Model identification and Smart structural vibration Control using H^∞ controller

M.Sridevi, P.Madhavasarma
Department of Electronics& Instrumentation Engineering,
SASTRA UNIVERSITY, Thanjavur- 613401 T.N, India,
Emails: spms1975@yahoo.co.in, mad_sarma@yahoo.co.in

Abstract- A smart structure plays a vital role in aerospace applications and robotics and other applications. It presents many challenging control problems due to their non-linear dynamic behavior. The Objective of this work is to design a controller that minimizes the structural vibration using H∞ controller. Vibration as a measured parameter has been used to evaluate model of a non-linear process (piezoelectric actuator and sensor) at different modes .The model was generated using an ARMAX technique. By selecting appropriate weighting functions H^∞ controller were designed based on mixed sensitivity approach using singular loop shaping method. The performance of H^∞ controller was compared with LQG controller based on vibration reduction. From the results it is observed that the H∞ controller is the best suited for smart structural process.

Index terms: H^∞ controller, ARMAX, smart structure, LQG, weighting function, loop shaping, weighting function,

I INTRODUCTION

In recent times structures that have self sensing action and reaction capabilities known as smart structures play a vital role in active control of vibration. There have been significant advances in modern control theory with robust controllers gaining widespread recognition in field of aerospace and robotics. The goal of robust analysis is to find a Multivariable Stability Margin (MSM) seen by the uncertainties using a proper non conservative and analytical tool. The H^∞ control strategy as compared to classical control techniques provides an advanced method and perspective for designing control systems. This is accomplished by shaping the

frequency response characteristics of the plant according to prespecified performance specifications in the form of frequency dependent weighting functions. Chopra [1] has used piezoelectric materials for sensing and actuation. Costain et al [2] have developed practical methods for vibration control of industrial equipment. Herman [3] has analyzed modeling of smart structure using piezoelectric material as sensor/actuator. Yang et al [4] have suggested on the location of sensor and actuator and there effects in control. Choi et al [5] have designed and implemented various control methods such as Variable Structure Control (VSC), Gaudenzi et al [6] has suggested genetic algorithm and fuzzy for piezoelectric bonded structures. Raymond et al [7] have determined the optimal constant Output Feedback Gains for linear multivariable Systems. Sridevi et al [8-9] have designed state and output feedback control for cantilever beam. Umapathy et al [10] have developed the periodic output feedback controllers for disturbance rejection for piezoelectric bonded structures. Brief discussion on H^{∞} control problems have been discussed by various authors. They have not discussed about weighting function selection for controller design [11-15].The present work aims at identification of model using ARMAX technique and design of H^{∞} controller that provides robust stability to structural uncertainties for the smart structure to suppress the fundamental vibration mode of a smart cantilever beam using MATLAB software.

II EXPERIMENTAL SETUP

The experimental set-up for smart structure control is shown in Figure 1. This consists of a cantilever beam made of aluminum bonded with one pair of collocated piezoelectric patches as sensor/actuator at the fixed end. To apply disturbance a piezoelectric patch is bonded at the free end of the beam. The conditioned piezo sensor output was given as analog input to dSPACE controller board. The control algorithm was developed using SIMULINK and implemented in real time on dSPACE system using RTW and dSPACE Real Time Interface tools. The controller output was directed to piezoelectric actuator through driving amplifier. The dimensions and properties of the beam and piezoelectric patches are given in Tables 1 and 2.

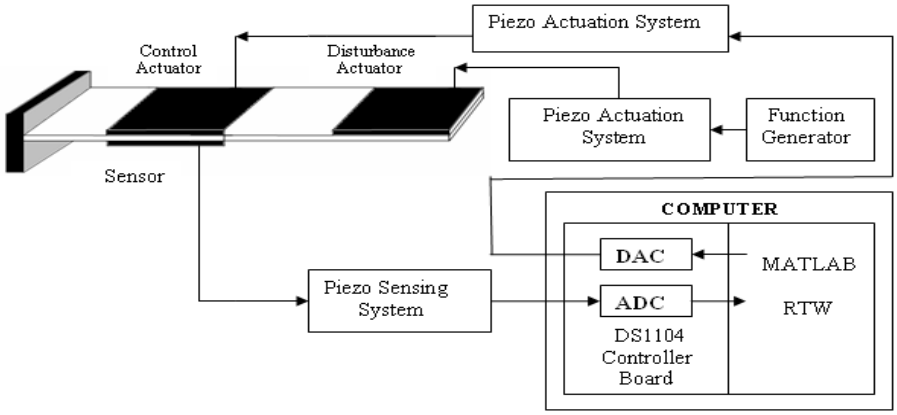


Figure 1 Experimental setup for smart structure control.

