

DESIGN OF A FOUR-WING HETEROGENEOUS FRACTIONAL-ORDER CHAOTIC SYSTEM AND ITS CIRCUIT SIMULATION

Xiaohong ZHANG, Qiang SUN and Peng CHENG

School of Information Engineering

Jiangxi University of Science and Technology, Ganzhou 341000

People's Republic of China

E-mail: xiaohongzh@263.net

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Abstract: Integer orders differential system is a special case of fractional-order differential system. Integer orders chaotic system that we usually study is ideally approximate to realistic chaotic system. Fractional-order chaotic system has broader and changeable values of order and more complex dynamical behavior than integer order chaotic system. Thus, fractional-order differential equation can describe the nonlinear characteristics of actual chaotic system more exactly, which has more prominent research meanings and application value. This paper designs a new four-wing four-dimensional heterogeneous fractional-order chaotic system, when the values of fractional-order $q_i (i = 1,2,3,4)$ ***are not identical(*** $q_1 = q_2 = 0.9, q_3 = q_4 = 0.8$ ***, in step size of 0.1), the attractors of this chaotic system will all show four-wing shapes in any direction. After analysis this chaotic system's stability and existence, this paper also introduces a nonlinear state feedback controller, and adopts the chain shape circuit to conduct experiment simulation through Multisim software 10.0. The results of circuit simulation and Matlab numerical operation have the same chaotic attractor phase diagram. This demonstrates the effectiveness of this four-wing four-dimensional heterogeneous fractional-order chaotic system's design and the feasibility of the feedback controller in the circuit; meanwhile, it provides referable bases for the application in actual circuits.***

Index terms*:* Fractional-order differential system, chaotic system, heterogeneous fractional-order, circuit simulation, feedback control.

I. INTRODUCTION

There are nonlinear phenomena of various forms of exercise in nature, while chaos is a specific form of exercise of nonlinear dynamical system [1]. Chaos theory reveals the unity of order and inorder and the unity of integral dimension and fractional dimension of nature. Since Lorenz discovered the first chaotic attractor in three-dimensional autonomous system in 1963, people have continually discovered new chaotic systems, such as Chen system, Lü system, Liu system, improved generalized Lorenz system and so on [2-4] , and then people conducted in-depth studies on these systems and established had corresponding chaotic dynamical theoretical systems[5-7].

In the process of studying integer orders chaotic system, people found that there were abundant dynamical behaviors when the order has a fraction [8]. Since integer orders chaotic system is all the idealization of realistic system, it is can reveal more inherent features and nonlinear laws of complex systems by using fractional orders and it is can also exactly depict systems' actual physical significance and engineering phenomena. Therefore, it is more meaningful to study fractional-order chaotic system.

This paper constructs a four-wing four-dimensional heterogeneous fractional-order chaotic system and the chaotic attractor will show four-wing shapes in any direction. Through studying the system's fractional order's dynamical behavior, the paper conducts numerical simulation based on improved Adama-Bashforth-Moulton method [9-13] and designs chain shape fractional-order chaotic circuit based on frequency-domain approximation method [14-17]. At last, the paper realizes the four-dimensional fractional-order chaotic system through Multisim software 10.0 circuit simulation platform.

II. THE BASIC THEORY OF FRACTIONAL-ORDER CALCULUS

a. The Definition of Fractional-order Calculus

In the developing process of fractional-order calculus theory, a lot of scholars put forward various different definitions of fractional-order calculus. However, there are mainly

Grunwald-Letnikov fractional-order calculus definition, Riemann-Liouville fractional-order calculus definition and Caputo fractional-order calculus definition [1,9].

The most common used definition is Riemann-Liouville fractional-order derivative, which is:

$$ {}_aD_t^q x(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_a^t \frac{x(\tau)}{(t-\tau)^{q-n+1}} d\tau, (n-1 \le q \le n) \tag{1} $$

In the formula, $\Gamma(\cdot)$ is Gamma function, ${}_aD_t^q$ is fractional-order calculus, ${}_aD_t^q x(t)$ is calculus operator. The definition of Gamma function is:

$$ \Gamma(v) = \int_0^\infty e^{-t} t^{v-1} dt, \mathrm{Re}(v) > 0 \tag{2} $$

b. Estimation-correction Numerical Value Numeration Method

There are mainly two categories of numeration methods of fractional-order calculus: one kind is frequency-domain algorithm and another kind is time-domain algorithm. The paper adopts the estimation Estimation-Correction numerical value Numeration Method of fractional-order calculus equation, that is, generalized Adama-Bashforth- Moulton algorithm which provides great convenience for directly computing calculus equation.

