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# ON THE GLOBAL STABILITY OF FRACTIONAL FEEDBACK NONLINEAR SYSTEMS WITH INTERVAL MATRICES OF POSITIVE LINEAR PARTS AND APPLICATION TO ELECTRICAL CIRCUITS

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The global stability of fractional multi-inputs multi-outputs continuous-time nonlinear feedback systems with interval matrices of positive linear parts and application to electrical circuits is investigated. New sufficient conditions for the global stability of fractional nonlinear systems are given. The new stability conditions are applied to nonlinear electrical circuits and demonstrated on a simple example of a fractional nonlinear feedback system with a positive linear part.

**Keywords:** global stability, fractional order, positive nonlinear system, electrical circuit.

## 1. Introduction

In positive systems inputs, state variables and outputs take only nonnegative values for any nonnegative inputs and nonnegative initial conditions (Berman and Plemmons, 1994; Farina and Rinaldi, 2000; Kaczorek, 2002). Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, and water and atmospheric pollutions models. A variety of models having positive behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc. An overview of state of the art in positive systems theory is given in several monographs and papers (Berman and Plemmons, 1994; Farina and Rinaldi, 2000; Kaczorek, 2002; 2011b; Kaczorek and Rogowski, 2015; Mitkowski, 2008).

Mathematical fundamentals of fractional calculus are presented by Kaczorek (2011b), Kaczorek and Rogowski (2015), Ostalczyk (2016) and Podlubny (1999). Positive fractional linear systems have been investigated by Busłowicz and Kaczorek (2009), Kaczorek (2019a; 2016; 2010; 2011a; 2012; 2011b; 2015b; 2020), Kaczorek and Rogowski (2015), Ruszewski (2019) and Sajewski (2017a; 2017b). Positive linear systems with different fractional orders have been addressed by Kaczorek (2010; 2011a) and Sajewski (2017b). Descriptor positive systems have been analyzed by Borawski (2017a) and Kaczorek (2012). Linear positive electrical circuits with state feedbacks have been addressed by Borawski (2017a) as well as Kaczorek and Rogowski (2015) and the stability of nonlinear systems by Kaczorek (2020) and Borawski (2017b). The global asymptotic stability of fractional differential equations has been investigated by Sene (2020) while the generalized Mittag-Leffler input stability has been discussed by Sene and Srivastava (2019). The global stability of nonlinear systems with negative feedbacks and positive not necessary asymptotically stable linear parts has been investigated by Kaczorek (2015a; 2019b). Application of modern big capacitors in nonlinear electrical circuits has caused a need for analysis of fractional order nonlinear electrical circuits.

In this paper, the global stability of fractional nonlinear multi-input multi-outputs systems with positive linear parts with interval matrices and application to nonlinear electrical circuits will be addressed.

The paper is organized as follows. In Section 2, the basic definitions and theorems concerning positive fractional linear systems are presented. The stability of interval positive fractional linear systems is discussed in Section 3. New sufficient conditions for the global of fractional multi-input multi-outputs nonlinear feedback systems with positive linear parts and interval matrices are established in Section 4. A procedure for computation of the matrix determining the class of nonlinear elements of the system, along with an example of nonlinear feedback systems with positive linear parts and interval matrices, is presented in Section 5. In Section 6, positive fractional nonlinear electrical circuits are discussed as an example of application of Theorem 6. Concluding remarks are given in Section 7.

The following notation will be used: $\mathbb{R}$, the set of real numbers; $\mathbb{R}^{n\times m}$, the set of $n \times m$ real matrices; $\mathbb{R}_+^{n\times m}$, the set of $n \times m$ real matrices with nonnegative entries and $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$; $M_n$, the set of $n \times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries); $I_n$, the $n \times n$ identity matrix.

## 2. Preliminaries

Consider the fractional continuous-time linear system

$$\frac{d^\alpha x}{dt^\alpha} = Ax + Bu, 0 < \alpha < 1, \tag{1a}$$

$$y = Cx, \tag{1b}$$

where $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}^m$, $y = y(t) \in \mathbb{R}^p$ are the state, input and output vectors, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and

$$\begin{aligned} \frac{d^\alpha x(t)}{dt^\alpha} &= \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot{x}(\tau)}{(t-\tau)^\alpha} d\tau, \\ \dot{x}(\tau) &= \frac{dx(\tau)}{d\tau}, \end{aligned} \tag{1c}$$

is the Caputo fractional derivative while

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt, \quad Re(z) > 0, \tag{1d}$$

is the gamma function (Kaczorek, 2011b; Kaczorek and Rogowski, 2015; Podlubny, 1999).

