DE GRUYTER OPEN
International Conference KNOWLEDGE-BASED ORGANIZATION
Vol. XXII No 3 2016

RESEARCHES UPON THE CAVITATION PHENOMENON

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Abstract: *We use from in distribution of the equations for a non viscous, incompressible liquid. Using the Dirac filtering property some integral equations were obtained, giving the liquid velocity and pressure fields due to the bubble's implosion. The difficulties derive from the fact that the system of differential equations with partial derivates is non-linear and the distribution in the spherical coordinates depends on several parameters. The most important criterion, decisive in the cavitation process, is the one in which the vaporization pressure intervenes* p_v.

Keywords: pressure, cavitation phenomenon, discontinuity surface, the Π theorem.

1. Introduction

The main problem of the hydrodynamics of cavitation implosion of a single bubble consists in pressure and velocity fields determination, including the collapse velocity of the bubble wall.

The bubble surface is a discontinuity surface, and the over pressure produced by a bubble collapse are very great (many thousands bar) facts that suggests the opportunity of the distribution theory use.

2. Theoretical problems

Let's write the equations of the mechanics of the light fluid at distribution.

It is considered a closed surface $S(\bar{r},t)=0$ which delimits an internal domain D_i from an external domain D_e, unlimited, filled with fluid.

The local form of the continuity equation, valid in the absence of the discontinuous surfaces, is:

$$\frac{\partial \rho}{\partial t}+div(\rho\bar{v})=0 \tag{1}$$

e choose $\theta(\bar{r},t)$ to be the characteristic function of the D^i domain defined by:

$$\theta(\bar{r},t)=\theta(x,y,z,t)=\begin{cases}1 & \text{for } (x,y,z)\in D\\ 0 & \text{for } (x,y,z)\notin D\end{cases} \tag{2}$$

we obtain:

$$\begin{aligned}\rho &= \rho^{(i)}\theta+\rho^{(e)}(1-\theta)\\ \bar{v} &= \bar{v}^{(i)}\theta+\bar{v}^{(e)}(1-\theta)\end{aligned} \tag{3}$$

assuming that all the functions are $C^2(D^i)$ and $C^2(D^e)$. We use the formula:

$$\frac{\partial\theta(\bar{r},t)}{\partial t}=(\bar{d}\cdot\bar{n})\delta_s \tag{4}$$

$$grad\,\theta(\bar{r},t)=-\bar{n}\,\delta_s \tag{5}$$

where $\bar{d}$ represents the local velocity of the mobile surface $S(\bar{r},t)=0$ and δ_s is the Dirac distribution associated with this surface.

By introducing (2) and (3) in (1) and using (4) and (5) according to the vectorial analysis calculations we obtain:

DOI: 10.1515/kbo-2016-0100
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$$0=\frac{\partial\rho}{\partial t}+div(\rho\bar{v})=\left\{\frac{\partial\rho^{(i)}}{\partial t}+div\left(\rho^{(i)}\bar{v}^{(i)}\right)\right\}\theta+$$

$$+\left\{\frac{\partial\rho^{(i)}}{\partial t}+div\left(\rho^{(e)}\bar{v}^{(e)}\right)\right\}(1-\theta)+\left[\rho(\bar{v}-\bar{d})\bar{n}\right]_S$$

Due to the fact that for each of the domains Di and De, the continuity equation remains valid, the formulas between brackets are null. Consequently, in distributions, the form of the continuity equation is valid anywhere $D^i\cup S(\bar{r},t)\cup D^e$ and it is:

$$\frac{\partial\rho}{\partial t}+div(\rho\bar{v})=\left[\bar{\rho}(\bar{v}-\bar{d})\bar{n}\right]_S\delta_s \qquad (6)$$

Analogously (4), the Euler equation in the formula stated by Helmholtz, valid in the absence of the discontinuous surfaces has the following formula:

$$\frac{\partial\bar{v}}{\partial t}+grad\frac{(\bar{v})^2}{2}-\bar{v}xrot\bar{v}=\bar{f}^{(m)}-\frac{1}{\rho}gradp \qquad (7)$$

and in the presence of the discontinuous surface S:

$$\frac{\partial\bar{v}}{\partial t}+grad\frac{(\bar{v})^2}{2}-\bar{v}xrot\bar{v}=$$

$$=\bar{f}^{(m)}-\frac{1}{\rho}gradp+\left[\rho\bar{v}(\bar{v}\bar{d})\bar{n}\right]_S\delta_s+\left[p\right]_S\bar{n}\delta_s$$

valid in $D^i\cup S\cup D^e$, where $\bar{f}^{(m)}$ represents the field of the massic forces and $\left[p\right]_S$ is the pressure scale which separates the two domains.

Observation: it is known from (1) that the effect of the viscosity in the cavitation phenomenon is very low, that is when considering the Euler equations, valid for the ideal fluids (light), is legitimate.

Hypothesis

It is considered an unlimited, incompressible, light fluid in which there is a spherical cavitation bubble, with the initial radius R_0, empty on the inside. The fluid movement of the implosion of the bubble is considered irrational $rot\vec{v}(\vec{r},t)=0$ and the massic forces field is unvalued $\vec{f}^{(m)}=0$.

Using the spherical coordinates, originated in the centre of the bubble, symmetrically, is enough for the study of the radial direction, reducing to a one-dimensional problem.

The closed surface $S(\vec{r},t)=0$ is assimilated by a sphere with the equation r = R(t), the local velocity of the surface becomes $\dot{R}(t)$, representing the implosion velocity of the bubble wall.

The mathematic pattern

In the spherical coordinates, the Dirac distribution (4) associated with the surface $S(\vec{r},t)=0$ $(r=R(t))$ is:

$$\delta_{S(r,t)}=\frac{\delta(r-R(t))}{4\pi R^2(t)}$$

In addition, in the spherical coordinates, the form of the vectorial differential operators which intervene in equation (6) and (7) is used:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v(r,t)\right)=\rho[v(r,t)-R(t)]\frac{\delta(r-R(t))}{4\pi R^2(t)} \qquad (8)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial r}\left[\rho v\left(v-\dot{R}(t)\right)\right]=\frac{\delta(r-R(t))}{4\pi R^2(t)}+\left[p\right]_S\frac{\delta(r-R(t))}{4\pi R^2(t)} \qquad (9)$$

$\lim_{r\to+\infty}p(r,t)=p_\infty$ where p_∞ is a constant

$\lim_{r\to+\infty}v(r,t)=0$,

$$\rho(r,t)=\rho_e\cdot H_{R(t)}(r)=\begin{cases}\rho_e & \text{pentru } r\geq R(t)\\ 0 & \text{pentru } 0<r<R(t)\end{cases}$$

ρ_e is for the density of the fluid, $H_{R(t)}(r)$ is Haviside's unit step function of the R(r) parameter.

Observation: $\frac{\partial \rho}{\partial t} = 0$

The implosion velocity of the bubble $r = R(t)$ is given by Rayleight's classic formula (non-linear differential equation) obtained in (2) especially using the kinetic energy theorem:

$$R(t) = \sqrt{\frac{2}{3}\frac{p_\infty}{\rho_e}\left[\left(\frac{R_0}{R(t)}\right)^3 - 1\right]} \qquad (10)$$

where $R_0 = R(t)_{t=0}$.

Calculus

The difficulties derive from the fact that: the system of differential equations with partial derivates is non-linear and the distribution in the spherical coordinates depends on several parameters.

The only advantage is that the equations are linked, that is from (8) may result $v(r,t)$ and then, knowing $v(r,t)$, from (9) may result $\rho(r,t)$.

In order to apply the filtration property of the Dirac distribution:

$$\int_{-\infty}^{+\infty} f(x)\upsilon(x - x_0)dx = f_{x_0}$$

under:

$$\int_{-\infty}^{+\infty} f(r,t)\rho(r - R(t))dr = f(R(t),t)$$

in the equations (8) and (9) we artificially extend the definition domain of the functions $p(r,t)$ and $v(r,t)$:

$$\tilde{p}(r,t) = \begin{cases} p(r,t) & \text{pentru } r \geq 0 \text{ si } t \geq 0 \\ 0 & \text{pentru } r < 0 \text{ si } t < 0 \end{cases}$$

analogously for $v(r,t)$.

