Using 3D tools to teach relational reasoning

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Abstract

Proper reasoning is at the core of argumentative competence, and is therefore part of every philosophical training. This paper argues that relational reasoning is rooted in our abilities to deal with three-dimensional spaces. That explains the fact that we reason better when relating premises in a spatial way, and that implies that teaching this type of reasoning should focus on spatial relations. This claim is argued for by using psychological findings and evolutionary theory, and an experiment with high school pupils.

Key words

exaptation, embodied cognition, relational reasoning, spatial layouts

Introduction

This paper focusses on the skill of *relational reasoning*, a type of deduction. Although conditional (if, then) and categorical (if all As are B, and Bs are C, then all As are C) are probably more visible deductive inferences in philosophical argumentation, the role of relational reasoning should not be underestimated. Our philosophical assessments are rife with examples like [1].

[1] If explanation A is better than explanation B, and B is better than explanation C, than A is better than C.

The basic rules leading to validity in deductive inferences are typically within the reach of secondary education pupils. The methods most often used in Dutch secondary education all take the following approach: the rules for validity are laid out (whether in natural language or in the shape of formal logic), and the student is asked to apply these to linguistically presented arguments. This works, but if we understand how reasoners actually make the inference from premises to conclusion, we might be able to improve our teaching on reasoning.

Teaching relational reasoning seems to be about teaching logical rules, or about teaching pupils to use their visual imagination, but a strong argument can be made that reasoning has more to do with spatial skills. In such a view, creating mental images, or adhering to rules of deduction, are detours, actually impeding reasoning performance. I argue that a *spatial* conception of relational reasoning allows teachers to directly activate pupils' spatial skills, and thereby enhance their reasoning skills more efficiently.

Section 1 presents the main contenders that psychologically explain deductive reasoning, and argues that the *space to reason theory* comes out as the most promising. This theory claims that our spatial skills form the root of our reasoning skills. In Section 2, I will give reasons to interpret the space to reason theory as a theory on embodied cognition, explaining *why* our reasoning skills are rooted in our spatial skills. Based on the theoretical foundation laid by the first sections, Section 3 describes a pilot study that was conducted with 137 high school pupils. This study both tests the hypothesis that was argued for in the first two sections, and points at the applicability of spatiality in teaching practices. This paper thus aspires to contribute to the growing body of work on embodied learning. Section 4 concludes, and offers some guidelines for educational implementation.

1. What we do when we reason about relations

Relational reasoning has been extensively studied in cognitive psychology, and there are currently three competing descriptions of how we infer from stated relations.[1] The first states that people (should) apply the rules of logic, much like the methods in Dutch education suggest. We will call this the *rule following theory* (RFT). The second theory claims that people primarily use some sort of mental images. Reasoners 'see' the premised state of affairs, in their 'mind's eye', and are able to infer a conclusion from this mental picture. We will call this theory the *visual imaging theory* (VIT). A third option, the *space to reason theory*, argues that we *seem* to use rules, or images, or

[1] There are other theories available. Probabilistic theories claim that people do not follow rules, or manipulate internal images, but are - mostly intuitively- estimating probabilities. If the conclusion is (feels?) probable in the light of the premises, people evaluate it as valid (Oaksford & Chater 2020). Another theory is based on dual-process accounts of human cognition. We use System-1 for fast thinking, and System-2 for deliberate thinking (Evans & Stanovich 2013). However, I do not consider these in this paper, since dual-process theories still have to explain what System-2 actually does when engaged, and probabilistic theories can be reduced to System-1 (Gazzo Castañeda, Sklarek, Dal Mas & Knauff 2023).

both, but that we actually employ something called *spatial layouts* when we infer. We process premises into a *spatial array*, based on the relevance of the stated relations, and infer from there.

Notwithstanding RFT's ideal of proper deduction, the fact is that people just don't follow rules like a computer might do. That's not a remark on the fallibility of humans, lamenting their poor performance, it's a remark on how we actually reason. RFT implies that reasoning gets more difficult when more steps are needed. Neither the order of steps, or the number of possible conclusions, should make any difference. But, in fact, they do. Empirical findings showing that the order of the premises and the number of possible outcomes are of great significance for reasoner's performance. Consider [2] and [3] (based on Knauff 2013):

[2] The Porsche is parked to the left of the Ferrari.

The Ferrari is parked to the left of the Hummer.

The Hummer is parked to the left of the Dodge,

The Dodge is parked to the left of the Beetle.

Is the Porsche parked to the left or to the right of the Beetle?

[3] The Dodge is parked to the right of the Ferrari.

The Hummer is parked to the left of the Beetle.

The Hummer is parked to the right of the Porsche.

The Dodge is parked to the right of the Porsche.

Is the Dodge parked to the left or to the right of the Hummer?

We can quickly ascertain that the answer to [2] is 'to the left', but [3] is much harder. For a rule-following system, these challenges are on a par, since they each need exactly the same set of steps to solve (five), however, human reasoners have a significantly much harder time dealing with [3]. The processing of the premises requires that the reasoner keeps different relations in mind, before integrating them into a coherent set (premise 1 and 2, for instance, are unrelated), the premises allow several possible coherent sets (the Dodge is to the right of the Porsche, but it can also be to the right of the Hummer, and even be to the right of the Beetle), and there is no valid conclusion (the Dodge can be to the right or to the left of the Hummer). Such properties *do* make a difference for us (Ehrlich & Johnson-Laird 1982).

Theories that argue that humans form mental pictures of the state of affairs presented by the premises can deal with the problems reasoners encounter in [3]. According to VIT, reasoners must work harder to create a mental image when some premises are unrelated, when several situations are possible, or when the argument is inconclusive because it puts more load on working memory (Baddeley 1997). VIT is also substantiated by neuroimaging results which consistently show activation of the primary visual cortex (V1) in reasoning tasks, and by subjects' reports on what they do when solving reasoning problems. However, Markus Knauff developed a series of experiments which demonstrate that visualisation actually *impedes* reasoning performance.

> [I]n several experiments, we found that the ease of visualization impaired reasoning: participants were significantly slower with these problems [the ones that were easy to visualize] than with the other sorts of problems. In other experiments, we also found that visualization results in lower reasoning accuracy: people make more errors with visual problems than with the other sorts of problems … This is called the *visual impedance effect*. (Knauff 2021, p. 738)

But how can we accommodate neuroimaging data and test subjects' reports? Fangmeier et al. (2006) were able to pry apart the different phases a reasoner goes through. In phase 1, the test subjects process the first premise, and visual areas of the brain, such as V1, are activated. In the second phase, the test subjects have to integrate all premises into a coherent idea of the situation, and activity in areas associated with memory and executive control are added to the activation in visual cortices. During phase 3, where the test subjects had to validate the offered conclusion, all brain activity in the visual areas disappeared, and areas in the dorsolateral prefrontal cortex (associated with executive control), and the spatial areas of the posterior parietal cortex, were activated. This means that visual processing occurs when processing the premises, but plays no part in the actual deductive activity, because '[o]nly in the third stage is essential deductive work carried out; the first two stages are merely normal processes of comprehension and description' (Johnson-Laird & Byrne 1991, p. 36). These findings explain the occurrence of visual processing activity during reasoning tasks, and yet keep such processing out of the inference itself. The less reasoners get distracted by irrelevant imagistic detail, the better they reason. In fact, Hamburger et al. (2018) demonstrated that people reason better when, using transcranial magnetic stimulation, their visual cortices were hindered. That spatial layouts are not reported is easily explained, because humans have no conscious access to processes in the parietal cortex.

The space to reason theory (SRT) accommodates the data. SRT builds on Johnson-Laird's mental models theory (Johnson-Laird & Byrne 1991). Knauff (2006) describes this theory as follows:

> According to model theory, the spatial representation captures one situation that is possible, given that the premises are true … Like a spatial diagram, the model's parts correspond to the parts of what it represents, and its structure corresponds to the structure of the reasoning problem … According to the model theory, reasoning with this model relies on processes that inspect and validate the model … As computational models suggest, the inspection process can be functionally described as a shift of a spatial focus that checks the cells of a spatial array and 'knows' from the scan direction the relation between two objects in the array. (p. 139)

SRT complements mental models theory with neurological data. The phases Fangmeier et al. (2006) distinguished point toward the construction of a mental model, and the actual inference as a spatial inspection process.

Let us see how SRT constructs the process of tackling a reasoning task. In phase 1, reasoners create something that's commonly called a 'mental representation' of the first premise. It is contested what such representations really are, but the dominant view is that they are symbolic entities, having both representational and syntactic properties. That means that they are about something (they display *intentionality*), and are suited for mechanistic processing as neuronally implemented tokens (Fodor 1987). In this view, often called *cognitivism,* reasoning is the processing of these entities that are symbolic, or *amodal,* meaning that they are not in any way shaped by what they represent. In Section 2, I will argue that it is likely that representations are, in fact, constitutively connected to modal, i.e. not symbolic, capacities.[2]

In phase 2, reasoners have to integrate all premises into a coherent representation of the stated situation. That explains why unrelated premises, multiple options for relating the relata, and inconclusiveness of the situation, cause trouble for human reasoners. A reasoner has to revise and evaluate the premised situation in a complicated way. In phases 1 and 2, cortices associated with linguistic and imagistic processing are activated, as well as areas for memory and control, because the reasoner is processing sensory input and actively creating a layout. Take [4] as example:

[4] The Dodge is to the left of the Beetle

The Beetle is to the left of the Porsche

Is the Dodge to the left of the Porsche?

In tackling [4], a reasoner processes the premised situation, and creates something like [5]:

[5] Dodge Beetle Porsche

When answering the question, the reasoner has to make a *spatial* judgement, and that explains the activation of cortical structures dedicated to spatial skills in phase 3. They have to judge whether the item Dodge is to the left of the item Porsche. This goes for relations that are explicitly spatial, such as 'being to the left of', as well as for relations that are not explicitly spatial, such as 'being smarter than'. Many researchers have convincingly argued that we routinely map nonspatial relations onto spatial arrangements (Gattis &Holyoak 1996; Halford, Wilson & Phillips 2010; Knauff et al. 2004; Tversky 2005). For instance, when engaging with [6], a reasoner arranges the relata on a spatial scale like [7]:

2 I will not discuss the ontological status of mental representations here. For my present purposes, it suffices to use 'representation' to stand for any kind of informational entity that plays a role in reasoning.

[6] Mary's smarter than John

John's smarter than Harry

[7] Mary

John

Harry

because we typically rate 'more' as 'higher'.[3] We employ (cortically recruit) our skill in creating and evaluating spatial arrangements, to be competent in relational reasoning. Now that we surmise that people reason spatially, we must investigate why that is the case.

2. Why we use spatiality when we reason about relations

When one kicks a ball around we can understand that the motoric activity, afforded and constrained by the particular morphology our body, and the ball, are part and parcel of what's happening there. However, imagining, remembering, and reasoning, to name a few phenomena, are seen as activities that are strictly internal, meaning that they happen in our inner space, often called 'mind', and often equated with neural processes in the brain (Clark &Toribio 1994; Thagard 2020). One of the ramifications of this cognitivist view is that true cognitive processes have nothing to do with the contingencies of our body. The RFT is closely related to this view. Cognitivism thus claims that cognition is amodal, meaning that its functioning is not shaped, caused or even slightly influenced by the specificities of our senses, physiology, motoric activities, etc. Of course, the outside world, including the body, is causally relevant as source of input, and arena for output. However, it is not part of cognition itself.

Knauff himself stresses that spatial layouts are amodal (Knauff 2013, p. 7). For SRT, that claim is understandable, since it wants to distinguish itself from VIT, which is highly modal, in some sense. However, it is hard to see how the 'spatiality' of SRT's account of reasoning can be disconnected from our specific spatial capacities.

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[3] According to conceptual metaphor theory (Lakoff & Johnson 2003), we map non-physical and nonspatial features, properties, relations, etc. onto physical and spatial features, properties and relations. In doing so, we understand affairs that we have not (and, possibly, cannot have) experienced in terms of affairs we have experienced. Because we are intimately familiar with spatial relations, we understand (on a conceptual level) and express (on a linguistic level) the non-experiential target (ratings of intelligence) in terms of the familiar source (space).

Phylogenetically and ontogenetically, dealing with space is needed long before dealing with linguistically presented reasoning tasks comes around. Evolution favours efficiency, and facing cognitive tasks with less expenditure of resources will get favoured. Using action, body and environment is more efficient than facing the same tasks with only internal processing, and will get favoured. These considerations provide arguments against cognitivism, and make way for so-called E-accounts of cognition (the 'E' standing for embodied, extended, embedded, enactive, ecological, etc). E-accounts are quite diverse, but are united in their opposition to cognitivism. They argue that there is not much room to spare between thinking and doing, and that humans routinely form systems which consist of neural and extra neural elements and actions. The locus of cognition is then not solely the brain, but the brain plus rest of body, plus environment, plus activities performed in that environment (see Gallagher 2023 for a recent and concise overview). This paper's position could be categorised as one in embodied cognition, in the sense (the early) Gallagher understands it: it is about the extent to which cognitive processes are shaped 'prenoetically' (Gallagher 2005, p. 2) by the fact that we are embodied.[4]

The next step in my argument is this: dealing with linguistically presented arguments demands the skill to deal with language. The advent of language is much too recent to evolutionarily *adapt* physiological structures to deal with it.[5] We are capable of dealing with language through *exaptation*, a process in which a physiological structure that was adapted for a specific function (e.g. feathers that were adapted to provide insulation and protection), gets a second function (enabling flight). Crucially, because they still have to perform the original function, structural properties remain as unchanged as possible, and influence the newer function (Anderson 2010). If feathers would be optimised for flight, but thereby became worthless as insulators, evolution would rule against it. At the same time, the specifics of insulating feathers influence the function of enabling flight. SRT points us in this direction: our capacity for dealing

4 Researchers have started to develop e-cognition based applications in educational settings (review in Skulmowski & Rey 2018, also Macrine & Fugate 2022). If learning is primarily about enhancing abilities to use the whole system of brain, body and world, it makes sense to look at the relations between the constituent elements of that system. This paper sits close to the notion of the development of skill as it is understood in ecological psychology: cognition is for doing, and learning is enhancing the ability to do things (Rietveld, Denys & Van Westen 2018).

5 It is assumed that humans started using language in any serious sense of that word around 150,000 years ago. In evolutionary terms, this is recent. The process of adaptation needs many more cycles of selection to allow for the emergence of new anatomical structures. Recent functions are therefore typically caused by exaptation (Gould & Vrba 1980).

with relational reasoning seems to be so tightly connected to our capacity for dealing with three-dimensional space that we, in all likelihood, got our relational reasoning capacities, through exaptation, from our spatial capacities. That would explain why relational reasoning is much like arranging things in space.

The cognitivist view cries against this: a reasoner reads (or hears) the asserted premises, encodes these perceptual inputs into symbolic representations of the stated situation, applies the rules of inference to these (owing to the syntactic properties of those symbolic entities), and, finally, expresses the results of this process in a perceptible judgement on validity. In this view, reasoning is an intracranial activity that gets 'fed' input by the senses, and delivers motoric activity as its output (Hurley 1998). Reasoning goes astray because of en- or decoding errors (Polk & Newell 1995), or because of incorrect processing (Rips 1994). On a semantic level, the premises contain information (whatever that may be), namely claims on hypothetical states of affairs (e.g. 'Mary is smarter than John' AND 'John is smarter than Harry'), and the argument suggests a relation between these states of affairs, often leading to a conclusion ('Mary is smarter than Harry'), which carries content, i.e. information about the hypothetical world conjured by the premises. Skill in reasoning is determined by having neural pathways that quickly make the right connections. By practicing, preferred neural pathways get strengthened (Hebbian learning), and one gets better at reasoning.

However, research into so-called *grounded cognition* argues that our concepts are thoroughly connected to embodied experience (Barsalou 2020). Behavioural and neurological findings demonstrate that we engage with verbs and concrete nouns in the same way that we engage with actual action and objects.[6] For instance, we activate the motoric system for kicking when we encounter the word 'kick' (Hauk et al. 2004), and people's physical reaction is closely connected to the *description* of the physical movement they get presented with For example, test subjects are quicker to push a lever if the situation describes an action away from the observer, as in 'she walked out of the room' (Glenberg & Kaschak 2002). Several theories argue that even our abstract words and concepts are grounded in bodily experience in this way (see overview in

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[6] It should be noted that the experimental findings that are referred to in this paragraph are contested. Hauk et al's (2004) findings are, for instance, criticised in Mahon and Caramazza (2008), and Glenberg and Kaschak's (2002) in Goldinger et al. (2016). The latter paper also offers a critical review of empirical findings from other researchers in grounded cognition. However, the discussion is ongoing (e.g. Barsalou 2016).

Kiefer & Harpaintner 2020). The most extensively elaborated of these theories is the *conceptual metaphor theory*, which argues that most, if not all, of our ways of talking (and thus conceptualising [7]) consist of metaphors whose sources are embodied experiences of, mostly, space and motion (Lakoff & Johnson 2003). [8] Grounded cognition theories provide a strong argument for viewing our thinking as constitutively based on our basic embodied capacities (Khatin-Zadeh et al. 2023). The mechanism of exaptation we discussed earlier explains why our reasoning skills are shaped by our spatial skills. That also explains why using external spatial layouts enhances reasoning performance; if a reasoner bypasses visualisation and rule-following, they use the underlying enabling mechanism with little or no distraction.

It turns out that reasoning isn't some highly disembodied activity. Reasoning is shaped by our descent and bodily history. Reasoning depends, modally, on our acquaintance with three-dimensional space. In the next sections I describe an experiment I designed and discuss the implications of this view for teaching practices.

3. An experiment on the spatiality of relational reasoning

If SRT's analysis is right, spatial layouts are central in relational reasoning. Steering reasoners away from hindrances and pointing them directly at spatial layouts would, according to SRT, enhance their reasoning performance. Explicitly instructing human reasoners to use their posterior parietal cortex will not work. Humans have no conscious access to processes in that area. What *would* work, is priming reasoners by making reasoning tasks explicitly spatial, thereby creating or strengthening neural pathways that tap as directly as possible into those areas. This leads to the following hypothesis: experience with tackling relational reasoning challenges by spatially manipulating external objects will enhance reasoning skills better than experiences. I designed a pilot study to test this.

137 first grade high school pupils were divided into five groups (their standard classes, labelled A, B, C, D, and E), were instructed on validity (video, 2:46 min), and took Test 1 in which they had to decide whether a conclusion validly followed from

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7 Not everybody is convinced that our words / sentences are on a par with our concepts / propositions. This is not the place to delve into that discussion. This paper simply joins the majority of researchers that thinks there is, at least, a very intimate connection between words and concepts.

8 In a way, SRT can be seen as complementary to conceptual metaphor theory. If our thinking is based on our sensorimotor acquaintance with the world, SRT presents a description of how relational thinking is based on our innate and acquired skills to deal with this three-dimensional world.

stated relations between up to 5 entities. The questions ranged from easy to challenging. [8] provides an example.

[8] The green car is more expensive than the blue car

The red car is less expensive than the yellow car

The yellow car is less expensive than the green car

The black car is more expensive than the blue car

The blue car is more expensive than the red car

Does it follow that the yellow car is more expensive than the blue car?

Since our hypothesis claims that relational reasoning *always* involves spatial relating, even if obscured by imagery or rule-following, we randomly mixed situations (is there only one possible spatial arrangement, or are there several?), counterfactual and factual relations (dogs are bigger than horses, or horses are bigger than dogs), valid and invalid conclusions, abstract and concrete entities (X and Y, or horse and cat), and spatial and non-spatial relations (being to the left of, or being smarter than). Pupils were allowed to use pen and paper.

After this test, Group A practiced for 90 minutes with relational reasoning challenges, supervised by a teacher who refrained from advising. Subjects in Group A were allowed to use anything they could get their hands on.

Group B got to view a video (1:48 mins) which explains the use of drawing in solving relational arguments. Such drawings are versions of visualising the relations in a spatial way. The group practiced for 90 minutes, guided by a teacher.

Group C got to view a video (1:29 mins) which explains the use of logical rules in solving relational arguments. Several rules were suggested, e.g. [9].

[9]

~~A is harder than B~~			*ignore*
B is easier than C	*replace with*	C is harder than B	
C is easier than A	*replace with*	~~A is harder than C~~	*ignore*
D is easier than B	*replace with*	B is harder than D	

Transitivity: if C is harder than B, and B is harder than D, then C is harder than D

The group practiced for 90 minutes, guided by a teacher who discouraged the use of other strategies.

Groups D and E got to view a video (1:34 mins) which explains the use of cards in solving relational arguments. The groups practiced for 90 minutes, guided by a teacher. This teacher handed each pupil 5 tokens (drywipe cards) and an erasable marker, and instructed on how to use these (write relata on cards, spatially arrange cards to express stated relations).

After this, all groups took Test 2, in which they used the same methods, tools and strategies they had practiced with. The questions on this test were comparable to those in Test 1.

After a short break (20 minutes), all groups took Test 3 (questions comparable to Test 1 and Test 2), in which they were only allowed to write their answers on an answering form. They were instructed that any other markings anywhere would render the test invalid.

Didactical wisdom says that training improves performance, and that alignment or lack thereof between practice and test influences test results. All groups received training after Test 1, and should therefore perform better at Test 2. Test 3, however, constrains possible strategies and tool use, and is therefore not aligned with the training of Groups B, C, D, and E (and possibly Group A). All else being equal, results should therefore look like this:

[10]

Group A: Test 1 < Test 2 ? Test 3

Group B, C, D, and E: Test 1 < Test 2 > Test 3

The results of Tests 1, 2 and 3, on a 10-point scale, show that *not* everything was equal:

[11]

		1	**2**	**3**	**4**	**5**	**6**
	# pupils	Test 1	Test 2	Test 3	Difference Test 1 – 2:	Difference Test 2 – 3	Difference Test 1 – 3
Group A (No instructions)	26	6.85	7.85	6.31	1.00	-1.54	-0.54
Group B (Drawing)	28	7.64	8.29	7.14	0.82	-1.14	-0.32
Group C (Rule-following)	28	7.38	6.00	5.81	-1.27	-0.65	-1.57
Group D (Cards)	27	7.26	8.74	7.63	1.48	-1.11	0.37
Group E (Cards)	28	5.93	8.30	7.00	2.37	-1.30	1.07

The effect of training (column 4) does not show up in Group C, suggesting that rule-following is inferior to whatever strategies pupils came up with in Test 1. The misalignment effect (column 5) does show up in all groups. That effect also explains why Groups A to C scored worse on Test 3 (column 6), even after training. Remarkably, not only did Groups D and E make more progress than the other groups from Test 1 to Test 2, they also scored higher on Test 3 than on Test 1, suggesting that training with the cards cancels the misalignment effect.

4. Discussion and concluding remarks

It's hard to see how the cognitivist' view of relational reasoning could account for these results. Whether one practices by notating steps in a rule-following procedure, by drawing, or by manipulating tokens, can only make a negative difference, being a distraction. The embodied view this paper advocates *can* explain the differences that showed up in the test. That reasoning abilities improve seems to be the effect of priming the reasoner to use spatial layouts.

There is a vast and growing body of work on spatial thinking and education (e.g. Ishikawa & Newcombe 2021; Uttal & Cohen 2012), most of which is not explicitly based on e-accounts of cognition. So why argue that embodiment is key? There are two points to be made on this issue. First of all, the vast majority, if not all, of the extant work on spatial thinking involves the mental visualisation of spatial properties, or of spatially presented nonspatial properties. When looking at reasoning, however, the arguments against VIT apply here. On the other hand, this paper's argument is no account of visualisation, but an account of how we spatially handle linguistically presented relations. That means that this paper's argument is on a different track.

More importantly, however, the extant work does not explain *why* spatial skills are as effective as they seem to be in a range of cognitive tasks. It is claimed that visualisation somehow presents the relevant information in a way that simplifies processing (cf. Barker-Plummer et al. 2017 for an extended analysis), but there is no discussion of why *spatial* processing would be effective. The fact that it is does not pre-empt, but supports this paper's argument.

Even though the test is just a pilot, it validates many teacher's long-standing intuition that actually *doing* something has a big effect, even when learning something as seemingly abstract as logic. The results also suggest that the seemingly disembodied activity of relational reasoning is, in fact, very much like moving things in space, and therefore very much an embodied activity.[9]

In conclusion, I share some practical advice:

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[9] The experiment was a pilot, and the results are only suggestive of the supposed effects at most. It might, for instance, be claimed that reasoners commonly *do* use rule-following effectively. It is just that Group C got the wrong kind of rules to follow for these particular test subjects. There could also be too many variables in this set-up: how can we guarantee that all groups applied themselves at the same level throughout the experiment? More and better research is needed to really substantiate the claimed effects.

- Consider how many items you want students to relate, and get that number of tokens, plus one or two, per student. They should, for instance, be able to express the ambiguity of the A-C relation as something like [12].

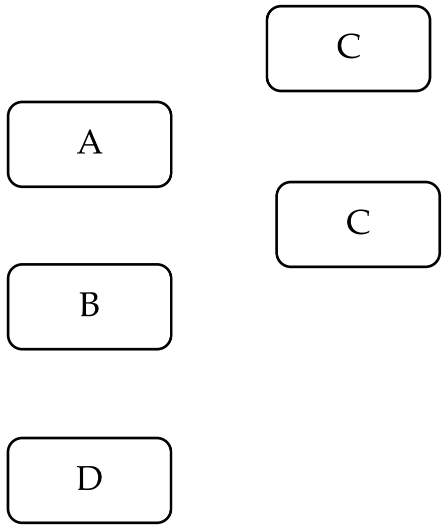
[12] A is harder than B

B is easier than C

C is harder than D

D is easier than B

Is C harder than A?



I used drywipe cards and whiteboard markers, but anything that you can carry around by the dozens, that's writeable, and preferably reusable, will do.

- Instruct pupils to write just enough letters to distinguish between the items. First of all, this will save time, and secondly, this will -further- eliminate distractions by world-involving knowledge.

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