J. Hydrol. Hydromech., 63, 2015, 1, 29–37
DOI: 10.1515/johh-2015-0001

# Hydrodynamic modeling of stratified estuary: case study of the Jadro River (Croatia)

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**Abstract:** The Jadro River with total length of 4.3 km and average annual discharge of 7.9 $m^3 s^{-1}$ is a relatively small river on the east coast of the Adriatic Sea, close to Split. Field campaign measurements were made to estimate salt intrusion in the Jadro estuary in July 2012. This measurement confirmed the stratified character of the estuary where fresh water flows in a thin layer over denser sea water. Furthermore, a numerical model was set up for simulating unsteady stratified flow without mixing between the layers. The model is applied for the Jadro River and field measurements are used for calibration. In addition, the steady state of stratification within the estuary is analyzed by a box model which assumes mixing between layers. Results of the numerical and the box models were compared. The flushing time estimated with the box model is approximately 1.5 day for summer steady conditions. Numerical analysis however shows that the residence time is much larger owing to flow unsteadiness.

**Keywords:** Estuary; Salinity; Numerical model; Box model; Jadro; Croatia.

## INTRODUCTION

Estuaries of Croatian rivers debouching into the Adriatic Sea are usually stratified (Legović, 1991; Ljubenkov, 2006). This condition typically occurs in seas with relatively small tides, such as the Mediterranean Sea (Ibanez et al., 1997; Kurup et al., 1998; Sierra et al., 2004). The most comprehensive measurement of this phenomenon on the east Adriatic coast have been conducted on the Neretva estuary, due to great economic and agricultural importance of that region for the Republic of Croatia. In addition, the process of salinization of a river estuary is of interest not only from a hydrodynamic point of view, but also for environmental, biological and chemical aspects (Kasai et al., 2010; Wolanski et al., 2006).

Ljubenkov (2006) developed a numerical model to simulate the unsteady stratified flow in a river mouth and applied it to the Neretva River. However, this model is quite general and is here applied to the Jadro River estuary.

Several authors applied commercial hydrodynamic software packages to the study of stratified estuaries. For example, Sierra et al. (2004) simulated the salt wedge dynamics in the Ebro River estuary by means of the MIKE 21 package, developed by the Danish Hydraulic Institute (DHI, 2011). The estuary was treated as a water body with two layers of different characteristics. Funahashi et al. (2013) applied the Delft3D-Flow model developed by Delft Hydraulics (DELTARES, 2011), to the Yura River estuary (Japan). However, complex 2-D and 3-D hydrodynamic commercial models are often expensive and therefore inaccessible. In addition, they often require very complex and expensive field measurements. Previous experience of Ljubenkov (2006) and Ljubenkov and Vranješ (2012) shows that a 1-D model, as that presented in this work, can be successfully applied to investigate salt wedge dynamics.

Besides a numerical approach, we also apply the two layer box model developed by Vieira and Bordalo (2000) for the Douro River estuary (Portugal), based on Knudsen's hydrographic theorem (Dyer, 1997). The results of these two different approaches, applied to the Jadro River, are here compared together and with field observations. Field campaign measurements were in fact made in the summer period, which is a critical period with the strongest salinization. Measurements are essential for both the calibration of numerical model and box model calculation, as well as for assessment the degree of stratification. The Jadro estuary turns out to be stratified in summer period when fresh water discharges are relatively small.

## STUDY AREA

The Jadro River flows from its source in the foothills Mosor through an alluvial valley and eventually debouches in the Adriatic Sea through the Bay of Kaštela (43° 32' 05" E, 16° 28' 33" E), close to Split (Croatia). The total length of the watercourse is 4.3 km (Fig. 1). Through the urban area of Solin the river splits into several arms.

The topographic catchment of the Jadro is relatively small and covers about 22 $km^2$. Even though the actual hydrological catchment is possibly much higher, its boundaries have not yet been reliably established. The complexity of groundwater flow and catchment size ensures continuity of river source throughout the year. The source is located at a level of 34.2 m above mean sea level. The discharge of the Jadro River is measured at the gauging (hydrologic) station of Majdan.

Several smaller streams and two larger tributaries, Poklinovac and Rupotina, contribute to the flowing discharge. However, the tributaries have in general a torrential character and, hence, feed the Jadro occasionally, during the rainy season, bringing significant amounts of sediment. In the summer period the Poklinovac and Rupotina streams are mostly dry.

The average annual discharge of the Jadro River measured at the Majdan station is 7.9 $m^3 s^{-1}$ (1961–2010), the highest mean annual flow being 12.8 $m^3 s^{-1}$ (1970) and the minimum 5.1 $m^3 s^{-1}$ (1983). Within a year there is a considerable variation of the flow (Fig. 2). Thus, the largest flows occur from November to March, with averages larger than 10 $m^3 s^{-1}$. The minimum mean monthly flows are observed in summer (July, August and September) and attain values of 3.3 $m^3 s^{-1}$, 2.9 $m^3 s^{-1}$ and 3.7 $m^3 s^{-1}$, respectively. It should be noted that even during the winter months relatively small flow rates can occur with monthly averages less than 4 $m^3 s^{-1}$.

