

INVITED ADDRESS

STRATEGIES FOR DEALING WITH COMPLEX DEDUCTIVE PROBLEMS: COMBINING AND DIVIDING

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The present paper addresses the basis for suppression effects observed in conditional inference. Previous research has shown that when an additional premise is included, the invalid inferences "denial of the antecedent" and "affirmation of the consequent" are suppressed (Rumain, Connell, & Braine, 1983). Byrne (1989) has shown that a similar suppression effect can occur in valid inferences (modus Ponens and modus Tollens). We briefly present the results of a series of experiments that show that strategic factors related to task and presentation format may affect the presence of suppression effects. We also present an introspective experiment (thinking aloud procedure) that clarifies the usage of two different strategies, the integration and the amendment strategy. The results are discussed with respect to the mental logic and mental model theories of reasoning.

Introduction

People constantly make inferences in daily life: when they are reading books, when they are listening or speaking to someone, when they are thinking about their environment, and so on... It is this ability of making inferences that makes us understand the world around us. Although in conversation, for example, many things are left unsaid, people are able to make sense of it, or discover contradictions in the conversation.

Some of these inferences are very easy to make for almost everybody. The

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following inference known as *Modus Ponens* is an example of such an easy inference:

If she has an essay to write, then she will study late in the library.

She has an essay to write.

Therefore, she will study late in the library.

Many psychological theories of reasoning postulate that people are equipped with formal rules of inference akin to those of a logical calculus (see e.g., Braine, 1978; Braine, Reiser, & Rumin, 1984; Johnson-Laird, 1975; Osherson, 1975; Rips, 1983, 1994). These theories are syntactic theories: They claim that deductive reasoning consists of the application of inference rules to the form of the premises and conclusion of an argument. We will mention them in the following as Mental Logic theories.

Consider again the problem above. According to Mental Logic theories, our mind contains a rule for *Modus Ponens*:

If p then q .

p

q

This rule or reasoning schema matches the form of the problem above. Therefore, the inference can be made promptly:

She will study late in the library.

Consider, however, the following problem:

If she has an essay to write, then she will study late in the library.

She will not study late in the library.

The correct solution to this *Modus Tollens* problem is:

She has not an essay to write.

According to Mental Logic theories, there is no rule in our mind corresponding to this kind of problem. Therefore, it is only indirectly that we can come up with the correct solution:

1. If p then q [logical form of premise 1]
2. Not- q [logical form of premise 2]
3. p [by hypothesis]
4. q [by *Modus Ponens* from 1 and 3]
5. q and not- q [by conjunction from 4 and 2]
6. Not- p [by *reductio ad absurdum* from 4 and 7]

The latter problem requires more reasoning steps and is used by the Mental Logic theories to explain the data of many studies that show that *Modus Ponens* inferences are easier to solve than *Modus Tollens* inferences. Consider, however, the following problem:

If she has an essay to write, then she will study late in the library.

She does not have an essay to write.

Many participants conclude based on these premises:

She will not study late in the library.

This conclusion, which is called the fallacy of *denying the antecedent*, is incorrect: If the conditional is interpreted as a true conditional, then it is possible that there are alternative conditions from which the consequent "she will study late in the library" could be implied. Of course, if the conditional is interpreted as a biconditional (e.g., If and only if she has an essay to make, then she will study late in the library), then the conclusion "she will not study late in the library" is correct.

A Mental Logic theorist could propose that reasoners are also equipped with invalid inference rules (see e.g., Von Domarus, 1944). However, most of them explain reasoning errors as comprehension errors (see e.g., Braine & Rumin, 1983; Marcus & Rips, 1979; Markovits, 1985; Rumin, Connell, & Braine, 1983). The major premise of the denial of the antecedent problem above would invite its obverse (see Rips, 1994, for an alternative invited inference):

If she does not have an essay to write,
then she will not study late in the library.

Reasoners can come up with the conclusion "she will not study late in the library" if they apply the Modus Ponens rule to the obverse of the conditional and the categorical premise. Rumin et al. (1983) tested this hypothesis by giving participants a possible alternative conditional. Consider again the problem above, but now accompanied with a *so-called alternative conditional*:

If she has an essay to write, then she will study late in the library.

If she has some textbooks to read,
then she will study late in the library.