Table 1: Properties and dimensions of the Aluminum beam

Length (m)	l	0.3
Width (m)	b	0.0127
Thickness (m)	t_b	0.0023
Young's modulus (Gpa)	E_b	71
Density (kg/m^3)	ρ_b	2700
Natural frequencies (Hz)	f	31.7, 200
Damping ratio	ξ	0.0524
Damping constants	α , β	2.8676 , 4.5231x10^{-4}

Table 2: Properties and dimensions of piezoelectric sensor/actuator

length (m)	l_p	0.0765
Width (m)	b	0.0127
Thickness (m)	t_a	0.005
Young's modulus (Gpa)	E_p	47.62
Density (kg/m^3)	ρ_p	7500
Piezoelectric strain constant (m V^{-1})	d_{31}	-247x10^{-12}
Piezoelectric stress constant (V m N^{-1})	g_{31}	-9x10^{-3}

III MODEL IDENTIFICATION

Identification and control is gaining wide spread recognition due to accurate modeling obtained through system identification techniques [16-20]. The actual output of the system and predicted model output is shown in Figure 2.The state space model of the identified system in continuous domain is

$$A=\begin{bmatrix}-1350 & -480300\\ 1 & 0\end{bmatrix};B=\begin{bmatrix}1\\0\end{bmatrix};C=\begin{bmatrix}0.002 & -0.1313\end{bmatrix};D=[0]$$

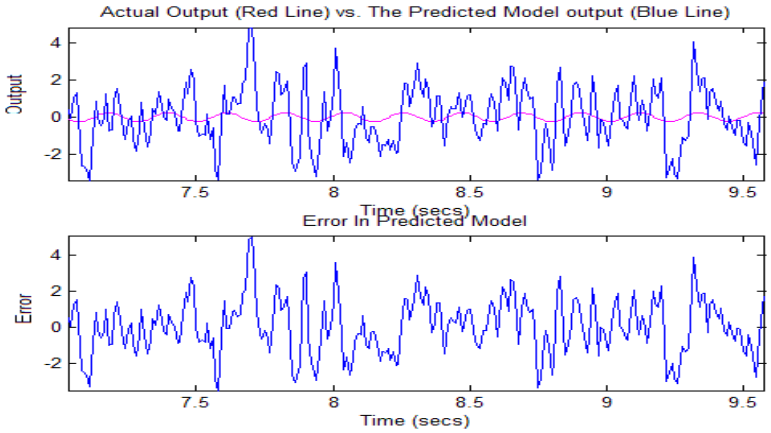


Figure 2 Actual and Predicted model output and Error in predicted model

.

IV KALMAN FILTER BASED LQG CONTROLLER DESIGN

The main criterion that accounts in the design of the estimator is the filter design and covariance matrices. If the covariance matrix is too low, the gain will also be low. Since the gain is the factor that determines the change in error of state estimates it has to be correctly calculated. The kalman filter is an estimator that produces three types of outputs given a noisy measurement sequence and the associated models. It estimates the state x (t) from noisy measurements z (t).It is almost like an implicit solution of equations, since state is not measured directly. The models can be thought of means to implicitly extract x(t) from z(t).Also the Kalman estimator can be thought of as a measurement filter that accepts the noisy sequence {z(t)} and produces the output $\tilde{\{}\hat{z}(t/t)\}$.Finally the estimator can be thought of as a whitening filter, that accepts noisy correlated measurements {z(t)} and produces

uncorrelated or white equivalent measurements {e(t)}, the innovation sequence. The operation of a Kalman filter algorithm can be viewed as a predictor-corrector algorithm. First, at time t , before receiving the measurement {z(t)}, with the previous filtered estimate $\tilde{x}$(t-1/t-1) and covariance P(t-1/t-1) it finds the best estimate of the state, based on (t-1) data samples. This is the "prediction phase" of algorithm. The state space model is used to predict the state estimate $\tilde{x}$(t-1/t-1) and associated error covariance P(t/t-1). Once the prediction based on the model is completed it calculates the innovation covariance and Kalman gain G. As soon as the measurement at time t, that is z(t), becomes available, the innovation e(t) is determined. Now this the correction phase of the innovation. The old or predicted, state estimate $\tilde{x}$(t/t-1) is used to form the filtered ,or corrected, state estimate $\tilde{x}$(t/t) and P(t/t).Here the error, or innovation, is the difference between the actual measurement and the predicted measurement $\tilde{Z}$(t/t-1).The innovation is weighted by the gain G(t) to correct the old state estimate(predicted) $\tilde{x}$(t/t-1).The associated error covariance is corrected as well. The algorithm then awaits the next measurement at time (t+1). In the absence of measurement the state space model is used to perform the prediction, since it provides the best estimate of the state .The first term of the predicted covariance P (t/t-1) relates to uncertainty in predicting the state using the model. The second term indicates the increase in error covariance, due to the contribution of the process noise. The corrected covariance equation indicates the predicted error covariance equation or the uncertainty due to prediction, which is decreased by the update, there by producing the corrected error covariance P (t/t).The controller gain is designed through the selection of controller poles through root locus analysis for the smart structure model

