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SIMULTANEOUS ROUTING AND FLOW RATE OPTIMIZATION IN ENERGY–AWARE COMPUTER NETWORKS

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The issue of energy-aware traffic engineering has become prominent in telecommunications industry in the last years. This paper presents a two-criteria network optimization problem, in which routing and bandwidth allocation are determined jointly, so as to minimize the amount of energy consumed by a telecommunication infrastructure and to satisfy given demands represented by a traffic matrix. A scalarization of the criteria is proposed and the choice of model parameters is discussed in detail. The model of power dissipation as a function of carried traffic in a typical software router is introduced. Then the problem is expressed in a form suitable for the mixed integer quadratic programming (MIQP) solver. The paper is concluded with a set of small, illustrative computational examples. Computed solutions are implemented in a testbed to validate the accuracy of energy consumption models and the correctness of the proposed traffic engineering algorithm.

Keywords: MINLP, MIQP, network optimization, green networking, fairness, multi-criteria.

1. Introduction

Energy dissipation has recently become an important problem in telecommunication networks. Efforts to reduce power consumption follow two directions—the design of more efficient equipment as well as the development of energy-aware network control strategies and protocols. These topics are mutually related to a large extent, because control strategy design is based on physical layer capabilities. The paper presents a formulation of a two-criteria computer network control problem. The first criterion is energy consumption, and the second one is the quality of service and service sustainability, which is our main concern.

As has been observed, substantial savings can be achieved by flow consolidation in periods of diminished traffic load, provided that the network devices are equipped with power-saving technologies (Mahadevan *et al.*, 2009; Chiaraviglio *et al.*, 2009). The presented model is aimed at determining the optimal routing and bandwidth allocation so as to fulfill quality of service requirements and save as much energy as possible. The problem is expressed in terms of mixed integer nonlinear programming, and solved for small scale network examples.

The energy consumption models were identified on the basis of real network equipment. Finally, the presented models and techniques are evaluated in a laboratory network for validity of assumptions, accuracy and efficacy.

The rest of this paper is structured as follows. The next section presents the motivation and related work. Then the power consumption model of a Linux router is developed. In Section 4, the mixed-integer formulation of the simultaneous routing and flow rate optimization problem is introduced and thoroughly studied. In Section 5, we consider some computational examples. Section 6 concludes the paper.

2. Motivation and related work

Methods for increasing energy efficiency of computer networks have gained much attention in the scientific

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publications over the last years. Initial efforts were aimed at the assessment of energy characteristics of network equipment and at building elementary models (Chabarek *et al*., 2008; Qureshi *et al*., 2009; Bolla *et al*., 2009; Vasić and Kostić, 2010). Upon this knowledge, some local, i.e., concerning a single device, strategies were built (e.g., Nedevschi *et al.*, 2008; Bolla *et al.*, 2010; 2014). However, it is possible to save even more energy by employing network-wide solutions. The reasons behind this are twofold: first, the traffic load tends to follow periodic patterns; second, the networks are to some extent redundant to increase reliability. Controlling the whole network allows switching off or reducing the performance of some devices while demands of users are served by the remaining equipment (Bianco *et al*., 2007; Roy, 2008). Such a task resembles traditional network design (Pióro *et al*., 2001) or QoS provisioning problems (e.g., Jaskóła and Malinowski, 2004; Malinowski *et al*., 2010), with the performance index covering mostly energy consumption of the network. The resulting problems typically involve a large number of binary variables to compute routes and flow rates, so their application is limited to relatively small networks. To reduce the complexity, linear models are applied (Chabarek *et al*., 2008; Restrepo *et al*., 2009; Chiaraviglio *et al*., 2011). Other approaches are to exploit the hierarchical structure of the equipment, e.g., bundled links (Fisher *et al*., 2010) or multiple line cards (Idzikowski *et al*., 2010). It must be noted that such decomposition cannot reduce the complexity of the problem; however, it allows building efficient heuristics.

A study of energy-aware traffic engineering in a TCP/IP network was recently undertaken by Niewiadomska-Szynkiewicz *et al*. (2014). The authors presented the proposition for a traffic engineering system, in which paths are chosen so as to satisfy given demands while the total energy dissipated by network equipment is minimized. Total power consumption is incorporated into the objective function of a binary linear programming problem, while the demand matrix is represented by a set of constraints.

We propose instead an approach in which flow rates are represented by variables whose values are determined jointly with routing. The quality of service is taken into account via the second optimization criterion and included in the objective function as a nonlinear component, which represents a penalty for not achieving the assumed flow rate. The combined joint routing and flow rate optimization mixed-integer nonlinear programming problem (MINLP) has to be solved, which leads to the feasible solution, even when the demand is greater than the capacity offered by the network. This property is achieved through the reduction of flow rates, which is accepted by a model taking into account elasticity of demand.

Moreover, in some cases a minor reduction may allow accommodating the traffic in a smaller number of links, having a great impact on the power consumption.

The problem of simultaneous routing and flow rate optimization in data networks was addressed by Bertsekas and Gallager (1992). They noticed that, if a flow rate is modeled as a decision variable instead of a constant, the problem formulation must contain an incentive to allocate some bandwidth to flows. Otherwise, if a routing problem is solved and the link cost is an increasing function of the link rate, the optimal bandwidth allocated to flows would always be zero. They proposed a two-criteria optimization problem, with a specific convex and decreasing function being a penalty for not achieving an eligible flow rate. The second criterion was a total link cost, which was minimized. The authors discussed thoroughly the optimality conditions and parameters of the penalty function, which determine the optimal choice of rates and paths. The discussion is, however, not applicable to the energy aware network, because the link power cannot be well described by means of a continuous function. It is then not consistent with the assumption that link costs must be expressed in terms of differentiable functions of flow rates.

Bertsekas and Gallager (1992) assumed multi-path routing, which means that each flow can be split among more than one path. It was shown there that, by only adding a new variable, this problem can be transformed to a standard multipath routing problem (i.e., with constant flow rates) and in the convex case solved efficiently by any continuous, nonlinear solver. Such standard multicommodity flow problems can be also solved in a distributed, asynchronous way, which was first shown by Tsitsiklis and Bertsekas (1986) as well as Bertsekas and Tsitsiklis (1989).

