DOI: http://doi.org/10.4038/sljp.v23i2.8090

INSTITUTE OF PHYSICS – SRI LANKA

Research Article

Studies on solitary waves in non-isothermal plasma

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ABSTRACT

In single and two-temperature non-isothermal electron plasma along with warm positive ion, warm negative ion and warm positron, propagation of non-linear ion-acoustic solitary waves are investigated under the variation of different plasma parameters. In this paper, we have studied the first, second and third order compressive non-linear solitary wave solutions along with their respective amplitudes by a very familiar approach known as 'tanh – method' with existence condition for solitary wave solutions. Again we are trying to compare the amplitudes between isothermal and non-isothermal two-temperature electron plasma. In addition to this, for two-temperature non-isothermal electron plasma - variation of energy connected with those amplitudes, fast and slow ion-accoustic phase velocity and profiles of Sagdeev potential for temperature variation of positive and negative ions are critically discussed. All the results obtained are represented graphically.

Key Words: Non-isothermal electron, isothermal electron, Pseudopotential method, drift motion, higher order amplitudes, energy and phase velocity.

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1. INTRODUCTION

In Plasma Physics, the important non-linear wave structure "Solitons" are produced due to the exact balance of non-linearity and dispersion. Physicists normally represent this solitary waves theoretically by the Kortweg-de-Vries (KdV) or Modified KdV equation and the Sagdeev pseudopotential equation which were supported experimentally by some renowned scientists. According to the characteristics of solitary waves, some of these are classified as dark solitons, bright solitons, envelope solitions, dressed solitons and like those in presence of isothermal or non-isothermal electron plasma. Besides this, another important kind of plasma, namely, "negative ion plasma" when included in the system, will drastically change the property of the soliton nature. During the last few years, "negative ion plasma" attracted much attention for their importance in laboratory experiments and space plasma observations. It is found by space plasma observations that considerable number of negative ions exist in the earth's ionosphere, neutral beam sources, plasma processing reactors and in low temperature laboratory experiments. Thus negative, ion plasma is inevitable for our consideration. This negative ion together with positive ion, positron with two-temperature and single temperature isothermal electron plasma have been studied theoretically and experimentally by many authors[1-12] for the propagation of ion-acoustic solitary waves. After showing the characteristics of solitary waves in the plasmas, they have reviewed the behaviour of the solitons in various types of plasma models making a close relationship between theoretical and experimental results. Later on, some physicists[13-14] investigated experimentally the existence and nature of the said solitary waves in a multicomponent plasma with that negative ions. On the other hand, in non-isothermal plasma, Schamel[15-16] first studied the ion-acoustic solitary waves. Considering this concept of non-isothermal electron plasma, Kalita and Bujarbarua[17] extended the works of previous authors for higher order effects in ion-acoustic solitary waves. In realistic situation, since the negative ions together with positive ions in the plasma possess some finite temperature[18-19] , they will affect the nature of solitary waves. The plasma with negative ions and positive ions exhibit both compressive and rarefactive solitons in isothermal case, whereas non-isothermal plasma gives different result. It is observed that, in the presence of a small percentage of non-isothermality, the solitary wave equation exhibits both kinds of solitons. Tagare and Reddy[20] studied higher order non-linearity in presence of negative ions, positive ions and non-isothermal electrons. Due to resonant electrons, the plasma actually behaves non-isothermally and gives more interesting results. Das et al[21] and others[22-23] investigated the effects of two-temperature non-isothermal electrons[24] for formation of solitons. Majumdar et al[25] studied higher order contributions to ion-acoustic solitons with two types of cold positive ions and two-temperature non-isothermal electrons by Bogoliubov – Mitropolsky method from which they got a modified solution of $Sech^4$ type with first, second and third order amplitudes. Chattopadhyay et al[26] studied first and second order solitary wave solutions in single temperature non-isothermal electron plasma for cold positive and cold negative ions with second order W-type soliton. Following Chattopadhyay et al[27] , the present author took warm positive ions and warm negative ions with drifts, warm positron in non-isothermal plasma and obtained two types of phase velocities (V_F and V_S) from the existence condition of solitary wave solution. In this paper first, second and third order compressive solitary wave solutions are obtained by the well-known ' tanh – method' with their corresponding amplitudes for single and two-temperature non-isothermal electron[28-29].The third order solutions are known as the spiky and explosive solitary wave solutions[29] .This theoretical observations highlight the occurrence of spiky solitary waves along with the explosion of the solitary waves related to satellite observations in interplanetary space plasmas. In addition to this phenomena, the said first, second and third order amplitudes are obtained similarly for two-temperature isothermal plasma in the presence of positron and we have tried to compare these amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma.

The purpose of this paper is to establish a correlation for comparison of amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma graphically, loss of energy due to change of amplitude in two-temperature non-isothermal plasma by graphical approach. Slow and fast mode phase velocities of that solitary waves are analysed in detail with graphical representation and finally nature of Sagdeev potential well is determined graphically by its profiles.