$$x_{(j+1)} = \Phi_j x_{(j)} + Bw_j \tag{1}$$

A discrete measurement is given by:

$$Z_{j+1} = H_{(j+1)} x_{(j+1)} + v_{j+1} \tag{2}$$

Where $j = 0,1,2.....n$

w_j = white noise with zero mean represented by Q,

Q = Process Noise

v_j = sensor noise represented by R

The algorithm is as follows:

One step Estimate Prediction:

$$\hat{x}_{j+1} = \Phi_j x_{(j/j)} \tag{3}$$

Measurement update:

$$x_{(j+1)/(j)} = x_{(j+1)/(j)} + G_{(j+1)} E_{(j+1)} \text{ here,} \tag{4}$$

$$E_{(j+1)} = Z_{(j+1)} - H_{(j+1)} x_{(j+1/j)} \tag{5}$$

Estimate error covariance prediction:

$$P_{(j+1)/(j)} = \Phi_{(j)} P_{(j/j)} \Phi'_j + BQB' \text{P} \tag{6}$$

Innovations Measurement Residual Variance:

$$\alpha_{j+1} = H_{(j+1)} P_{(j+1/j)} H'_{(j+1)} + R \tag{7}$$

Kalman's gain:

$$G = P_{(j+1)/(j)} * H'_j * \alpha^{-1}{}_{(j+1)} \tag{8}$$

Estimate Error Measurement Update:

$$P_{(j+1/j+1)} = P_{(j+1/j)} - G_{(j+1)} H_{(j+1)} P_{(j+1/j)} \tag{9}$$

Filter output:

$$P_{(j/j)} = E(x_j - x_{j/j})(x_j - x_{(j/j)})' \tag{10}$$

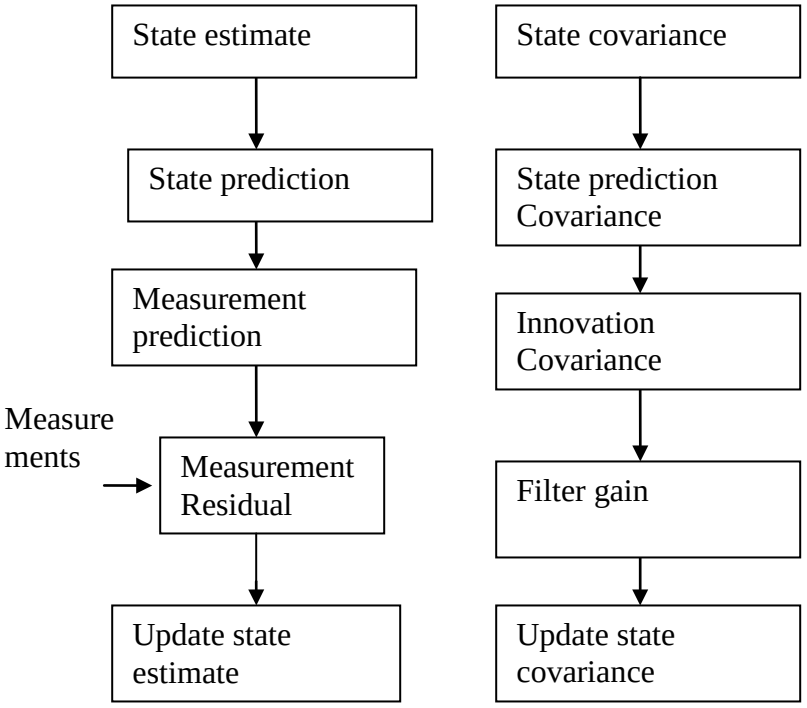
Predictor:

$$P_{(j+1/j)} = E(x_{j+1} - x_{j+1/j})(x_{j+1} - x_{(j+1/j)})' \tag{11}$$

Estimator output:

$$x_{j+1/j+1} = x_{(j+1)/j} + G_{j+1}E_{j+1} \tag{12}$$

The one cycle representation of state estimation using kalman filter is as follows.