To any one of nonlinear dynamical systems with original value, when $q \in (m-1, m)$, if its fractional-order calculus equation:

$$ \begin{cases} \dfrac{d^q x}{dt^q} = f(t, x(t)), & 0 \le t \le T \\ x^{(k)}(0) = x_0^{(k)}, & k = 0,1,...,m-1 \end{cases} \tag{3} $$

Its Volterra calculus equation will be described as:

$$ x(t) = \sum_{k=0}^{m-1} x_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau, x(\tau)) d\tau \tag{4} $$

Equation (3) and equation (4) are equivalent, if $h = \frac{T}{N}, t_n = nh, n = 0,1,\cdots, N \in Z^+$, formula (4) will may be scattered as:

$$x_h(t_{n+1}) = \sum_{k=0}^{m\text{-}1} \frac{t_{n+1}^k}{k!} x_0^{(k)} + \frac{h^q}{\Gamma(q+2)} f(t_{n+1}, x_h^p(t_{n+1})) + \frac{h^q}{\Gamma(q+2)} \sum_{i=0}^{n} a_{i,n+1} f(t_i, x_h(t_i)) \tag{5}$$

According to the different fragments of the i , which will form difference equation. The value of correction coefficient in formula (5) will be:

$$a_{i,n+1} = \begin{cases} n^{q+1} - (n-q)(n+1)^q & i = 0 \\ (n-i+2)^{q+1} + (n-i)^{q+1} - 2(n-i+1)^{q+1} & 0 \le i \le n \\ 1 & i = n+1 \end{cases} \tag{6}$$

The initial estimation approximation of fractional-order dynamical system's variety in formula (5) is described as:

$$x_h^p(t_{n+1}) = \sum_{k=0}^{m-1} \frac{t_{n+1}^k}{k!} x_0^{(k)} + \frac{1}{\Gamma(q)} \sum_{i=0}^{n} b_{i,n+1} f(t_i, x_h(t_i)) \tag{7}$$

In which estimation coefficient $b_{i,n+1}$ is

$$b_{i,n+1} = \frac{h^q}{q}((n-i+1)^q - (n-i)^q) \tag{8}$$

Compared with common numeration method to solve fractional-order differential equation, estimation-correction method can be easily used to numerical solution of any fractional-order differential equation and it provides great convenience for solving the question of fractional-order calculus.

III. THE CONSTRUCTION AND NUMERICAL SIMULATION OF FOUR-WING FOUR-DIMENSIONAL HETEROGENEOUS FRACTIONAL-ORDER CHAOTIC SYSTEM

a. The Construction of Four-wing Heterogeneous Fractional-order Chaotic System

To construct a simple four-wing four-dimensional integer orders chaotic system, its differential equation is described as:

$$\begin{cases} \dfrac{dx}{dt} = -ax + yz + bw \\ \dfrac{dy}{dt} = cy - xz \\ \dfrac{dz}{dt} = xy - dz \\ \dfrac{dw}{dt} = xz - ew \end{cases} \tag{9}$$

To change formulation (9) into fractional-order form, its differential equation will be described as :

$$\begin{cases} \dfrac{d^{q_1}x}{dt^{q_1}} = -ax + yz + bw \\ \dfrac{d^{q_2}y}{dt^{q_2}} = cy - xz \\ \dfrac{d^{q_3}z}{dt^{q_3}} = xy - dz \\ \dfrac{d^{q_4}w}{dt^{q_4}} = xz - ew \end{cases} \tag{10}$$