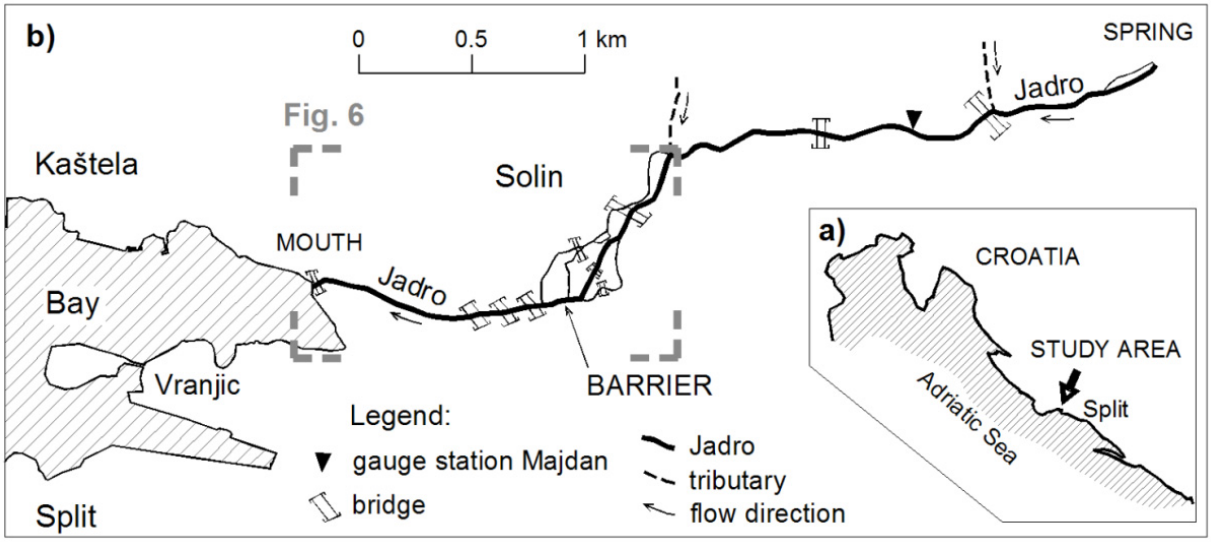



**Fig. 1.** Location map of the Jadro River.

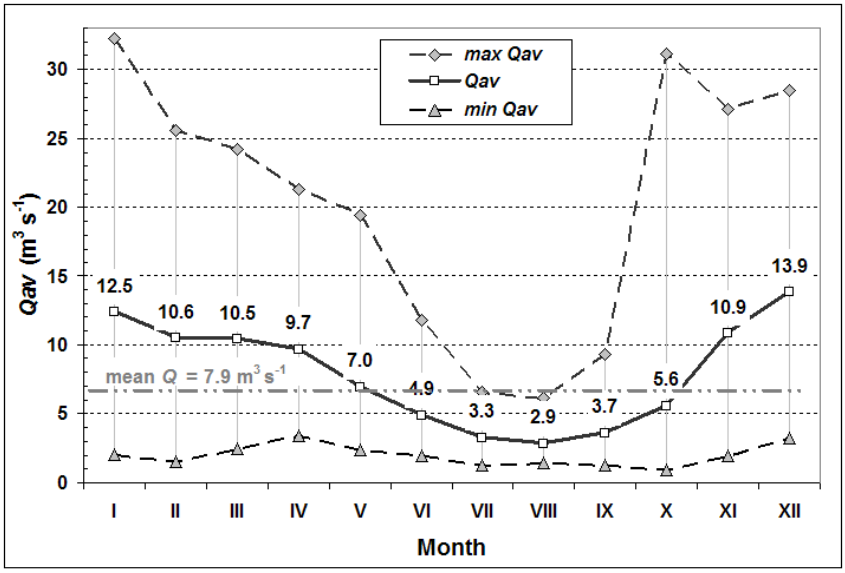



**Fig. 2.** Monthly discharges of the Jadro River at gauging station Majdan (in the period 1961–2010) (*Qav* is average discharge, max*Qav* is maximum mean monthly discharge and min*Qav* is minimum mean monthly discharge).

## MEASUREMENTS

For the purposes of this study, field measurements were made in July 2012. The vertical profiles of temperature, conductivity, salinity and pH for the whole flow depth were made at three locations of the Jadro estuary, namely at the mouth, at a railway bridge (location 0+750 m i.e. distance from the mouth) and at an old bridge (0+910) sections (Fig. 3). The hydrological conditions associated to the day of measurement correspond to normal summer conditions, with a flow at the station Majdan of 2.95 $m^3 s^{-1}$, and a sea level at the river mouth of 0.44 m a.s.l. (11h).

The estuary thus is stratified, with a thickness of the upper layer (fresh water) varying from 0.5 m at the mouth to 0.65 m at the upstream boundary. A narrow zone located between the upper and the lower layer (thickness of about 0.2 m) ensures the transition from fresh water to salt water. Therefore, a sharp interface (halocline) between layers was assumed in the numerical approach. The transition zone is characterized also by a sudden change of the other relevant parameters (temperature and density) such that halocline approximately coincides with thermocline (temperature) and pycnocline (density).

## MATHEMATICAL MODELS

Two approaches are pursued for the analysis of the Jadro estuary. The first is based on a two-layer numerical model in which a sharp interface without mixing is assumed to separate the layers. The second approach is based on a box model and includes interfacial mixing.

### Numerical model

The two-layer mathematical model was developed for unsteady stratified flow without mixing through the interface such that density is constant in each layer (Arita and Jirka, 1987; Hodgins et al., 1977; Schijf and Schonfeld, 1953).

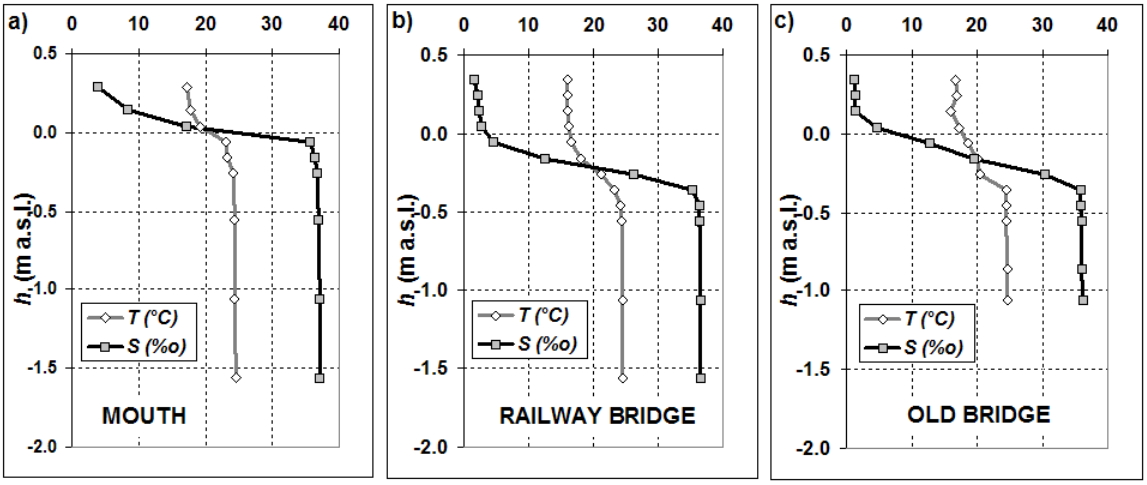



**Fig. 3.** Field measurements (July 26th, 2012): a) mouth (chainage 0+000), b) railway bridge (0+750), c) old bridge (0+910).