**Definition 1.** (*Kaczorek 2011b; Kaczorek and Rogowski, 2015*) The fractional continuous-time linear system (1) is called (internally) positive if $x(t) \in \mathbb{R}_+^n$, $y(t) \in \mathbb{R}_+^p$, $t \geq 0$ for any initial conditions $x(0) \in \mathbb{R}_+^n$ and all inputs $u(t) \in \mathbb{R}_+^m$, $t \geq 0$.

**Theorem 1.** (Kaczorek 2011b; Kaczorek and Rogowski, 2015) *The fractional continuous-time linear system (1) is positive if and only if*

$$A \in M_n, \quad B \in \mathbb{R}_+^{n\times m}, \quad C \in \mathbb{R}_+^{p\times n}. \tag{2}$$

**Definition 2.** (*Kaczorek 2011b; Kaczorek and Rogowski, 2015*) The positive fractional continuous-time system (1) for $u(t) = 0$ is called asymptotically stable if

$$\lim_{t\to\infty} x(t) = 0 \quad \text{for any} \quad x(0) \in \mathbb{R}_+^n. \tag{3}$$

**Theorem 2.** (Kaczorek 2011b; Kaczorek and Rogowski, 2015) *The fractional positive continuous-time linear system (1) for $u(t) = 0$ is asymptotically stable (the matrix is Hurwitz) if and only if one of the following equivalent conditions is satisfied:*

1. *All coefficient of the characteristic polynomial*

$$\begin{aligned} p_n(s) &= \det[I_n s - A] \\ &= s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \end{aligned} \tag{4}$$

*are positive, i.e., $a_i > 0$ for $i = 0, 1, \ldots, n-1$.*

2. *There exists a strictly positive vector $\lambda^T = [\lambda_1 \quad \cdots \quad \lambda_n]^T, \lambda_k > 0, k = 1, \ldots, n$, such that*

$$A\lambda < 0 \quad or \quad \lambda^T A < 0. \tag{5}$$

**Theorem 3.** *The fractional positive system (1) is asymptotically stable if the sum of entries of each column (row) of the matrix A is negative.*

*Proof.* Using (5), we obtain

$$\begin{aligned} A\lambda &= \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ldots & \vdots \\ a_{n1} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} \\ &= \begin{bmatrix} a_{11}\lambda_1 + \cdots + a_{1n}\lambda_n \\ \vdots \\ a_{n1}\lambda_1 + \cdots + a_{nn}\lambda_n \end{bmatrix} \\ &= \begin{bmatrix} a_{11} \\ \vdots \\ a_{n1} \end{bmatrix} \lambda_1 + \cdots + \begin{bmatrix} a_{1n} \\ \vdots \\ a_{nn} \end{bmatrix} \lambda_n < \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \end{aligned} \tag{6}$$

and the sum of entries of each column of the matrix $A$ is negative since $\lambda_k > 0$, $k = 1, \ldots, n$. The proof for rows is similar. ∎

## 3. Stability of fractional interval positive linear systems

Consider the interval fractional positive linear continuous-time system

$$\frac{d^\alpha x}{dt^\alpha} = Ax, \quad 0 < \alpha < 1, \tag{7}$$

where $x = x(t) \in \mathbb{R}^n$ is the pseudo-state vector and the matrix $A \in M_n$ is defined by

$$\widehat{A} \leq A \leq \bar{A} \quad \text{or, equivalently,} \quad A \in [\widehat{A}, \bar{A}]. \tag{8}$$

The interval asymptotic stability of linear systems has been investigated by Kharitonov (1978).

**Definition 3.** The interval fractional positive system (7) is called asymptotically stable if the system is asymptotically stable for all matrices $A \in M_n$ satisfying the condition (8).