In which system parameters $(a,b,c,d,e) = (8,5,12,60,4)$, to system (10), $\nabla V = \dfrac{\partial \dot{x}}{\partial x} + \dfrac{\partial \dot{y}}{\partial y} + \dfrac{\partial \dot{z}}{\partial z} + \dfrac{\partial \dot{w}}{\partial w} = -a + c - d - e = -48 < 0$. As a result, the system is dissipative and converges with index rate $e^{-a+c-d-e} = e^{-48}$, and the system will level off to an attractor as time goes on.

b. Four-wing Four-dimensional Heterogeneous Fractional-order Chaotic System's Numerical Simulation

To conduct discretization on the fractional-order chaotic system by adopting the estimation-correction numerical value numeration method of fractional-order calculus equation, the discretization result is:

$$\begin{cases} x_{n+1} = x_0 + \dfrac{h^{q_1}}{\Gamma(q_1+2)}((-ax^*_{n+1} + y^*_{n+1}z^*_{n+1} + bw^*_{n+1}) + \sum_{j=0}^{n} a_{j,n+1}(-ax_j + y_j z_j + bw_j)) \\ y_{n+1} = y_0 + \dfrac{h^{q_2}}{\Gamma(q_2+2)}((cy^*_{n+1} - x^*_{n+1} * z^*_{n+1}) + \sum_{j=0}^{n} a_{j,n+1}(cy_j - x_j z_j)) \\ z_{n+1} = z_0 + \dfrac{h^{q_3}}{\Gamma(q_3+2)}((x^*_{n+1}y^*_{n+1} - dz^*_{n+1}) + \sum_{j=0}^{n} a_{j,n+1}(x_j y_j - dz_j)) \\ w_{n+1} = w_0 + \dfrac{h^{q_4}}{\Gamma(q_4+2)}((x^*_{n+1} * z^*_{n+1} - ew^*_{n+1}) + \sum_{j=0}^{n} a_{j,n+1}(x_j z_j - ew_j)) \end{cases} \tag{11}$$

We can solve the fractional-order differential equation's numerical value of the fractional-order chaotic system's arbitrary order, as long as we choose a chaotic system's initial value and the value of arbitrary fractional-order q. Here, we choose the initial value of state variable $(x_0, y_0, z_0, w_0) = (2,1,1,2)$ $q_1 = q_2 = 0.9$, $q_3 = q_4 = 0.8$, and through program numeration, we work out the attractor phase diagram of chaotic system (10), as shown in figure 1.

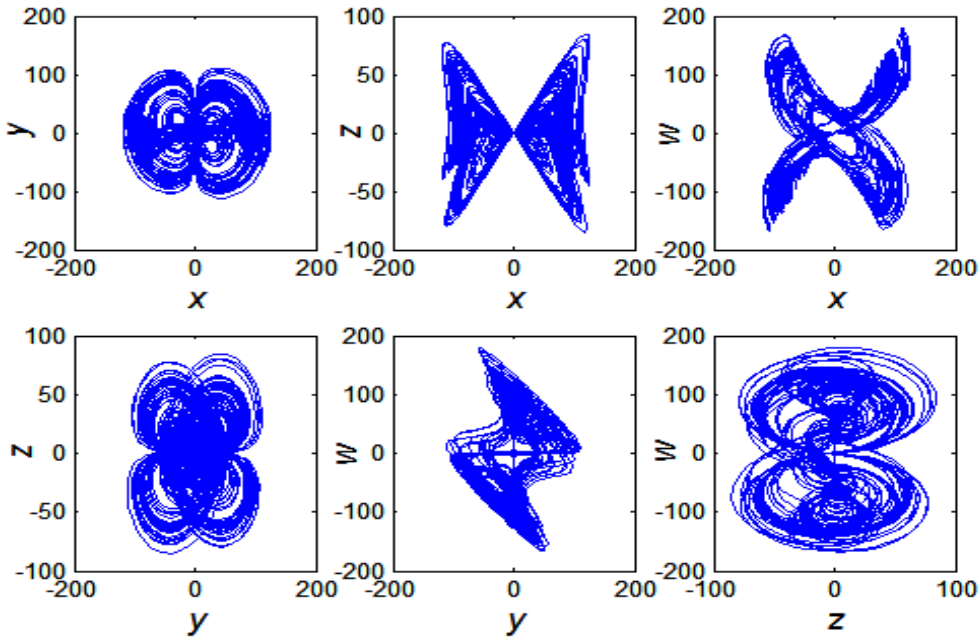


Figure 1. Phase diagram of four-wing four-dimensional heterogeneous fractional-order chaotic system's numerical simulation

