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CALCULATION OF FAILURE PROBABILITY OF CONSTANTLY LOADED CANTILEVER BEAM BY MONTE CARLO METHOD

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In this paper, we demonstrate a probabilistic approach to the design of structures on a cantilever beam with constant load. Individual variables in the mathematical model are not represented deterministically by their specific values but randomly by probability distributions. Normal distribution is used for all random variables. The resulting probability of failure is calculated using a simple Monte Carlo method, for which a brief overview is also provided in this article. Such a probabilistic proposal is the subject matter of newly emerging field Reliability of Structures.

Keywords: failure probability, constant load, cantilever beam, Monte Carlo method, reliability of structures

The conventional approach to the design of engineering structures is deterministic where all variables entering the mathematical model are given by a single value. The development of computers and new mathematical methods allows the transition to probabilistic methods where each variable in calculation is represented randomly. Such random calculation inputs better reflect the real behaviour of technical systems. Therefore, the probabilistic approach is currently a very promising trend of current mechanics. Input load, material properties and geometry is determined by a histogram, or using an appropriate probability distribution and its parameters. The result of computation tends to random variable again, mostly failure probability. Such a proposal described in general terms is the essence of the rapidly growing field Reliability of Structures.

This approach is shown on a simple example of constantly loaded cantilever beam using the Monte Carlo method. In this article, we present a brief overview of the origin and use of this method.

Material and methods

Monte Carlo method

The Monte Carlo method is a general term describing a group of mathematical methods using random numbers for calculation (Melchers, 1987). The application of this method is not limited only to probabilistic problems but it enables solving deterministic problems as well. The only requirement is the problem solution described by probability density function. An example of probabilistic solution is generating stochastic processes directly related to probability density functions. Examples of deterministic tasks solved by the Monte Carlo method are evaluation of multidimensional integrals and solving systems of equations (Marek et al., 2003).

The solution itself consists in generating random numbers according to respective distribution densities of random variables and a multiple evaluation of function of such random variables (Frydrýšek, 2010). At a high number of repetitions, the evaluation of these simulation results based on the law of large numbers and based on the validity of central limit theorem converges to an exact solution. In many cases, it is possible to predict the statistical error of the result and to determine based on it the number of simulations to obtain the required accuracy.

The Monte Carlo method has become in many areas a standard tool for solving various technical problems. Whereas when in use it is necessary to generate a huge amount of simulations (often millions), its application is closely related to the development of modern computers (Schneider, 1997). Their growth of calculation speed and capacity enables the progress of the method itself. The Monte Carlo simulation is useful not only for scientists and researchers but for its easy comprehension and high compatibility with the thinking of engineers, it is very suitable for teaching purposes. It is quite easy to demonstrate on it various theoretical methods.

Buffon's needle (Krejsa, 2012) is considered the first example of using this simulation method. An experiment from the 18th century estimated the value of Ludolf number π. Buffon was randomly throwing a needle of length l on the paper with row spacing d, and the number π was estimated using the expression $\pi = (2 \cdot l) / (p \cdot d)$, where p was the proportion of trials in which the needle intersected lines to the total number of trials. The name of this method originated at the end of World War II in the production of atomic bomb among members of the Manhattan Project, who were Enrico Fermi, Robert Oppenheimer and Stanislav Marcin Ulam. It was named according to the world-famous casino in Monte Carlo in the Monaco principality. The principle of a ball in a roulette that chooses a random number into which it falls closely resembles the generation of random numbers.

The basic scheme of a simple or direct Monte Carlo method consists of several steps (Páleš, 1996):

1. Generation of individual random variables according to their respective probability distributions (for example: geometry of structure, material properties, load).
2. Evaluation of the mathematical function of limit state, which is formed by random variables.

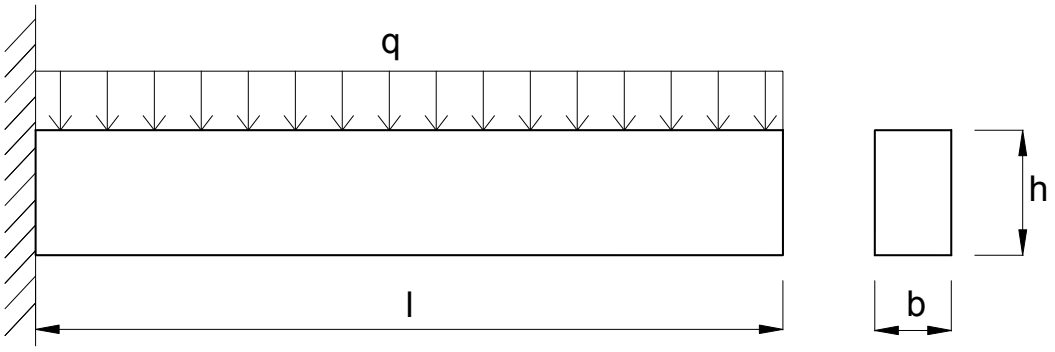


Figure 1 Cantilever beam and its parameters

3. Determination of the failure probability of structure as a proportion of the number of simulations for which the limit state is less than zero to the total number of simulations.