She does not have an essay to write.

If reasoners are given such problems, they will not conclude:

She will not study late in the library.

The fact that the fallacy of denying the antecedent can be suppressed is taken as evidence that we do not possess invalid inference rules, but that reasoners make denial of the antecedent conclusions because they translate the original premises.

A similar way of thinking is used to explain the making and suppression of another fallacy: the fallacy of *affirming the consequent*. Consider the following problem:

If she has an essay to write, then she will study late in the library.

She will study late in the library.

Many participants conclude based on these premises:

She has an essay to write.

Again, this fallacy can be suppressed by adding an alternative antecedent, which is taken as evidence that we do not possess invalid inference rules, but that reasoners make affirmation of the consequent conclusions because they

translate the original premises. According to Byrne (1989), the above argument has an interesting consequence: If valid inferences could be suppressed, then neither would reasoners possess valid inference rules. In a first experiment, she presented some participants the major premise, accompanied with a so-called *additional conditional*, for example:

If she has an essay to write, then she will study late in the library.

If the library remains open, then she will study late in the library.

She has an essay to write.

For such a problem, reasoners do not come up with the logically correct Modus Ponens conclusion:

She will study late in the library.

In other words, Byrne showed that Modus Ponens can be suppressed, and in a similar way, Modus Tollens can be suppressed. She concluded that this suppression effect challenges the assumption of Mental Logic theories that formal rules of inference such as Modus Ponens are part of our mental logic.

Byrne (1991) argues that an alternative theory, namely, the Mental Model theory (e.g., Johnson-Laird, 1983) can explain the suppression of the valid inferences. According to the Mental Model theory, reasoning consists of three main stages. First, the premises are understood: A mental model of the situation they describe is constructed on the basis of their meaning and of any relevant general knowledge triggered during the process of interpretation. Second, reasoners formulate a conclusion based on the model. People will only draw conclusions that convey some information that was not explicitly asserted by the premises. Third, a search is made for alternative models of the premises in which the putative conclusion is false. If there is no such model, then the conclusion is valid; that is, it must be true given that the premises are true. If there is such a model, then it is necessary to return to the second stage to determine whether there is any conclusion that holds over all the models constructed so far. The theory's essential processing assumption is that the more models that have to be constructed the harder the inferential task will be. This principle has nowadays been supported by studies of syllogistic reasoning (Johnson-Laird & Bara, 1984), spatial and temporal reasoning (Byrne & Johnson-Laird, 1989; Schaeken, Johnson-Laird, & d'Ydewalle, 1996a, 1996b; Vandierendonck & De Vooght, 1996), propositional reasoning (Johnson-Laird, Byrne, & Schaeken, 1992, 1994), and reasoning with multiple quantifiers (Johnson-Laird, Byrne, & Tabossi, 1991).

We will illustrate the model theory by considering the representation of a conditional such as:

If there is a circle on the card, then there is a square on the card.

The initial models of this assertion are as follows:

[circle] square

...

Reasoners realize that both shapes may be present (first model). They also realize that the assertion is consistent with other possibilities (second model). The three dots accordingly signify a wholly *implicit* model, that is, one that has no explicit content. Reasoners need to note that the implicit model represents cases where the antecedent does not hold: The square brackets represent that the antecedent is exhaustively represented in the explicit model, and so it cannot occur in the implicit model (see Johnson-Laird & Byrne, 1991).

Given the initial models of the conditional, consider the following categorical premise:

There is a circle.

It picks out the situation represented in the first model and eliminates the second model. The remaining model:

[circle] square

yields the Modus Ponens conclusion:

There is a square.

The valid deduction Modus Tollens has the following sort of premises:

If there is a circle, then there is a square.

There is not a square.

The categorical premise eliminates the first model. Therefore, no obvious conclusion can be drawn, since the second model is implicit at first. Accordingly, "nothing follows" is the most frequent error that reasoners make given these premises. However, a valid deduction can be made. In order to do this, reasoners have to flesh out the implicit model. In the case of a conditional interpretation, the models are:

¬circle square

¬circle ¬square

The categorical premise calls for the elimination of the models containing the squares. Therefore, only one model is consistent with all the information thus far:

¬circle ¬square

That model yields the Modus Tollens conclusion:

There is not a circle.