IV. THE CIRCUIT SIMULATION OF FOUR-WING FOUR-DIMENSIONAL HETEROGENEOUS FRACTIONAL-ORDER CHAOTIC SYSTEM

Fractional-order calculus is the extension of integer orders calculus theory. By using fractional-order differential operator, we can more exactly describe all kinds of dynamical phenomena and features in real world. The concrete implementation of fractional-order chaotic system in analog circuit makes it possible to transfer the design concept from theory to application. Circuit output features can reflect the system's all kinds of dynamical phenomena more objectively.

In the process of designing fractional-order circuit, we usually design corresponding integer orders chaotic circuit firstly and then replace the capacitance in integer orders chaotic circuit with fractional-order module (unit circuit), thereby constructing fractional-order chaotic circuit. Based on bode plot frequency-domain approximation method, W. M. Ahmad designed the bode plot approximation numeration result chart when maximum error was $2dB$ and put forward the approximation formulation in frequency-domain when transfer function $q = 0.1 \sim 0.9$ (in step size of 0.1).

a. The Design of Chain Shape Simulation Circuit

According to circuit theory, the approximation formulation of q, from 0.1 to 0.9, in reference [1], bode plot approximation chart, can be realized by the complex frequency-domain of the chain shape equivalent circuit between figure 2(a) A and B. When $q = 0.9$, it can be worked out that the approximation formula of $1/s^q = 1/s^{0.9}$ is:

$$\frac{1}{s^{0.9}} = \frac{2.2675(s+1.292)(s+215.4)}{(s+0.01292)(s+2.154)(s+359.4)} \tag{12}$$

In formula (12), $s = jw$, it is complex frequency and the chain shape circuit unit is shown as figure 2(b). Transfer function $F(s)$ between A and B is:

$$F(s)=\frac{1/C_1}{s+1/R_aC_1}+\frac{1/C_2}{s+1/R_bC_2}+\frac{1/C_3}{s+1/R_cC_3} \tag{13}$$

Taking $C_0=1uF$, since $F(s)C_0=1/s^{0.9}$, we can reach:

$R_a=62.84M\Omega, R_b=250K\Omega, R_c=2.5K\Omega, C_1=1.23uF, C_2=1.83uF,\ \ C_3=1.1uF$.

When $q=0.8$, we can reach that the approximation formulation of $1/s^q=1/s^{0.8}$ is:

$$\frac{1}{s^{0.8}}=\frac{5.3088(s+0.1334)(s+2.371)(s+42.17)(s+749.9)}{(s+0.01334)(s+0.2371)(s+4.217)(s+74.99)(s+1334)} \tag{14}$$

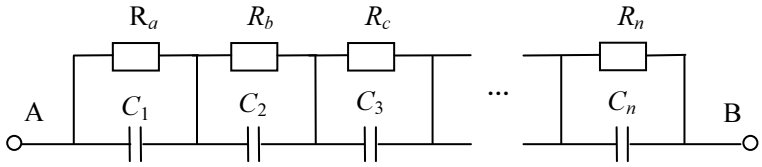
The chain shape circuit unit is shown as figure 2(c), and transfer function $F(s)$ between A and B is:

$$F(s)=\frac{1/C_1}{s+1/R_aC_1}+\frac{1/C_2}{s+1/R_bC_2}+\frac{1/C_3}{s+1/R_cC_3}+\frac{1/C_4}{s+1/R_dC_4}+\frac{1/C_5}{s+1/R_eC_5} \tag{15}$$

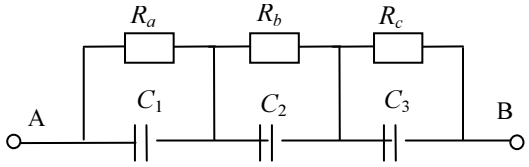
Since $F(s)C_0=1/s^{0.8}$, we can reach:

