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Research Article

Optical pulse propagation in inhomogeneous fibers in the anomalous dispersive regime: dispersion-managed bright solitons

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Abstract

We study the nonlinear electromagnetic wave propagation in inhomogeneous fibers in the anomalous dispersive regime. In order to include the inhomogeneous physical effects, the nonlinear Schrödinger (NLS) equation, which governs the solitary pulse propagation in optical fiber, is modified by adding terms for phase modulation and fiber power loss/gain. Lax pair construction and Hirota transformation are introduced in order to bilinearize and solve the modified inhomogeneous NLS equations. A general integrability condition is arrived for gain and loss cases. The modified NLS equations in the anomalous dispersive regime are then Hirota bilinearized, and exact bright solitons solutions are obtained. The analytical soliton solutions are also obtained. The results show that in both gain and loss cases the areas of the pulse envelopes remain preserved during the propagation in the fibers, demonstrating that bright solitary wave propagation is maintained in the cores. The results are discussed in detail.

Keywords: Optical soliton; Nonlinear Schrödinger equation; Horota transformation; Self-phase modulation.

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1. INTRODUCTION

Modern communications rely on fast digital optical fiber systems, though there is, however, a draw back to using fiber solitons as the current electronic amplifiers and switches just cannot operate quickly enough for the short pulses and high speeds that optical solitons can provide [1]. The idea of optical fiber communications is to inject laser pulses into a fiber. Information is coded within these pulses. In order to increase the amount of information we can send in one second, we can make the pulses as short as possible and inject many pulses per second, but there is a limit of the width of laser pulses. When a pulse propagates along a fiber, its width increases: the narrower the pulses, the more rapid the increase. This effect is called dispersion. We then can imagine that if we inject too many short pulses into a fiber, they will overlap after propagating over some distance. We then will not be able to distinguish between pulses, and information will have been lost. Fortunately, there is also another counter-effect which shortens the width of a pulse. This effect is called nonlinearity. If the intensity of a laser pulse is strong enough, this nonlinear effect can be in an active balance with the dispersion effect. The result is a pulse that can keep its shape for a long propagation distance, even including small disturbance or perturbation. These steady pulses are called optical solitons. Solitons are a special breed of optical pulses that can propagate through an optical fiber undistorted for tens of thousands of kilometers. The key to soliton formation is the careful balance of the opposing forces of chromatic dispersion and self-phase modulation (SPM) [2].

Solitons refer to the functional form of the specific solutions of the underlying nonlinear equation that describes light propagation in nonlinear optical media. The fundamental property of solitons that makes them attractive for optical communications is that solitons maintain their shape upon propagation and even exhibit particle behavior by surviving collisions with one another. A wide applicability of the soliton equation implies soliton phenomena which are common in various fields of physics. This is the essence of soliton physics. Solitons appear in almost all branches of physics, such as hydrodynamics, plasma physics, nonlinear optics, condensed matter physics, low temperature physics, particle physics, nuclear physics, biophysics and astrophysics [3].

An optical soliton is a pulse that travels without distortion due to dispersion or other effects. They are a nonlinear phenomenon caused by SPM, which means that the electric field of the wave changes the index of refraction seen by the wave (Kerr effect). The SPM causes a red shift at the leading edge of the pulse. Solitons occur when this shift is cancelled due to the blue shift at the leading edge of a pulse in a region of anomalous dispersion (bright soliton), resulting in a pulse that maintains its shape in both frequency and time. Solitons are therefore an important development in the field of optical communications. Here, in this work, an inhomogeneous nonlinear Schrödinger (NLS) equation with varying parameter for phase modulation in the anomalous (bright soliton) dispersive regime with both gain and loss (damping) have been considered and exactly solved. In order to solve eigenvalue problem, Lax pair and Hirota transformation have been introduced and used.

2. SOLITON SOLUTIONS, RESULTS AND DISCUSSION

We start with the governing equation, after taking the inverse Fourier transform and replacing $(\omega-\omega_0)$ by the differential operator $i(\partial/\partial t)$, in uniform optical fibers [3]

$$\frac{\partial A}{\partial Z}+\beta_1\frac{\partial A}{\partial t}+i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}=i\gamma|A|^2A, \tag{1}$$

where A is slowly varying amplitude of the pulse envelop, the parameters $\beta_1=1/v_g$ and β_2 include the effects of group velocity dispersion (GVD) to first and second orders, respectively, v_g is the group velocity associated with the pulse, γ governs the effects of SPM and Z is the propagation distance along the fiber.