A review of earlier works on cross-layer optimization may be found in the extensive methodological work of Chiang *et al*. (2007). In the last years, this approach has been successfully applied to optimize service delivery in service-oriented environments (Callaway *et al*., 2010), and to control congestion and media access in multihop wireless networks (Chen *et al*., 2011).

Multipath routing leads, however, to TCP packets reordering, which contributes to QoS deterioration, so herein it is assumed that single path routing is required.

The integrated single path routing and network flow control problem, which is much more important from the practical point of view, was first addressed by Jaskóła and Malinowski (2004) and independently by Wang *et al*. (2005). In both cases the resulting formulations were mixed-binary-continuous and nonlinear. Such problems are much more complicated and difficult to solve than in the multipath case. In particular, if solely Lagrange relaxation is applied to solve such an MINLP problem, inevitably a duality gap appears (Li and Sun, 2006), resulting in suboptimality of the solution (Wang *et al*.,

2005; Chiang *et al*., 2007). A very similar model to that of Jaskóła and Malinowski (2004) was later proposed by Tian *et al*. (2008) for multi-rate ad hoc networks. As for the algorithm to obtain the solution, the generalized benders decomposition was suggested. Wang *et al*. (2011) provide formulae to estimate the performance gap between multi- and single-path solutions of joint optimization of transmission rates and paths, and propose an iterative projection method that, combined with a branch-and-bound technique, may be used to obtain a near optimal single-path problem solution.

In our paper we show that a simpler formulation—without a double set of flow conservation law based equations (on both routing and flow variables)—can be used to solve a more complex problem (namely, involving an energy cost component) than the one considered by Jaskóła and Malinowski (2004) as well as Tian *et al*. (2008).

3. Electrical energy dissipation in typical IP network devices

The power consumed by a typical network device exhibits some nonlinear dependency on the carried traffic. Several computational models (Qureshi *et al*., 2009; Bolla *et al*., 2009) consist of at least two components—constant and load related, and may be hence reduced to the following formula:

$$\pi(p) = \pi_0 + \sum_{i \in I} \pi_i(p_i), \tag{1}$$

where I is the set of all network device ports, p is a vector consisting of loads $p_i, i \in I$, carried by ports of a network device, π_0 represents a fixed part of the power, which is consumed irrespectively of the traffic traversing a device by fans, bus and other common functions, e.g., management. The traffic dependent part of the power $\pi_i(p_i)$, consumed by the port i itself and data plane functions attributed to it, is assigned to the ports in dependence on the fraction of the whole traffic served by each of them. If the energy consumed by, e.g., the network processor of the device exhibits some dependence on the load, it is assumed that it can be related to ports and included in $\pi_i(p_i)$.

Currently available network devices are generally not equipped with advanced power-saving functions. In consequence, their energy consumption does not depend much on the network load. Recent efforts tend to increase the power proportionality of network equipment by means of transition into a low power state during idle periods (*low power idle*—LPI) and frequency scaling techniques. The traffic-independent part of the power, i.e., π_0 in (1), apart from the improvement of device electronics, may also be reduced through *smart sleeping* allowing to turning off parts of device for some longer period. For a detailed review of mentioned solutions, see, e.g., the work of Bolla *et al*. (2011). The adoption of results in the industry is relatively slow, with the power saving Ethernet IEEE 802.3az (IEEE, 2012) being the most important standard.

To further leverage the newly emerging features, it is reasonable to build a network-wide algorithm, employing routing or traffic engineering techniques for flow aggregation in periods of a reduced traffic load. This paper presents the rationale for the introduction of such techniques.

The most convenient way to test such methods is to implement them using off-the-shelf equipment—typically, PC computers with multiport network adapters and open source routing software. Although Linux routers are not widely used in industry, they have multiple advantages as laboratory equipment over specialized devices. Through the functionality implemented in additional software packages, it is possible to enable and control energy saving mechanisms, like, e.g., frequency scaling governors in a Linux kernel. It is also possible to alter open-source software to implement new ideas.

The results of measurements presented below were collected using a software router based on a PC computer equipped with Linux OS and a 4-port 1 Gb/s Ethernet card. It is important to note that the energy consumed by such a setup is influenced by other elements, like, e.g., hard disks, which take marginal or no part in transferring the network traffic. This is the reason why the π_0 component may be relatively high. Furthermore, the dependence on the network load should be mostly attributed to frequency scaling and idle mechanisms of the Linux kernel. The numerous experiments employing various relations of flows possible to attain using up to four interfaces have shown that the decomposition of the variable part of power into a sum of $\pi_i(p_i)$ is possible—the overall error was not higher than approximately 5%. Figure 1 presents measured values of power assigned to a single interface. It must be reminded that Fig. 1 shows only the variable part of the consumed power while the fixed, common for the whole device, part π_0 was estimated at 31W.

The shape of $\pi_i(p_i)$ is fairly steep for small rates of traffic, which may be explained by the behaviour of the default frequency governor algorithm in Linux which, after reaching some threshold, switches to maximum processor frequency. The maximum rate achieved, 812 Mb/s, represents speed limit $\bar{p}_i$ in the tested setup and is probably a consequence of the grade of hardware used.

The shape of the curve in Fig. 1 makes it rational to use a piecewise linear approximation. By such a procedure it is possible to model the power consumption in the most important range, i.e., approximately. from 50 Mb/s to the maximum speed. The approximation line may be defined by Eqn. (2) below and is presented in Fig. 1:

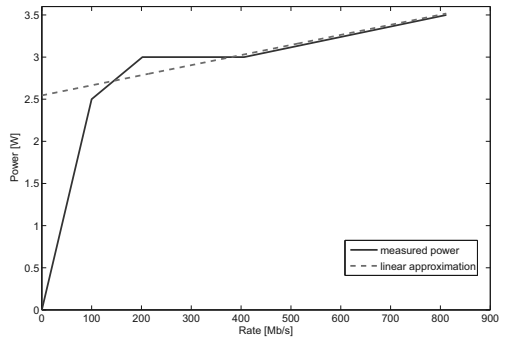

Fig. 1. Variable part of power attributed to a single port: $\pi_i(p_i)$, and its piecewise linear approximation.

