

Enlargement and Reduction Characteristics of Facsimile Transmission Copies

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A research project was designed to determine the extent of enlargement and reduction of documents transmitted by different facsimile machines. In addition, the scope of the study was expanded to answer the following questions: *Does the transmitting or receiving machine have a greater influence on the enlargement/reduction of faxed documents? Is the degree of enlargement/reduction consistent from one transmission to another? Do significant differences exist in the degree of enlargement/reduction from one page to another in multi-page facsimile transmissions? Is the degree of enlargement/reduction equal along the horizontal and vertical axes?* Receiving machines were found to have a greater influence on the output size of faxed documents relative to the sending machines. Considerable changes in size were observed between originals and faxed copies, between separate pages of multi-page facsimile transmissions and within individual faxed documents. Although extreme differences in size were uncommon, faxed copies ranged from 12% smaller to 2.9% larger than the original template. Document examiners should be aware of these facts when inspecting faxed documents.

Introduction

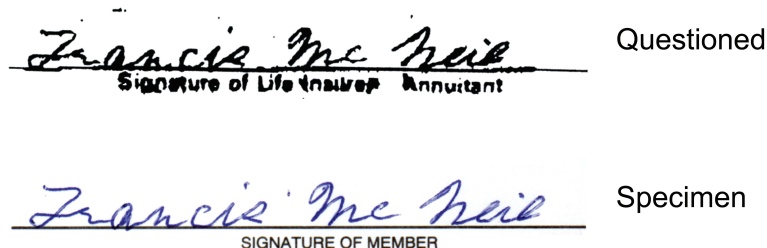
Faxed business contracts, purchase agreements, invoices and purchase orders are often submitted to private and government laboratories. Questions arise not only with respect to the contents of these faxed documents but also whether they were actually transmitted on their purported dates. Document examiners are well acquainted with the loss of image quality or detail in facsimile copies, but these are not the only issues that must be considered.

It is generally accepted that facsimile machines often reduce the width and length of faxed documents to suit the printing parameters of receiving machines. Dimensional changes between original documents and faxed copies have been acknowledged by Casey-Owens (1990), McComb (1991), Miller (1997), Welch (1998) as well as Lindblom and Purdy (1998). Some sheet-fed receiving facsimile machines compress transmitted documents to provide enough space for a Transmit Terminal Identifier (TTI) and/or

Receive Terminal Identifier (RTI) on the printed copies. The latter can appear at either the top or bottom of the page depending on the particular fax machine. Adjusting the settings on some models can also change the size of facsimile transmitted documents.

Welch (1998) reported, "The different lengths recorded at the left and right edges of many received faxes indicate that the image is 'out-of-square'. In many of those instances, the diagonals are approximately equal suggesting that the image is a regular trapezium, rather than a rectangle, but to such a small extent it is not apparent to the naked eye." He also reports, "Size changes were very small, a few percent reduction or increase is the most that the survey revealed. This is only a little larger than the size change of nearly one percent which might be caused in the width of office paper simply by changes in ambient humidity."

A recent case examined by one of the authors involved a faxed document bearing a contested signature. While reviewing the specimens, one



Signatures taken from two different documents



The faxed signature was enlarged 107% and superimposed with the original. They are offset slightly to demonstrate their registration.

Figure 1. Example of a signature transfer in a facsimile fabrication.

exemplar exhibited a striking similarity to the questioned signature. Although both were alike, the exemplar signature was obviously larger than the faxed version (Figure 1). After the faxed document was enlarged approximately seven percent, the two signatures registered when superimposed. Since this was greater than the degree of reduction reported by Welch, it raised the possibility that the questioned document was faxed several times.

A research project was designed to measure enlargement or reduction in faxed documents and to answer the following questions:

- Does the transmitting or receiving machine influence the enlargement or reduction?
- To what extent does the size vary between the original and the faxed version?
- Is the degree of enlargement or reduction consistent from one transmission to another?
- Do significant differences exist with respect to the degree of enlargement or reduction from one page to another in multi-page transmissions?
- Is the degree of enlargement or reduction equal along the horizontal and vertical axes?