Equation (8) becomes:

$$\int_{-\infty}^{+\infty} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho(r,t)\tilde{v}(r,t)\right)dr = \qquad (8')$$

$$= \int_{-\infty}^{+\infty} \frac{\tilde{\rho}(r,t)}{4\pi R^2(t)}\left[\tilde{v}(r,t) - \dot{R}(t)\right]\rho(r - R(t))dr$$

For the left part of the equation (8') several calculus are necessary:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v(r,t)\right) = \frac{1}{r^2}r^2\frac{\partial}{\partial r}\left(v(r,t)\right) - \frac{1}{r^2}2rv(r,t)$$

because the density is 0 for $r < R(t)$, the inferior integration limit is $R(t)$ not $-\infty$

using the asymptomatic condition

$\lim_{r\to+\infty} v(r,t) = 0$

$$\int_{R(t)}^{+\infty} \frac{\partial}{\partial r}\tilde{v}(r,t)dr = \tilde{v}(r,t)\Big|_{R(t)}^{+\infty} = 0 - v(R(t),t)$$

The equation (8') becomes:

$$\rho e\left[-\tilde{v}(R(t),t) + 2\int_{R(t)}^{+\infty} \frac{v(r,t)}{r}dr\right] = (11)$$

$$= \frac{\rho(R(t),t)}{4\pi R^2(t)}\left[\tilde{v}(r(t),t) - \dot{R}(t)\right]$$

but for $r \geq R(t)$ $\quad \begin{cases} \tilde{v}(r,t) = v(r,t) \\ \rho(R(t),t) = \rho e \\ v(R(t),t)\dot{R}(t) \end{cases}$

it results: $\int_{R(t)}^{+\infty} \frac{v(r,t)}{r}dr = \frac{1}{2}\dot{R}(t)$

where $\dot{R}(t)$ and $R(t)$ are known from (10). It can be observed that the integral depends only on the velocity values at the bubble wall. By applying the same treatment as before to the equation (9), we obtain:

$$\int_{R(t)}^{+\infty} \frac{\partial}{\partial t}\left(v+\frac{v^2}{2}\right)dr+\frac{1}{\rho e}\int_{R(t)}^{+\infty}\frac{\partial p}{\partial r}dr=$$

$$=\rho e\left[v\left(R(t),t\right)\left(v\left(R(t),t\right)-\dot{R}(t)\right)\right]\cdot\frac{1}{4\pi R^2(t)}+\frac{p\left(R(t),t\right)}{4\pi R^2(t)}$$

because $v\left(R(t),t\right)=\dot{R}(t)$ the first formula of the right element is null.

$$\int_{R(t)}^{+\infty}\frac{\partial p}{\partial r}dr=p_\infty-p\left(R(t),t\right)$$

$$\int_{R(t)}^{+\infty}\frac{\partial}{\partial t}\left(v+\frac{v^2}{2}\right)dr+\frac{p_\infty}{\rho e}=p\left(R(t),t\right)\cdot\left[\frac{1}{4\pi R^2(t)}+\frac{1}{\rho e}\right] \quad (12)$$

3. Conclusions

Used in distributions, the equations form in the fluid mechanics and the filtration property of the Dirac distribution, several integral formulas regarding the cavitational implosion are obtained.

The mathematic pattern, which only describes the fluid movement, can only indicate something about the hydrodynamic effects of the cavitational implosion, the thermal and electrochemical effects, experimentally presented, can be analogously analyzed.

The most important criterion, decisive in the cavitation process, is the one in which the vaporization pressure p_v intervenes.

The cavity length does not change significantly, the liquid – vapor interface is smooth and has a glossy aspect along a short distance from the leading edge. At the end of the cavity it breaks partially into small bubbles. As the cavity expands, the liquid – vapor interface becomes distorted, wavy and unstable, yielding to breakup and unsteadiness. At this stage significant variations of the location of the cavity closure point are observed while shedding vapor structures called „cloud" cavitatin. This process induces high - level pressure pulses and is known to be one of the most destructive forms of cavitation.

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