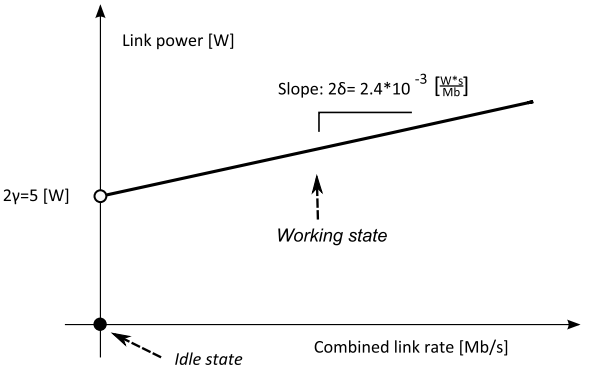

Fig. 2. Model of the power consumed by a link. "Combined link rate" is a total rate of all flows traversing the link in both directions.

$$\pi_i(p_i) = 1.2 \cdot 10^{-3} p_i + 2.5. \tag{2}$$

4. Formulation of the problem

4.1. Model of the network. All the subsequent discussions apply to a network based on gigabit Ethernet, which is a full-duplex technology. Therefore, it can be represented by a digraph $(N, \mathcal{A})$, where N is the set of nodes, $\mathcal{A}$ is the set of arcs.

Let A be the matrix containing the description of the topology of the network, that is,

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{A}, \text{ where } i,j \in N, \\ 0 & \text{otherwise.} \end{cases}$$

For convenience, let us also define L, the set of link labels, and a one-to-one mapping $\kappa : \mathcal{A} \to L$. Each network link $l \in L$ is characterized by its capacity c_l.

The traffic is described in terms of arrival rates on network links, the so-called fluid model (Bertsekas and Gallager, 1992).

The term "traffic flow" or simply "flow" is used here in the meaning adopted in IETF documents (Brownlee *et al.*, 1999; Rajahalme *et al.*, 2004), namely, some sequence of packets sharing the same source, destination address, and possibly other classifiers, like the flow label or the port number.

The following notation related to the traffic flows is used:

- W : the set of flows; each flow $w \in W$ is related to a directed pair of nodes $(s(w), d(w))$: the source node $s(w) \in N$ and the destination node $d(w) \in N, s(w) \neq d(w)$.
- x_w : the rate achieved by the flow w.
- $\bar{x}_w, \underline{x}_w$: box constraints on x_w.

Flow based, single-path routing is assumed in the article. This means first that the routing mechanism must use information about flow membership of datagrams and preserve this information throughout the network. Second—a flow cannot be split across different links in any node. These characteristics can be met, for example by multi protocol label switching or policy based routing. In consequence, there is no need for a separate description of the path rate in the model, as it is equal to the corresponding flow rate.

The routing is represented by a binary matrix; i.e., b is the routing matrix built of elements

$$b_{wl} = \begin{cases} 1 & \text{if } l \in r_w, \\ 0 & l \notin r_w, \end{cases}$$

$l \in L, w \in W$, where $r_w \in \mathcal{P}_w \subseteq 2^L$, r_w is the path of flow w, $\mathcal{P}_w$ is the set of all potential paths connecting $s(w)$ with $d(w)$.

4.2. Model of energy consumption. The discussion in Section 3 deals with energy consumption of a single port. Each network link engages two ports on both ends, so the power consumed by a link is a sum of power attributed to related ports, and is best described by the expression (3) below and illustrated in Fig. 2. The constant component π_0 from Eqn. (1) is omitted here. It is currently impractical to physically turn off the whole router, so it is pointless to include π_0 in the optimization model.

It was observed during experiments that the model is independent of the direction of the traffic, i.e., the traffic outgoing from and incoming to the port can be summed up. The power related to the connection between nodes i, j is the sum of the power dissipated by the corresponding ports:

$$\overbrace{\gamma e_{\kappa(i,j)} + \delta \sum_{w \in W} x_w (b_{w,\kappa(i,j)} + b_{w,\kappa(j,i)})}^{\text{port } i}$$
$$+ \overbrace{\gamma e_{\kappa(j,i)} + \delta \sum_{w \in W} x_w (b_{w,\kappa(j,i)} + b_{w,\kappa(i,j)})}^{\text{port } j}, \quad (3)$$

where

$$e_{\kappa(i,j)} = \begin{cases} 1 & \text{if } \exists w \in W : (b_{w,\kappa(i,j)} x_w > 0), \\ 1 & \text{if } \exists w \in W : (b_{w,\kappa(j,i)} x_w > 0), \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

is a binary variable denoting whether the port of node i, connected by the link (i, j) with node j, is working or idle, and γ, δ are coefficients of the power consumption model (see Fig. 2).

The expression (3) is equivalent (by the regrouping of components) to

$$\gamma(e_{\kappa(i,j)} + e_{\kappa(j,i)}) + 2\delta \sum_{w \in W} x_w b_{w,\kappa(i,j)} + 2\delta \sum_{w \in W} x_w b_{w,\kappa(j,i)}. \quad (5)$$

The total power dissipated by the network is then expressed by

$$P(x) = \gamma \sum_{l \in L} e_l + 2\delta \sum_{w \in W} \sum_{l \in L} b_{wl} x_w, \quad (6)$$

where x is the vector of all flow rates built of $x_w, w \in W$.

4.3. Quality of service component. The QoS related term being a component of the objective function represents a penalty for not achieving an assumed flow rate $\bar{x}$. It reflects the intention of the operator to fulfill the networks primary objective, which is to carry the traffic with certain quality characteristics.

The QoS function can be somehow related to the monetary cost of violating a service level agreement with a customer. It can be also chosen arbitrarily, so as to reflect preferences of a decision-maker, as was done in this case.