Methods & Materials

Ten facsimile machines located within the city of Ottawa were selected for this study. These machines were produced by several manufacturers and used various types of printing media, some being sheet and others roll fed (Table 1). Their suggested retail prices also cover a wide range from under \$200 to more than \$4500 (USD). All of the machines involved in this research produced TTI's on the faxed document, while only machine 3 registered RTI's. The presence of the RTI may have an impact on the amount of reduction found on faxes received by the Pitney-Bowes model.

A three-page template was developed to aid in measuring the size of transmitted documents. Each page contained three horizontal, three vertical and two diagonal lines (Figure 2). Each of the three horizontal lines was 17.8 cm long and the length of each vertical line was 24.1 cm. The two diagonal lines measured 30.0 cm.

Each machine faxed the three-page template to the nine other machines in the study. Once they were collected, the following measurements were recorded from each template received:

Machine	Manufacturer	Model	Paper Type	Sheet or Roll Fed	Print Method	Price
1	Panasonic	KX-F280	Thermal	Roll	Thermal	\$600.00
2	Brother	675	Thermal	Roll	Thermal	\$499.00
3	Pitney-Bowes	9820	Plain	Sheet	Laser	\$4,000.00
4	Canon	L 785	Plain	Sheet	Laser	\$4,295.00
5	Sharp	UX 103	Thermal	Roll	Thermal	\$699.00
6	Panasonic	KX-F700	Thermal	Roll	Thermal	\$499.00
7	Canon	7500	Plain	Sheet	Laser	\$4,795.00
8	Sharp	UX 510	Plain	Sheet	Inkjet/Bubblejet	\$175.00
9	Ricoh	3000 L	Plain	Sheet	Laser	\$2,995.00
10	Sanyo	100	Thermal	Roll	Thermal	< \$250.00

Table 1. Facsimile machines used in the study¹

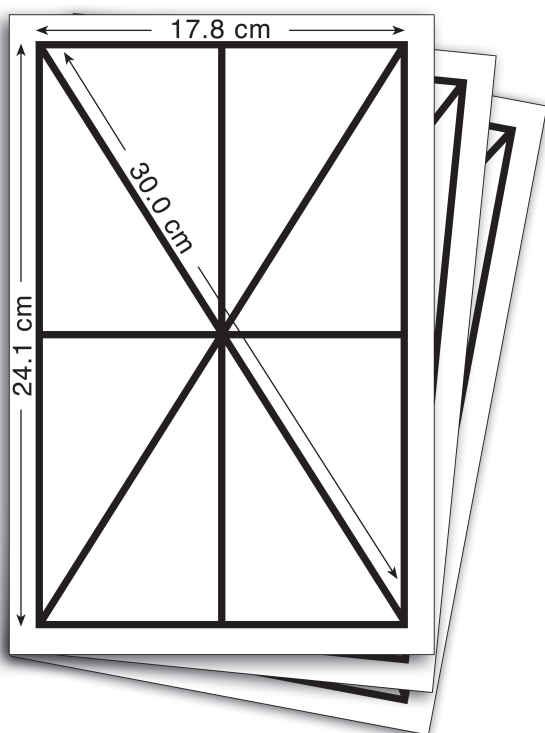


Figure 2. Three-page fax template consisting of three identical pages sent to each fax machine.

- the lengths of the three horizontal lines (top, center and bottom),
- the lengths of the three vertical lines (left, center and right), and,
- the lengths of the two diagonal lines on each page.

To minimize subjective error, three people independently measured each parameter to an accuracy of ± 0.05 cm. The average of each set of three independent measurements was used to determine the following values:

- the difference in enlargement or reduction along the horizontal and vertical axes of each page,
- the difference in enlargement or reduction within the three pages of each transmission,
- the minimum and maximum enlargement or reduction values for all transmissions,
- the variation in enlargement or reduction within the templates sent by each machine, and,
- the variation in enlargement or reduction within the templates received by one machine.

Descriptive statistics were applied to organize and summarize the data. This involved calculating the percent enlargement or reduction for each transmission and determining the range of values

(i.e., the maximum and minimum size of transmitted documents). Inferential statistics were then used to quantify some observations and answer specific questions regarding the data.

Data for each sending machine was broken into 10 separate groups. A similar approach was taken for the transmitting machines, yielding a total of 20 groups. Width, length and diagonal mean measurement data sets were then developed for the 9 transmissions in each group. Highlighted rows and columns in Table 2 graphically illustrate two such data sets. The green cells represent one data set of 9 mean vertical values corresponding to transmitting facsimile machine 2. The red cells represent 9 mean values corresponding to receiving facsimile machine 2.

The variance² of mean values was calculated to determine the extent of fluctuation within each data set. A standard t-test³ was applied to the transmitting and receiving machine mean value variances. The null hypothesis used for this study assumed that the sending and receiving machines have an equal effect on the enlargement/reduction of facsimile transmitted documents.

ANOVA (Analysis of Variance) was also used to compare the absolute mean values of enlargement or reduction factors. ANOVA⁴ is analogous to the t-test in determining if differences between mean values are significant.

Results

Once the three horizontal lines and three vertical lines on each transmitted page were measured, the average values for the width and length of each pair of sending/receiving fax machines were calculated. A few transmissions between machines either failed or were incomplete. In one case, the lower portion of one page of the template sent by fax machine 8 was not received by machine 3. Consequently, the vertical lines L1, L2 and L3 could not be measured. Asterisks in Tables 2 and 3 denote such missing values.

The average lengths of vertical lines (L1, L2, L3) printed by the receiving fax machines appear in Table 2. Values fall within the range 21.2 cm to 24.8 cm. Some receiving machines consistently reduced or enlarged the size of the transmitted

Mean Vertical Measurements												
Original size 24.1 cm		Transmitting Fax										Variance ^T
		1	2	3	4	5	6	7	8	9	10	
Receiving Fax	1	N/A	24.1	24.2	24.1	24.4	24.2	24.1	24.7	24.0	24.4	0.0478
	2	24.3	N/A	24.1	24.0	24.3	24.1	24.1	24.6	23.9	24.3	0.0436
	3	23.3	23.3	N/A	23.5	23.3	23.8	23.4	(*)	23.7	22.6	0.131
	4	24.2	24.1	24.1	N/A	24.3	24.0	24.0	24.5	23.9	21.8	0.640
	5	24.4	24.4	24.4	24.4	N/A	24.3	24.3	24.8	24.3	24.6	0.0275
	6	24.0	24.0	24.0	23.9	24.1	N/A	23.9	24.4	23.8	24.1	0.0294
	7	22.9	23.3	23.1	23.4	23.3	23.5	N/A	23.1	23.6	22.4	0.132
	8	22.2	22.0	22.1	22.0	22.2	22.1	22.0	N/A	21.9	21.2	0.093
	9	23.3	23.6	23.6	23.9	23.7	23.8	23.7	23.5	N/A	22.7	0.128
	10	24.1	23.8	24.0	23.9	24.1	24.0	23.9	24.3	23.8	N/A	0.0261
Variance ^R		0.565	0.509	0.520	0.484	0.510	0.443	0.479	0.371	0.473	1.57	

Table 2. Average lengths (in cm) of three vertical lines on the received templates. The green cells represent one data set of 9 mean vertical values corresponding to transmitting facsimile machine 2. The red cells represent 9 mean values corresponding to receiving facsimile machine 2. The right column represents the variance (Variance^T) between transmitting machine values for each receiving machine. Similarly, the bottom row shows, the variance (Variance^R) between receiving machine values for each transmitting machine. The asterisk denotes a missing value.

template. For example, vertical lines on templates received by fax machine 3 varied from 23.8 cm (-1.2%) to 22.6 cm (-6.2%). Templates received by machine 8 varied from 22.2 cm (-7.9%) to 21.2 cm (-12.0%) and exhibited the greatest range.

Those machines that consistently reduced the size of transmitted documents were all sheet-fed models. Receiving machine 5, a thermal roll-fed model, enlarged the size of transmitted templates in each instance. The average lengths of the vertical lines ranged from 24.8 cm (+2.9%) to 24.3 cm (+0.8%).

The average lengths of horizontal lines (W1, W2, W3) printed by the receiving facsimile machines are shown in Table 3. Receiving facsimile machines 3 and 8 consistently reduced the widths of transmitted templates. The average width of templates printed by machine 3 varied from 17.7 cm (-0.6%) to 16.6 cm (-6.7%) while facsimile machine 8 reduced their average width from 16.5 cm (-7.3%) to 15.7 cm (-11.8%). Once again, machines 3 and 8 are sheet-fed models.

The right column, Variance^T , of Tables 2 and 3 represent the variance between transmitting machine values for each receiving machine. Similarly, the bottom row of these tables show

Variance^R , the variance between receiving machine values for each transmitting machine. In each instance, Variance^T indicates the values for transmitting machines are more consistent. The range of Variance^T is larger than the variance within each data set. For Variance^R , the situation is substantially different. The range of Variance^R is quite comparable to the variance observed in the individual data sets. Furthermore, *F-tests* (see endnote 4) confirmed that the variances of the receiving machine were statistically different. By way of example, calculations applied to the variance values for the received templates yielded $F = 36.2$, $F_{crit} = 2.0$ and $p = 0$ (1.8E-24). In contrast, calculations applied to the transmitting machine data resulted in scores of $F = 1.0$, $F_{crit} = 2.0$ and $p = 0.43$. Since F is greater than F_{crit} and p is much less than 0.05, the data for receiving machines is statistically significant. These results indicate that, generally, receiving machines distort faxed documents to a greater extent than transmitting machines.

Figure 3 shows that the mean measured vertical values tend to be more widely scattered when a facsimile machine is transmitting documents than when it is receiving documents. When facsimile

Mean Horizontal Measurements												
Original size 17.8 cm		Transmitting Fax										Variance^T
		1	2	3	4	5	6	7	8	9	10	
Receiving Fax	1	N/A	17.8	17.7	17.7	18.0	17.9	17.8	17.8	17.7	17.8	0.01
	2	17.8	N/A	17.7	17.8	18.0	17.9	17.8	17.8	17.7	17.9	0.00944
	3	17.2	17.4	N/A	17.5	17.3	17.7	17.4	(*)	17.5	16.6	0.108
	4	17.9	17.9	17.7	N/A	18.0	17.9	17.8	17.8	17.7	16.1	0.345
	5	17.8	17.9	17.7	17.8	N/A	17.8	17.8	17.8	17.7	17.9	0.005
	6	17.8	17.8	17.7	17.7	18.0	N/A	17.8	17.8	17.7	17.8	0.00861
	7	17.8	17.8	17.7	17.8	18.0	17.8	N/A	17.8	17.6	17.8	0.0111
	8	16.4	16.4	16.2	16.3	16.5	16.4	16.4	N/A	16.2	15.7	0.0569
	9	17.8	17.8	17.7	17.8	18.0	17.8	17.8	17.8	N/A	17.9	0.00694
	10	17.9	17.9	17.7	17.8	18.0	17.8	17.8	17.8	17.7	N/A	0.00944
Variance^R		0.247	0.238	0.250	0.239	0.275	0.230	0.220	0.00	0.243	0.789	

Table 3: Average lengths (in cm) of three horizontal lines on the received templates. The asterisk denotes a missing value. The right column, (Variance^T), represents the variance between transmitting machine values for each receiving machine. Similarly, the bottom row, (Variance^R), shows the variance between receiving machine values for each transmitting machine.