The presentation of the paper is arranged in the following manner:

In sec.2, the Sagdeev pseudopotential function $[\psi(\phi)]$ for single and two temperature non-isothermal electron plasma are calculated . The critical values of the velocity of solitary waves are found from the existence condition of solitary wave solutions. Fast and slow mode phase velocities are also discussed in this section. First, second and third order compressive solitary wave solutions with their corresponding amplitudes are calculated in sec. 3(a). In sec. 3(b), the same first, second and third order amplitudes are calculated similarly for two-temperature isothermal plasma. The loss of energy for first, second and third order amplitudes related to two-temperature non-isothermal electron plasma are also calculated here. Sec. 4 contains a comprehensive discussion and concluding remarks are given in sec.5.

2. FORMULATION

The normalised basic equations for a collisionless, unmagnetized warm plasma consisting of warm positive ion, warm negative ion, warm positron and warm non- isothermal electron in unidirectional propagation are

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial x}\left(n_\alpha u_\alpha\right)=0 \tag{1}$$

$$\frac{\partial u_\alpha}{\partial t}+u_\alpha\frac{\partial u_\alpha}{\partial x}+\frac{\sigma_\alpha}{Q_\alpha n_\alpha}\frac{\partial p_\alpha}{\partial x}=-\frac{Z_\alpha}{Q_\alpha}\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial p_\alpha}{\partial t}+u_\alpha\frac{\partial p_\alpha}{\partial x}+3p_\alpha\frac{\partial u_\alpha}{\partial x}=0 \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_e-\sum_\alpha Z_\alpha n_\alpha \ -n_p \tag{4}$$

In this case $n_\alpha, u_\alpha, \sigma_\alpha, Q_\alpha, p_\alpha, Z_\alpha, n_e, \phi$, n_p, t and x are, respectively, the number density , velocity, temperatures, mass ratios of negative to positive ions, pressure, charge, concentration of non-isothermal electrons, electrostatic potential, concentration of positron, time and distance.

The charge neutrality condition of the plasma is

$$\sum_\alpha n_{\alpha 0}Z_\alpha \ +n_{p0} \ =n_{e0} \tag{5}$$

Here $\sigma_\alpha=\frac{T_\alpha}{T_e}$ [T_α = Temperature of positive (negative) ion, T_e = Temperature of electron], Q_α= $\frac{m_\alpha}{m_i}$ [m_α = mass of positve (negtive) ion, m_i = mass of positive ion], Q_α = 1 and $Z_\alpha=1$ for positive ion (α = i), $Q_\alpha=Q$ and $Z_\alpha=-Z$ for negative ion (α = j).

The above equations (1) to (4) are normalised by the following ways: densities n_α, n_e, n_p by equilibrium value n_0, velocities u_α by $\sqrt{\frac{KT_e}{m_\alpha}}$ where m_α is the mass of ion and K is the

Boltzmann constant, pressure p_α by $p_0 = n_0 T_\alpha$, potential by $\frac{KT_e}{e}$, time by $\sqrt{\frac{m_\alpha}{4\pi e^2 n_o}}$ and the distance by the Debye length $\sqrt{\frac{KT_e}{4\pi n_o e^2}}$.

The boundary conditions are

$$n_\alpha \to n_{\alpha o}, u_\alpha \to u_{\alpha o}, p_\alpha \to 1, n_e \to 1, n_p \to \chi \text{ and } \phi \to 0 \text{ at } |x| \to \infty .$$

In equations (1) to (4) we have used the well-known transformation $\eta = x - Vt$, where V is the velocity of the solitary wave .

After simplifying some steps and following Chattopadhyay[30] we get finally from equations (1) to (4)

$$\frac{d^2\phi}{d\eta^2} = n_e - \frac{1}{2}\Sigma Z_\alpha \sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{3\sigma_\alpha}} \left[\sqrt{(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}})^2 - \frac{2Z_\alpha \phi}{Q_\alpha}} - \sqrt{(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}})^2 - \frac{2Z_\alpha \phi}{Q_\alpha}} \right] - \chi e^{-\sigma_p \phi} \quad (6)$$

$$\text{Where } n_\alpha = \frac{1}{2}\sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{3\sigma_\alpha}} \left[\sqrt{(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}})^2 - \frac{2Z_\alpha \phi}{Q_\alpha}} - \sqrt{(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}})^2 - \frac{2Z_\alpha \phi}{Q_\alpha}} \right] \quad (7)$$

For real n_α , the following restrictions on ϕ is

$$-\frac{Q}{2Z}\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 < \phi < \frac{1}{2}\left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 \quad (8)$$

The normalised positron density (n_p) is $n_p = \chi\, e^{-\sigma_p \phi}$ (9)

Where $\sigma_p = \frac{T_e}{T_p}$ is the temperature ratio of T_e (electron temperature) and T_p (positron temperature), χ is the positron density at $\varphi = 0$.