Equation (1) is a nonlinear differential equation and can be reduced to the usual form of NLS equation by marking the following transformation of variables

$$\left.\begin{aligned} T&=(t-\beta_1 z)/T_0\\ z&=Z/L_D\\ u&=\sqrt{|\gamma|L_D}\,2A \end{aligned}\right\}, \tag{2}$$

where T_0 is a temporal scaling parameter and is taken to be the input pulse width, and $L_D=T_0^2/|\beta_2|$ is the dispersion length. In terms of the variables, defined in Eq. (2), Eq. (1) takes the form, so-called NLS equation (which governs the solitary pulse propagation in the fiber)

$$i\frac{\partial u}{\partial z}-s\frac{\partial^2 u}{\partial T^2}\pm 2|u|^2 u=0\,, \tag{3}$$

where $s = sign(\beta_2) = \pm 1$ stands for the sign of the GVD parameter. Let $\beta_2 = \beta$ for clarity. β is related to the frequency dependence of group velocity and is positive (negative) for normal dispersion (anomalous dispersion). This means that $s = +1(-1)$ is for normal (anomalous) dispersion i.e. for dark (bright) solitons.

Equation (3) can also be written in a simple form

$$i\,u_z - s\,u_{tt} + 2|u|^2\,u = 0\,, \tag{4}$$

where $u(z,t)$ is the envelope of the axial electric field, subscripts z and t denote partial derivatives with respect to these variables.
If β is negative ($\beta < 0$) the NLS equation (Eq. (4)) becomes

$$i\,u_z + u_{tt} + 2|u|^2 u = 0 \tag{5}$$

The general solution of Eq. (5) can be obtained as

$$u(z,t)=2\,\mu_{20}\;e^{-2i\left[\mu_{10}\,t+2\int_0^z\left(\mu_{10}^2-\mu_{20}^2\right)dz\right]}\operatorname{sech}\left[2\left(\mu_{20}\,t+4\int_0^z\mu_{10}\mu_{20}dz\right)+\delta\right] \tag{6}$$

where μ_{10} and μ_{20} are constants, and δ is the integration constant. Equation (6) is the exact bright one soliton solution (1SS) in homogeneous optical fiber. Equation (6) is three-dimensionally plotted in Fig. 1. Figure 1 shows that there is a conserved peak in a constant background, and this is the bright soliton without fiber loss/gain and phase modulation.
In the derivation of Eq. (4), it is assumed that fiber is uniform. Considering homogeneous fiber medium, Eq. (4) has widely been studied by a number of authors (e.g. [4-6]). However, in an actual fiber, the core medium is, in general, not uniform or homogeneous [7,8]. This inhomogeneity arises mainly due to imperfection or defects in fiber core medium and fluctuations in core/cladding radius. Therefore, the study of nonlinear wave

propagation in inhomogeneous media is of great interest in the area of fiber optics communications [9].

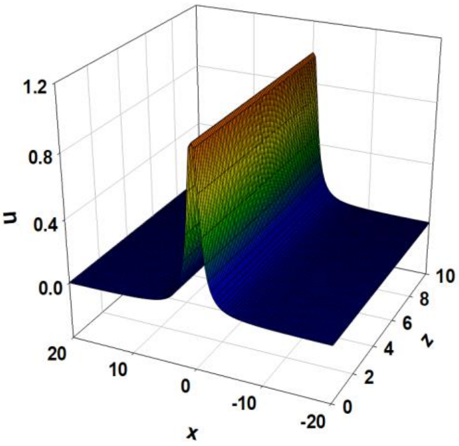

Figure 1: Amplitude profile for the bright 1SS in homogeneous fiber. $|u(z,t)|$ is plotted as a function of z and *t* (a 3D plot of Eq. (6)).

In case of inhomogeneities in the fiber medium, the NLS equation can be modified and be written in a general form

$$i\,u_z - s\,u_{tt} + 2|u|^2 u = \varphi(z)t^2\,u \pm i\,g(z)u \tag{7}$$

where $\varphi(z)$ is phase modulation and $g(z)$ is the fiber gain (loss) for $g > 0$ $(g < 0)$.

