle and subitizing skills. The teaching activities were designed to support their transition from the level of perceptual to conceptual subitizing and to follow their development in this process. With both clinical interviews and teaching experiment activities, it was aimed to reveal the learning trajectories that the children reach and cannot reach regarding subitizing.

### *3.1 Screening interview-1: Counting and number conservation*

In order to reveal whether children conserved numbers and they could keep track of the objects or not while counting, the related activities were carried out in the first session. As a result of the interviews, it was found that neither Eren nor Beren acquired the number conservation. When Eren was asked to count the objects in a group containing four or five objects he could not make one-to-one

correspondence. He could only count the objects correctly for the groups containing three or fewer objects. While comparing the number of objects in small sets, for example, four teacups and four teaspoons Eren counted each group separately saying that each group had four objects. However, when the spoons are moved away from each other, although Eren knew the number of both groups is four, saying that the number of spoons was more than that of cups. For the groups with four objects, Beren managed to make one-to-one correspondence and said that the number of objects was the same in both groups. However, for the groups containing five objects, she did not comprehend this fact and relied on the gap between the objects. When Beren was asked to count a group of 4-5 objects, she did not manage to make one-to-one correspondence and she often skipped counting an object or counted it twice.

*3.2 Screening interview-2: Subitizing*

The purpose of this activity was to determine the schemas that the children when subitizing. The children saw number cards (Figure 1) for approximately 1-2 seconds. Then they were asked to say the number of the dots on the cards, explain how they saw the dots, and draw what they saw. It was found that both Eren and Beren managed to recognize the absolute number of objects for two and three dots. When asked to explain how they saw these dots they count the numbers 1, 2, 3 using their fingers. It is an evidence for that they used perceptual subitizing. Because they both used a visual image for the numbers. When Eren was shown a card with domino four orientation and was asked the question "How many dots have you seen?", he said that there were five dots. Teacher-researcher followed up this activity with the questions, "How do you know it? Can you draw what you have seen?", he drew four dots in a linear arrangement. When he was shown five dots in the linear and pentagonal arrays, he said that there were six dots and drew a linear array containing seven dots. When he was given five dots in an X arrangement, he said that there were four dots and drew four dots. Beren also managed to subitize for the cards containing two and three dots. However, when she was shown three dots in the linear and triangular arrays, she drew all of them in a linear array. Beren managed to do subitizing for the cards containing four dots and drew what she saw using a square array. However, for the arrays containing five dots, she drew the arrays containing six, seven, or eight dots again using the linear arrays (Figure 2).

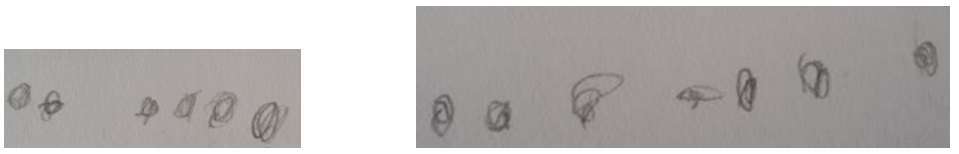


*Figure 2.* Beren's drawings of five-dot shapes in pentagonal and X arrays.

*3.3 Analysis of the teaching experiment*

In the first session of the teaching experiment, Eren was shown a card containing three dots in a triangular shape. He said that there were three dots. When he was asked to draw what he saw, he drew three dots in a triangular shape. Then, he asked to arrange three-dot-cards to understand which features of the number was employed by Eren. He arranged two cards with three dots in the triangular and linear orientations. Eren was asked to describe the similarities and differences between these two cards. He described the former as "Here there is a shape similar to the roof of the house." In the second session, Beren was shown three dots in an L array (two dots in a linear array and one dot on the right side of the bottom). Beren said that she saw three dots when she was asked to draw it, she said that it was like an "armchair" and produced the following shape with four dots (Figure 3). Beren could not explain how many dots she saw for a different orientation of three dots and tried to draw what she saw depending on the shape of the dots (Figure 4).

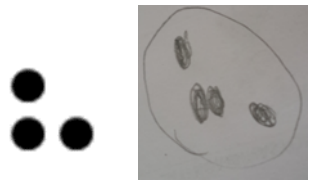


*Figure3*



*Figure 4*

The IPS activity includes observational abstractions by taking advantage of the visual motion or the shape of the dots. Eren and Beren relied upon empirical abstractions when subitizing, given responses such as "It looks like an armchair." or "It looks like a roof." and subitized them based on the shape of the dots. Another evidence of their involvement in the IPS activity is that they thought that the larger gap between items comparatively as increasing the quantity of the set. For instance, in the fourth session, Beren said that there were more dots in the orientation where the gap between the counter was larger. However, Beren managed to say that there were four dots when these dots were close to each other. In this process, it was seen that Beren opened her four fingers one by one depending on the visual image of the dots and relied on the counting action in cases where she could not subitize the objects. Beren's incorrect answer for the four dots in the square orientation where larger gaps were created between the counters showed she could not define the number of subgroups.