$R_a=37.85M\Omega, R_b=1.754M\Omega, R_c=0.17M\Omega, R_d=0.017M\Omega, R_e=0.0018M\Omega, C_1=1.98uF,$

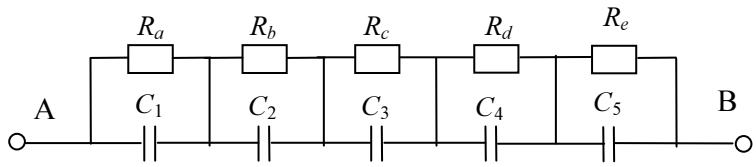
$C_2=2.4uF, C_3=1.39uF, C_4=780nF, C_5=420nF$.



(a) The realized chain shape unit circuit of $1/s^q$



(b) The realized chain shape unit circuit of $1/s^{0.9}$



(c) The realized chain shape unit circuit of $1/s^{0.8}$

Figure 2. The chain shape unit circuit of fractional-order $1/s^q$

Based on the above analyses of chain shape unit circuits of order 0.9 and order 0.8, the designed chain shape circuit schematic diagram ($q_1 = q_2 = 0.9$, $q_3 = q_4 = 0.8$) is shown as figure 3. When the output coefficient of multiplier AD633 in circuit schematic diagram 3 is 0.1, $R_1 = R_{10} = R_{19} = R_{22} = 120K\Omega$, $R_{11} = R_{12} = R_{20} = R_{21} = R_{29} = R_{30} = 100K\Omega$,

$R_2 = 24K\Omega, R_6 = 10K\Omega, R_{18} = 2K\Omega, R_{28} = 30K\Omega, R_{31} = 15K\Omega$.

b. The Analyses of Circuit Simulation's Result

By using Multisim software 10.0 to conduct simulation on the four-dimensional fractional-order chaotic system, we can reach the phase diagram of formula (10) from every direction and it is shown as figure 4(a)~(f).

Through experimental verification of numerical simulation and circuit simulation, it shows that these circuit simulation diagrams have the same chaotic attractor. It verifies the existence of four-wing four-dimensional heterogeneous fractional-order chaotic system designed by this paper and this heterogeneous fractional-order chaotic system can be realized by actual electronic circuit, which lays the certain theoretical foundation of application in actual projects.