In the optimization model, the QoS is represented by $\sum_{w \in W} Q_w(x_w)$, i.e., the total cost of violating a quality of service of flows in W. Each $Q_w(x_w)$ is a convex and continuous function, decreasing with respect to the carried traffic in the interval $[\underline{x}_w, \overline{x}_w]$. It is attains a minimum (zero) at $\overline{x}_w$, the point at which user expectations are fully satisfied.

The convexity of $Q_w(x_w)$ is associated with the belief that small deviations from the nominal throughput $\Delta = \bar{x}_w - x_w$ are perceived by network users as relatively harmless, while large deviations should be avoided. It is reflected by the shape of the QoS violation cost function. The slope of the curve becomes steeper as the rate x_w approaches zero (see Fig. 3).

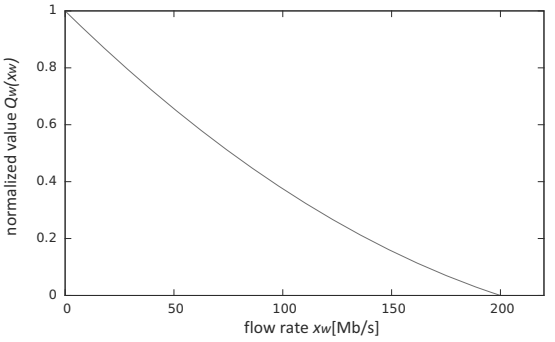

Fig. 3. QoS violation cost function Q_w for $\bar{x} = 200$.

Fairness. Another reason for adopting the nonlinear cost function is the fairness of the solution when a network is subject to congestion.

Let us consider a simple example of two flows w_1 and w_2 with demands $\bar{x}_1, \bar{x}_2 \geq c$ competing for the same bottleneck link with capacity c, represented by the constraint $x_1 + x_2 \leq c$. If the system minimizes a linear, decreasing objective function of x, the solution would be any pair (x_1, x_2) satisfying the equation $x_1 + x_2 = c$. The biased allocation $(0, c)$ belongs to the set of optimal solutions as does the equalized, fair solution $(c/2, c/2)$.

The above example illustrates the problem of ambiguity, and possibly also unfairness of the allocation. The fairness of the solution can be regarded as another optimization criterion. It is a desirable trait of the bandwidth allocation vector, but contributes to a deterioration in terms of the remaining criteria.

Among the most popular fairness definitions are proportional fairness and max-min fairness.

The proportionally fair source rate vector is the solution of a total utility maximization problem (see the discussion by Kelly (1997)):

$$\max_x \sum_{w \in W} \log(x_w)$$

subject to

$$\sum_{w \in W} b_{wl} x_w \leq c_l, \quad \forall l \in L,$$
$$x \geq 0.$$

If the optimal allocation vector $\hat{x}$ exists, for any other feasible vector x^* the aggregate of proportional changes is not positive:

$$\sum_{w \in W} \frac{x_w^* - \hat{x}_w}{\hat{x}_w} \leq 0.$$

In turn, a bandwidth allocation is max-min fair (see, e.g., Bertsekas and Gallager, 1992; Mo and Walrand, 2000), if any rate cannot be increased, without decreasing

some other rate which is already less than, or equal to, this rate. The max-min fair allocation would be achieved (Kelly, 1997) by rescaling the bandwidth units so that every $c_l < 1$, and by solving the utility maximization problem

$$\max_x \sum_{w\in W} -(-\log(x_w))^{\phi}, \qquad \phi \to \infty,$$

subject to

$$\sum_{w\in W} b_{wl}x_w \le c_l, \quad \forall l \in L,$$

$$0 \le x \le 1.$$

It is not possible to solve the above stated problem by using general nonlinear programming solvers. There exist, though, finite sequential algorithms, of which the most basic one is presented by Bertsekas and Gallager (1992).

This algorithm starts with the all-zero rate vector and increases equally the rates on all paths together until saturation of one or more links. This guarantees that each session using a saturated link has the same rate as every other session using that link. In the next steps, all sessions not using the saturated links are incremented equally in rate until one or more new links become saturated. When all sessions pass through at least one saturated link, the algorithm stops.

Both the fairness criteria are dependent on the particular network conditions. In the case of proportional fairness, the flow rate depends on the number of congested links it is crossing. The max-min fair allocation depends on the worst bottleneck on the flows path. It is then clear that max-min and proportionally fair allocations depend not only on a traffic pattern, but also on routing and link capacities, or otherwise—network provisioning.

From the user's point of view, all the internal complexities of the network, expressed here as capacity constraints in the network optimization model, are not visible, and in fact irrelevant. Jain *et al*. (1984) proposed then a measure of fairness based solely on the allocation vector:

$$\mathcal{J}(x) = \frac{(\sum_{w\in W} x_w)^2}{|W|\sum_{w\in W} x_w^2}. \tag{7}$$

Equation (7) better reflects the user's expectations, and can be used for benchmarks. It takes value $\mathcal{J}(x) = 1$ when all participants receive an equal share, and $\mathcal{J}(x) = 1/|W|$ in a winner-takes-all distribution.

From the above discussion on fairness it is clear that there is no consensus among the researchers on the choice of a fairness definition. Moreover, imposition of any additional fairness criterion leads to a solution that is either equal or worse in terms of the original criteria.

Much weaker than discussed above, a simplistic rule is proposed here to avoid bias of allocations:

Principle 1. *Equal rates are achieved by flows which, simultaneously, are assigned the same cost function, are bounded by the same box constraints, and are affected by exactly the same bottlenecks in the network (i.e., they are associated with exactly the same set of active capacity constraints).*

Theorem 1. *Let us consider the bandwidth allocation problem*

$$\min_x \sum_{w\in W} f(x_w), \tag{8}$$

$$\sum_{w\in W} b_{wl}x_w \le c_l, \qquad \forall l \in L, \tag{9}$$

$$0 \le x_w, \le \bar{x}_w \quad \forall w \in W. \tag{10}$$

If f is a strictly convex function, the solution $\hat{x}$ of the problem (8)–(10) conforms to Principle 4.3.