In a PSS activity, children can subitize for the small subgroups of two or three objects. However, they cannot subitize the entire composite group. In addition,

in identifying the subgroups several factors such as the symmetrical aspects and color of the items, the gap between the items, and the orientations of the items are important. Like in the IPS activities students whose number comprehension is not sufficiently developed subitize subgroups taking into account the properties of them (symmetry, color, position, patterns, etc.) (MacDonald, 2013). In the eighth session, Beren was shown a card with three dots in a triangular arrangement and a dot in the upper left of the triangle (Figure 5). She was asked to tell how many dots there were on the card and then to show what she remembered using the counters. Beren placed the counters to form subgroups from 1 and 4 dots. When she was given another chance to see the card for the second time in one second, she recognized that for the subgroup with three dots she used four dots, and she removed the fourth one. The dialogue in this activity is given as follows:
Showing the following card:



*Figure 5*

*Teacher-researcher: How many dots did you see Beren?*
*Beren: I don't know.*
*Teacher-researcher: Then can you show the dots you saw on the card using the counters?*
*Beren made the following representation by placing four counters in a square arrangement and one counter in a separate way (Figure 6).*
*Teacher-researcher: Would you like me to show the card to you again?*
*Beren: Hı hı. (Shaking her head)*
*When Beren was shown it again and asked to see how many dots she saw, she took one of the counters and arranged it correctly (Figure 7).*
*Teacher-researcher: How many counters are there, Beren?*
*Beren: 3 and 1.*
*Teacher-researcher: Then, how many dots are there on this card?*
*Beren: 3 and 1.*

As can be seen from the dialogues above it is evidence that Beren performed a PSS activity. Although she could tell the number of dots in the subgroups, she did not realize that she can reach the total number of dots in the set with the combination of these subgroups.



*Figure 6*



*Figure 7*

Beren was shown another card consisting of five dots in which four dots were in the square arrangement and one dot on the right side and was asked to tell how many dots she saw and to draw what she saw (session fifteen). Beren could not say that there were five dots. Instead, she drew four circles in a linear array. When Beren was given a second chance to see the card for 1-2 seconds, she recognized that there was another dot and added another dot to the linear array. Although Beren performed subitizing for the subgroups, she was not aware of the fact that she can use subgroups to describe the total composite group.
A similar observation was made for Eren. When he was given five dots in the subgroup of 2-2-1 he could not say that there were five dots. However, when he was asked to create what he saw using the counters, he produced subgroups of dots in the form of 2-2-1 (Figure 8). The symmetrical properties of the dots are very effective for the students to rely on the PSS activity and to recognize the subgroups.

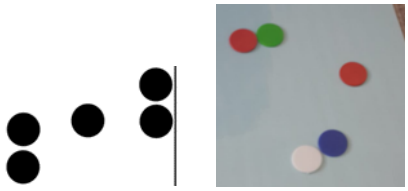


*Figure 8*

In PAS activity Eren and Beren described the subgroups and then the composite group. Beren employed PAS activity in the process. When the card in the figure below was shown to Beren and was asked how many she saw, she answered 2-2. When she was asked to show what she saw using the counters as illustrated in Figure 9 she produced the subgroups using two red and two blue counters. However, she could not subitize for the composite group. When asked again how many dots she saw in the group she replied she saw 2-2. She pointed out that the red and blue counters separately and said “Here there are two dots, and here there are two dots.” It shows that Beren carried out the PDS activity.