The Kalman filter based LQG controller designed and implemented Using MATLAB software. The simulation results for open loop and closed loop responses are shown in Figure3and Figure 3a for smart cantilever beam

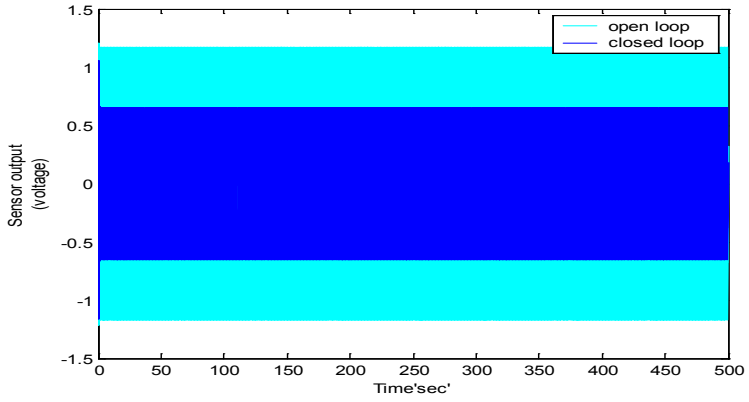


Figure3Open loop and closed loop Responses for 490sin29.1t

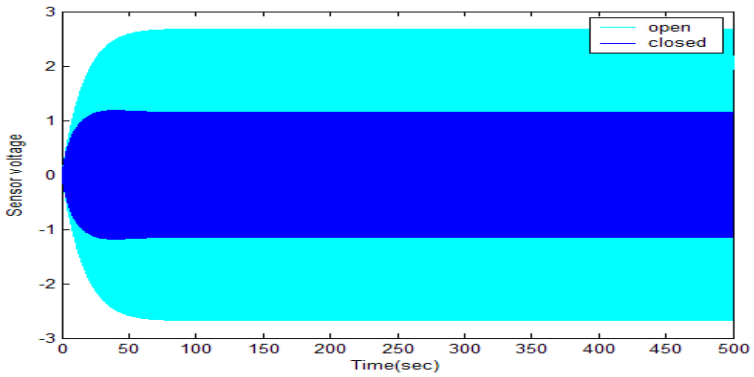


Figure3 a Open loop and closed loop Responses for 690Sin1200t

V H^{∞} CONTROLLER IMPLEMETATION

The H^{∞} control strategy as compared to classical control techniques provides an advanced methods and perspective for designing control systems. This is accomplished by shaping the frequency response characteristics of the plant according to prespecified performance specifications in the form of frequency dependent weighting functions. The goal of robust analysis is to find a Multivariable Stability Margin (MSM) seen by the uncertainties using a proper nonconservative and analytical tool. The H^{∞} theory provides a direct, reliable procedure for synthesizing a controller which optimally satisfies singular value loop shaping specifications. F(s) transfer function, then Sensitivity function S(s) is

$$S(s) = (I + L(s)^{-1}), \tag{13}$$

The complementary sensitivity function T(s) is

$$T(s) = L(s)(I + L(s))^{-1} \tag{14}$$

R(S) is represented as

$$R(s) = F(s)(I + L(s))^{-1} \tag{15}$$

$$L(s) = G(s)F(s) \tag{16}$$

The effect of additive and multiplicative uncertainties on sensitivity and complementary Sensitivity function are shown below

$$\bar{\sigma}(\Delta_M(j\omega)) = \frac{1}{\bar{\sigma}(T(j\omega))} \tag{17}$$

Perturbation, then greater the stability margin. Similarly,

$$\bar{\sigma}(\Delta_A(j\omega)) = \frac{1}{\bar{\sigma}(R(j\omega))} \tag{18}$$

The stability margins of the control system via singular value are

$$\bar{\sigma}(R(j\omega)) \le \left| w_2^{-1}(j\omega) \right| \tag{19}$$

$$\bar{\sigma}(T(j\omega)) \le \left| w_3^{-1}(j\omega) \right| \tag{20}$$

Where w_2 and w_3 are weightages on the control and output signal respectively.

The representation of the plant with the weighting functions is shown in Figure 4.

.