Proof. The Lagrange function for (8)–(10) is

$$\begin{aligned}\mathcal{L}(x,\lambda,\theta,\phi) &= \sum_{w\in W} f_w(x_w) + \sum_{l\in L}\lambda_l\Big(\sum_{w\in W} b_{wl}x_w - c_l\Big)\\ &\quad + \sum_{w\in W}\theta_w(x_w - \bar{x}_w) - \sum_{w\in W}\phi_w x_w,\end{aligned} \tag{11}$$

where λ, θ, ϕ are the vectors of Lagrange multipliers, λ consists of the elements $\lambda_l \ge 0, l \in L$ and θ, and ϕ consist of respectively $\theta_w \ge 0$, $\phi_w \ge 0$, $w \in W$.

From the Karush–Kuhn–Tucker optimality conditions, necessarily

$$\begin{aligned}&\frac{\partial\mathcal{L}(\hat{x},\hat{\lambda},\hat{\theta},\hat{\phi})}{\partial x_w}\\ &= \frac{\mathrm{d}f(\hat{x}_w)}{\mathrm{d}x_w} + \sum_{l\in r_w}\hat{\lambda}_l + \hat{\theta}_w - \hat{\phi}_w = 0, \quad \forall w \in W,\end{aligned} \tag{12}$$

$$\hat{\lambda}_l\Big(\sum_{w\in W} b_{wl}\hat{x}_w - c_l\Big) = 0, \qquad \forall l \in L, \tag{13}$$

$$\hat{\theta}_w(\hat{x}_w - \bar{x}_w) = 0, \quad \forall w \in W, \tag{14}$$

$$\hat{\phi}_w\hat{x}_w = 0, \quad \forall w \in W, \tag{15}$$

$$\hat{\lambda} \ge 0, \quad \hat{\theta} \ge 0, \quad \hat{\phi} \ge 0, \tag{16}$$

where $\hat{\lambda}, \hat{\theta}, \hat{\phi}$ are optimal Lagrange multipliers.

Let us consider a subset of flows $\mathcal{W} \subseteq W$, such as those described in Principle 1. They share links in their paths, some of those links being congested. Disjoint parts of the paths $r_w : w \in \mathcal{W}$ are not congested (have some slack on it), although they may be shared with other flows

$w \notin \mathcal{W}$. Moreover, the flow rates are subject to identical upper box constraint: $\forall w \in \mathcal{W}, \bar{x}_w = \bar{v}$.

For every $w \in \mathcal{W}$, $\sum_{l \in r_w} \hat{\lambda}_l$ is identical, because from (13) all $\hat{\lambda}_l$ which are not common for every r_w are equal to zero.

Let us consider two cases:

1. *All box constraints are inactive:*

   From (14)–(16), if $\forall w \in \mathcal{W}$ box constraints (10) are inactive, i.e., $\hat{x}_w \in (0, \bar{x}_w)$, then, from (14), $\forall w \in \mathcal{W}, \hat{\theta}_w = 0, \hat{\phi}_w = 0$.

   From (12), for every $w \in \mathcal{W}$, $f'(\hat{x}_w)$ has the same value. If $f(x_w)$ is strictly convex, then $f'(x_w)$ is strictly increasing, and, for every two arguments m, n: $f'(m) = f'(n) \Rightarrow m = n$. Hence every flow w is assigned equal rate $\hat{x}_w$.

2. *Any box constraint is active:*

   If any of the box constraints (10) is active, the following holds:

$$\exists w \in \mathcal{W} : \hat{x}_w = \bar{v} \Rightarrow \forall w \in \mathcal{W} : \hat{x}_w = \bar{v},$$

$$\exists w \in \mathcal{W} : \hat{x}_w = 0 \Rightarrow \forall w \in \mathcal{W} : \hat{x}_w = 0.$$

By contradiction, if for some $w \in \mathcal{W}$, the optimum $\hat{x}_w = \bar{v}$, and for some other flow $u \in \mathcal{W}$ the optimal rate would be $\hat{x}_u < \hat{x}_w$, then, from (14),(16), $\hat{\theta}_w \geq 0$, $\hat{\theta}_u = 0$, from (15): $\hat{\phi}_w = 0$ (unless $\bar{v} = 0$, which is trivial). From (12),

$$f'(\hat{x}_w) + \hat{\theta}_w = f'(\hat{x}_u) - \hat{\phi}_u,$$

where $\hat{\phi}_u \geq 0$. Hence $f'(\hat{x}_u) \geq f'(\hat{x}_w)$, while $\hat{x}_u < \hat{x}_w$; but this is contradictory to the fact that f' is strictly increasing. Similar reasoning can be conducted for $\hat{\phi}_w$ and a lower box constraint. ■

4.4. Simultaneous optimization of the quality of service and energy consumption. After the discussion on the model of energy consumption, and general remarks on the decision maker's and the user's preferences regarding the quality of service, a two criteria mixed integer network problem of simultaneous optimal bandwidth allocation and routing can be formulated:

$$\min_{e,b,x} \alpha \sum_{w \in W} Q_w(x_w) + (1-\alpha)\big(\gamma \sum_{l \in L} e_l + 2\delta \sum_{w \in W} \sum_{l \in L} b_{wl} x_w\big), \quad (17)$$

where e is the vector containing all elements $e_l, l \in L$.

The first component of the objective function is related to the quality of service and was defined in Section 4.3, while the second component represents the total power dissipated by the network, as described by the formula (6). The weighing coefficient $\alpha \in (0, 1)$ can be altered to emphasize any of the objectives.

The solution is subject to flow conservation law equations:

$$\sum_{i \in N} a_{ij} b_{w,\kappa(i,j)} - \sum_{k \in N} a_{jk} b_{w,\kappa(j,k)} = \begin{cases} -1, & \forall w \in W, j = s(w), \\ 1, & \forall w \in W, j = d(w), \\ 0, & \forall w \in W, j \neq s(w), j \neq d(w), \end{cases} \quad (18)$$

which assure that the same amount of traffic related to flow w enters and exits each node, with the exception of source and destination nodes. Additionally, these equations enforce a single path routing. This means that each source–destination pair is served by exactly one path.