Byrne (1991) argues that the Mental Model theory can explain the suppression effect. According to her, reasoners will integrate the two premises, dependent on the meaning of that premise and the general knowledge. If the antecedent of the second premise is understood as an additional requirement, then it will be conjoined with the antecedent of the first premise (Byrne, 1991):

p q r

...

Consequently, reasoners will suppress Modus Ponens conclusions: Affirmation of the first antecedent by itself is not sufficient for drawing the Modus Ponens conclusion. If the antecedent of the second premise is understood as an alternative requirement, then the premises will be interpreted such that they are individually sufficient:

$$\begin{array}{cc} p & r \\ q & r \end{array}$$

...

As a result, reasoners will suppress making denial of the antecedent argument in problems with an alternative antecedent: Denial of the first antecedent alone is not sufficient for deciding that "not-r" is the case, because the model indicates that "r" can still hold in the model with "q". The same line of reasoning explains the suppression of the valid Modus Tollens conclusion in case of an extra additional antecedent, and the suppression of the invalid affirmation of the consequent conclusion in case of an extra alternative antecedent.

In this paper, we will focus on this explanation of Byrne (1991). In the first part of this article, we report an overview of previous research on this topic, which forced us to change our view on the nature of the suppression and also on the nature of the underlying reasoning processes (Dieussaert et al., submitted). In the second part, we introduce a new study that gives us a more detailed insight about the processes going on.

Part 1: An Overview of Former Studies

In this part, we give a compact overview of previous research (Dieussaert et al., submitted). It seems necessary to us to describe these studies in order to appreciate the qualities of the 'think aloud' experiment as a way to reveal the existence of strategies.

In a first experiment, we used exactly the same design as Byrne (1989) did in her first experiment, but translated the data in Dutch. In this experiment the findings of Byrne (1989) were replicated, that is, the suppression of conditional inferences was reconfirmed. Indeed, reasoners made less denial of the antecedent and less affirmation of the consequent inferences (which are invalid inferences) when the major premise was accompanied by an alternative antecedent. Reasoners made also less Modus Ponens and Modus Tollens inferences (which are valid inferences) when an additional antecedent accompanied the major premise. However, there is a serious problem with both our Experiment 1 and the first experiment of Byrne (1989). For example, given the following Modus Ponens problem:

If she has an essay to write, then she will study late in the library.

If the library stays open late, then she will study late in the library.

She will not study late in the library.

O She has an essay to write.

O She has not an essay to write.

O You don't know whether she has an essay to write or not.

The answer alternatives deal with the inferences that can be made with respect to the first conditional only. We tried to solve these problems in a subsequent experiment, in which reasoners could choose their answers among many answer alternatives based on both premises. They were combined in several ways. Because we enlarged the palette of answer alternatives, many reasoners gave a complex answer. Indeed, they did not only give information about the first conditional, but they tried to give information about both the conditional premises. It even seems to be the case that they tried to reason conditionally. Indeed, in a somewhat broader definition, we can describe a "conditional" answer as one in which at least a part of the answer is in accordance with the normal conditional pattern. This means that the single conditional answer is extended with another element mentioned in one conditional premise that enlarges the scope of that single conditional answer or makes it narrower. For example, given an affirmation of the consequent problem as:

If it rains, then Marianne gets wet.

If she takes a walk, then Marianne gets wet.

Marianne gets wet.

The conclusion 'it rains' may be extended with 'and she makes a walk', which makes the scope narrower in this case. If we take into account such a broader definition, we did not find suppression in Experiment 2. Conditional reasoning with two conditionals is not the same as reasoning with one conditional. Making the Modus Ponens argument means, considering the two conditionals, confirming the consequent (and doing something with the antecedent of the second conditional premise). Similarly, making the Modus Tollens argument means denying the antecedent (and doing something with the antecedent of the second conditional premise); making the denial of the antecedent argument means denying the consequent (and doing something with the antecedent of the second conditional premise); and making the affirmation of the consequent argument means confirming the antecedent (and doing something with the antecedent of the second premise). It appears that, if reasoners are given a less arbitrarily set of choices in a forced choice paradigm, the suppression phenomenon disappears. Indeed, reasoners choose or provide answers that show clear signs of conditional reasoning.

Our results show that it is not the case that the first premise of the argument was rendered false by the additional premise, so that people

simply refuse to use it in reasoning, as Savion (1993) claimed. They also show that reasoners did not reject the first premise of the argument (see Bach, 1993). Reasoners take into account the first premise and they reason with this premise. However, some of them change their conclusion considering the second (additional or alternative) premise; some others may combine the two premises, as Byrne (1989) proposed. Politzer and Braine (1991) responded also to the experiments of Byrne (1989) by stating that the procedure caused the participants to doubt the truth of one of the premises due to its perceived inconsistency with other premises. However, Byrne (1991) showed data that falsify this account. Our data confirmed Byrne (1991).

Some of the answers we observed are difficult to explain by the current theories of human deductive reasoning. Consider the following denial of the antecedent problem:

If it is raining, then she gets wet.

If it is snowing, then she gets wet.

It is not raining.

Many reasoners conclude based on these premises:

She doesn't get wet, or it is snowing.

The Mental Logic theories (e.g., Rumain et al., 1983) and also Byrne (1989, 1991) propose that reasoners integrate the two conditionals in one disjunctive conditional, such as:

If it is raining or snowing, then she gets wet.

Byrne (1991) argued that reasoners would combine the first conditional and the second conditional into a disjunctive antecedent. This would lead to the following models:

p	r
q _{alt}	r
...	

Consequently, "r" can still hold even if "not-p" is the case. However, the conclusion "not-r or q_{alt}" can only be made if the implicit model above are fleshed out. Johnson-Laird et al. (1992) claim that considering multiple models is difficult because of the load on working memory. They say that as soon as the capacity of working memory is exceeded, reasoners are unlikely to reach a conclusion that depends on multiple models. The answer "not-r or q_{alt}" requires multiple models with three atomic propositions, which is an enormous amount of information for working memory (see also Schaeken & Johnson-Laird, submitted). Therefore, the explanation of these results in terms of the Mental Model theory is not as straightforward as Byrne (1989) might have suggested.

By reanalyzing the results of our second experiment, a vague idea about the processes that could be going on was created. Some participants seemed

to start reasoning with the categorical premise and the first major premise. Based on these two premises, they formulated a putative conclusion. Next, they controlled if they had to amend this conclusion in the light of the second major premise (which is an additional or an alternative premise). We named this process of reconsidering a former conclusion '*amendment strategy*', in contrast with the '*integration strategy*' as proposed by Byrne. Consider, for example, the following Modus Ponens problem in the additional condition:

If she has an essay to write, then she will study late in the library.

If the library stays open, then she will study late in the library.

She has an essay to write.

Combining the first conditional with the categorical premise leads to the Modus Ponens conclusion:

She will study late in the library.

Next, they control if they can hold this conclusion with respect to the second premise. Therefore, we would expect many participants to add to their conclusion the amendment:

... if also the library stays open.

in a production task.

We tested this hypothesis explicitly in another study, which in fact consisted of three related experiments. The content, design and procedure of one of these experiments was the same as the former experiments, except that a production task was used.

In another of these experiments, we presented the conditionals on a computer screen in two stages. We presented the first conditional together with a categorical premise and asked the participants to give a conclusion. Next, we gave the second conditional and asked the participants whether they wanted to change their putative conclusion, and if so, how. Consider the following Modus Ponens problem:

First stage:

If she has an essay to write, then she will study late in the library.

She has an essay to write.

What follows necessarily?

[Conclusion by the participant]

Second stage:

If the library stays open late, then she will study late in the library.

Would you change your conclusion "[conclusion by participant]" in the light of the following information, and if so, how?

Final conclusion by the participant]

We hypothesized that such a presentation would induce a strategy in which participants would first form a putative conclusion based on one conditional and the categorical premise, and if necessary would amend a

second part including information about the second conditional premise.

In the third of these related experiments, we presented the participants integrated conditionals such as:

If she has an essay to write and the library stays open,
then she will study late in the library.

We hypothesized that such a presentation would elicit reasoning from integrated conditionals.

The results confirmed our predictions. Indeed, almost all answers in the second experiment were in line with the amendment strategy and almost all answers in the last one were in line with the integration strategy. Moreover, to explain all the answers observed in the production task, one has to take into account the answers of both these experiments. Of all answers, 73% was correctly predicted as a pure integration answer, a pure amendment answer or a possible integration/amendment answer.

However, we must admit that some of our predictions were not confirmed: in the last experiment, we observed answers which we classified as amendment answers. It is only if the reasoning progresses in two stages that these answers are explainable. This clearly shows that at this moment we do not know enough about the strategic and interpretative components, which play a crucial role in reasoning.

Indeed, reasoning researchers must pay more attention to different strategies that are used. Previous research (see e.g., Roberts, 1993; Roberts, Gillmore, & Wood, 1997) shows that reasoners use different strategies in tackling a deductive reasoning problem. This aspect makes it sometimes difficult to distinguish between the actual basic reasoning process from the strategies that overly and obscure them. Our experiments revealed at least two different strategies, used by the participants, that is, the integration strategy, which was proposed by Byrne (1989, 1991) and Romain et al. (1983) and the amendment strategy.

Part 2: An Introspective Experiment

The former series of experiments only provided indirect evidence for two strategies used to solve complex problems. More than 70% of the answers in Experiment 3 of Dieussaert et al. (submitted), were predicted by the use of the integration strategy, the amendment strategy or both. Other answers could be explained 'ad hoc' by one of the two strategies. An intriguing finding was the appearance of amendment answers in the experiment that was supposed to induce the integration strategy. The load on working memory (Johnson-Laird et al., 1992; Schaeken et al., submitted) can explain this finding.

We now present another experiment in which we try to trace more directly whether the inferred amendment and integration strategies are ecologically valid strategies. Ericsson and Simon (1984) argued in favor of 'think aloud' studies. In such studies, participants are not asked to remember, report, and interpret anything, but they are asked to externalize explicit thought by verbalizing it. Evans (in press) emphasized that the obvious limitation of this method is that only explicit processes can be investigated: Implicit processes cannot be verbalized (see also, Johnson-Laird, Savary, & Bucciarelli, in press). This critique doesn't apply to the strategies we are interested in, because these are clearly explicit processes (for a recent discussion of the role of strategies in deductive reasoning: see Schaeken, De Vooght, Vandierendonck, & d'Ydewalle, in press). However, a potential danger of this technique is that thinking aloud changes the underlying reasoning process. Therefore, we will first compare the answers produced by the participants with those in the previous experiment (see Table 1).

Method

Design and material. Every participant solved the same 12 problems; three problems with a different content for each of the four conditional syllogisms: Modus Ponens, Modus Tollens, denial of the antecedent and affirmation of the consequent. The 'denial of the antecedent' and 'affirmation of the consequent' problems included two major conditionals. The second conditional contained an *alternative* antecedent, for example:

If it is raining, then Marianne gets wet.

If it is snowing, then Marianne gets wet.

Each of these alternative antecedents is a sufficient condition for the consequent.

The 'Modus Ponens' and 'Modus Tollens' problems had also two major conditionals, but in this case the second conditional contained an *additional* antecedent, for example:

If it is raining, then Marianne gets wet.

If she takes a walk, then Marianne gets wet.

The contents were based on the results of a pilot study. Eighteen voluntary participants were asked to mark on a one to nine scale how alternative (1) versus additional (9) they thought a specific antecedent was within a problem. For example:

If Jan goes out fishing, then there is fish on the menu tonight.

a. If Jan buys a fish in the store,
then there is fish on the menu tonight.

b. If Jan catches a fish, then there is fish on the menu tonight.

Item *a* as well as item *b* had to be scored on the scale. The order of the

Table 1. The various answers for the four conditional inferences are mentioned above each single line. 'p' is the first antecedent, 'q' is the second (alternative or additional) antecedent, 'r' is the consequent. Below the single line is marked whether the answer is predicted amendment answer (am), integration answer (int), or could be both (int/am). Table 1 further contains the percentages of the various answers of the previous experiment (Experiment 3.1 in Dieussaert et al., submitted) and of the present Introspection study. The four conditional arguments are separated by a double line.

Modus Ponens	r		r if (also) q	-r if -q	both. rifq&-rif-q	nothing follows	r and q		other	
	am	int/am	am	int/am	both. rifq&-rif-q	nothing follows	r and q	other		
additional.	Previous	62	25	2	3	0	7	0	1	
	Introspection	57	25	0	0	1	6	9	3	
Modus Tollens	-p/-q		-p and -q	-p or -q	maybe p follows	other				
	am	int/am	am	int/am	maybe p follows	other				
additional.	Previous	36	30	30	0	4	4			
	Introspection	41	17	28	4	4	4			
Denial of the antecedent	-r		r if q	-r if -q	both. rifq&-rif-q	-r unless q	q and r	nothing follows	other	
	am	int/am	am	int/am	both. rifq&-rif-q	-r unless q	q and r	nothing follows	other	
alternative.	Previous	43	4	3	15	19	0	4	13	
	Introspection	29	13	0	18	19	4	10	6	
Affirmation of the consequent	p/q		p and q	p or q		nothing follows	other			
	am	int/am	am	int/am	nothing follows	other				
alternative.	Previous	10	12	66	5	7				
	Introspection	6	9	73	9	4				

Note. The other answers contain answers given in less than 3% of the cases or answers that weren't directed to the problem.

sixteen premises was randomized. From these eight pairs, the three pairs of premises with a high alternative score for the alternative part and a high additional score for the additional part were chosen for this experiment. Their respective scores were:

If it is raining, then Marianne gets wet.

Alternative: 2.8 If it is snowing, then Marianne gets wet.

Additional: 6.6 If she goes out for a walk,
then Marianne gets wet.

If Jan goes out fishing, then there is fish on the menu tonight.

Alternative: 1.9 If Jan buys a fish in the store,
then there is fish on the menu tonight.

Additional: 8.1 If Jan catches a fish,
then there is fish on the menu tonight.

If she goes out working,
then the woman next door takes a housekeeper.

Alternative: 2.2 If she has a child,
then the woman next door takes a housekeeper.

Additional: 7.4 If she earns enough money,
then the woman next door takes a housekeeper.

The mean score for the alternative items is 2.8, while the mean score for the additional items is 7.4.

Procedure. Participants were tested individually. At the beginning of the experiment, the following instruction was given to each participant: "We are interested to find out how people think when they try to solve problems. That's why we will give you some problems and ask you to 'think aloud' while you are solving the problem. *Always start with reading the problem aloud.* Please, say everything that comes to your mind **ALOUD** while you are thinking about the problem. If you remain silent for a few seconds, I will encourage you to speak aloud again. Take your time to think about each problem. When you reach a final conclusion, in other words, when the problem is finished according to you, *you have to repeat the conclusion* once more very clearly and *put the sheet* with your problem on it *aside*. After that, I will give you another problem until the experiment is finished."

After they read the instructions, there was some time to ask questions about the procedure. The participant solved each of the 12 problems in a randomized order. When the 'think aloud' experiment was over, we asked participants how they thought they solved the problems. If their answer was too vague, we explained them the amendment and the integration strategy, and asked whether they were in favor of one or the other (or none of them).

Participants. Twenty-three first year students of the University of Leuven, department of Psychology, participated in the experiment as a partial fulfillment of a course requirement. Since every student understood and followed the instructions, none of them was excluded from the analysis. One participant didn't answer one denial of the antecedent problem, therefore he couldn't be scored on this item, and was excluded for this particular item only.

Results and Discussion

Table 1 presents the percentages of inferences made from the four types of problems in this introspection experiment and in Experiment 3.1 of Dieussaert et al. (submitted). The pattern of the data in the introspection experiment clearly parallels the data of previous research, where the participants produced their answers without given alternatives. Indeed, it is shown that the specific procedure of 'thinking aloud' hasn't altered the reasoning process of the participants.

Again a high amount of the answers (75%) was predicted (see Table 1 for predictions; see Dieussaert et al., submitted, for a more extensive overview). Of all answers, 40 % were predicted as pure amendment answers, 4 % as pure integration answers and 31 % as possible integration/amendment answers. This distribution equals the distribution of the previous experiment. Therefore, this 'think aloud' experiment reconfirms our previous findings, but it also yields additional information: With this 'think aloud' procedure we can investigate the processes underlying the answers of the participants. We will use this direct evidence to reinforce our claim about the strategies. We will now discuss the data for each of the four problem types separately.

Modus Ponens. The largest answer category contains the logically valid conclusion "r" (57%). The broad conditional conclusion of the form "r if q" (25%), together with the broad conditional conclusions "r and q" (6%), and "q" (9%) represent the answers in which one has taken the second premise into account. When we take a closer look at the data, these answers can be linked to the different strategies. The "r and q" answers, for example, are clearly the result of the use of the integration strategy. Participants gave answers such as: "She gets wet, because it rains, while she is walking". This is also true for the "q" answers (e.g., "If she takes a walk, then she gets wet, but so, Marianne is walking and she gets wet because it rains and if it rains, then Marianne gets wet, so she is taking a walk"). This explanation may look a bit confusing, but it becomes understandable when one keeps in mind that 'raining', 'getting wet' and 'taking a walk' should go together.

It seems difficult to explain the logically correct "r" result when the

integration strategy is used: In that case, 'p' as well as 'q' is a necessary condition, for 'r' to be fulfilled. However, it is possible that people in the first stage only consider the first conditional premise and the categorical premise and infer a conclusion based on this information. In a second stage they may take the second conditional premise into account, but in that case, it is still possible that they don't change their former conclusion. E.g., "So she takes a domestic help, but this doesn't mean that she earns enough money, but one of the two conditions are fulfilled, so she takes a domestic help." or "John goes out fishing, so there will be fish on the menu, whether he catches one or not."

However, when the participants prefer to change their conclusion inferred in the first stage, they may give answers as "r if q". E.g., "But this doesn't necessarily mean that she earns enough money, because that is uncertain, so we don't know it for sure, oh yes, if she works outdoors, then she earns money, so there is a domestic help, so she can take one, but if she doesn't earn enough money, she can't take one, so it is not sure that she earns enough money, so it may be both, she may take a domestic help or not."

Modus Tollens. With an additional antecedent, we typically find three large answer categories: "not p/not q" (41%), "not p and not q" (17%) and "not p or not q" (28%), from which the second answer can't count as an integration answer. Since people usually give little explanation when making this inference, it is difficult to extract the difference between amendment and integration answers. But in cases as the following, we can easily see the difference between the integration and the amendment strategy: "If John goes fishing and if he catches a fish, then there is fish on the menu tonight, but there is no fish on the menu..." versus "John didn't go fishing, because always when he goes fishing, he eats fish at night, even if he didn't catch one..." (amendment)

Denial of the antecedent. We find a whole range of answers with an alternative antecedent. Only one of them is a simple conditional conclusion "not r" (29%), while in the other answers the second antecedent is taken into consideration, which makes them broad conditional answers: "r if q" (13%), "both, r if q and not r if not q" (18%), "not r unless q" (19%). Logically correct answers are given in 10% of the cases. Again, some answers clearly reveal the use of the amendment strategy, while others result from the use of the integration strategy.

The "not r" as well as the "not r unless q" answers typically reveal amendment answers. In a first stage, participants may conditionally conclude "not r". In a second stage, they may opt to extend this conclusion with 'unless q' or they may leave the original conclusion as it was. The

following answers are typical examples of these two possibilities: "Hmm, she works outdoors, then she takes a domestic help, so she doesn't take a domestic help, if she has a baby, then she takes a domestic help, so she doesn't have to take a domestic help unless she has a baby, that is the conclusion" (not r unless q) and "She doesn't need a domestic help because she can take care for the baby herself, and still clean the house." (not r)

On the contrary, the "both, r if q and not r if not q " answer is considered to be an integration answer. It would be unlikely to consider the two possibilities ' r if q ' and ' $\text{not } r$ if not q ' if one extends a conclusion of the first stage of reasoning with another part. E.g., "It depends on her having a baby or not. If she has no baby, then she won't take a domestic help, but if she has then she will take a domestic help. So, the decision to take a domestic help depends on her having a baby or not". However, it is still possible that the use of the amendment strategy leads to an answer of this kind. This is certainly true for the answer " r if q ", which is an example of both strategies: E.g., "It depends on whether it is snowing or not, whether Marianne gets wet." (integration) and "So, there is no fish on the menu, only if he went to the store, then there is fish on the menu tonight" (amendment)

Affirmation of the consequent. With the alternative antecedents, one large conditional answer category " p or q " (73%) appears. This can be explained by the integration as well as the amendment strategy. Again, in the data, the difference between the two becomes clear. The following conclusion is a typical example of an amendment answer. One can clearly distinguish the two stages of reasoning. The participant forms a conclusion in the first stage. Then there is a short pause, and the participants extend the first conclusion. "Necessarily follows that he went fishing, ... or he bought fish in the store". The following conclusion is a typical example of an integration answer. "So, the two cases in which the woman next door takes a domestic help, are 'or she has a baby', 'or she works outdoors', conclusion: she has a baby or she works outdoors or both." It is clear that the participant takes into consideration the two possibilities at once.

The answer " p and q " (9%) can only be explained when an amendment strategy is used. E.g., "So she may have a baby, but in fact it is, but she can also take a domestic help even if she doesn't work outdoors, but she works outdoors, so probably she has a baby and she goes working outdoors." The two stage reasoning is again clearly apparent in this example.

Another broad conditional answer is " p/q " (6%) which is an amendment answer as explained for the equivalent Modus Tollens answer "not p / not q ". Finally, 9% of the participants gave a logically correct answer ("nothing follows").

Question. After the series of answers the participants gave, we asked them informally how they thought they solved the problem. This seemed a very difficult task for most participants. Most of them gave confused answers, or said that they 'just did it'. Two clear examples for every strategy are worth citing here: 'I looked at the third sentence and then I went back, for example John doesn't go fishing, so if he doesn't go fishing, there is no fish on the menu, but he may still buy the fish, so, there may still be fish on the menu' (Amendment) or 'First I read, rather schematic, it is like a image, so then I see that for example she gets wet is the same for both of them, and snow and rain isn't equal, but it resembles each other, and then you have 'it doesn't rain', and then go back to where something about rain is given. What is given in the following sentence and then read that sentence, so in fact you look at the second premise and you see whether it changes something about the conclusion of it' (Amendment) 'First I've read without thinking, and then I first read the third line, to see from which situation to start, and then I look in the other sentences if there is something, some extra information about the situation, and then I finally take the sentence that is appropriate for this problem, or I combine both sentences, such that I can conclude a kind of necessary conclusion, and if not I also tell it' (Integration) 'I first read it and with the given facts, I see if there are several things contradicting each other or not, like for example, if John goes fishing and he catches a fish, than there is fish, and if there is no fish on the menu, than he didn't catch fish or he didn't go fishing, so trying to find out what happened.' (Integration)

General Conclusion

Contrary to the hypothesis of Byrne (1991), at least some of the reasoners do not combine the information of the two conditionals and do not give a conclusion based on the combined premise. Indeed, some of the reasoners seem to have reasoned in two stages. In the first stage, they form a putative conclusion on the basis of the first conditional and the categorical premise, and in the second stage, they possibly amend the putative conclusion in the light of the information in the second premise.

The results of the 'think aloud' experiment parallel the answer pattern we observed in former experiments (Dieussaert et al., submitted). However, contrary to the former experiments, which provided only indirect evidence for strategies, the present experiment gives us more direct evidence about the underlying strategies. The results are clear: Participants use the two proposed strategies, and they tend to give some specific answers according to the strategy they used. Moreover, the last experiment shows that the differences between the strategies aren't strictly related to one individual. A

participant may solve problems of the same type in a different way, but a particular problem (e.g., Modus Ponens, rain-problem) is not always solved by one of the two strategies. Therefore, the strategies do not seem to be related to specific problems, but rather to specific answers. It still is not clear why people change their strategy over the several problems. We expect that current studies on strategies will lead to new insights and viewpoints on this topic (see Schaeken, De Vooght, Vandierendonck, & d'Ydewalle, in press).

We now briefly return to the discussion of the introduction between Mental Logic and the Mental Model theory. We argue, in defense of the Mental Model theory, that the amendment strategy resembles the basic principle of the Mental Model theory. People reason by constructing one or more mental models, they formulate a putative conclusion and test this conclusion, by searching for alternative models that falsify the putative conclusion. Furthermore, if reasoners work with each of the two major premises separately, they minimize the number of atomic propositions they have to consider simultaneously. This is in accordance with the claim that reasoners try to minimize the load of working memory. Nevertheless, when a strategy can be easily integrated within one theory, it doesn't mean that it cannot be integrated within another one.

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