The advantage of the method is its easy understandability, possibility to work with the function of limit state as a random variable, and to compile for it for example a histogram. The disadvantage is in the necessity of a large number of simulations in the evaluation of small failure probability, which can lead to a significant slowdown of calculation for complicated mathematical functions.

Application of Monte Carlo method

First, we present the required mathematic relationships for using the Monte Carlo method (Beck and Da Rosa, 2006). In case of a simple direct simulation, random variables $\boldsymbol{X}$ (indication in bold letters represents the vector of random variables) are generated according to the distribution of probability density function $f_{\boldsymbol{x}}(\boldsymbol{x})$. We introduce the so-called indicator function $I_{[g(\boldsymbol{x})]}$ that for each evaluation of limit state takes value 1 for failure and value 0 for reliability, thus:

$$I_{[g(\boldsymbol{x})]} = 1 \text{ for } g(\boldsymbol{x}) < 0$$
$$I_{[g(\boldsymbol{x})]} = 0 \text{ for } g(\boldsymbol{x}) \geq 0 \quad (1)$$

Then, the probability of failure is calculated by multiple integration over the area of all random variables:

$$P_f = \int_{\text{all } \boldsymbol{x}} I_{[g(\boldsymbol{x})]} \cdot f_{\boldsymbol{x}}(\boldsymbol{x})\, d\boldsymbol{x} \quad (2)$$

Its mean value is computed using the summation of indicator function:

$$E(P_f) = \frac{\sum_{i=1}^{n} I_{[g(\boldsymbol{x})]}}{n} \quad (3)$$

where:
n – the number of performed simulations

Simulation result is loaded with statistical error, which can be expressed by the variance of estimated mean value of failure probability:

$$\text{Var}\,(P_f) = \frac{\sum_{i=1}^{n} (I_{[g(\boldsymbol{x})]} - E(P_f))}{n \cdot (n-1)} \quad (4)$$

Building structures mostly have a low probability of failure. Therefore, with the simple Monte Carlo method, it is usually necessary to perform a large number of simulations in order to keep the variance within the required limits. Variance (4) decreases with increasing the number of simulations.

Results and discussion

We apply the Monte Carlo method to a simple model of the cantilever beam loaded constantly. Figure 1 shows the calculated beam with indication of its parameters, too.

As known from mechanics and the theory of elasticity, bending moment for this case of constraint and load is given by the formula:

$$m = \frac{q \cdot l^2}{2} \quad (5)$$

Section modulus of the rectangle in Figure 1.

$$w = \frac{b \cdot h^2}{6} \quad (6)$$

Table1 **Distribution** parameters of random variables

Variable	Mark	Distribution	Mean	Standard deviation	Unit
X_1	q	normal	1.15	0.03335	kg cm^{-1}
X_2	l	normal	60	0.6	cm
X_3	b	normal	4	0.12	cm
X_4	h	normal	1	0.03	cm
X_5	R	normal	3,600	298.8	kg cm^{-2}

Source: Liu and Kiureghian, 1997

When R means the beam resistance, then the resulting limit state function may be written in the form:

$$G(\boldsymbol{X}) = R - \frac{m}{w} = R - \frac{3 \cdot q \cdot l^2}{b \cdot h^2} = X_5 - \frac{3 \cdot X_1 \cdot X_2^2}{x_3 \cdot x_2^2} = 0 \qquad (7)$$

$G(\boldsymbol{X})$ is entered with a capital letter here because it is a random variable, while the small letter $g(\boldsymbol{x})$ in Equations (1)–(4) is a concrete implementation of limit state and thus being a deterministic variable. For individual random variables, we have chosen normal probability distributions according to Table 1.

We have simulated the individual variables $X_1 - X_5$ with the given parameters and we programmed Equations (3) and (4). Then, it was enough to change the number of simulations, and we ended with the results shown in Table 2.

Table 2 Mean and variance of failure probability for different numbers of simulations

Simulations – n	$E(P_f)$	Var (P_f)
10	0.1	9×10^{-3}
100	0.12	9.386667×10^{-4}
1,000	0.114	8.957912×10^{-5}
10,000	0.0997	8.081893×10^{-6}
100,000	0.10286	8.278875×10^{-7}
1,000,000	0.103045	8.29027×10^{-8}

As can be seen from the second column of Table 2, the values of failure probability constantly refine with the number of simulations. With increasing order of simulations the number of decimal places of $E(P_f)$ increases, which is consistent with the interpretation of Vacareanu et al. (2007). Having ten times the number of simulations in the first column of Table 2, the magnitude of Var (P_f) decreases sequentially in each row by one order, resulting in a more accurate value of failure probability (Frydrýšek, 2010; Krejsa, 2012). The mean value of failure probability has stabilized at number 0.103. The result is the same as in the work of Beck and Da Rosa (2006); however, we have reached it with a greater number of simulations only. These authors reported only n = 10,000 to reach the result but does not mention the variance and statistical error of this value.

Conclusion

We have presented on the practical example the probabilistic approach to the design of the technical system. For the cantilever beam with a constant load, probability distributions of relevant random variables were input data. The mathematical model was evaluated using the simple Monte Carlo method. The result is failure probability the value of which is continuously refined through increasing number of simulations.

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