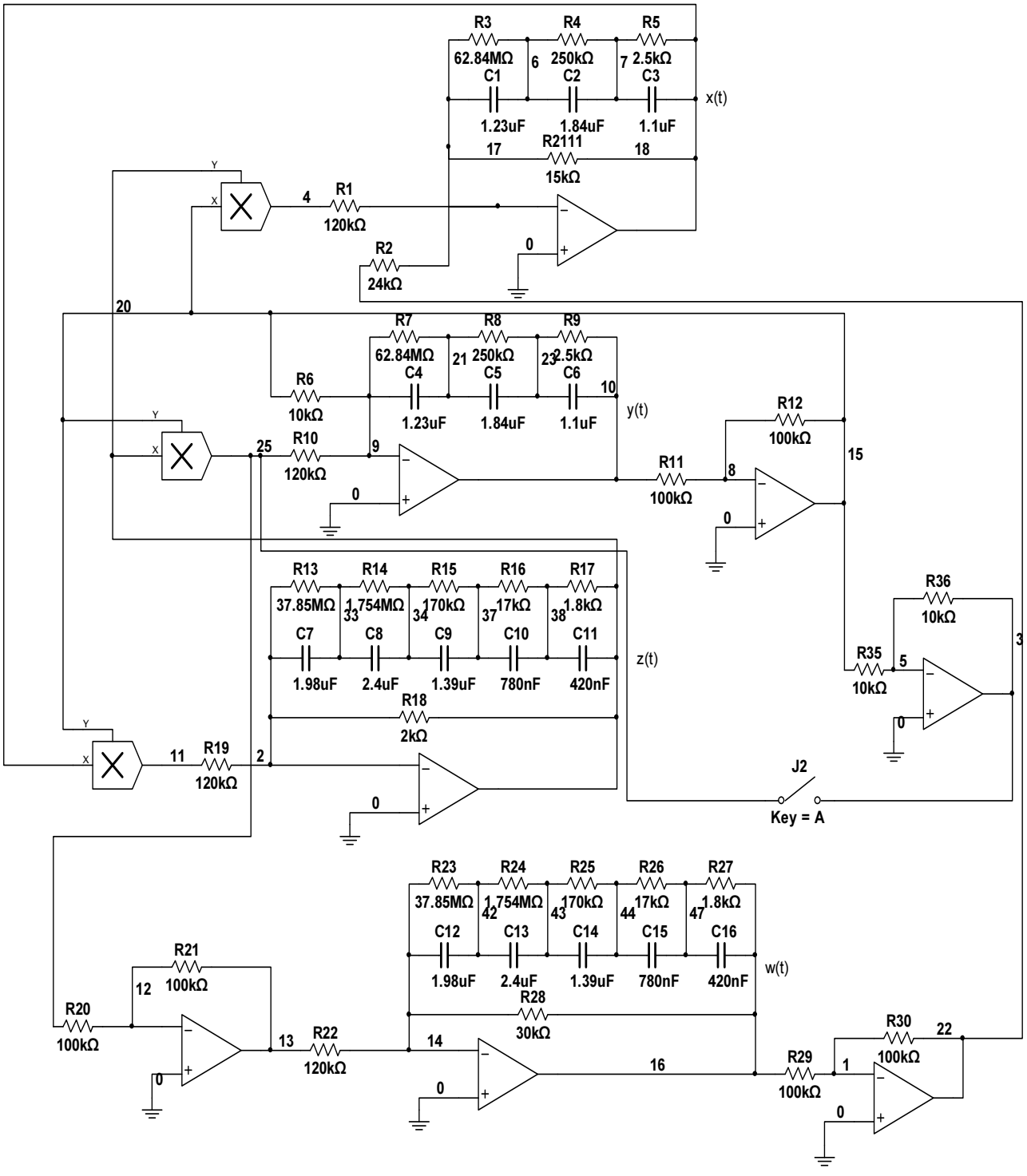
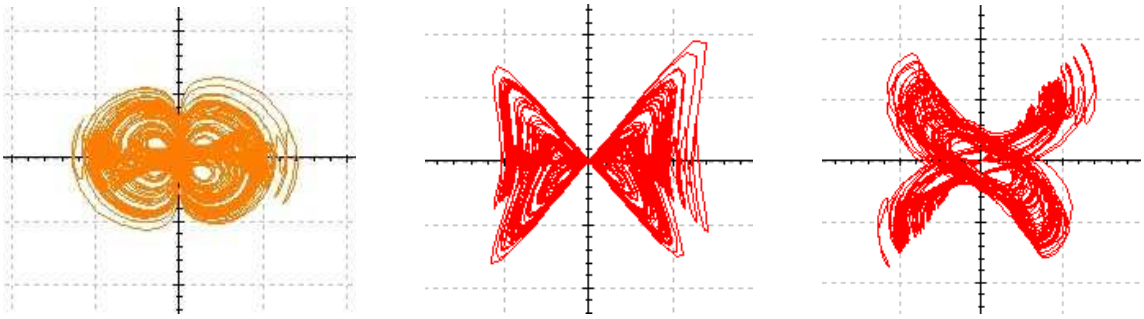
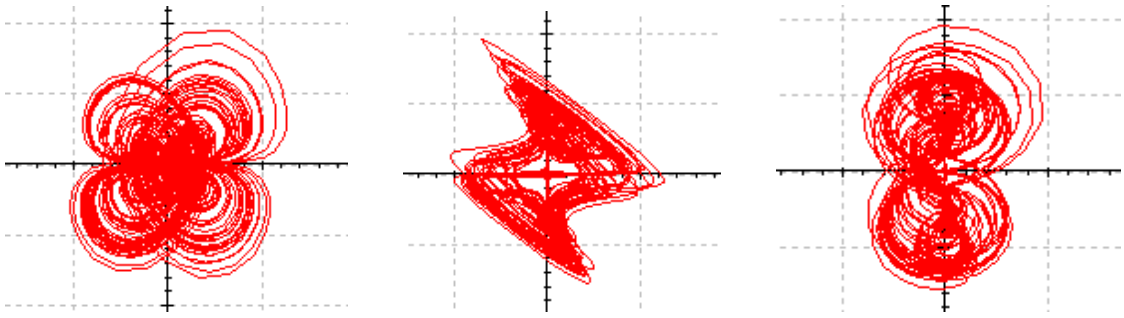