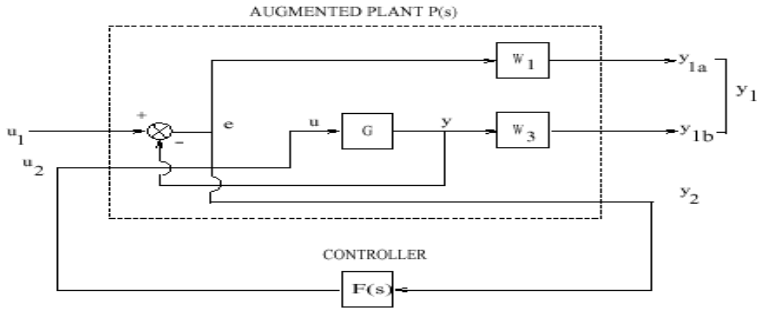


Figure 4Augmented plant

The shape of uncertainty regions defined by the amplitude ratio and phase shift bounds does not have a simple geometry and therefore cause difficulties during analysis. This problem is overcome by specifying a disk, large enough to encapsulate the original uncertainty region. The disk has only one parameter, its radius, $\tilde{l}_a(\omega)$, which is a function of frequency. That is, at each frequency value, there will be a corresponding disk. It is expected that $\tilde{l}_a(\omega)$ will increase with frequency, as it is easier to characterize the process when they are near the steady state. Although the disks provide a convenient means of describing uncertainties, their use results in more conservative description of uncertainties. Thus use of disks of radius $\tilde{l}_a(\omega)$ yields a description of a set $\Im$ of processes about the nominal model and this can be written mathematically as:

$$\Im = \left\{ G_p : \left| G_p(j\omega) - \tilde{G}_p(j\omega) \right| \le \tilde{l}_a(\omega) \right\}$$

where G_p is the real process, $\tilde{G}_p$ is the nominal model of the process and $\tilde{l}_a(\omega)$ is the radius of the disk used to describe uncertainties between the real and the nominal process at each frequency value. In other words , any member of the set $\Im$ satisfies :

$$G_p(j\omega) = \tilde{G}_p(j\omega) + l_a(j\omega)$$

where $\tilde{l}_a(\omega)$ is the uncertainty associated with the model $\tilde{G}_p(j\omega)$ and is known as the *additive uncertainty*. Further the additive uncertainty is bounded according to :

$$\left| l_a(j\omega) \right| \le \tilde{l}(j\omega)$$

Alternately, the plant uncertainties can be modeled using *multiplicative uncertainty* description as below :

$$\Im = \left\{ G_p : \frac{\left| G_p(j\omega) - \tilde{G}_p(j\omega) \right|}{\left| \tilde{G}_p(j\omega) \right|} \le \tilde{l}_m(j\omega) \right\}$$

Here , any member $\Im$ satisfies

$$G_p(j\omega) = \tilde{G}_p(j\omega)(1 + l_m(j\omega))$$

where $l_m(j\omega)$ is the multiplicative uncertainty description that has the following bounds

$$\left| l_m(j\omega) \right| \le \tilde{l}_m(j\omega)$$

The additive and multiplicative uncertainty bounds are related by the following equation :

$\tilde{l}_a(j\omega) = \left| \tilde{G}(j\omega) \right| \tilde{l}_m(j\omega)$ .One of the performance objectives of controller design is to keep the error between the controlled output and the set-point as small as possible, when the closed-loop system is affected by external signals. Thus, to be able to asses the performance of a particular controller, we need to be able to quantify the relationship between this error, the process and the controller. One such quantifying measure is the *sensitivity function* $\varepsilon(s)$ and its counter part *complementary sensitivity function* $\eta(s)$ .The sensitivity function $\varepsilon(s)$ relates the effect of disturbance d(s) on the process output Y(s). The complementary sensitivity function $\eta(s)$ relates the effect of the set-point R(s) on the process output Y(s). Thus, for a conventional feedback control system, these functions are defined as follows:

$$\varepsilon(s) = \frac{Y(s)}{d(s)} = \frac{1}{1 + G_C(s)G_P(s)}$$
$$\eta(s) = \frac{Y(s)}{R(s)} = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)}$$

Also $\eta(s) = 1 - \varepsilon(s)$. For perfect disturbance rejection, $\varepsilon(s) = 0$ and for perfect set-point tracking $\eta(s) = 1$. Both these function values are bound between 0 and 1.

If there is process noise N(s) $\neq 0$, then

$$\eta(s) = \frac{Y(s)}{R(s) - N(s)} = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)}$$

$\eta(s)$ is also affected by noise N(s). In this case, $\eta(s)$ has to be made small so as to reduce the influence of random inputs. In other words we want $\eta(s) = 0$ or equivalently $\varepsilon(s) = 1$. This means that good set-point tracking and disturbance rejection has to be traded off against the noise suppression. Design of controller involves the selection of the weighting functions satisfying singular value loop specifications and the stability margins. The weighting functions over the desired frequency range are not related directly to performance characteristics. Numerous trial weighting functions are required in order to obtain desired performance characteristics. The weighting functions to the error signal, control signal, output signal are represented as w_1, w_2 and w_3 respectively. The weighting function w_1 is used to reshape the frequency response characteristics and is chosen in such away that w_1 act as a low pass filter. No weightage is given to control signal w_2. The weighting function w_3 is decided based on the multiplicative uncertainty present in the plant. The plant is augmented with the decided weighting functions. The H^∞ controller is designed for the augmented plant. The H^∞ controller F(s) obtained is given below in equation 21

$$F(s) = \begin{bmatrix} -3.1760 & 239.0253 & 0.0552 & -0.7667 & 0.0011 \\ -160.4203 & -3.1763 & -0.2562 & -0.0046 & -0.002 \\ 3.5205 & -2.6571 & -599.28 & 191.59 & -5.2468 \\ -0.2384 & 0.1797 & 0.1295 & -13.01 & 0.3564 \\ -0.0091 & -0.0014 & 0.0112 & -0.3121 & 0 \end{bmatrix} \quad (21)$$