By the condition (5) of Theorem 2, the fractional positive system (7) is asymptotically stable if there exists a strictly positive vector $\lambda > 0$ such that the condition (5) is satisfied.

For two fractional positive linear systems

$$\frac{d^\alpha x_1}{dt^\alpha} = \widehat{A}x_1, \widehat{A} \in M_n, \quad 0 < \alpha < 1, \tag{9a}$$

and

$$\frac{d^\alpha x_2}{dt^\alpha} = \bar{A}x_2, \bar{A} \in M_n, \quad 0 < \alpha < 1, \tag{9b}$$

there exists a strictly positive vector

$$\lambda \in \mathbb{R}_+^n \quad \text{such that} \quad \widehat{A}\lambda < 0 \quad \text{and} \quad \bar{A}\lambda < 0 \tag{10}$$

if and only if the systems (9) are asymptotically stable.

**Remark 1.** According to Theorem 2, in a general case, it is not necessary to find the same vector $\lambda \in \mathbb{R}_+^n$ for two different systems. However, for interval systems, it is reasonable for choose one vector $\lambda \in \mathbb{R}_+^n$ for (9a) and (9b).

**Theorem 4.** *If the matrices $\widehat{A}$ and $\bar{A}$ of the fractional positive systems (9) are asymptotically stable, then their convex linear combination*

$$A = (1-q)\widehat{A} + q\bar{A} \quad \textit{for} \quad 0 \le q \le 1 \tag{11}$$

*is also asymptotically stable.*

*Proof.* By the condition (5) of Theorem 2, if the fractional positive linear systems (9) are asymptotically stable, then there exists a strictly positive vector $\lambda \in \mathbb{R}_+^n$ such that

$$\widehat{A}\lambda < 0 \quad \text{and} \quad \bar{A}\lambda < 0. \tag{12}$$

Using (5) and (12), we obtain

$$A\lambda = [(1-q)\widehat{A} + q\bar{A}]\lambda = (1-q)\widehat{A}\lambda + q\bar{A}\lambda < 0$$

for $0 \le q \le 1$. Therefore, if the positive linear systems (9) are asymptotically stable, then their convex linear combination (11) is also asymptotically stable. ∎

**Theorem 5.** *The interval positive system (7) is asymptotically stable if and only if the positive linear systems (9) are asymptotically stable.*

*Proof.* By the condition (5) of Theorem 2, if the matrices $\widehat{A} \in M_n$, $\bar{A} \in M_n$ are asymptotically stable, then there exists a strictly positive vector $\lambda \in \mathbb{R}_+^n$ such that (5) holds. The convex linear combination (11) satisfies the condition $A\lambda < 0$ if and only if (12) holds. Therefore, the interval system (7) is asymptotically stable if and only if the positive linear systems (9) are asymptotically stable. ∎

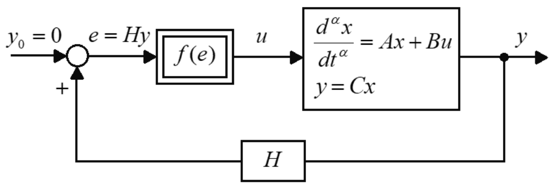


Fig. 1. Fractional nonlinear feedback system.

**Example 1.** Consider the fractional interval positive linear continuous-time system (7) with the matrices

$$\widehat{A} = \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} -4 & 3 \\ 2 & -5 \end{bmatrix}. \tag{13}$$

Using the condition (5) of Theorem 2, we choose $\lambda = [1\ 1]^T$ and obtain

$$\widehat{A}\lambda = \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix} < 0 \tag{14a}$$

and

$$\bar{A}\lambda = \begin{bmatrix} -4 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix} < 0. \tag{14b}$$

♦

Therefore, the matrices (13) are Hurwitz.