2.1 Fiber gain

The modified NLS equation (Eq. (7)) with phase modulation and fiber gain ($g > 0$) in the anomalous dispersive regime ($\beta < 0$) reduces to

$$iu_z + u_{tt} + 2|u|^2 u = \varphi(z)t^2\,u + ig(z)u \tag{8}$$

Introducing variable transformation as [10],

$$u(z,t) = U(z,t)\exp\left(-\frac{igt^2}{2}\right) \tag{9}$$

Using above variable transformation in Eq. (8), we get

$$\exp\left(-\frac{igt^2}{2}\right)\left[i\frac{\partial U}{\partial z} + \frac{U}{2}t^2\frac{dg}{dz} + \frac{\partial^2 U}{\partial t^2} - 2igt\frac{\partial U}{\partial t} - igU + g^2t^2U + 2|U|^2U\right] =$$

$$\exp\left(-\frac{igt^2}{2}\right)\left[\varphi(z)t^2U + ig(z)U\right]$$

Or, $$i\frac{\partial U}{\partial z}+\frac{U}{2}t^2\frac{dg}{dz}+\frac{\partial^2 U}{\partial t^2}+2|U|^2U-2igt\frac{\partial U}{\partial t}-2igU-\left(g^2+\varphi\right)Ut^2=0 \tag{10}$$

The Lax pair associated with Eq. (10) is derived as,

$$\frac{\partial\psi}{\partial t}=\mathrm{P}\psi \qquad \frac{\partial\psi}{\partial z}=Q\psi \qquad \psi=\left(\psi_1\psi_2\right)^T \tag{11}$$

where $\mathrm{P}=\begin{pmatrix}-i\Delta & U\\ -U & i\Delta\end{pmatrix}$

and $Q=2i\Delta^2\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}+2\Delta\begin{pmatrix}-igt & U\\ -U^* & igt\end{pmatrix}+i\begin{pmatrix}|U|^2 & \frac{\partial U^*}{\partial t}-2igtU^*\\ \frac{\partial U^*}{\partial t}+2igtU^* & -|U|^2\end{pmatrix}$.

Here U^* is the complex conjugate of U and Δ is the variable spectral parameter given by $\Delta=\Delta_0\exp\left(2\int_0^z g\,dz\right)$.

The compatibility condition $\frac{\partial\mathrm{P}}{\partial z}-\frac{\partial Q}{\partial t}+\left[\mathrm{P},Q\right]=0$ gives,

$$i\frac{\partial U}{\partial z}+\frac{\partial^2 U}{\partial t^2}+2|U|^2U=2igt\frac{\partial U}{\partial t}+2igU \tag{12}$$

By comparing Eqs. (10) and 12), we find that Eq. (8) is completely integrable with integrability condition [11,12]

$$\frac{dg}{dz}-2\left(\varphi+g^2\right)=0 \tag{13}$$

for the exact solutions.

The Hirota derivative operators for the system are

$$D_z^m D_t^n b(z,t).c(z,t)=\left(\partial_x-\partial_{x'}\right)^m\left(\partial_t-\partial_{t'}\right)^n b(z,t)c(z',t')|_{x'=x,t'=t} \tag{14}$$

with the transformation in the form,

$$U=\frac{b}{c} \tag{15}$$

where $b(z,t)$ and $c(z,t)$ are complex and real functions respectively. Using the above transformation in Eq. (12), we obtain

$$\left(iD_z+D_t^2-2igtD_t-2ig\right)(b.c)=0\,, \qquad D_t^2(c.c)=2b^2 \tag{16}$$

The multi-soliton solutions can be obtained by the following perturbation expansions of b and c:

$$b = \sigma b_1 + \sigma^3 b_3 + \sigma^5 b_5 + \sigma^7 b_7 + \cdots\cdots\cdots \tag{17}$$

$$c = 1 + \sigma^2 c_2 + \sigma^4 c_4 + \sigma^6 c_6 + \cdots\cdots\cdots\cdots \tag{18}$$

where σ is an expansion coefficient. For 1SS only the first two terms in Eqs. (17) and (18) respectively are needed [3,12]. So, the bright 1SS can be obtained by the following perturbation expansions of b and c:

$$b = \sigma b_1 \tag{19}$$

$$c = 1 + \sigma^2 c_2 \tag{20}$$

Plugging Eqs. (19) and 20) in Eq. (16) then collecting the terms (coefficient of $\sigma, \sigma^2, \sigma^3$ and σ^4) with same power of σ, we obtain:

$$\begin{aligned}
&\left(iD_z + D_t^2 - 2igtD_t - 2ig\right)(b_1.1) = 0 \\
&D_t^2\left(1.c_2 + c_2.1\right) = 2|b_1|^2 \\
&\left(iD_z + D_t^2 - 2igtD_t - 2ig\right)(b_1.c_2) = 0 \\
&D_t^2\left(c_2.c_2\right) = 0
\end{aligned} \tag{21}$$