The H^{∞} controller obtained after performing model reduction is giving in equation 22

$$F(s)=\begin{bmatrix} -12.98 & 0.28 & -0.0518 & 0.3313 \\ -0.2801 & -6.2403 & 195.7742 & 0.0036 \\ -0.0518 & -195.7742 & -0.2165 & 0.0007 \\ -0.3313 & 0.0036 & 0.0007 & 0 \end{bmatrix} \quad (22)$$

The Norm of the cost function is T_{y1u1}=0.987. The open loop and closed loop responses with H^{∞} controller are shown in Figures 5 to 9 the frequency response characteristics of open loop and closed loop are shown in Figure10.

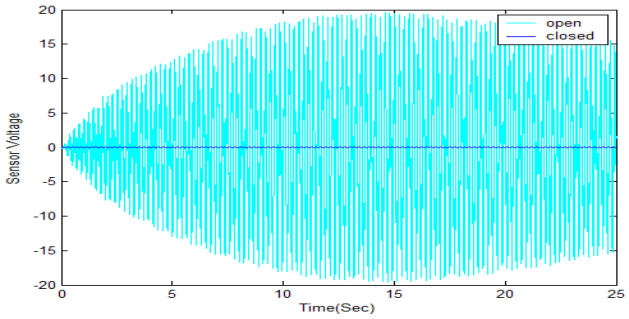


Figure 5 Responses of open loop and closed loop for 490sin29.1t

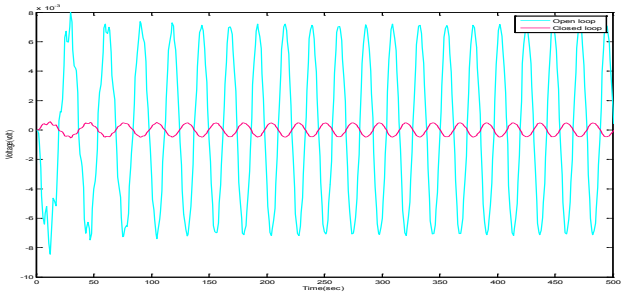


Figure 6 Open loop and closed loop responses for 1sin29.1t

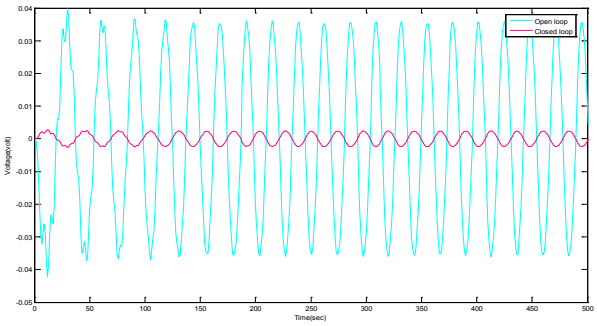


Figure 7 Open loop and closed loop responses for 5sin29.1t

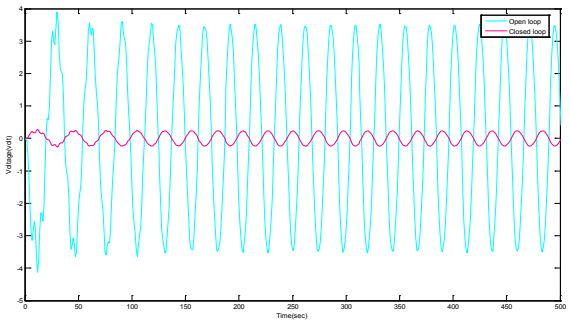


Figure 8 Open loop and closed loop responses for 690sin29.1t

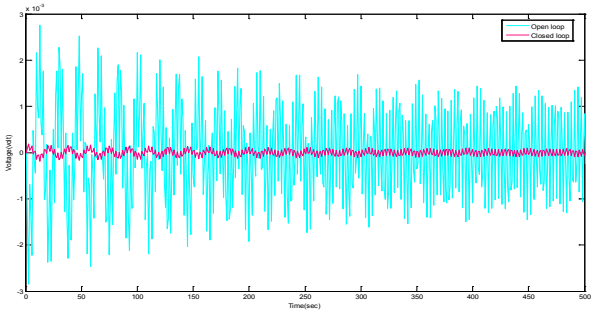


Figure 9 Open loop and closed loop responses for 1sin1200t

The H-inf controller for the identified beam is given in equation 23

$$F(s)=\begin{bmatrix} -3.1760 & 239.0253 & 0.0552 & -0.7667 & 0.0011 \\ -160.4203 & -3.1763 & -0.2562 & -0.0046 & -0.002 \\ 3.5205 & -2.6571 & -599.28 & 191.59 & -5.2468 \\ -0.2384 & 0.1797 & 0.1295 & -13.01 & 0.3564 \\ -0.0091 & -0.0014 & 0.0112 & -0.3121 & 0 \end{bmatrix} \tag{23}$$

The H-inf controller obtained after performing model reduction is given in 24