## 4. Global stability of fractional nonlinear feedback systems

Consider the fractional nonlinear feedback system shown in Fig. 1, which consists of a fractional positive linear part, a nonlinear element with the matrix characteristic $u = f(e)$ and a feedback with the positive gain matrix $H$. The fractional linear part is described by the equations

$$\frac{d^\alpha x}{dt^\alpha} = Ax + Bu, \quad 0 < \alpha < 1, \quad y = Cx, \tag{15a}$$

where $x = x(t) \in \mathbb{R}_+^n, u = u(t) \in \mathbb{R}_+^m$, $y = y(t) \in \mathbb{R}_+^p$ are the state, input and output vectors, and

$$\begin{aligned} &A \in [\widehat{A}, \bar{A}] \in M_n, \\ &B \in [\widehat{B}, \bar{B}] \in \mathbb{R}_+^{n\times m}, \\ &C \in [\widehat{C}, \bar{C}] \in \mathbb{R}_+^{p\times n}. \end{aligned} \tag{15b}$$

The matrix characteristic of the nonlinear element satisfies the condition

$$u_i = f(e_i) \le k_{i1}e_1 + \ldots + k_{ip}e_p, \quad i = 1, \ldots, m \quad \text{or} \quad u \le Ke, \tag{16a}$$

where

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}, \quad K = \begin{bmatrix} k_{11} & \cdots & k_{1p} \\ \vdots & \ddots & \vdots \\ k_{m1} & \cdots & k_{mp} \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ \vdots \\ e_p \end{bmatrix}. \tag{16b}$$

**Definition 4.** The fractional nonlinear positive system is called globally stable if it is asymptotically stable for all nonnegative initial conditions $x(0) \in \mathbb{R}_+^n$.

The following theorem gives sufficient conditions for the global stability of the fractional positive nonlinear system.

**Theorem 6.** *The fractional nonlinear system consisting of a positive asymptotically stable linear part described by (15a) with the interval matrices (15b), a nonlinear element satisfying the condition (16), and a feedback with the positive gain matrix $H \in \mathbb{R}_+^{m\times p}$ is globally stable if there exists a matrix $K$ with positive entries such that the sum of entries of each column (row) of the matrix*

$$\begin{aligned} &(1-q)\widehat{A} + q\bar{A} + BKHC \\ &\quad = \begin{cases} \widehat{A} + \widehat{B}K_1H\widehat{C} \in M_n & \text{for} \quad q = 0, \\ \bar{A} + \bar{B}K_2H\bar{C} \in M_n & \text{for} \quad q = 1, \end{cases} \end{aligned} \tag{17}$$

*is negative.*

*Proof.* The proof will be derived with the use of the Lyapunov method (Lyapunov, 1963; Leipholz, 1970). As the Lyapunov function $V(x)$ we choose

$$V(x) = \lambda^T x \geq 0 \quad \text{for} \quad x \in \mathbb{R}_+^n, \tag{18}$$

where $\lambda$ is a strictly positive vector, i.e., $\lambda_k > 0$, $k = 1, \dots, n$. Using (18) and (15), we obtain

$$\begin{aligned} \frac{d^\alpha V(x)}{dt^\alpha} &= \lambda^T \frac{d^\alpha x}{dt^\alpha} = \lambda^T(Ax + Bu) \\ &= \lambda^T(Ax + Bf(e)) \leq \lambda^T(A + BKHC)x \end{aligned} \tag{19}$$

since $u = f(e) \leq Ke = KHCx$.

From (19) it follows that $(d^\alpha V(x))/dt^\alpha < 0$ if the sum of entries of each column (row) of the matrix (17) is negative (Theorem 3) and the fractional nonlinear positive system is globally stable. ∎

## 5. Procedure and an example

To find the maximal $K$ satisfying the condition (17) for the fractional nonlinear positive system, the following procedure can be used.

**Procedure 1.**

*Step 1.* Using the matrices $\widehat{A}$, $\widehat{B}$, $\widehat{C}$ of the positive linear system and the matrix $H$, compute the matrix $K_1$ such that the sum of all entries of each column (row) of the matrix

$$\widehat{A} + \widehat{B}K_1H\widehat{C} \tag{20}$$

is negative.

If $mp > n$, then we choose $mp - n$ nonnegative entries of the matrix $K$ and the remaining entries (components of vector $k$) are computed as the solution of the linear matrix equation

$$Gk = h, \tag{21}$$

where the matrix $G$ and the column vector $h$ are defined by the sum of entries of each column (row) of the matrix (20).