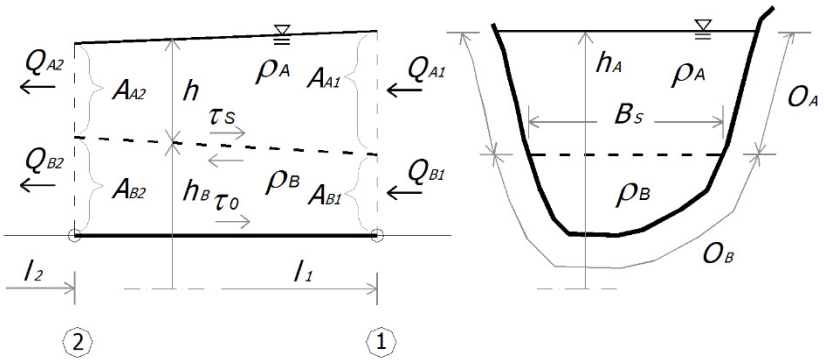



**Fig. 4.** The scheme of the two-layer numerical model ($Q_A$ is fresh water discharge, $Q_B$ is saltwater discharge, $h_A$ and $h_B$ are elevations, $A_A$ and $A_B$ are cross-sectional areas, $\rho_A$ and $\rho_B$ are water densities, $\tau_S$ is the interfacial shear stress, $\tau_0$ is the river bed shear stress).

The scheme of the two layer system, considered in the model and describing the lower reaches of a microtidal estuary when the salt wedge has already passed through it, is presented in Fig. 4. Field measurements show that indeed the estuary is stratified into two layers with nearly uniform salinity. Actually, there is a low salinity in the upper layer, gradually decreasing upstream (Fig. 3) and an almost constant sea water salinity in the lower layer. In particular, salinities were about 4‰ at the water surface and 37‰ in the lower layer at the river mouth, and 2‰ at the surface and 36‰ in the lower layer at the old bridge section. Therefore, the density $\rho_A$ of the upper layer has approximately the same value of fresh water while the density in the lower layer $\rho_B$ corresponds to sea water density. Even though some mixing takes place through the interface, this exchange is very low and almost negligible for salt wedge dynamics (Christodoulou, 1986). Therefore, upward entrainment and mixing across the interface are here ignored. Since the transition layer between the upper (fresh) water layer and the lower sea water is very narrow, approximately 0.2 m (Fig. 3), a sharp interface is assumed in the following analysis. The variables of the problem are the elevations $h_A$ and $h_B$ of the water surface and interface which divides the fluid into two layers with distinct density and discharges $Q_A$ and $Q_B$ (Fig. 4). Fresh water discharge $Q_A$ is always directed downstream. On the other hand, the discharge of the lower layer $Q_B$ can be positive or negative depending on the sea water moves upstream or downstream of the estuary under the influence of tides and river flow. The mathematical model is based on the principles of conservation of mass and momentum. Conservation of mass within a "control volume" was set separately for the upper and lower layer of the stratified flow, giving rise to two continuity equations. Each of them provides a relationship between the cross-sectional area of flow ($A$) and the discharge ($Q$) at every point of the stratified layer, having set to 1 the coefficient accounting for the non-uniform distribution of the velocity across the section. The other two equations are derived from conservation of momentum in the longitudinal direction, set for each layer. Detailed description of the derivation of these equations can be found in Schijf and Schonfeld (1953), Abbott (1979), Anderson et al. (1984). The unsteady flow in a well stratified estuary is described by the following four differential equations:

continuity equation for the upper layer

$$\frac{\partial A_A}{\partial t} + \frac{\partial Q_A}{\partial x} = 0 \tag{1}$$

continuity equation for the lower layer

$$\frac{\partial A_B}{\partial t} + \frac{\partial Q_B}{\partial x} = 0 \tag{2}$$

momentum equation for the upper layer

$$\frac{\partial Q_A}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q_A^2}{A_A}\right)+gA_A\frac{\partial h_A}{\partial x}+\frac{1}{\rho_A}\tau_0 O_A+\frac{1}{\rho_A}\tau_S B_S=0 \qquad (3)$$

momentum equation for the lower layer

$$\frac{\partial Q_B}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q_B^2}{A_B}\right)+\frac{\rho_A}{\rho_B}g\frac{\partial}{\partial x}(hA_B)-\frac{\rho_A}{\rho_B}gh\frac{\partial A_B}{\partial x}+ \\ +gA_B\frac{\partial h_B}{\partial x}+\frac{1}{\rho_B}\tau_0 O_B-\frac{1}{\rho_B}\tau_S B_S=0 \qquad (4)$$

Hereafter, the subscripts $A$ and $B$ will denote the upper (fresh water) and lower (sea water) layer, respectively. $A_A$ and $A_B$ are cross-section areas of flow. $O_A$ and $O_B$ are wetted perimeters, and $\rho_A$ and $\rho_B$ are water densities (Fig. 4). Moreover, $B_S$ is the interface width, $\tau_S$ is the interfacial shear stress, $\tau_0$ is the river bed shear stress, $g$ is acceleration due to gravity, $h_A$ is the surface elevation of the upper water layer, $h_B$ is the interface elevation, and $h$ is the thickness of the upper layer.