$$F(s)=\begin{bmatrix} -12.98 & 0.28 & -0.0518 & 0.3313 \\ -0.2801 & -6.2403 & 195.7742 & 0.0036 \\ -0.0518 & -195.7742 & -0.2165 & 0.0007 \\ -0.3313 & 0.0036 & 0.0007 & 0 \end{bmatrix} \tag{24}$$

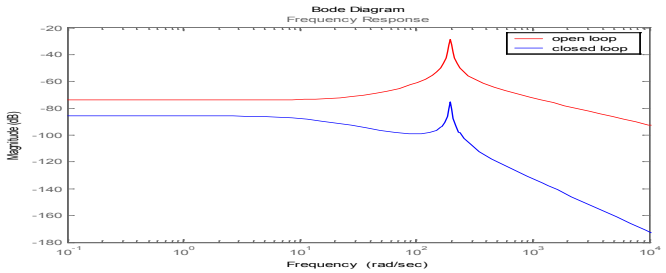


Figure 10 Open loop and closed loop responses in frequency domain

The importance of the cost function described here is it guarantees that certain desirable properties are sought, or are maintained if they already exist, such as good stability margins and low output sensitivity. The plot of cost function, Sensitivity function, Complementary Sensitivity function, closed loop response for step input for the smart canti lever beam are shown in Figure 11. The cost sensitivity function for beam is less than 1db, sensitivity function is 42 db and complementary sensitivity function is 27.3 db

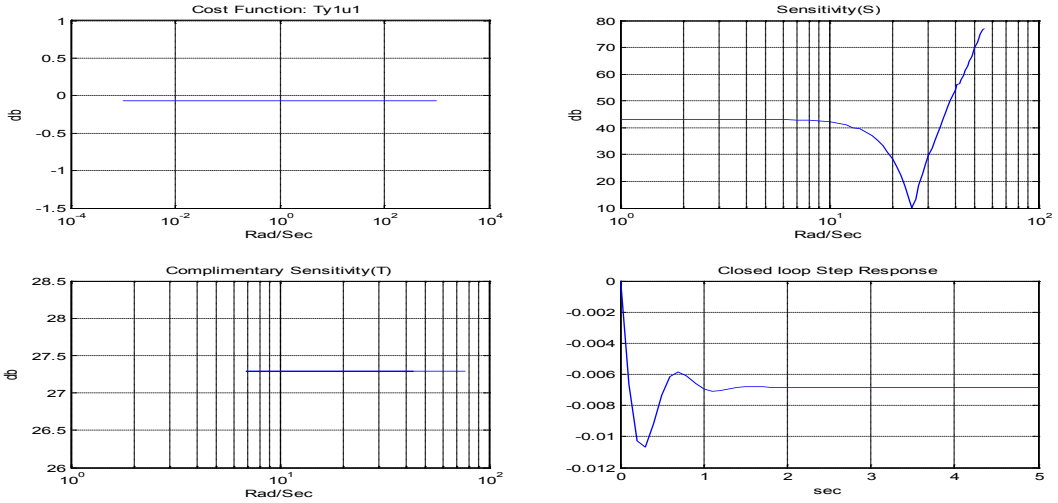


Figure11 Responses of cost, Sensitivity function 'S',
Complementary sensitivity function 'T' and closed loop response for Step input.

The open loop and closed loop responses for the cantilever beam system was tested for various excitation signals.Table3 shows the performance analysis of the controllers for vibration suppression at various amplitude and frequencies. The amplitude of vibration was reduced to approximately varying from 92 percent to 97 percent for H infinity controller and 50 percent to 64 percent for LQG controller.