Solving the set of equations (Eq. (21)). We conveniently assume $b_1 = e^{v+\mu}$ and $c_2 = e^{v+\mu+(v+\mu)^*}$ where $(v+\mu)^*$ is the complex conjugate of $(v+\mu)$. After some algebra, Eq. (12) yields

$$\begin{aligned}
U(z,t) = {} & 2\varphi_{20} \exp\left(2\int_0^z gdz\right) e^{-2i\left[\left\{\varphi_{10}\exp\left(2\int_0^z gdz\right)\right\}t + 2\int_0^z \left(\phi_{10}^2 - \varphi_{20}^2\right)\left(\exp\left(2\int_0^z gdz\right)\right)^2 dz\right]} \\
& \operatorname{sech}\left[2\left\{\varphi_{20}\left(\exp\left(2\int_0^z gdz\right)\right)t + 4\int_0^z \varphi_{10}\varphi_{20}\left(\exp\left(2\int_0^z gdz\right)\right)^2 dz\right\} + \Omega\right]
\end{aligned} \tag{22}$$

where φ_{10}, φ_{20} and Ω are constant.

Using Eqs. (9) and (22), we obtain the variable transformation as

$$u(z,t) = 2\varphi_{20}\exp\left(2\int_0^z g dz\right) e^{-2i\left[\left\{\varphi_{10}\exp\left(2\int_0^z g dz\right)\right\}t + 2\int_0^z \left(\phi_{10}^2-\varphi_{20}^2\right)\left(\exp\left(2\int_0^z g dz\right)\right)^2 dz\right] - \frac{igt^2}{2}}$$

$$\operatorname{sech}\left[2\left\{\varphi_{20}\left(\exp\left(2\int_0^z g dz\right)\right)t + 4\int_0^z \varphi_{10}\varphi_{20}\left(\exp\left(2\int_0^z g dz\right)\right)^2 dz\right\} + \Omega\right] \quad (23)$$

Let, $v = -2i\left\{\varphi_1 t + 2\int_0^z \left(\varphi_1^2 - \varphi_2^2\right)dz\right\}$ and $\mu = 2\left(\varphi_2 t + 4\int_0^z \varphi_1\varphi_2 dz\right) + \Omega$, where Ω is an integration constant, with $\varphi_1 = \varphi_{10}\exp\left(2\int_0^z g dz\right)$ and $\varphi_2 = \varphi_{20}\exp\left(2\int_0^z g dz\right)$.

Equation (23) can be written as

$$u(z,t) = 2\varphi_2 e^{\left(v - igt^2/2\right)}\operatorname{sech}\mu \quad (24)$$

Equation (24) is the exact bright 1SS of Eq. (8) in inhomogeneous fiber with fiber gain and phase modulation. Equation (24) is plotted three- and two-dimensionally in Figs. 2 and 3. As can be seen, from Fig. 2, the pulse amplitude increases as *z* increases. This is due to the admittance of the terms $\varphi(z)$ and $g(z) < 0$ in Eq. (7), which also shows that the soliton amplitude grows at the same rate as the respective power amplitude with inclusion of phase modulation and gain [13]. From Fig. 3, it can clearly be seen that the pulse amplitude increases as *z* increases.

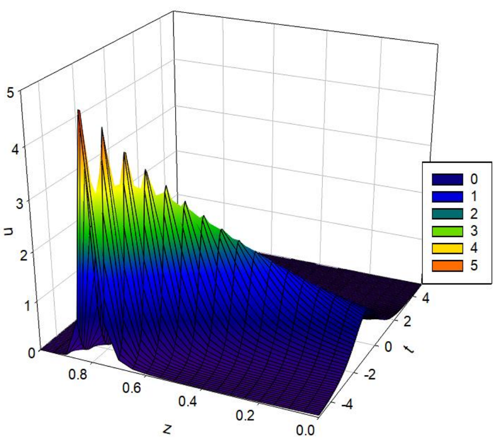

Figure 2: Amplitude profile for the bright 1SS in inhomogeneous fiber (with gain i.e. $g > 0$): $|u(z,t)|$ as a function of *z* and *t* (a 3D plot of Eq. (24) with $\varphi_{20} = 0.5,\ g = 0.1z^{-1}$ *and* $\Omega = 0$).