The bed frictional stress is related to boundary layer dynamics, see e.g. (Officer, 1976) and is usually modeled through a relation of the form (Arita and Jirka, 1987):

$$\tau_0=c_f\,\rho\frac{v^2}{2} \qquad (5)$$

where $c_f$ is the river bed friction coefficient. Water velocities for upper and lower layers are:

$$v_A=\frac{Q_A}{A_A} \qquad (6)$$

$$v_B=\frac{Q_B}{A_B} \qquad (7)$$

Numerous research works (Arita and Jirka, 1987; Sargent and Jirka, 1987) show that modeling interfacial friction is a very difficult task. Generally, the interfacial friction coefficient varies along the salt wedge and depends on local flow properties such as velocity and density profiles, mixing rates across the interface, etc. Arita and Jirka (1987) showed that good results can be obtained by assuming a constant interfacial friction along the wedge and provided a diagram for estimating it. Nevertheless, it does not exist a commonly accepted diagram for interfacial friction coefficient, analogous to the Moody diagram for wall friction (Arita and Jirka, 1987). In this study, the interfacial shear stress $\tau_S$ was expressed as:

$$\tau_S=c_{fs}\,\rho\frac{\Delta v^2}{2} \qquad (8)$$

where $c_{fs}$ is the interface friction coefficient, $\rho$ is the average density within the two layers, and $\Delta v$ is the difference between upper and lower layer velocities:

$$\rho=\frac{1}{2}(\rho_A+\rho_B),\quad \Delta v=v_A-v_B=\frac{Q_A}{A_A}-\frac{Q_B}{A_B} \qquad (9)$$

Due to the wide range of flow parameters and friction coefficients in real estuaries, Arita and Jirka (1987) suggested that each model has to be calibrated with available data, because only measurements give sufficient details for initial model formulation and for the final validation. In this study, the constant friction coefficients along the wedge were estimated through calibration. In addition, wind stress at the water surface has been ignored. Sierra et al. (2004) in fact showed that wind action is not a dominant factor for salt wedge dynamics, even though it can play non-negligible secondary role.

The river bed bathymetry is described by non-uniform polygonal cross-sections. Indeed, taking into account properly the river geometry is very important for salt wedge dynamics. It is one of the major advantages of numerical approach compared with analytical approach based on laboratory experiments and/or theory of stratified flow with constant cross-section.

The system (1) – (4) is a non-linear system of partial differential equations containing four variables $h_A = h_A(x,t)$, $h_B = h_B(x,t)$, $Q_A = Q_A(x,t)$ and $Q_B = Q_B(x,t)$. This set of equations does not have analytical solution and, hence, has to be solved numerically, e.g. through finite element method (FEM). This numerical method is described in numerous research works and literature (Anderson et al., 1984; Vallabhan and Asik, 2011; Zienkiewicz and Taylor, 2000) and has been successfully applied to real river flow calculation in many practical problems. The domain is divided into linear finite elements, connected by nodal points. In this study, a Galerkin form of FEM was used and the governing equations were translated into a set on non-linear algebraic equations for the unknown elevations and discharges at the nodes of the computational grid. In the time integration, a standard θ-scheme was used. The resulting non-linear system is solved by the Newton-Raphson iterative method. The whole computational model was written in FORTRAN.

Various numerical methods (i.e., finite difference method, finite volume method, etc.) can be used for solving differential equations. Each method has its own advantages and disadvantages. The main advantage of the FEM is the easy handling of the complex domain geometry and element properties (Vallabhan and Asik, 2011). Since, the Jadro estuary has non-uniform geometry with mild changes, such as relatively gentle hydrologic variations and subcritical (slow) flow, especially in the summer period, the Galerkin FEM was applied with the θ-scheme time discretization. Some discontinuous fronts and similar sudden changes of flow parameters are not expected in the estuary.

Boundary conditions associated to the governing equations consist of discharges for both layers at the upstream (landward) boundary and elevations at the downstream (seaward) boundary. Initial conditions include setting elevations and discharges in each node of the computational mesh at the beginning of the calculation.

**Box model**

Interaction of fresh and salt water in an estuary is generally a very complex process, as explained in many research works (Arita and Jirka, 1987; Hodgins et al., 1977; Ralston et al., 2010; Schijf and Schonfeld, 1953; Sierra et al., 2004). In the case of strongly stratified estuaries, the salt circulation can be described with a relatively simple model, since salt transport results from a balance between horizontal and vertical advection of salt. Under this assumption some simple calculation schemes can be set up to determine the mean circulation. These schemes are based on conservation of volume and salt, i.e. on the so called Knudsen´s hydrographical theorem (Dyer, 1997; Vieira and Bordalo, 2000).

Estuary is split into boxes along in the longitudinal direction, with two layers in the vertical direction containing saline (bottom) and brackish (top) water, respectively. A small amount of mixing is allowed through vertical advection. A sketch of this box model and the related hydraulic components is presented in Fig. 5.

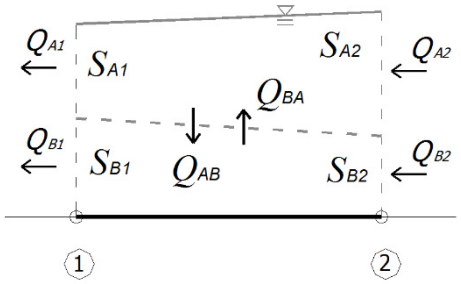



**Fig. 5.** The scheme of the box model ($Q_A$ is fresh water discharge, $Q_B$ is saltwater discharge, $Q_{AB}$ and $Q_{BA}$ are vertical interfacial fluxes, $S_A$ and $S_B$ are salinities).

The conservation equations for water and salt fluxes in the two layers are:
continuity equation for the upper layer

$$Q_{A1} + Q_{BA} = Q_{A2} + Q_{AB} \tag{10}$$

continuity equation for the lower layer

$$Q_{B1} + Q_{AB} = Q_{B2} + Q_{BA} \tag{11}$$

salt flux for the upper layer

$$Q_{A1} \cdot S_{A1} + Q_{BA} \cdot S_{BA} = Q_{A2} \cdot S_{A2} + Q_{AB} \cdot S_{AB} \tag{12}$$

salt flux for the lower layer

$$Q_{B1} \cdot S_{B1} + Q_{AB} \cdot S_{AB} = Q_{B2} \cdot S_{B2} + Q_{BA} \cdot S_{BA} \tag{13}$$

Here, $Q_{Ai}$ and $Q_{Bi}$ are horizontal fluxes, ($i$ = 1, 2 denoting the upstream and the downstream box boundary, respectively). In addition, subscript $AB$ denotes vertical flux from the $A$ layer to the $B$ layer, and $BA$ refers to opposite. The quantities $S_{AB}$ and $S_{BA}$ denote mean salinities in the upper and lower layer, such that $S_{AB} = (S_{A1} + S_{A2})/2$ and $S_{BA} = (S_{B1} + S_{B2})/2$.