*Step 2.* Using the matrices $\bar{A}$, $\bar{B}$, $\bar{C}$ of the positive linear system and the matrix $H$, compute the matrix $K_2$ such that the sum of all entries of each column (row) of the matrix

$$\bar{A} + \bar{B}K_2H\bar{C} \tag{22}$$

is negative.

*Step 3.* Find the desired $K$ such that the matrices (20) and (21) are Hurwitz.

**Remark 2.** The necessary and sufficient conditions for Theorem 2 can be also used to compute the entries of the matrix $K$ (Kaczorek, 2020). Usually, in this case the computations are more complicated and the procedure has the following form.

**Procedure 2.**

*Step 1.* Using the condition 1 of Theorem 2, find the value of $k_1$ for which the matrix

$$\widehat{A} + k_1BC \in M_n \tag{23}$$

is asymptotically stable.

*Step 2.* In a similar way, find the value of $k_2$ for which the matrix

$$\bar{A} + k_2BC \in M_n \tag{24}$$

is asymptotically stable.

*Step 3.* Find the desired value of $k$ as

$$k = \min(k_1, k_2). \tag{25}$$

**Example 2.** Consider the fractional nonlinear feedback system shown in Fig. 1 with the interval matrices of the positive linear part,

$$\begin{aligned} &\widehat{A} = \begin{bmatrix} -3 & 1 \\ 2 & -4 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} -4 & 2 \\ 3 & -5 \end{bmatrix}, \\ &\widehat{B} = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.4 \end{bmatrix}, \\ &\widehat{C} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \end{aligned} \tag{26}$$

and the gain matrix

$$H = \begin{bmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix}. \tag{27}$$

♦

Using Procedure 1, we derive what follows.

*Step 1*. Using (20) and (26), we obtain

$$\widehat{A} + \widehat{B}K_1 H\widehat{C} = \begin{bmatrix} f_1 & f_2 \\ f_3 & f_4 \end{bmatrix},$$

$$\begin{aligned} f_1 &= 0.128k_{11} + 0.064k_{12} + 0.064k_{21} + 0.032k_{22} - 3, \\ f_2 &= 0.192k_{11} + 0.256k_{12} + 0.096k_{21} + 0.032k_{22} + 1, \\ f_3 &= 0.032k_{11} + 0.016k_{12} + 0.096k_{21} + 0.048k_{22} + 2, \\ f_4 &= 0.048k_{11} + 0.064k_{12} + 0.144k_{21} + 0.192k_{22} - 4. \end{aligned} \tag{28}$$

The sum of entries of the first column of the matrix (28) is $0.16k_{11} + 0.08k_{12} + 0.16k_{21} + 0.08k_{11} - 1$, and the sum of entries of the second column of the matrix (28) is $0.24k_{11} + 0.32k_{12} + 0.24k_{21} + 0.32k_{11} - 3$.

Assuming $k_{11} = k_{12} = 1$ and solving the system of linear inequalities

$$\begin{cases} 0.16k_{11} + 0.08k_{12} + 0.16k_{21} + 0.08k_{22} - 1 < 0, \\ 0.24k_{11} + 0.32k_{12} + 0.24k_{21} + 0.32k_{22} - 3 < 0, \end{cases}$$

we obtain $k_{21} < 1.5$ and $k_{22} < 6.5$.

Therefore, the maximal $K_1$ for which the matrix (20) is Hurwitz is $K_1 = \begin{bmatrix} 1 & 1 \\ 1.49 & 6.49 \end{bmatrix}$.

*Step 2*. Using (24) and (26), we obtain

$$\bar{A} + \bar{B}K_2 H\bar{C} = \begin{bmatrix} f_1 & f_2 \\ f_3 & f_4 \end{bmatrix},$$

$$\begin{aligned} f_1 &= 0.2k_{11} + 0.1k_{12} + 0.12k_{21} + 0.06k_{22} - 4, \\ f_2 &= 0.3k_{11} + 0.4k_{12} + 0.18k_{21} + 0.24k_{22} + 2, \\ f_3 &= 0.08k_{11} + 0.04k_{12} + 0.16k_{21} + 0.08k_{22} + 3, \\ f_4 &= 0.12k_{11} + 0.16k_{12} + 0.24k_{21} + 0.32k_{22} - 5. \end{aligned} \tag{29}$$

The sum of entries of the first column of the matrix (29) is $0.28k_{11} + 0.14k_{12} + 0.28k_{21} + 0.14k_{22} - 1$, and the sum of entries of the second column of the matrix (29) is $0.42k_{11} + 0.56k_{12} + 0.42k_{21} + 0.56k_{22} - 3$.

Assuming $k_{11} = k_{12} = 1$ and solving the system of linear inequalities

$$\begin{cases} 0.28k_{11} + 0.14k_{12} + 0.28k_{21} + 0.14k_{22} - 1 < 0, \\ 0.42k_{11} + 0.56k_{12} + 0.42k_{21} + 0.56k_{22} - 3 < 0, \end{cases}$$

we obtain $k_{21} < 0.428$ and $k_{22} < 3.285$.

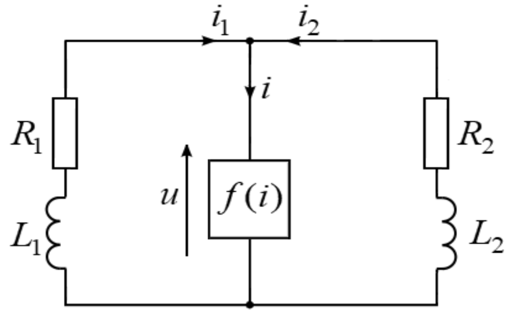


Fig. 2. Electrical circuit.

Therefore, the maximal $K_2$ for which the matrix (25) is Hurwitz is

$$K_2 = \begin{bmatrix} 1 & 1 \\ 0.427 & 3.284 \end{bmatrix}.$$

*Step 3.* Thus, the maximal $K$ for which the matrices (28) and (29) are Hurwitz is

$$K = \begin{bmatrix} 1 & 1 \\ 0.427 & 3.284 \end{bmatrix}.$$

## 6. Positive fractional nonlinear electrical circuits

In this section, the new sufficient stability conditions will be applied to positive fractional nonlinear interval electrical circuits.

**Example 3.** Consider the nonlinear fractional electrical circuit shown in Fig. 2 with the given interval resistances $\widehat{R}_1 = 3$, $\bar{R}_1 = 3.2$, $\widehat{R}_2 = 2$, $\bar{R}_2 = 2.1$, inductances $\widehat{L}_1 = \bar{L}_1 = \widehat{L}_2 = \bar{L}_2 = 1$, the characteristic $u = f(i)$ (Fig. 3) satisfying the condition

$$0 \le \frac{f(i)}{i} \le k, \tag{30}$$

and with the feedback gain $h = 0.5$. Find the value of $k$ for which the nonlinear fractional electrical circuit is globally stable.

Using Kirchhoff's laws, we may write, for the fractional electrical circuit, the equations

$$R_1 i_1 + L_1 \frac{d^\alpha i_1}{dt^\alpha} = u, R_2 i_2 + L_2 \frac{d^\alpha i_2}{dt^\alpha} = u, \tag{31a}$$

which can have the form

$$\frac{d^\alpha}{dt^\alpha}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + Bu, \tag{31b}$$

where

$$A = \begin{bmatrix} -\frac{R_1}{L_1} & 0 \\ 0 & -\frac{R_2}{L_2} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix}. \tag{31c}$$

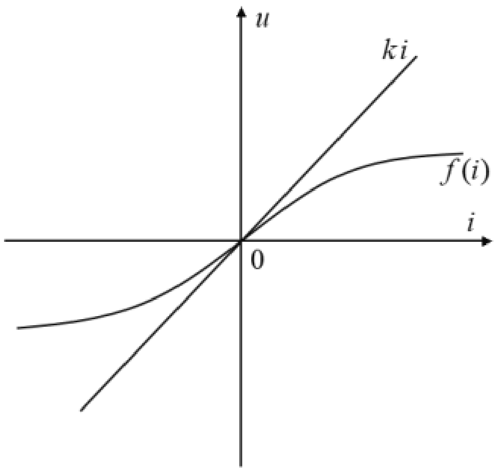


Fig. 3. Characteristic of a nonlinear element.

As the output $y$ of the electrical circuit we choose

$$y = i_1 + i_2 = C \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}. \tag{31d}$$

♦

Applying Procedure 1, we obtain the following.