The variable e must be set according to its definition given in (4),

$$e_{\kappa(i,j)} \geq b_{w,\kappa(i,j)}, \quad \forall i \in N, \forall j \in N, \forall w \in W, \quad (19)$$

$$e_{\kappa(i,j)} \geq b_{w,\kappa(j,i)}, \quad \forall i \in N, \forall j \in N, \forall w \in W. \quad (20)$$

The variable $b_{wl}, \forall w \in W, \forall l \in L$ represents routing and is binary by definition, while $x_w, \forall w \in W$ is positive and limited:

$$b_{wl} \in \{0, 1\}, \quad (21)$$

$$0 \leq \underline{x}_w \leq x_w \leq \overline{x}_w, \qquad \forall w \in W. \quad (22)$$

The link capacity constraint is a nonlinear inequality:

$$\sum_{w \in W} b_{wl} x_w \leq c_l, \quad \forall l \in L, \quad (23)$$

which, as nonconvex, is not accepted by most solvers. Fortunately, it can be transformed to its linear counterpart, by means of a substitution described in Section 4.5. The resulting mixed integer optimization problem with quadratic objective function and linear constraints is solved by means of CPLEX or Gurobi many times faster than when using general nonlinear solvers.

4.5. Elimination of nonlinearity from constraints. Nonlinearity from the constraint (23) can be eliminated by substituting this inequality with a subsequent set of linear inequalities:

$$\sum_{w \in W} y_{wl} \leq c_l, \quad \forall l \in L, \quad (24)$$

$$y_{wl} \geq x_w - M(1 - b_{wl}), \quad \forall w \in W, \quad \forall l \in L, \quad (25)$$

$$y_{wl} \geq 0, \quad \forall w \in W, \quad \forall l \in L. \quad (26)$$

An auxiliary variable $y_{wl} = b_{wl} x_w, w \in W, l \in L$, denotes the part of a traffic rate in the link l assigned to

the flow w. The constant M can be chosen arbitrarily; however, it has to be greater than the greatest x_w in the optimized system:

$$M > \bar{x}_w, \quad \forall w \in W. \tag{27}$$

The objective function can be thus rewritten as

$$\min_{e,b,x,y} \alpha \sum_{w\in W} Q_w(x_w) + (1-\alpha)\big(\gamma \sum_{l\in L} e_l + 2\delta \sum_{w\in W}\sum_{l\in L} y_{wl}\big), \tag{28}$$

where y is the vector containing all elements $y_{wl}, l \in L, w \in W$.

The validity of the proposed equivalence can be proved by contradiction.

Theorem 2. *Given that $\alpha \in (0,1)$ and the link energy function is strictly increasing with respect to its rate, the optimization problem (17)–(23) is equivalent to the problem consisting of the objective function (28), the constraints (18)–(22) and (24)–(26).*

Proof. Let us prove that for

$$(\hat{e},\hat{b},\hat{x},\hat{y}) = \arg\min_{e,b,x,y} \alpha \sum_{w\in W} Q_w(x_w) + (1-\alpha)\big(\gamma \sum_{l\in L} e_l + 2\delta \sum_{w\in W}\sum_{l\in L} y_{wl}\big), \tag{29}$$

with the constraints (18)–(22) and (24)–(26), the following equation holds:

$$\hat{y}_{wl} = \hat{b}_{wl}\hat{x}_w, \quad \forall (w,l). \tag{30}$$

By contradiction, assume first that

$$\exists (w,l) : \hat{y}_{wl} < \hat{b}_{wl}\hat{x}_w. \tag{31}$$

Two cases are considered:

(a) $\hat{b}_{wl} = 0$:

The inequality (31) takes the form $\hat{y}_{wl} < 0$, which is inconsistent with the constraint (26).

(b) $\hat{b}_{wl} = 1$:

From (31), $\hat{y}_{wl} < \hat{x}_w$, which is contrary to the constraint (25): $\hat{y}_{wl} \geq \hat{x}_w$ (after substitution $\hat{b}_{wl} = 1$).

In turn, assume now that

$$\exists (w,l) : \hat{y}_{wl} > \hat{b}_{wl}\hat{x}_w. \tag{32}$$

Two cases are considered:

(a) $\hat{b}_{wl} = 0$:

From (32), $\exists (w,l) : \hat{y}_{wl} > 0$. Then, without violating any constraint, the objective function (28) can be improved by a solution

$$(\hat{e},\hat{b},\hat{x},\hat{y}_{-(wl)}, y^*_{wl} = 0),$$

where $\hat{y}_{-(wl)}$ denotes all the elements of $\hat{y}$ apart from the element $\hat{y}_{wl}$. The reason is that $y^*_{wl} < \hat{y}_{wl}$ and the objective function is increasing with respect to y_{wl}.

It is sufficient to analyze the constraints (24)–(26), because (18)–(21) are unaffected by the change in y. The constraint (25) is met, assuming that M was chosen according to (27) and $\hat{x}_w$ satisfies (22).

(b) $\hat{b}_{wl} = 1$:

From (32), $\exists (w,l) : \hat{y}_{wl} > \hat{x}_w$.

Now the variable y_{wl} can be altered so as to improve the objective function (28):

$$(\hat{e},\hat{b},\hat{x},\hat{y}_{-(wl)}, y^*_{wl} = \hat{x}_w).$$

The constraint (24) is then still satisfied, because $(\hat{e},\hat{b},\hat{x},\hat{y})$ is feasible, and $y^*_{wl} < \hat{y}_{wl}$, so

$$\sum_{k\in W\setminus w} \hat{y}_{kl} + y^*_{wl} < \sum_{k\in W} \hat{y}_{kl} \leq c_l, \quad \forall l \in L.$$

The constraints (25) and (26) are also satisfied when $\hat{b}_{wl} = 1$, $y^*_{wl} = \hat{x}_w$ and (22) holds.

Equation (30) is then always true at the optimum. ■

The above transformation is more economical than the one presented by Bisschop (2007). It introduces two additional constraints instead of four.