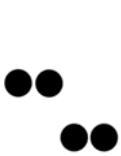




*Figure 9*

In the twelfth session, Beren played the camera game. In this game, Beren was shown the dots placed on the camera screen image shot and asked to tell how many dots she saw. She was also asked to draw or demonstrate what she saw or remembered using counters. A screenshot was shown to Beren in Figure 10. Following dialogue between Beren and the researcher-teacher occurred as follows:





*Figure 10* *Figure 11*

*Teacher-researcher: How many dots did you see?*
*(Beren tries to remember the dots and their orientations and opens her two fingers under the table.)*
*Beren: I saw five dots.*
*Teacher-researcher: So can you recreate the image which you see on the camera with counters? How did you see it?*
*Beren: Yes. (She makes the model given in Figure 11 using the counters.)*
*Beren: (Showing the model she produced) I saw two dots, two dots, and one dot.*

Because of the gap between the dots, Beren could not subitize the total composite group. Supportive teaching sessions were held for Beren to grasp the subgroups. In this process, 2 red and 2 yellow dots are shown to Beren in a square arrangement. Beren said that she saw four dots on this card, and when asked how she saw it, she replied 2-2. She also opened her four fingers and grouped then two by two and replied, "I saw it like this." (Figure 12).




*Figure 12*

It shows that Beren could subitize for four dots and used the clustered items in explaining how she saw them. It can be suggested that Beren managed to perform the PDS activity because she relied on the color of the items. She could not subitize when the dots were not given in clustered subgroups.

## Conclusion, discussion, and suggestions

The purpose of this study was to describe how two preconserver children's subitizing activity changed over time. Based on the findings from this study, for Eren and Beren, subitizing activities were found to be perceptually limited. Specifically, it was found that four types of perceptual subitizing emerged to explain how symmetry, the gap between items, color of items, and canonical patterns promoted strategies that children relied on when constructing number understanding. During the teaching experiment, although these students carried out the PAS and PDS activities that required the separating and combining numbers and seeing the relationship between the subgroups and the composite groups, they used perceptual units in this process. The relationship between the number conservation and the conceptual subitizing activities requires the coordination of thinking structures related to both ordering and classification. However, it was found that the students could not move to the PCS activity and the conceptual subitizing activities. Therefore, although the PAS and PDS activities were carried out to Eren's and Beren's conceptual subitizing skills, they just used the physical properties (color, symmetry, etc.) and spatial patterns (proximity/distance, symmetry, etc.) of the dots. In other words, the ability of children to describe the cluster of items as subgroups and then the composite group or to describe the composite group and then subgroups were based on physical characteristics and spatial patterns of the dots. Eren and Beren found it difficult to perform perceptual subitizing if the number of dots on the cards was more than 5, if the dots were given in the same color and if the gap between the dots were not equidistant. Cognitively, Piaget (1965) explains that for children to conserve number serial and classification thinking structures need to be coordinated simultaneously. When Eren and Beren could not subitize the

subgroups perceptually, they relied on their serial thinking structure. This finding is consistent with the previous findings of the studies with the preschool children who did not have the ability of the number conservation (MacDonald 2013, 2015). For children to structure the numbers extensively, they must have two understandings (serial thinking and classification) (Piaget, 1965). The children's understanding of the classification allows them to recognize that a number is a combination of smaller numbers or is a part of bigger numbers. Children who acquired the serial and classification thinking structure understand that the number five is a continuation of the number four and it consists of the numbers such as four and one or two and three. The ability of children to coordinate these two skills allows them to abstract numbers in many different ways (MacDonald & Shumway, 2016). Given that Eren's and Beren's classification schemas were not sufficiently developed, they had difficulty in seeing subgroups to describe the composite group. The findings of the study indicate that children's subitizing skills are closely related to their number conservation development. A similar finding was reported in the studies by MacDonald (2013, 2015). The related research (MacDonald, 2013; MacDonald & Wilkins, 2019) suggests the necessity of number conservation for the development of children's conceptual subitizing skills.

When Eren and Beren were able to engage in a perceptual subitizing activity where subgroups were described with their features such as proximity, separation, symmetry, and color, this supports their perceptual subitizing. Perceived changes in color and density allowed them to attend to the subgroups. As the findings of the study suggest the children showed improvement in regard to the number conservation and perceptual subitizing during the teaching experiment, but they did not manage to have the ability to the conceptually subitize. As a result of the practices used in the study although Eren and Beren could not have clear progress each in their subitizing skills, there occurred some changes which improved their perceptual subitizing. But considering their initial inability when counting, subitizing, and conserving numbers, their development could be considered to be quite typical. It is not clear enough whether the reason for these children's lack of conceptual subitizing is due to the lack of the psychological background of the subitizing or the lack of their experience on subitizing. To understand this, a longer-term teaching experiment should be conducted with them, and they should be provided with more experience to support their transition process from perceptual subitizing to conceptual subitizing. When designing mathematical games and assessments for young children, being aware of different types of subitizing categories may provide better support children's number understanding and subitizing. Also parents' engagement in the children's mathematics education process is necessary (Deringöl, 2019) to make children gain more experience with numbers.

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