*Step 1*. Using $\widehat{A}$, $B$, $C$ and $h = 0.5$, we compute $k_1$ such that the sum of all entries of each column of the matrix

$$\begin{aligned} \widehat{A} + k_1 hBC &= \begin{bmatrix} -\frac{\widehat{R}_1}{\widehat{L}_1} & 0 \\ 0 & -\frac{\widehat{R}_2}{\widehat{L}_2} \end{bmatrix} + k_1 h \begin{bmatrix} \frac{1}{\widehat{L}_1} \\ \frac{1}{\widehat{L}_2} \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -3 + 0.5k_1 & 0.5k_1 \\ 0.5k_1 & -2 + 0.5k_1 \end{bmatrix} \end{aligned} \tag{32}$$

is negative, and we obtain $k_1 < 2$.

*Step 2*. Similarly, using $\bar{A}$, $B$, $C$ and $h = 0.5$, we compute $k_2$ such that the sum of all entries of each column of the matrix

$$\begin{aligned} \bar{A} + k_2 hBC &= \begin{bmatrix} -\frac{\bar{R}_1}{\bar{L}_1} & 0 \\ 0 & -\frac{\bar{R}_2}{\bar{L}_2} \end{bmatrix} + k_2 h \begin{bmatrix} \frac{1}{\bar{L}_1} \\ \frac{1}{\bar{L}_2} \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -3.2 + 0.5k_2 & 0.5k_2 \\ 0.5k_2 & -2.1 + 0.5k_2 \end{bmatrix} \end{aligned} \tag{33}$$

is negative, and we obtain $k_2 < 2.1$.

*Step 3*. Therefore, the maximal $k$ for which the matrices (32) and (33) are Hurwitz is $k = k_1 = 2$.

If we apply Procedure 2, we obtain the following.

*Step 1.* Using the condition 1 of Theorem 2 and $\widehat{A}$, we have

$$\begin{aligned} &\det[I_2 s - (\widehat{A} + k_1 hBC)] \\ &\quad = \begin{vmatrix} s + 3 - 0.5k_1 & -0.5k_1 \\ -0.5k_1 & s + 2 - 0.5k_1 \end{vmatrix} \\ &\quad = s^2 + (5 - k_1)s + 6 - 2.5k_1. \end{aligned} \tag{34}$$

The fractional nonlinear electrical circuit is globally stable for $k_1 < 6/2.5 = 2.4$.

*Step 2.* In a similar way, using $\bar{A}$, we obtain

$$\begin{aligned} &\det[I_2 s - (\bar{A} + k_2 hBC)] \\ &\quad = \begin{vmatrix} s + 3.2 - 0.5k_2 & -0.5k_2 \\ -0.5k_2 & s + 2.1 - 0.5k_2 \end{vmatrix} \\ &\quad = s^2 + (5.3 - k_2)s + 6.72 - 2.65k_2, \end{aligned} \tag{35}$$

and the fractional nonlinear electrical circuit is globally stable for $k_2 < 6.72/2.65 \approx 2.536$.

*Step 3.* Using (35), we obtain

$$k = \min(k_1, k_2) = k_1 = 2.4, \tag{36}$$

and the fractional nonlinear electrical circuit is globally stable for $k_1 < 2.4$.

From a comparison of the results of Procedures 1 and 2 it follows that the second approach gives a less restrictive condition for the global stability of the fractional nonlinear electrical circuit.

The above deliberations can be easily extended to fractional electrical circuits with all interval parameters.

## 7. Concluding remarks

The global stability of multi-input multi-output continuous-time fractional nonlinear feedback systems with interval matrices of positive linear parts was investigated. Using the Lyapunov method for fractional positive systems, new sufficient conditions for the global stability of these systems were established (Theorem 6). Procedures for computation of the matrix $K$ defining the new stability conditions were proposed (Procedures 1 and 2) and demonstrated on a simple example of a nonlinear feedback electrical circuit with a positive interval linear part. The new stability conditions were applied to fractional positive nonlinear electrical circuits. The discussion can be extended to nonlinear positive discrete-time systems and to fractional nonlinear discrete-time ones. An open problem is extension to different orders positive fractional nonlinear systems.

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