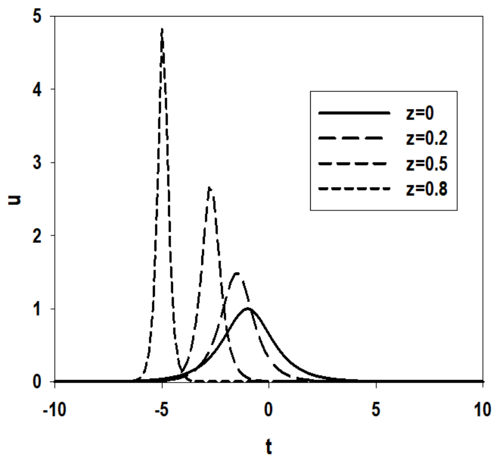

Figure 3: Amplitude profile for the bright 1SS in inhomogeneous fiber core (with gain i.e. $g > 0$): $|u(z,t)|$ as a function of *t* for different values of *z* (a 2D plot of Eq. (24).

2.2 Fiber loss

In phase modulation and fiber loss ($g<0$) in the anomalous dispersive regime ($\beta<0$), Eq. (7) can be written as

$$iu_z + u_{tt} + 2|u|^2 u = \varphi(z)t^2 u - ig(z)u \tag{25}$$

Introducing variable transformation as

$$u(z,t) = U(z,t)\exp\left(\frac{igt^2}{2}\right) \tag{26}$$

The eigenvalue problem associated with Eq. (25) is

$$\mathrm{P} = \begin{pmatrix} -i\Delta & U \\ -U & i\Delta \end{pmatrix}$$

and $Q = \Delta^2\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \Delta\begin{pmatrix} igt & U \\ -U^* & -igt \end{pmatrix} + i\begin{pmatrix} |U|^2 & \dfrac{\partial U^*}{\partial t} + 2igtU^* \\ \dfrac{\partial U^*}{\partial t} - 2igtU^* & -|U|^2 \end{pmatrix}$

Proceeding as before, it is found that the Eq. (25) is integrable with condition

$$\frac{dg}{dz} + 2\left(\varphi + g^2\right) = 0 \tag{27}$$

Following the same procedure, we obtain

$$\begin{aligned} U(z,t) &= 2\varphi_{20}\exp\left(2\int_0^z g\,dz\right) e^{-2i\left[\left\{\varphi_{10}\exp\left(2\int_0^z g\,dz\right)\right\}t + 2\int_0^z \left(\phi_{10}^2 - \varphi_{20}^2\right)\left(\exp\left(2\int_0^z g\,dz\right)\right)^2 dz\right]} \\ &\quad \mathrm{sech}\left[2\left\{\varphi_{20}\left(\exp\left(2\int_0^z g\,dz\right)\right)t + 4\int_0^z \varphi_{10}\varphi_{20}\left(\exp\left(2\int_0^z g\,dz\right)\right)^2 dz\right\} + \Omega\right] \end{aligned} \tag{28}$$

So, the bright soliton solution with fiber loss can be written as

$$\begin{aligned} u(z,t) &= 2\varphi_{20}\exp\left(2\int_0^z g\,dz\right) e^{-2i\left[\left\{\varphi_{10}\exp\left(2\int_0^z g\,dz\right)\right\}t + 2\int_0^z \left(\phi_{10}^2 - \varphi_{20}^2\right)\left(\exp\left(2\int_0^z g\,dz\right)\right)^2 dz\right] + \frac{igt^2}{2}} \\ &\quad \mathrm{sech}\left[2\left\{\varphi_{20}\left(\exp\left(2\int_0^z g\,dz\right)\right)t + 4\int_0^z \varphi_{10}\varphi_{20}\left(\exp\left(2\int_0^z g\,dz\right)\right)^2 dz\right\} + \Omega\right] \end{aligned} \tag{29}$$

Let, $v = -2i\left\{\varphi_1 t + 2\int_0^z \left(\varphi_1^2 - \varphi_2^2\right)dz\right\}$ and $\mu = 2\left(\varphi_2 t + 4\int_0^z \varphi_1\varphi_2\,dz\right) + \Omega$, where Ω is an integration constant with $\varphi_1 = \varphi_{10} \exp\left(2\int_0^z g dz\right)$ and $\varphi_2 = \varphi_{20} \exp\left(2\int_0^z g dz\right)$.

Equation (29) can be written as,

$$u(z,t) = 2\varphi_2 e^{\left(v+igt^2/2\right)} \operatorname{sech} \mu \tag{30}$$

Equation (30) is the exact bright 1SS of equation (25) in inhomogeneous fiber with fiber loss and phase modulation. Equation (30) is plotted three- and two-dimensionally in Figs. 4 and 5. Figure 4 depicts that the pulse amplitude decreases as *z* increases. This is due to the admittance of the terms $\varphi(z)$ and $g(z) < 0$ in Eq. (7) with opposite sign, which also shows that the soliton amplitude decays at the same rate as the respective power amplitude with inclusion of phase modulation and loss. This is also seen from 2D plot in Fig. 5.

As a result of chirping, the pulse broadens as it propagates along the fiber and as a result of damping; the amplitude or depth of the pulse reduces. Therefore, the solitary pulse amplitude is found to increase or decrease, depending on the sign of *g(z)* i.e. gain or loss, with same amount of broadening in the pulse width during its propagation such that the area of the pulse remains constant, in consistent with those obtained in earlier investigations [8,14].

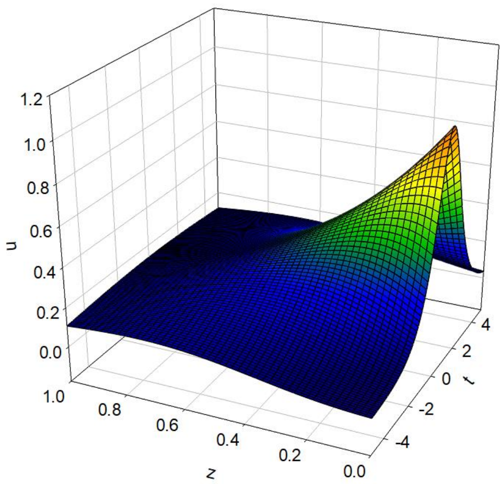

Figure 4: Amplitude profile for the bright 1SS in inhomogeneous fiber (with loss i.e. $g < 0$): $|u(z,t)|$ as a function of *z* and *t* (a 3D plot of Eq. (30) with $\varphi_{20} = 0.5,\ g = 0.1z^{-1}\ and\ \Omega = 0$).

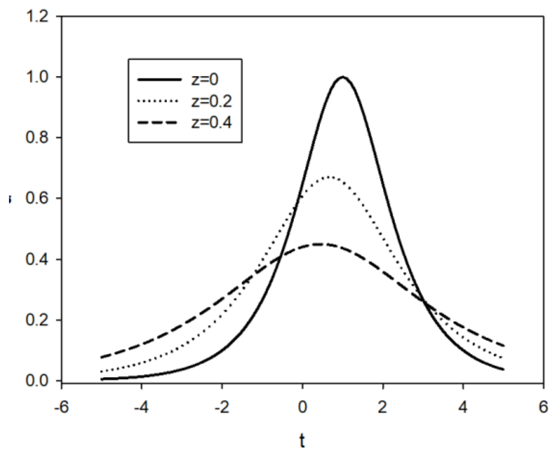

Figure 5: Amplitude profile for the bright 1SS in inhomogeneous fiber (with loss i.e. $g < 0$): $|u(z,t)|$ as a function of *t* for different values of *z* (a 2D plot of Eq. (30)).

3. CONCLUSIONS

The nonlinear wave propagation in inhomogeneous optical fibers was studied in the anomalous dispersive regime. We modified the NLS equation in order to include the inhomogeneous effects, i.e. phase modulation and fiber loss/ damping effects. The complete integrability conditions were derived for gain and loss cases and the modified NLS equations in the anomalous dispersive regime were exactly solved to construct bright soliton solutions. For the inhomogeneous case, we observed that the amplitude of the pulse is increased for gain and is decreased for damping as it propagates along the fiber. The pulse width was found to broadening at each stage of propagation such that the area of the pulse envelope remains conserved. The results were discussed.

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