4.6. Discussion on parameter selection. As the quality of service is generally a more important objective than energy saving, it is required that the flow rate be reduced for the sake of energy saving only in exceptional circumstances; for example, when a minor reduction and rerouting allow putting some interface into a sleep mode. It would be then undesirable if the marginal cost of the energy in the linear part of the energy model (see Section 4.2) were greater than the profit from improvement of the QoS. Otherwise, $\hat{x}_w$ would never reach $\overline{x}_w$, even if it is not constrained on any link on its path. This condition can be written as

$$\alpha \frac{\mathrm{d}Q_w(\bar{x}_w)}{\mathrm{d}x_w} + (1-\alpha)\frac{\partial P(\bar{x}_w)}{\partial x_w} \leq 0. \tag{33}$$

Assuming that x_w at the operating point is nonzero, the derivative

$$\frac{\partial P(x)}{\partial x_w} = 2\delta \sum_{l\in L} b_{wl} = 2\delta |r_w|, \tag{34}$$

where $|r_w|$ is the path length of the flow w. This means that the condition (33) is path-dependent, so the function Q_w, as well as parameter α, should be carefully tuned according to the network diameter.

Taking into account the above considerations, the following quadratic function is a justified choice for a model of user preferences:

$$Q_w(x_w) = \frac{-\mu_w \bar{x}_w + \xi_w}{\bar{x}_w^2} x_w^2 + \frac{\mu_w \bar{x}_w - 2\xi_w}{\bar{x}_w} x_w + \xi_w, \tag{35}$$

where the parameters are interpreted as follows: μ_w—a negative of slope of $Q_w(x_w)$ in $\bar{x}_w$, ξ_w—$Q_w(0)$.

In order to meet the strict convexity requirement for $Q_w(x_w)$, the following must hold:

$$\xi_w > \mu_w \bar{x}_w. \tag{36}$$

In order to meet the condition (33), μ_w has to be chosen so that

$$\mu_w \geq \frac{2\delta(1-\alpha)}{\alpha} \sum_{l \in L} b_{wl}. \tag{37}$$

The path lengths are not known prior to solving the optimization problem, so μ_w must be chosen arbitrarily. The conservative value is obtained by substituting $\sum_{l \in L} b_{wl}$ in Eqn. (37) with the diameter of the network.

5. Experiments

5.1. Comments on computations. The optimization problem comprising a quadratic objective function and some linear constraints, as the one formulated in previous sections, can be effectively solved by means of the CPLEX solver. It proved to be a few times faster than the solvers designed for general mixed-integer nonlinear programming.

The time necessary to obtain a solution depends mostly on the dimension of the problem, that is, the number of network nodes, links and traffic flows. The tested network consists of seven nodes, ten links, and ten flows. The related optimization problem has 230 continuous variables, 200 binary variables and 700 linear constraints. The computational complexity of MIP is non-polynomial, but in the case of examples of a limited size, as the ones analyzed in the paper, solutions were obtained in a split second.

5.2. Testbed network and testing procedures. Experiments were carried out in the test network (see Fig. 4) consisting of 7 PC computers with Linux OS acting as software routers. Each one was equipped with a four-core i7 processor and a four-port 1Gb/s Ethernet card. Additionally, two of them had fast Ethernet (100Mb/s) cards providing one link of a slower bandwidth. Power consumption was measured individually by general grade power meters assigned to computers. Data were gathered on-line during experiments and stored in a metering system database.

Paths computed by numeric algorithms were established in the testbed network by means of a policy-based routing (PBR). This way, a functionality of MPLS was emulated by standard Linux mechanisms. In order to force the computed flow rates, UDP traffic was generated on selected testbed nodes using `Iperf`—an open source traffic generator. Due to the quantisation of a buffer size used by `Iperf` and imperfections of network adapters, achieved data rates differed from calculated ones in the case of some flows; the error was, however, negligible, as it did not exceed 3%. After completing the experiment, the data rates reported by traffic generators were automatically collected and preprocessed.

5.3. Results of the experiments. The presented algorithms were evaluated in numerous computational experiments in order to gain insight into the characteristics of the problem. Some of the numerical results were then verified in the testbed in order to validate models of energy consumption, and to check whether they are suitable for the particular application for which they were created.

The presented examples illustrate the connection between parameters α, μ and ξ, and properties of the calculated solutions of the problem.

In the first run of the experiment, stress was put on the quality of service. The challenge was to allocate the capacity to ten flows. Each one declared the requested rate of 200 Mb/s, and was related to the specific source-destination pair of nodes.

The network topology, as well as link capacities, reflected those found in the laboratory network; however, it was observed that a link bandwidth of 1 Gb/s is never attained in practice. An adequate headroom was then assumed in calculations, and link capacities were set on the level of 812 Mb/s. Parameters of the objective function are gathered in Table 1, in the row "high quality".

The calculated paths are shown in Fig. 5. All the flows have achieved an equal rate of the required 200 Mb/s, which was adequately reflected by Jain's index of fairness (7).

In the second example, called "low energy", the weighing coefficient α was set to 0.5 to put equal emphasis on both criteria. As a consequence of that, the calculated power was reduced by 16.86 W, which is related to the rerouting of the flow no. 1 (see Fig. 6), and turning off some links which are no longer necessary. The quality of service dropped in this case. Some flow rates had to be reduced, because the declared traffic would not fit into the network under the new routing pattern. The most truncated flows rate was 148 Mb/s vs. the requested 200 Mb/s. The decision whether or

Table 1. Parameters of the objective function. In each experiment $\mu = 75 \cdot 10^{-4}$.

No.	Experiment	α	$\bar{x}$	ξ	Jain's index
1.	High quality	0.95	200	3	1
2.	Low energy	0.5	200	3	0.985
3.	Linear $Q(x)$	0.5	200	1.5	0.91
4.	Reduced demand	0.5	50	3	1

Table 2. Power consumption.

No.	Experiment	Calculated power [W]	Measured power[W]
1.	High quality	45.16	63.65
2.	Low energy	28.3	51.75
3.	Linear $Q(x)$	28.5	—
4.	Reduced demand	22.6	34.35

not it is acceptable belongs to a decision-maker, who can fine-tune parameters when the obtained performance is not satisfactory.

The third experiment, titled "linear $Q(x)$", was performed to illustrate the discussion (Section 4.3) on how the convexity of $Q(x)$ affects fairness. The parameter ξ was modified so that $Q(x)$ was linear—the condition (36) does not hold. In this case, power consumption is almost the same as in the previous experiment (see Table 2), and the routing is unchanged, but the fairness index has dropped substantially (Table 1, row 3). It turns out that one of the flows was truncated to 12 Mb/s, while the remaining flows were allocated 200 Mb/s.

In the fourth experiment, titled "reduced demand" a typical scenario was simulated, when a decision-maker wishes to reduce the offered capacity. In this case, despite rerouting (see Fig. 7), neither a number of active links nor the mean path length is changed, because apparently there is no potential for further routing optimisation. Nevertheless, power consumption is substantially reduced because flow rates are lower, which affects the linear part of power model.

5.4. Verification of power consumption models in the test network. The results of experiments number 1, 2 and 4 were transferred to the laboratory network.

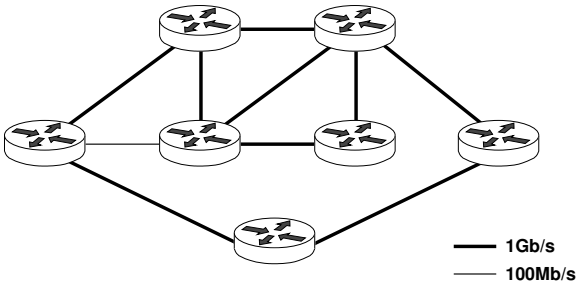

Fig. 4. Testbed network.

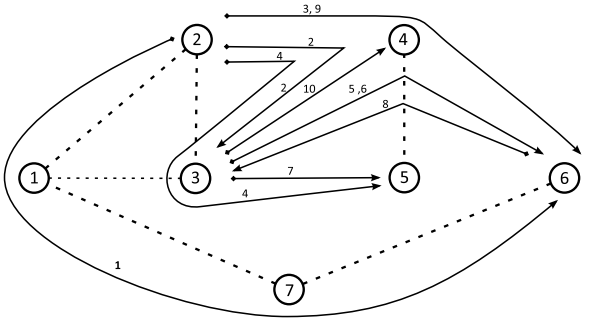

Fig. 5. Distribution of flows: high quality.

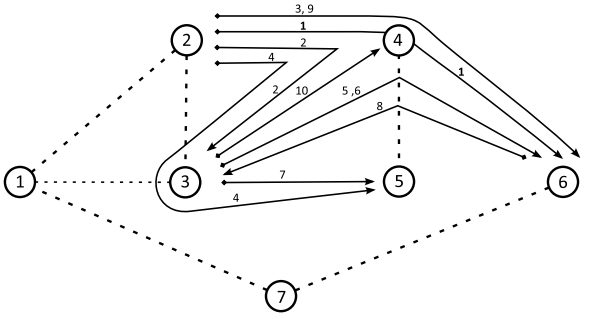

Fig. 6. Distribution of flows: low energy.

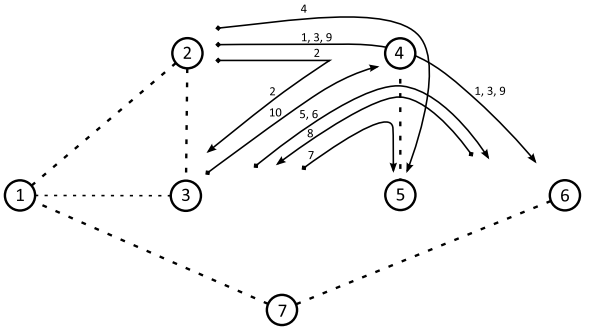

Fig. 7. Distribution of flows: reduced demand.

Calculated versus measured power dissipation values are summarized in Table 2. The invariable part of the energy consumed by computers, denoted with π_0 in (1), was not part of the model, and is subtracted from the measured values of the consumed power.

The real-life measured power consumption was greater than the calculated value. The estimation error ranged from 11.75 to 23.45 W and averaged at 17.9 W. The mean error is within 36% of the mean measured power. It must be noted, however, that the constant part of the power of each node, π_0, is about 31 W, which means that the estimation error is close to 6.7% of the whole power consumed by the network.

The absolute difference between the columns of Table 2 can be explained by additional activities of processors related to the tasks of traffic generation and measurement, which were performed during the experiments and were not covered by the model.

For verification of the above claim, additional experiments were conducted. The energy consumed by each node was measured separately. The results, confirmed that energy consumption of the nodes which acted solely as routers was modelled with a much greater accuracy than for the ones which acted also as traffic generators.

6. Conclusions

The paper presents a computational system created for traffic engineering in IP networks. It is aimed at simultaneous optimization of the quality of service and energy dissipation. It is made up of an optimization engine, models of user preferences and energy consumption.

The discussion carried out in the paper provides an analysis of the problem properties and covers the subject of the choice of model parameters. Some questions are left open for future research.

The traffic matrix must be provided by an external mechanism. This constitutes quite a difficult and errorprone task, because the procedure must take into account many factors: preferences of users and of the decision maker, and the topology and capacity of the network. The presented system is fortunately immune to erroneous input thanks to allowing some elasticity of demand.

The computational complexity of the mixed integer programming problem is non-polynomial, so in the case of big or heavily loaded systems some specialized, probably heuristic optimization must be used.

Power saving impact of the presented traffic engineering strategy is mostly dependent on the susceptibility of network device energy consumption to load changes and network topology. In the presented experiments, using contemporary off-the-shelf PC computers, in a specific topology, power consumption could be reduced by about 10% in response to a load change (experiment 1 vs. 4). It is, however, worth noting that in the second example the power is reduced by 4.2%, with only a minor deterioration of the quality of service, making advantage of traffic elasticity. Assuming that future specialized network devices will be able to decrease power consumed in idle mode even further, we believe that the presented model has great potential.

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