Figure 3. Chain shape circuit of four-wing four-dimensional heterogeneous fractional-order chaotic system



(a) *x*-*y* plane (b) *x*-*z* plane (c) *x*-*w* plane



(d) *y*-*z* plane (e) *y*-*w* plane (f) *z*-*w* plane

Figure 4. Circuit simulation phase diagram of four-wing heterogeneous fractional-order chaotic system

V. THE LINEAR FEEDBACK CONTROL OF CHAOTIC SYSTEM

If the fractional-order chaotic system is

$$\frac{d^q X}{dt^q} = F(X) \tag{16}$$

After adopting linear feedback, its controlled system is

$$\frac{d^q X}{dt^q} = F(X) - KX \tag{17}$$

of which $F = (f_1, f_2 ..., f_n)^T$ is nonlinear function's is N-dimensional vector $X = (x_1, x_2, ..., x_n)^T$ is N-dimensional state vector and $K = diag[k_1, k_2, ..., k_n]$ is linear

feedback control parameter, $0 \le q \le 1$. As to the four-dimensional chaotic system in this paper, if $k_1 = k_2 = k_4 = 0$, its controlled system will be described as formula (17). In the formula, coefficients $a = 10, b = 40, c = 2.5,\ \ d = 10, e = 1$, it is obviously that $S = (0,0,0,0)$ is a balance point of system (18) and system (18) linearizes its Jacobia matrix as formula (19) at S point.

According to the stability theory of fractional-order linear system, when k_2 is taken proper value, eigenvalue λ_i of controlled system (18) meets $\left|\arg(\lambda_i)\right| > \frac{q\pi}{2}$, in which $i = 1,2,3$, and controlled system (18) will gradually and steadily reaches balance point S. In this way, it realizes control of this four-wing four-dimensional heterogeneous fractional-order chaotic system.

$$\begin{cases} \dfrac{d^{q_1} x}{dt^{q_1}} = -ax + yz + bw \\ \dfrac{d^{q_2} y}{dt^{q_2}} = cy - xz \\ \dfrac{d^{q_3} z}{dt^{q_3}} = xy - dz - k_3 y \\ \dfrac{d^{q_4} w}{dt^{q_4}} = xz - ew \end{cases} \tag{19}$$

$$J = \begin{bmatrix} -a & 0 & 0 & b \\ 0 & c & 0 & 0 \\ 0 & 0 & -d & -k_3 \\ 0 & 0 & 0 & -e \end{bmatrix} \tag{20}$$

The feedback control circuit is shown as figure 5.

When circuit switch in Fig.5 is closed, we can observe state variables time waveform in Fig. 6 at any time t by fractional order chaotic circuit feedback controller.