The calculation starts at the most upstream box, where $Q_{A1}$ is equal to the river discharge, $S_{A1} = 0$ and $Q_{B1} = 0$, and proceeds downstream involving adjacent boxes in succession till the mouth.

An attempt for estimating flushing time was also made in this work. Flushing time is defined as the time required for replacing the existing water accumulated in the estuary by discharge (Dyer, 1997). In the case of a stratified estuary, flushing time can be thought as the time taken by a water particle entering into the estuary to pass through the whole salt wedge and to return back to sea through the upper layer. The concept of flushing time (residence time) is important for water quality analysis and ecology. For example, it controls the pollutant retention time, one of the factors that determines the exposure period to estuary organisms. If the water mass within the estuary is regularly and quickly exchanged with the sea, pollutants will also pass through quickly, possibly causing less harm than in a stagnant estuary. Vieira and Bordalo (2000) suggested the following estimate of flushing time:

$$T = V / R \tag{14}$$

where $V$ is the volume of the estuary and $R$ is the freshwater discharge.

Total flushing time ($T$) contains two components, $T_B$ and $T_A$, which refer to the lower and upper layer, namely:

$$T = T_A + T_B = \sum_j V_{Aj} / Q_{Aj} + \sum_j V_{Bj} / Q_{Bj} \tag{15}$$

where $j$ is the box ordering number, $V_A$ and $V_B$ are volumes of box compartments, $Q_A$ and $Q_B$ are the corresponding average box discharges, such that $Q_A = (Q_{AUPSTREAM} + Q_{ADOWNSTREAM})/2$ and $Q_B = (Q_{BUPSTREAM} + Q_{BDOWNSTREAM})/2$.

The Jadro River estuary has been described with 46 cross sections, with an average distance of about 25 m. Two extra cross sections were considered within the Bay of Kaštela to reduce possible errors in the downstream boundary condition (Hodgins et al., 1977; Sierra et al., 2004; Ralston et al., 2010). Cross sections represent nodes of the finite elements mesh used to discretize the governing equations. The mesh is composed by 48 nodes and 47 elements.

The downstream boundary of the model is set up within the sea, adjacent to the river mouth. The upstream boundary of the model is located on an existing cascade, about 1150 m upstream from the mouth (Fig. 6). In the correspondence of this section a concrete step of height 0.5 m and top elevation 0.00 m a.s.l. has been built to prevent the upstream intrusion of sea water.

The river width is 30 m at the upstream boundary and increases to 45 m at the estuary mouth. Average water depth is 3.6 m at the mouth and decreases to 1.0 m just downstream of the cascade.

The total inflow of fresh water is estimated from the nearest upstream hydrological station of Majdan (3+150). Various branches contribute to the overall river discharge in Solin, but all of them enter the system upstream of the model boundary. In summer, the fresh water flow in the Jadro River depends only on the discharge provided by the spring, fully registered at the hydrological station of Majdan. Therefore, the upstream boundary condition is assumed to be $Q_{A\ UPSTREAM} = f(Q_{\ MAJDAN})$. The upstream boundary flow is thus a function of time (Fig. 7) given in the form of polynomials and is imposed at the most upstream node of system (node 48). On the other hand, the upstream salt water flow is constantly zero ($Q_{B\ UPSTREAM} = 0$), since the existing cascade acts as barrier for the lower layer.

Generally, fresh water is delivered to the sea by the upper layer. At the river mouth, a complex mixing process then takes place, which is not specifically analyzed in this work. The thickness of the upper freshwater layer at the estuary mouth has to be properly defined. In order to reduce the impact of this downstream boundary condition, two extra cross sections were added.

At the downstream boundary the surface water level ($h_A$) is imposed. The hourly tidal record is taken from the nearby station of Split, such that $h_{A\ DOWNSTREAM} = h_{\ TIDE\ GAUGE}$ (Fig. 7). The mean sea level was 0.32 m a.s.l. Tides were diurnal, with a maximum tidal range from 0.45 m to 0.59 m a.s.l., and a minimum between 0.04 m a.s.l. and 0.24 m a.s.l. The daily amplitude varied from 0.23 m (27 Jul) to 0.44 m (23 Jul). Since the typical tidal range is less than 0.5 m, the Jadro River can be classified as micro tidal estuary (Ibanez et al., 1997). Therefore, the effects of tidal currents on the estuary physical characteristics (salinity and velocity) are relatively small (Funahashi et al., 2013). The downstream seawater level ($h_{B\ DOWNSTREAM}$) was estimated on the bases of field measurements, tidal records ($h_{A\ DOWNSTREAM}$) and freshwater flow ($Q_A$). The thickness of upper fresh water layer ($h_{A\ DOWNSTREAM} - h_{B\ DOWNSTREAM}$) was estimated to be 0.2 m, in the 7 day period 23$^{th}$ – 29$^{th}$ July 2012 considered in this study.