Table 3 Performance analysis of the controllers

S.No	**Controller**	**Input voltage (Volt)**	**Input frequency (Hz)**	**Output amplitude (Volts)**		**vibration Suppression (%)**
				Open loop	**Closed loop**	
1.	H^{∞} **controlle**r	1	29.1	0.007	0.0005	92.8
2.		5		0.037	0.002	94.5
3.		490		3.8	0.2	94.7
		1	1200	1.7	0.02	97
4.		5		0.005	0.002	97
		490		0.56	0.1	92
5	**LQG controller**	490	29.1	1.25	0.53	50
		690	1200	2.7	1	64

VI CONCLUSIONS

The second order modal was identified for the cantilever beam using ARMAX technique. H^{∞} Controller using Singular value loop shaping and LQG controller has been designed. The plant augmentation was carried using weighing functions which were based upon addititive and multiplicative uncertainties. The closed loop response for the designed controller obtained using MATLAB software. It is seen from the Table H^{∞} controller is better suited than LQG controller for this process

REFERENCES

[01] I.Chopra. Review of state of art of smart structures and integrated systems, AIAA Journal, 40(11), 2002, pp. 2145-2182.

[02] A. K Costain. J. M Robichaud, Practical methods for vibration control of industrial equipment (Bretechengg ltd Canada 2004, PP 1-8)

[03] Herman Shen M H, Analysis of beams containing piezoelectric sensors and actuators, Smart Materials, Structures, (3), 1994, pp. 439-447.

[04] Yang S. M, and Lee Y. J, Optimization of non-collocated sensor/actuator location and feedback gain in control systems Smart Materials, Structures, (8), 1993, pp. 96-102.

[05] Choi S.B, Chong C.C and Lee C.H Position tracking control of a smart flexible structure featuring a piezoflim actuator, Journal of Guidance, Control and Dynamics, 19(6), 1996, pp. 1364-1369.

[06] P Gaudenzi. E Fantini., K Vlasis. and J. Charis Gantes. Genetic algorithm optimization for the active control of a beam by means of PZT actuators, Journal of Intelligent Material Systems and Structures, 9, 1998, pp. 291-300.

[07] A, Raymond. On the determination of the optimal constant Output Feedback Gains for linear Multivariable Systems IEEE transactions on Automatic control, vol Ac-15:1 146-142. 1970.

[08] M Sridevi. J, Balasubramani. And M Umapathy. Design & Implementation of State and Output Feedback Control for Piezoelectric Bonded Beam Proceedings of ISSS 2005 International Conference on Smart Materials, Structures and Systems, Bangalore, 28th to 30th, 2005, Vol 1, pp SA 148 – SA 155

[09] M Sridevi., .M Umapathy, Vibration control of smart cantilever beam using DS1104, M.Tech Thesis NIT Trichy 2005.

[10] .M Umapathy, B Bandyopadhyay. Vibration control of flexible beam through smart structure concept using periodic output feedback System Science Journal, 26(1), 2000, pp. 43-66

[11] N Rubin. And D Limbeer.. H-inf identification for robust control In proceedings of American control conference Pages 2040-2044, Chicago,June 1994.

[12] J.C Doyle,. K Glover,, P.P Khargonekar, and Francis, B.A.State-Space Solutions 18 of Standard H2 and H-inf Control Problems IEEE Transactions on Automatic Control, Vol. AC-29, 831-847.

[13] J.L Crassidis, D.J Leo, D.J Inman, and D.J Mook. Robust Identification and Vibration Suppression of a Flexible Structure Journal of Guidance, Control, and Dynamics, Vol. 17, No. 5, 921-928.

[14] M. G Saponov, D. J. N Limebeer, and R. Y Chiang,. Simplifying the H-inf Theory via Loop-Shifting, Matrix-Pencil and Descriptor Concepts Int. J.Control, 50, No. 6, pp. 2467–2488.

[15] D Edberg. and A.Bicos, Design and Development of Passive and Active Damping Concepts for Adaptive Structures Conference on Active Materials and Adaptive Structures by G. Knowles, IOP Publishing Ltd., Bristol, UK, 377-382.

[16] R.N Juang. Applied System Identification Prentice Hall, Englewood Cliffs, NJ. 1994.

[17] P.Madhavasarma, S. Sundaram. Model based tuning of controller for non linear hemispherical tank processes. Instrum. Sci&Technol. 2007, 35, 681-689

[18] P.Madhavasarma, S Sundaram, Model based tuning of controller for non linear spherical tank processes. Instrum. Sci&Technol. 2008, 36, 420-431.

[19] P.Madhavasarma,; S. Sundaram,. Model based evaluation of controller using flow sensor for conductivity process. 2007, 36, 420-431. J. Sensors & Transducers, 79, 1164-1172.

[20] P.Madhavasarma, S. Sundaram, Leak Detection and Model Analysis for Nonlinear Spherical Tank Process Using Conductivity Sensor. *J. Sensors & Transducers*, 2008, *89*, pp. 71-76.

: