Hannu Nurmi

COMMENTS TO JONES

The Ambiguity of Proportional Representation

Introduction

In the preceding, H. S. Jones discusses one of the perennial themes in democratic theory, *viz.* the concept of representation. Jones pays particular attention to arguments for and against proportional representation (PR, for short) in the late 19th and early 20th century debates. Rather than comment on the arguments or Jones's analysis thereof, I concentrate on some assumptions that the proponents and opponents of PR apparently shared. To wit, it seems that the participants of the debates had a common view of what it means for an assembly to be composed in a proportional manner: it should be a miniature model of the electorate at large. In this brief note I shall take issue of the above common view by arguing that some fundamental conceptual problems remain unresolved in the very notion of PR.

The debate discussed by Jones seems to be based on following assumptions:

1. For any group of voters, it is always better to have more representation than less thereof.
2. The degree to which perfect proportionality has been achieved can be determined in a way that is independent of the voting procedure.
3. If the share of parliamentary seats assigned to a party corresponds

to its share of popular vote, then perfectly proportional representation has been achieved.

I shall endeavor to show that, although *prima facie* plausible, each one of these claims can be shown is erroneous. We shall begin with the first claim.

Schwartz's Paradox

Regardless of whether one considers parliaments as venues of public debate or of voting according to predetermined agendas, it intuitively seems that the parties are better off with more seats than less seats. The influence over the voting outcomes seems to increase – or at least not decrease – with additional seats. Similarly, the views of larger representative groups can be expected to be heard better than those of smaller ones. And yet, Schwartz (1996) has shown this to be an erroneous assumption. Consider the following situation involving three parties in a 100-member parliament.

Party 1	*Party 2*	*Party 3*
(30 seats)	(45 seats)	(25 seats)
status quo	nuclear power	coal power
coal power	status quo	nuclear power
nuclear power	coal power	status quo

The issue to be decided is the building of a large power plant. The government proposes the building of a nuclear power plant. Its second preference is the status quo (i.e. postponing the decision with the hope that some new energy saving innovations or new energy sources will be found) and the third the building of a coal power plant. Its views are supported by party 2. The preferences of other groups are listed above.

If the parliamentary amendment procedure is used, the "natural" agenda is to vote first on the coal vs. nuclear plant and confront then the winner of this vote with status quo. Since coal is preferred to nuclear power by parties 1 and 3, coal wins the first ballot. In the

second ballot it is defeated by status quo since parties 1 and 2 prefer status quo to coal power. Thus, status quo would seem to prevail. This is the worst outcome from party 3's view-point.

Suppose now that this party had somewhat less representation in the parliament. To be specific, assume that it had 13 instead of 25 seats and that the 12 seats it loses would be evenly distributed between parties 1 and 2. Thus parties 1 and 2 would now have 36 and 51 seats, respectively. This would make the nuclear plant the Condorcet winner, *i.e.* an alternative that defeats all its competitors by a majority of votes. *Eo ipso* the nuclear plant would be the outcome ensuing from the parliamentary vote. This outcome is preferred to the status quo by party 3. This, in turn, means that party 3 would be better off with less representation than with more representation. It is easy to see that all seat distributions that result from party 3 losing 12 or more seats to be equally divided between the other two parties give party 3 a better outcome (nuclear plant) than the original situation where status quo is the result. Thus, less representation is beneficial to party 3. Indeed, in this example party 3 is better off having no representation at all than having a quarter of seats of the parliament. Surely, this observation runs counter the conventional wisdom underlying the debate reported by Jones.

The phenomenon observed in the above table comes pretty close the no-show paradox (Fishburn and Brams 1983; Nurmi 1999). The paradox occurs whenever a group of voters gets a better outcome by not voting at all than by voting according to their preferences. In Schwartz's paradox the abstainers' ballots are redistributed evenly among the competitors. Thus, although not every instance of the no-show paradox is an instance of Schwartz's paradox, they are pretty close to each other. It has been shown by Moulin (1988) that vulnerability to the no-show paradox is fairly common among voting systems. All systems that necessarily elect a Condorcet winner when one exists, may result in a no-show paradox. On the other hand, there are also systems which are bad on both counts, i.e. may result in a no-show paradox and fail to elect a Condorcet winner. Two such systems are widely used: the plurality runoff and alternative vote[1]. We now turn to the meaning of proportionality or rather the profound ambiguity of the concept.

Proportionality of Opinions

The intuitive concept of proportionality connects the views of the members of the representative body with those of the electorate at large. According to this intuitive conception, the views of the electorate are determined by election results. But in typical elections, each voter can reveal very little of his/her opinions regarding candidates and/or parties. More often than not, he/she can only pick one alternative (party list or candidate) as his/her favorite. Sure, there are elections, notably of STV or AV variety, that allow for a richer expression of opinions. My point, however, is that assuming that the voters have opinions regarding all candidates or at least several of them, the notion of proportionality underlying current electoral systems becomes ambiguous. Consider an example.

30% of voters	35% of voters	25% of voters	10% of voters
A	C	D	B
B	B	B	C
C	A	A	D
D	D	C	A

Here we have four candidates competing for three seats in a constituency. Systems based on plurality or one person – one vote idea as well as STV would elect A, C and D. Yet, B is ranked first or second by all voters, while D is ranked last by 75% of them. The exclusion of B would seem unreasonable.

By a slight modification of the above table one may create a situation where plurality and STV systems end up with different outcomes. By assigning 35% of the voters to the leftmost group, 40% to the next one, 15 % to the next one and 10% to the rightmost group, we get different results with plurality based systems and STV. The former ends up with A,C and D, while STV results in A,B and C. It is not difficult to see that our notion of proportionality is crucially dependent on the voting system. This – along with voter preferences – determines what kind of seat distributions we consider proportional (see Nurmi 1985 and Baker 1996, for further discussion). Thus, it seems that the second assumption referred to

above poses problems of rather profound nature. It turns out that the very meaning of proportionality hinges on an implicit assumption of the social choice procedure to be used. Consequently, it may well happen that, given a preference profile, we may have several proportional outcomes (*e.g.* allocations of parliamentary seats to parties) depending on the underlying choice procedures (*e.g.* STV, Borda[2] count, plurality).

Proportionality of What?

Suppose now that nearly perfect proportionality has been achieved in the sense that there is an agreement as to what voting procedure is used in defining proportionality and, moreover, the seat distribution of parties corresponds closely to the distribution of support in the electorate. In other words, let us assume that the problem of the preceding section has been solved in a satisfactory manner. The voting body is then assumed to be a miniature model of the electorate at large in relevant respects. Surely our assumption is very strong, but nevertheless it can easily be seen that crucial problems remain open even if the proportionality of seat distribution were our sole *desideratum*.

Consider a voting body – say, a parliament – with 100 seats. Suppose, moreover, that on the basis of elections held, the perfectly proportional seat distribution would give party A 55 seats, party B 25 seats and party C 20 seats. The main role of parliaments is to enact laws and other norms. In passing legislation, the parliaments resort to collective decision making procedures. Very often the majority rule is being applied. In other words, one looks at decision alternatives in a binary fashion and at any given stage of the procedure chooses that alternative which is supported by a majority in a contest with another alternative (or set of alternatives).[3] It is clear that when the number of seats of a party exceeds that of the majority, it determines the winners in every pairwise contest. Thus, the influence of such a party over the legislative outcomes is decisive. In our example, party A clearly determines the outcome of every pairwise vote. Hence its control over the legislation is complete. With 55% of

the seats – and, by assumption, of the popular support – it controls 100% of the legislation.

On the basis of examples like the one just discussed one could argue that what one should distribute proportionally is not seats but voting power, *i.e.* influence over legislative outcomes. This, however, poses the question of measuring the latter. What the above example suggests is that the seat distribution is at times a poor proxy of voting power distribution. In particular under circumstances where one party is capable of dictating the voting outcomes, the distribution of seats to others is largely irrelevant. But is the negative conclusion valid in general, that is, are we in general entitled to the conclusion that the seat distribution gives a distorted picture of the voting power distribution?

The answer to this question depends on one's measure of voting power. There is a rich literature on different measures of *a priori* voting power. These measures equate voting power with the importance of a party when voting coalitions are assumed to form in specific ways. For example, perhaps the most widely used *a priori* voting power measure, the (normalized) Banzhaf index makes the simplifying – and often empirically false – assumption that all winning coalition are equally likely to form. Under this assumption and taking into account the seat distribution and the required number of votes to pass legislation it counts the number of winning coalitions in which a party is non-redundant. These coalitions are called swings of the party (*i.e.* the party swings a non-winning coalition into a winning one by joining it). Dividing the number of swings of a party with the total number of swings of all parties gives the Banzhaf index value of the party.

Banzhaf index is but one of many measures of *a priori* voting power. Its advantages and disadvantages *vis-à-vis* other similar measures has been a subject of debate for some time (see Felsenthal and Machover 1998, for a relatively recent over-view and analysis). Whatever its shortcomings, it is certainly more informative and useful measure of voting power than the practice of distributing seats to parties in proportion to their popular support. What this practice ignores is the fact that decisions in collective bodies are always made in accordance with decision rules. Typically these state the vote thresholds that have to be exceeded in order to pass new legislation. That a

measure which takes these thresholds into account is superior to the common practice, seems pretty obvious. What remains an open question – and a subject of an amazingly hot debate – is whether one should include more institutional detail into power measures. One such detail could be the existence of a spatial continuum along which the parties occupy more or less fixed positions. The traditional left-right continuum is an obvious candidate for such a spatial dimension. Whether it still binds the parties in their coalition behavior is, however, somewhat questionable.

Concluding Remarks

We have discussed three implicit assumptions underlying the 19th century debate on PR. Each one of them turns out to be either false or highly problematic. The first states that it is always better to have more representation than less is simply false as shown by Schwartz's paradox. The second assumption states that we can recognize a proportionally composed voting body when we see it. Our preceding discussion shows, however, that the notion of proportionality is highly procedure-dependent. What is a proportional distribution in the sense of one procedure may not be that in the sense of another method. The third assumption equates voting power distribution with seat distribution. It seems to be false in a wide variety of cases.

Notes

1 Alternative vote (AV) is simply the application of single transferable vote system to single-member constituency, whereupon the quota needed for election is 50%.

2 Borda count is a voting system based on similar ballots as STV. With k alternatives, each first rank gives an alternative k-1 points, each second rank k-2 points etc. The Borda winner is the alternative with the largest sum of points.

3 We have no space here for a discussion on variants of parliamentary voting procedures. The amendment procedure used *e.g.* in Finland, Sweden and the United States is binary in the sense of confronting

alternatives with other alternatives. The binariness of the successive procedure – which is used most widely in parliaments of our time – is based on comparing one alternative with all the other remaining alternatives. The reader is refereed to Riker (1982) and Rasch (1995) for discussion on parliamentary voting procedures.

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