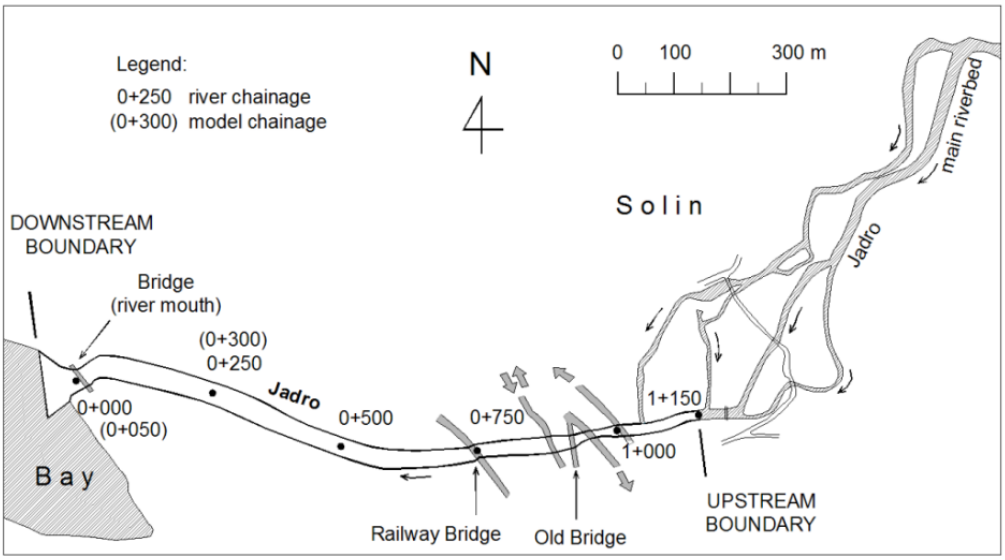



**Fig. 6.** The Jadro River estuary with numerical model boundaries.

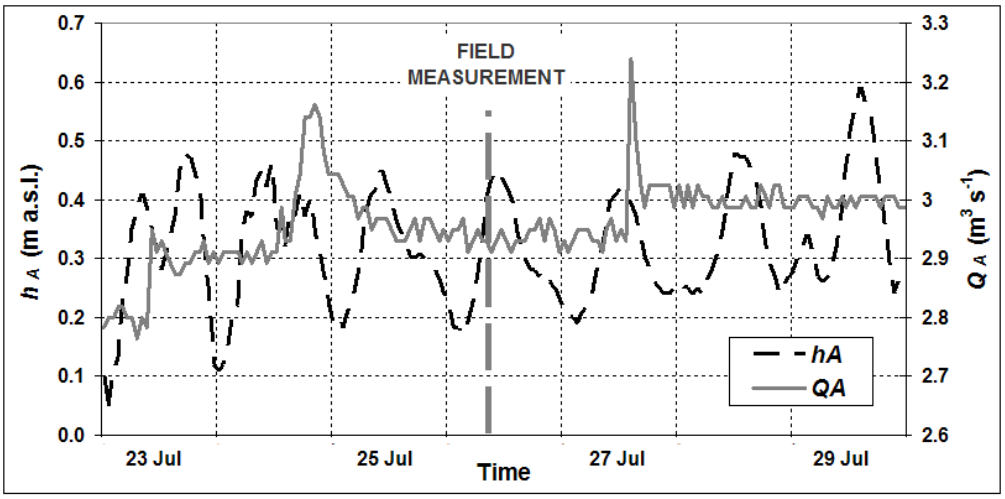



**Fig. 7.** Boundary conditions (from 23th to 29th of July 2012) ($Q_A$ is fresh water discharge, $h_A$ is surface water elevation).

Field measurements carried out in this period are used in order to simulate the stratified flow in non-stationary conditions and to calibrate model parameters. The integration time interval ($\Delta t$) was 45 s and a constant coefficient $\theta = 0.75$ (standard θ-scheme i.e. a mix of explicit and implicit Euler scheme) is assumed.

Adopted friction coefficients are $c_f = 0.003$ at the river bed and $c_{fs} = 0.002$ at the salt wedge interface. The interface elevations used for calibration and comparison purposes correspond to points with 20 ‰ (g $l^{-1}$) salinity in the constructed salinity – depth profiles (Fig. 3). Density of the upper fresh water is $\rho_A = 1000$ kg $m^{-3}$, and layer of salt water it is assumed $\rho_B = 1025$ kg $m^{-3}$.

The initial state (measured on July 20th, 2012) consists of water levels $h_A$ and $h_B$ for all nodes, and the corresponding steady state, such that the flow discharge is the same on all mesh elements of the top layer ($Q_A = 3$ $m^3$ $s^{-1}$), and identically zero in the lower salt layer ($Q_B$) (stopped salt wedge). The error associated to the estimated initial condition disappears in a very short period of time (a few hours), after which the initial conditions do not affect the solution any more.

## RESULTS

Fig. 8 shows the calculated longitudinal profiles of the water surface ($h_A$) and the interface ($h_B$) as compared to the corresponding measurements carried out the July 26th 2012, at 11:00 AM. Since, there is a relatively small fresh water discharge, water surface $h_A$ has a very small gradient. The considered surface water elevation (0.44 m a.s.l.) corresponds to high tide conditions. The calculated interface profile ($h_B$) fits satisfactory the experimental points. Therefore, the obtained solution provides a qualitatively good assessment of real halocline. During the 2012 field campaign the calculated freshwater discharge ranged between 2.95 $m^3$ $s^{-1}$ and 2.98 $m^3$ $s^{-1}$, at the three gauged cross sections; the corresponding salt water fluxes were estimated from the model to be between 0.002 and 0.044 $m^3$ $s^{-1}$ (Table 1). At the river mouth, the mean water velocity was about 0.2 m $s^{-1}$ in the upper layer and 0.001 m $s^{-1}$ in the lower layer.

The inflow of fresh water into the system varied from 2.76 to 3.24 $m^3$ $s^{-1}$, in the observed 7-day period. Therefore salt wedge intrusion was present in the whole period, reaching the upstream cascade. Only minor movement of the salt wedge under

the action of the tides is suggested by computations. The maximum water level was 0.59 m a.s.l. (July 29th at 14 h), and minimum 0.04 m a.s.l. (July 23th at 1 h).

The interface elevation at the river mouth varied between –0.40 m and 0.14 m a.s.l., in phase with the surface water elevation. Fresh water fluxes varied between 2.50 and 3.26 $m^3 s^{-1}$ while bottom fluxes varied from –0.99 to 1.08 $m^3 s^{-1}$ at the river mouth.

Fig. 9 shows water levels and fluxes calculated at the railway bridge. Extreme surface elevations were 0.04 and 0.59 m a.s. as it was mentioned previously. Interface elevation varied between –0.59 and –0.06 m a.s.l. Fresh water fluxes varied between 2.68 and 3.21 $m^3 s^{-1}$, while bottom fluxes varied in the range –0.19 to 0.20 $m^3 s^{-1}$. Hence, the lower layer fluxes fluctuate around zero with a corresponding speeds up to ±0.007 m $s^{-1}$ at this location.