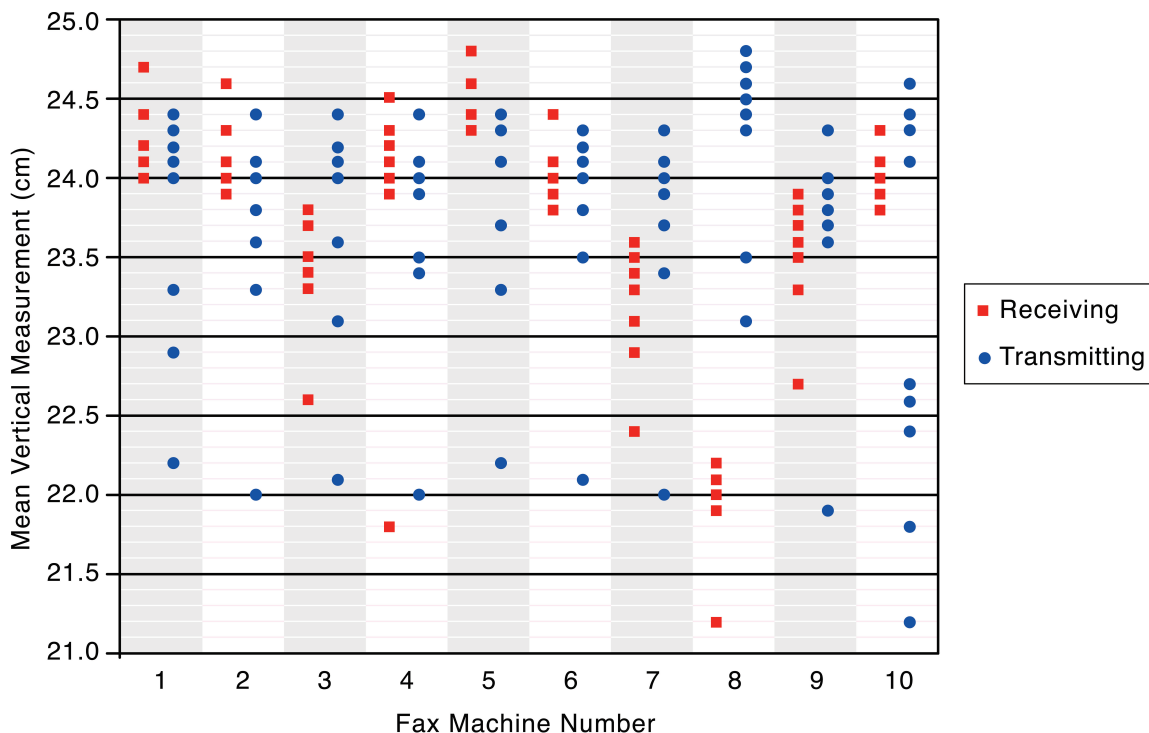


Figure 3. Distribution of Mean Vertical Measurements for the Transmitting and Receiving Machines.

machine 2 is the sending machine, the plotted values cover a much broader range than when it is the receiving machine. This coincides with the corresponding variance values 0.509 and 0.044 reported in Table 2. Data pertaining to transmissions between other fax machines exhibit a similar tendency.

The cluster graph shows tighter grouping of the receiving machine values (small Variance^T), and therefore greater consistency in the behavior of the transmitting machines. For example, the lengths of faxes received by machine 1 vary from 24.0 cm to 24.7 cm, whereas those transmitted by machine 1 range from 22.2 cm to 24.4 cm (large Variance^R).

The diagonals of templates received by facsimile machines 3, 7, 8 and 9 were all reduced in size as seen in Table 4. For example, diagonals printed by facsimile machine 8 displayed the maximum degree of reduction for all facsimile transmissions. The lengths of the diagonals on faxes produced by machine 8 ranged from 27.8 cm (-7.3%) to 26.4 cm (-12.0%). The minimum reduction observed among these four receiving machines was produced by machine 9 where values ranged from 29.0 cm (-3.3%) to 29.8 cm (-0.7%). Since the

diagonals are influenced by the lengths of the horizontal and vertical lines according to Pythagoras' theorem⁵, it is not surprising that the degree of enlargement or reduction for the diagonals in many instances was similar to that calculated for the horizontal and vertical lines.

This study also considered changes in the length and width within the three-pages of each transmission. For each page transmitted, differences between the maximum and minimum length and width values of the received templates were recorded. The histogram in Figure 4 shows that these inter-page variations ranged from 0.0 cm to as much as 1.5 cm. Most varied from 0.0 cm to 0.1 cm. Values greater than 0.1 cm were deemed to be significant as they are beyond measurement error and the influence of humidity on paper size. The results of this study demonstrate that page-to-page variations can be much greater than previously reported with the greatest deviations occurring along the vertical axis.