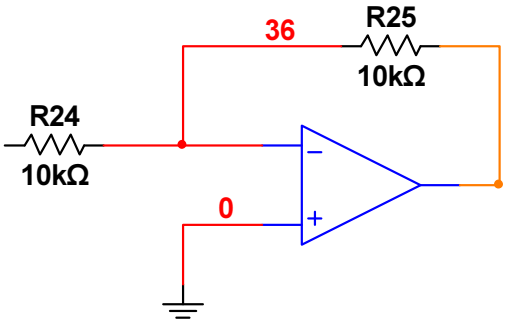
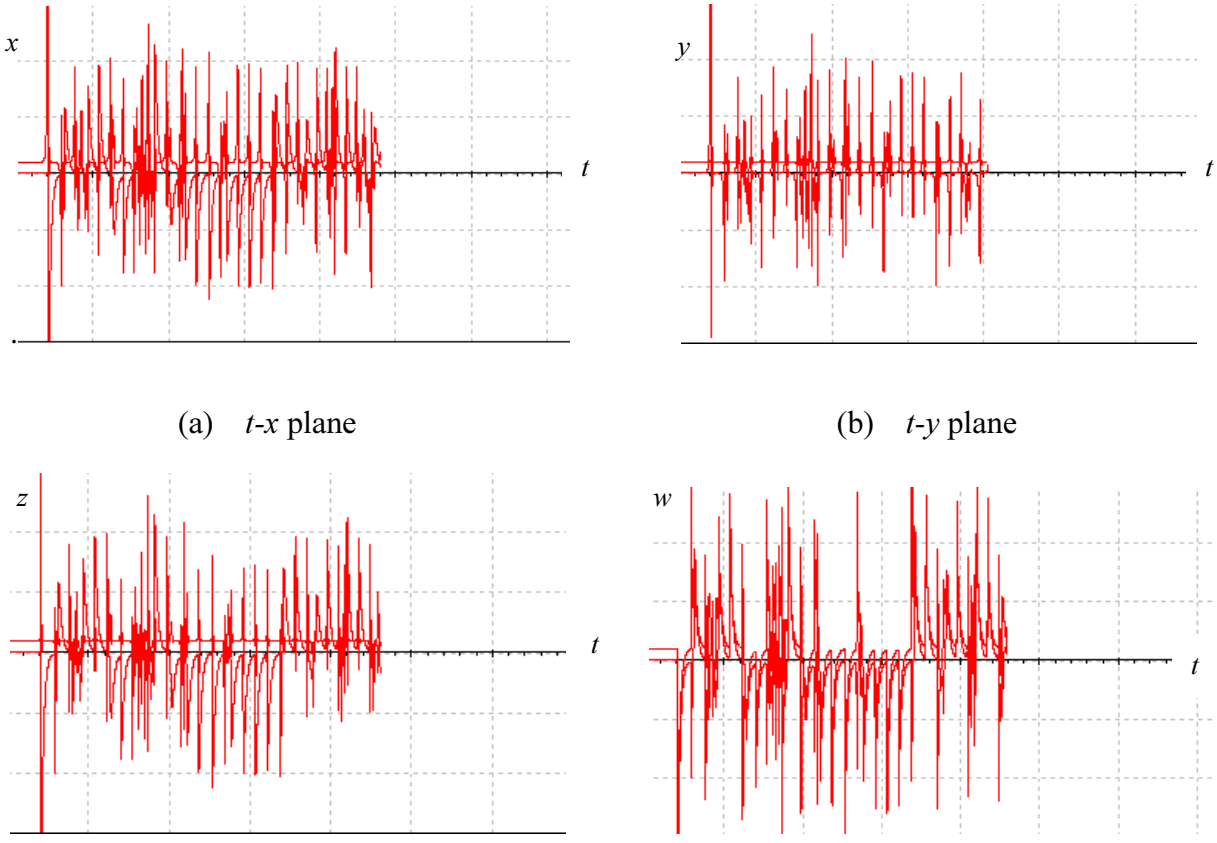


Figure 5. Feedback controller



(a) *t*-*x* plane (b) *t*-*y* plane

(c) *t*-*z* plane (d) *t*-*w* plane

Figure 6. State variable time waveform of feedback controller

Since the outcome diagram 4 of circuit experiment and the outcome diagram 1 of numerical simulation have the same chaotic attractor, it verifies the effectiveness and feasibility of this controller. Moreover, the designed controller has universality and it can be used in circuit realization and control of other fractional-order chaotic systems.

VI. CONCLUSION

This paper designs a four-wing four-dimensional heterogeneous fractional-order chaotic system and analyzes its stability and existence. The paper adopts chain shape circuit form to conduct circuit simulation on the fractional-order system through Multisim software. The results of circuit simulation and Matlab software numerical value numeration have the totally same chaotic locus and dynamical behavior, which verifies the effectiveness and reliability of the designed circuit. Meanwhile, the paper analyzes the feedback controller in circuit simulation. The results of circuit simulation and numerical simulation verify the effectiveness and feasibility of this controller and it has actual universality and application values in engineering field.

VII. ACKNOWLEDGMENTS

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