The steady state of estuary corresponding to measurement conditions (July 26th at 11 h) is analyzed by box principle. The Jadro reach is divided into three boxes corresponding to gauged cross sections. Salinity of each layer is calculated as an average value of measurements performed in the top layer above the narrow zone where transition from brackish to salt wedge takes place and separately in lower layer below the transition zone (Table 2). Upper layer salinity tends to increase in downstream direction due to vertical advection from bottom layer. On the other hand, lower layer salinity decreases in longitudinal direction achieving a maximum at the river mouth and a minimum at the upstream boundary, as a consequence of vertical advection from upper layer. Estimated freshwater horizontal discharges from box model are higher than corresponding numerical values (Table 2) due to interfacial mixing and vertical fluxes. The maximum difference is at river mouth, where $\Delta Q_A$ = 3.551 – 2.982 = 0.569 $m^3 s^{-1}$ i.e. 17%. Corresponding flows in the lower layer are up to 0.6 $m^3 s^{-1}$ and are directed upstream (negative values) owing to salt wedge intrusion. At the same time, interfacial salt fluxes are much stronger in the upward direction than in the downward direction (Table 3). These results confirm field measurements indicating a slight interfacial mixing in both directions, the interfacial fluxes being much lower than fresh water discharges $Q_A$.

The total flushing time of the Jadro estuary, calculated by Eq. 13, is 37.36 h. Most of the flushing time about 35.93 h is determined by the lower layer, due to very low salt water fluxes. The remaining 1.43 h is associated with the flow in the upper layer (Table 4). The salt wedge volume was much larger than fresh water one, the corresponding values being about 42.13 thousand $m^3$ and 16.96 thousand $m^3$, respectively. Generally, flushing time increases as the fresh water inflow decreases (Vieira and Bordalo, 2000). This calculation method corresponds to steady state conditions, assessed through field campaign measurement. It has to be stressed however that the salt

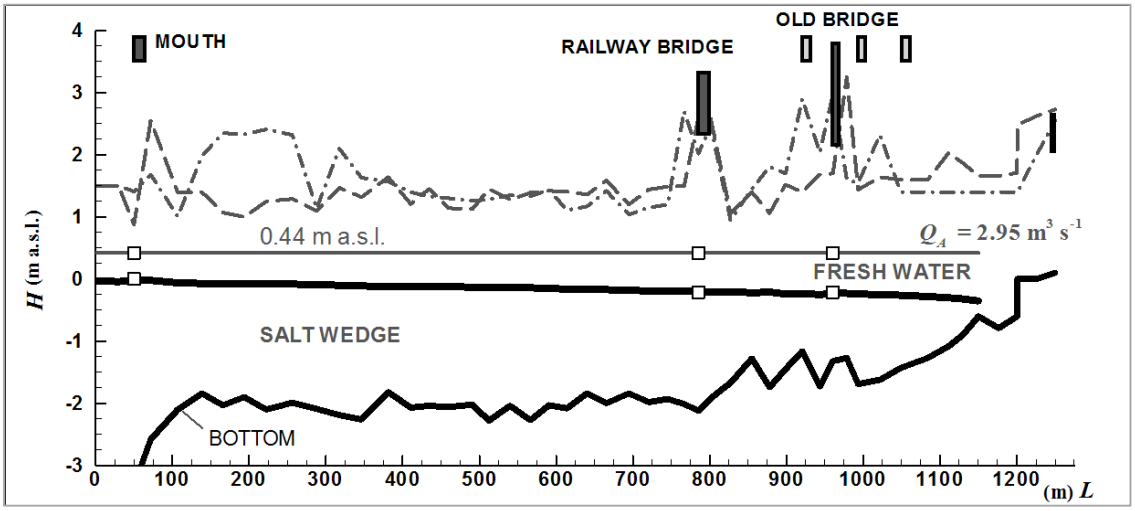



**Fig. 8.** Estimated water surface and interface at 11h on July 26th, 2012 ($Q_A$ is fresh water discharge).

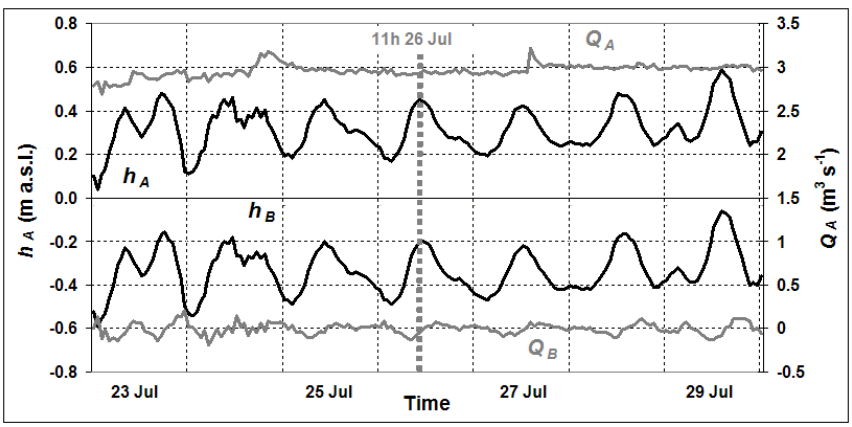



**Fig. 9.** Estimated water elevations and fluxes at railway bridge (from 23th to 29th of July 2012) ($Q_A$ is fresh water discharge, $Q_B$ is saltwater discharge, $h_A$ is surface elevation and $h_B$ is saltwater elevation).

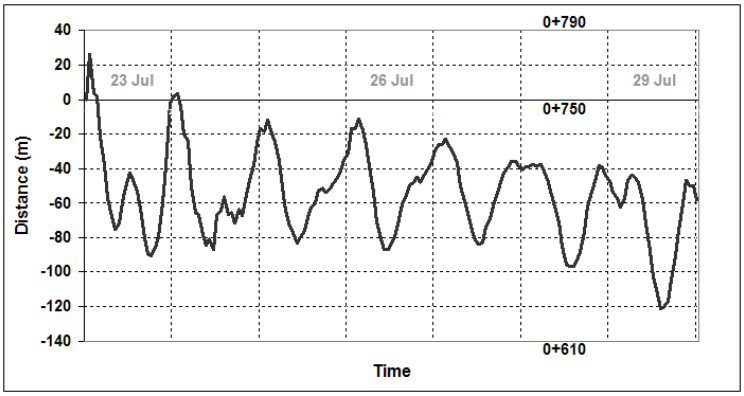



**Fig. 10.** Distance travel of salt water particle in the 7-day period (from 23th to 29th of July 2012).

**Table 1.** Horizontal fluxes according numerical model (11h in the 26th of July 2012).

| Cross section | | 0+000 | 0+750 | 0+910 | 1+150 |
|---|---|---|---|---|---|
| Upper layer | $Q_A$ ($m^3$ $s^{-1}$) | 2.982 | 2.958 | 2.952 | 2.950 |
| Bottom layer | $Q_B$ ($m^3$ $s^{-1}$) | –0.044 | –0.004 | –0.002 | 0.0 |