The final phase of this study involved analyzing variations in the length and width of template measurements within each transmitted page. Intra-page differences between the maximum and

Mean Diagonal Measurements											
Original size 30 cm		Transmitting Fax									
		1	2	3	4	5	6	7	8	9	10
Receiving Fax	1	N/A	30.1	30.0	30.0	30.3	30.1	30.0	30.4	29.9	30.3
	2	30.2	N/A	29.9	29.9	30.3	30.0	30.0	30.3	29.8	30.2
	3	28.9	29.1	N/A	29.3	29.1	29.7	29.2	(*)	29.4	28.1
	4	30.1	30.0	29.9	N/A	30.3	30.1	29.9	30.3	29.8	27.2
	5	30.3	30.3	30.2	30.1	N/A	30.2	30.2	30.6	30.1	30.4
	6	29.9	29.9	29.8	29.8	30.1	N/A	29.8	30.2	29.7	30.1
	7	29.0	29.4	29.2	29.4	29.3	29.5	N/A	29.2	29.5	28.6
	8	27.7	27.5	27.4	27.4	27.8	27.5	27.5	N/A	27.4	26.4
	9	29.4	29.6	29.5	29.8	29.7	29.8	29.7	29.5	N/A	29.0
	10	30.0	29.8	29.8	29.8	30.1	29.9	29.8	30.3	29.7	N/A

Table 4: Average lengths (in cm) of two diagonal lines on the received templates. The asterisk denotes a missing value.

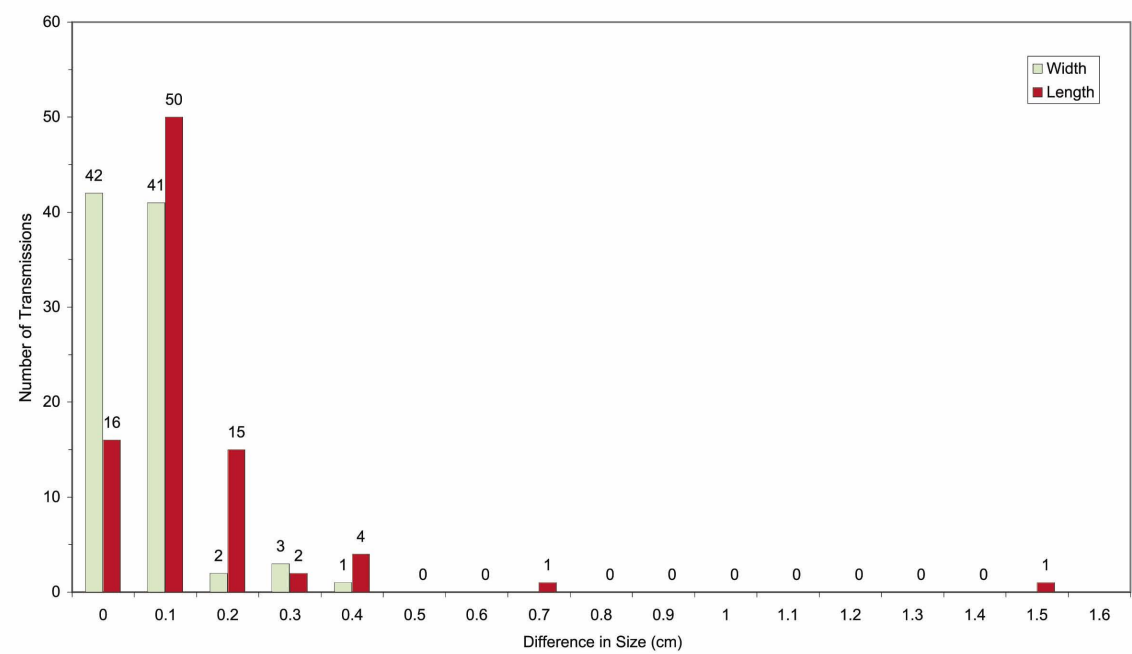


Figure 4. Histogram showing the maximum difference in length and width among the three pages of each transmission.