**Table 2.** Salinities and horizontal fluxes for box model.

| Cross s. | River mouth | | Railway bridge | | Old bridge | | Barrier | |
|---|---|---|---|---|---|---|---|---|
| Chainage | 0+000 | | 0+750 | | 0+910 | | 1+150 | |
| Layer | $S$ (‰) | $Q$ ($m^3$ $s^{-1}$) | $S$ (‰) | $Q$ ($m^3$ $s^{-1}$) | $S$ (‰) | $Q$ ($m^3$ $s^{-1}$) | $S$ (‰) | $Q$ ($m^3$ $s^{-1}$) |
| Upper | 6.23 | 3.551 | 2.72 | 3.190 | 2.03 | 3.127 | 0 | 2.95 |
| Bottom | 36.83 | –0.600 | 36.18 | –0.239 | 35.94 | –0.176 | 35.77 | 0 |

**Table 3.** Vertical fluxes for box model.

| No box | | | 3 | 2 | 1 |
|---|---|---|---|---|---|
| Upstream cross section | | | 0+750 | 0+910 | 1+150 |
| Downstream cross section | | | 0+000 | 0+750 | 0+910 |
| Total | $Q_{AB}$ | ($m^3$ $s^{-1}$) | 0.00850 | 0.00149 | 0.00086 |
| Specific | $q_{AB}$ | ($m^3$ $s^{-1}$ $m^{-1}$) | $1.13 \times 10^{-5}$ | $0.93 \times 10^{-5}$ | $0.36 \times 10^{-5}$ |
| Total | $Q_{BA}$ | ($m^3$ $s^{-1}$) | 0.369 | 0.065 | 0.177 |
| Specific | $q_{BA}$ | ($m^3$ $s^{-1}$ $m^{-1}$) | $0.49 \times 10^{-3}$ | $0.41 \times 10^{-3}$ | $0.73 \times 10^{-3}$ |

**Table 4.** Flushing time parameters.

| No box | | 3 | 2 | 1 | Total |
|---|---|---|---|---|---|
| Upstream cross s. | | | | | |
| Volume | ($m^3$) | 12553 | 2542 | 1869 | 16964 |
| Flushing time | (h) | 1.03 | 0.22 | 0.17 | 1.43 |
| Downstream cross s. | | | | | |
| Volume | ($m^3$) | 35790 | 4283 | 2058 | 42131 |
| Flushing time | (h) | 23.7 | 5.73 | 6.5 | 35.93 |

wedge is usually moving along the estuary and only rarely is arrested. Therefore, it is here suggested that this phenomena should be considered from an unsteady point of view. Fig. 10 presents the distance travelled upstream / downstream by a salt water particle in the analyzed period, estimated through the numerical approach. The selected salt water particle was located at the railway bridge (relative location 0, absolute 0+750) at the beginning of calculation (initial state). First, the particle moved upstream due to salt wedge intrusion. After, it moved downstream from the bridge. Upstream and downstream movement under tides is evident. At the end of the observation period, the water particle moved only 60 m from the bridge, in downstream direction. The total interval of movement varied from –120 m (0+630) to +25 m (0+775). Mass of salt water moves upstream downstream under the forcing provided by boundary conditions, fresh water flow and tides, and could be retained in the certain area of the estuary for a long time. Therefore, the residence time of a water particle in an estuary could be long lasting, depending on hydrological conditions.

Finally, salt wedge intrusion was estimated by numerical model for average boundary conditions, i.e. average annual fresh water discharge of 7.9 $m^3 s^{-1}$ at upstream boundary and mean sea level of 0.3 m a.s.l. at downstream boundary. For this steady state conditions, salt wedge penetrates about 1000 m into the estuary. It confirms that the estuary is stratified most of the year with deep salt wedge intrusion even upstream from old bridge (0+910).

The estimation presented in this work reproduces quite well the field data regardless it is relatively short estuary with sparse measurements. With regard to the model accuracy, it will be very interesting to apply some other numerical method for the stratified estuary and to compare results.

## CONCLUSION

This work presents an one-dimensional hydrodynamic model of a stratified estuary solved by using two different approaches, numerical and box model. The study area was the Jadro River estuary situated on the east Adriatic coast, near the city of Split (Croatia). A two-layer mathematical model was developed for unsteady stratified flow without mixing through the interface. Since, governing equations do not have analytical solution, a finite element method (FEM) based of Galerkin scheme was used to solve the problem. Numerical approach allows simulation of unsteady flow, which provides useful information on salt wedge dynamics. Field measurements were used for model calibration.

The box model was applied by assuming steady state conditions, to quantify intensity of mixing between the upper layer of fresh (brackish) water and the lower salt water. The box model suggests that the fluxes in the upper layer are larger than those estimated through the numerical model. Similar results are obtained for salt water fluxes. This result is a consequence of mixing through interface which is accounted for in the box approach. Interfacial mixing is much stronger in upward direction, from salt water to fresh water, than in the opposite direction. During summer, when the water flow is minimum the total flushing time for the Jadro estuary is about 1.5 day. The analysis carried out using the results of the numerical model shows that residence time of a water particle could be long lasting, depending on hydrological conditions i.e. fresh water discharge.

The results presented in this paper provide description of the Jadro River stratification in summer period, when the estuary salinization is the strongest. These results could be quite useful for extensive analysis of other estuarine processes such as ecological, biological etc.

Further field studies and measurements are however needed to validate the model using a different set of data, independent of that used for the present calibration and to better characterize the different hydrological regimes that can establish in the Jadro estuary. These will support researchers to understand the details of the salt wedge dynamics in response to freshwater inflow, tidal dynamics, meteorological forcing etc. such as general estuary processes.

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Received 19 November 2013
Accepted 5 August 2014