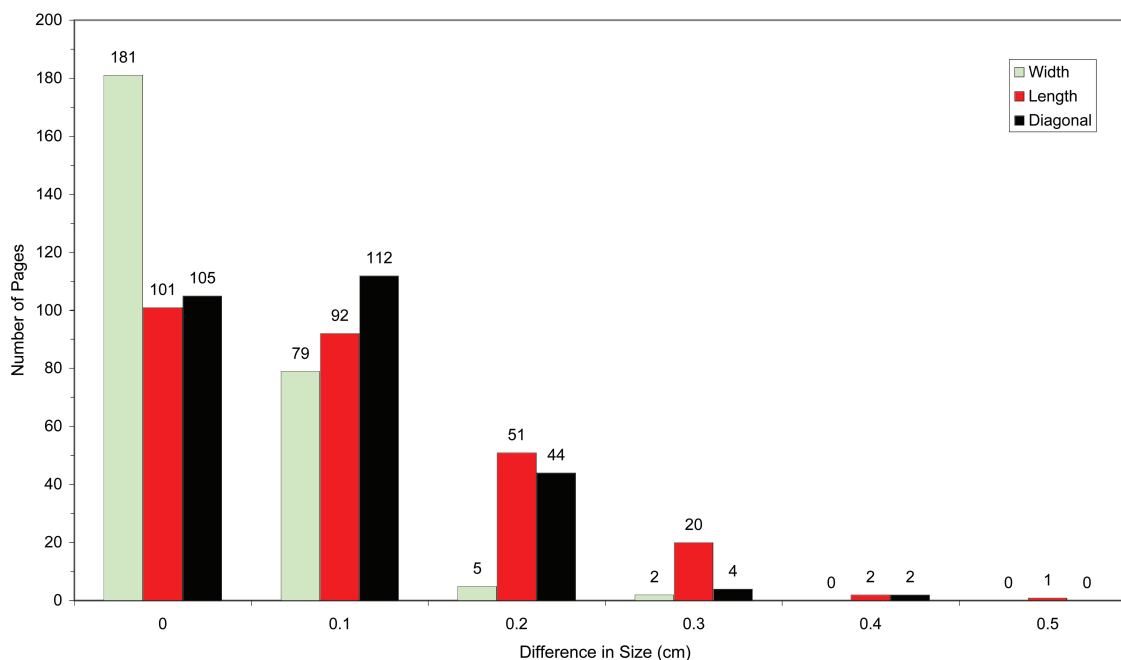


Figure 5. Histogram showing the maximum difference in length, width and diagonal dimensions within each transmitted page.

minimum values for the three widths (W1, W2, W3) and three lengths (L1, L2, L3) varied from 0.0 cm to 0.5 cm. While most values are in the 0.0 cm to 0.1 cm range, the histogram in Figure 5 shows a few dimensional changes of 0.2 and 0.3 cm. Substantial changes of 0.4 and 0.5 cm were rare and pertained only to length and diagonal measurements.

Of greater interest are differences between the two diagonals on the same template page. This indicates that the template's rectangular shape was distorted to an irregular trapezoid or a geometrical figure with, at most, two parallel sides. The diagonals on each of the facsimile transmitted pages were compared and their difference recorded. In 105 cases (39.3%) the two diagonals were equal. In 112 cases (41.9%) the two diagonals differed by 0.1 cm and in forty-four instances (16.5%) they differed by 0.2 cm. On four templates (1.5%) the diagonals differed by 0.3 cm and the greatest difference, 0.4 cm, occurred twice (0.75%).

Discussion

It has been reported that facsimile transmitted documents are usually smaller and occasionally larger than their original counterparts. The

results of this study support this axiom and clearly show that receiving facsimile machines contribute more to output size than transmitting machines. The underlying reasons why this occurs are unclear.

Five of the ten facsimile machines used in this study consistently enlarged or reduced the size of transmitted documents. Tables 2 and 3 show that documents received by facsimile machines 3, 7, 8 and 9 consistently decreased the size of the template along the vertical axis whereas fax machines 3 and 8 reduced dimensions along both axis. Receiving machine 5 was the only machine that produced a consistent enlargement and it did so only along the vertical axis. The maximum degree of enlargement/reduction in the horizontal, vertical and diagonal directions were 11.8%, 12.0% and 12.0% respectively.

Inter-page Variations

Instances where separate pages of a multi-page facsimile transmission have different sizes are often regarded with suspicion by document examiners. This lack of homogeneity raises the possibility that some pages of the transmission may have been faxed or copied more than the rest.

Although faxed documents are typically larger or smaller than their originals, significant changes also occurred within separate pages of a single transmission. These variations tended to be greater along the vertical rather than the horizontal axis. Differences as great as 1.5 cm were found in the vertical direction whereas the maximum recorded horizontal variation was 0.4 cm.

The maximum differences of length/width values between the faxed and original three-page templates are shown in Figure 4. These translate to reductions of 2.3% along the horizontal and 6.2% along the vertical axis. This observation has great importance when examining multi-page faxed documents. Spatial anomalies of this order do not provide conclusive proof that anything is amiss.

Although the source of variations in the size of facsimile transmitted documents has not been determined, it is interesting to note that the length of North American letter-size paper is approximately 1.3 times its width. Notwithstanding the resolution differences along the vertical and horizontal axis, it might be expected that locations along the vertical axis have a greater opportunity to vary than along the horizontal axis. The ability of facsimile machines to produce higher resolution along the horizontal rather than the vertical axis while operating in standard mode might also be a contributing factor. The different technologies (thermal, thermal-transfer, ink/bubble jet and laser) of the fax machines represent yet another factor that might influence the size of facsimile copies. Although the maximum degree of variation along the vertical axis was three times that encountered along the horizontal axis, an explanation for this phenomenon has not been determined.

Intra-page Variations

Intra-page variations represent differences in enlargement or reduction within single transmitted pages. These were found in many faxed templates with the most significant occurring along the vertical axis and on the diagonals. An example of this is shown in Table 5.

As seen in the table, even though page-to-page differences are greater, variations within a single page as great as 0.3 cm in the width of a regular sheet of paper and 0.5 cm along its length were found. This translates into changes of 1.7% and

2.1% along the horizontal and vertical axis respectively.

	Maximum horizontal variation (cm)	Maximum vertical variation (cm)
Page 1	0.1	0.1
Page 2	0.3	0.3
Page 3	0.0	0.5

Table 5. Variations in templates faxed from machine 5 to machine 6.

Suppose a questioned faxed document consists of a typed paragraph and signature separated by a large open area. The results of this study show that differences in the size of objects at the top and bottom of a faxed copy can vary as much as 2% without arousing suspicion. Although such variations in size might indicate a cut-and-paste alteration to a photocopied document, they are also within the range of enlargement/reduction of normal facsimile transmissions. This is perhaps the most significant discovery of this research. Other supporting evidence must be found before declaring a faxed document fraudulent simply because widely spaced objects differ in size by as much as 2%.

Conclusions

The results of this research bring into perspective issues concerning the lack of homogeneity in faxed documents. Although extreme differences in size are not as common as subtle changes, they do nevertheless occur. This should be reason for the document examiner to approach faxed documents with caution. The presence of one anomaly, such as the shortened length of a signature line, does not in itself constitute evidence of spuriousness.

Facsimile machines have a tendency to reduce and occasionally enlarge the size of transmitted documents. It is clear from the results of this study that the degree of enlargement or reduction can vary not only from one transmission to another but also within a single multi-page transmission as well as within individual pages. On some occasions, the enlargement or reduction can be quite broad. The data shows the behavior of a fax machine is, to some degree, unpredictable.

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Endnotes

1. Manufacturer's suggested retail price in USD circa 1998.

2. Variance,
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

3. This research employed a standard *t*-test,

$$t = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}}$$

with a null hypothesis of equal means at a significance value of 95 percent ($\alpha=0.05$). If the observed *t* is found to be above the critical value *t* (found in a *Table of Values of the t Distribution*, for a given α value) the difference between the means is significant at the specified level (α). Additionally, a very low *p* value (less than α) will confirm that the two means are statistically different, and there is evidence to reject the null hypothesis.

4. The primary tool used in ANOVA is the *F* statistic ($F \text{ value} = \text{Variation between means} / \text{Variation within means}$) where a critical value (F_{crit}) is compared against the observed *F* value. If the observed value is greater than the critical value the null hypothesis is rejected, and the differences among the obtained means are statistically significant. The *p* value (the smallest significance level at which the null hypotheses would be rejected) also helps to determine the statistical significance of the *F*-test. A *p* value ≤ 0.05 is considered statistically significant. Microsoft Excel was used to calculate both the *t* and *F* statistic.

5. The theorem that the sum of the squares of the lengths of the sides of a right triangle is equal to the square of the length of the hypotenuse – *Canadian Dictionary* (1997), International Thomson Publishing, Scarborough Ontario.