We now want to discuss first, second and third order solitary wave solutions and their corresponding amplitudes for single and two-temperature non-isothermal electron plasma.

The normalised electron density n_e for single temperature non-isothermal electron plasma is

$$n_e = 1 + \varphi - \frac{4}{3} b_1 \phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15} b_2 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 + \ldots\ldots\ldots\ldots \quad (10)$$

where $b_1 = \frac{1-\beta}{\sqrt{\pi}}$, $b_2 = \frac{1-\beta^2}{\sqrt{\pi}}$, $\beta = \frac{T_{ef}}{T_{et}}$, $0 < b_1 < \frac{1}{\sqrt{\pi}}$ and $0 < b_2 < \frac{1}{\sqrt{\pi}}$

T_{ef} is the constant temperature of free electrons and T_{et} is the constant temperature of trapped electrons and β is their ratio.

By equations (6), (9) and (10) we get

$$\frac{d^2\phi}{d\eta^2} = 1+\varphi - \frac{4}{3}b_1\phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15}b_2\,\phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 + \ldots\ldots\ldots\ldots$$

$$-\frac{1}{2}\Sigma Z_\alpha\sqrt{\frac{Q_\alpha\, n_{\alpha0}^3}{3\sigma_\alpha}}\left[\sqrt{(V-u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha0}}})^2-\frac{2Z_\alpha\phi}{Q_\alpha}}-\sqrt{(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\, n_{\alpha0}}})^2-\frac{2Z_\alpha\phi}{Q_\alpha}}\right]$$

$$-\chi e^{-\sigma_p\phi} \qquad (11)$$

Again the normalised electron density n_e for two-temperature non-isothermal electron plasma is

$$n_e = 1+\phi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{3}{2}}}\phi^{\frac{3}{2}}+\frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{\left(\mu+\nu\beta_1\right)^2}\phi^2-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{5}{2}}}\phi^{\frac{5}{2}}+\frac{1}{6}\frac{(\mu+\nu\beta_1^3)}{\left(\mu+\nu\beta_1\right)^3}\phi^3-\cdots$$

(12)

Where $b_l = \frac{1-\beta_l}{\sqrt{\pi}}$, $b_h = \frac{1-\beta_h}{\sqrt{\pi}}$, $b_l^{(1)} = \frac{1-\beta_l^2}{\sqrt{\pi}}$, $b_h^{(1)} = \frac{1-\beta_h^2}{\sqrt{\pi}}$, $\beta_1 = \frac{T_{el}}{T_{eh}}$, $\beta_l = \frac{T_{el}}{T_{el,t}}$, $\beta_h = \frac{T_{eh}}{T_{eh,t}}$

μ , ν , β_l , β_h and β_1 are respectively the unperturbed number density of low temperature and high temperature electrons , temperature ratio of free and trapped electrons in low temperature, temperature ratio of free and trapped electrons in high temperature, temperature ratio of free electrons in low and high temperature .

when $\beta_l \to \beta$, $\mu \to 1$, $\nu \to 0$, $\beta_h \to 1$, then concentration of two-temperature non-isothermal electron plasma reduces t o non-isothermal single temperature electron plasma.

We can now write the differential equation for positive and negative ion from equations (6),(9) and (12)

$$\frac{d^2\phi}{d\eta^2}=$$

$$1+\phi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{3}{2}}}\phi^{\frac{3}{2}}+\frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{\left(\mu+\nu\beta_1\right)^2}\phi^2-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{5}{2}}}\phi^{\frac{5}{2}}+\frac{1}{6}\frac{(\mu+\nu\beta_1^3)}{\left(\mu+\nu\beta_1\right)^3}\phi^3-\cdots$$

$$-\frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left[\left\{\left(V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2-2\phi\right\}^{\frac{1}{2}}-\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2-2\phi\right\}^{\frac{1}{2}}\right]$$

$$+\frac{ZQ^{\frac{1}{2}}n_{j0}^{\frac{3}{2}}}{2\sqrt{3\sigma_j}}\left[\left\{\left(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2+\frac{2Z\phi}{Q}\right\}^{\frac{1}{2}}-\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^2+\frac{2Z\phi}{Q}\right\}^{\frac{1}{2}}\right]$$

$$-\chi e^{-\sigma_p\phi} \tag{13}$$

Equation (11) or (13) can be written in the following energy equation:

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2+\psi(\varphi)=0 \tag{14}$$

Where $\psi(\varphi)$ is the Sagdeev pseudopotential function.

The Sagdeev pseudopotential function $\psi(\varphi)$ for single temperature non-isothermal electron plasma is

$$\psi(\varphi)=\left[-\varphi-\frac{1}{2}\phi^2+\frac{8}{15}b_1\phi^{\frac{5}{2}}-\frac{1}{6}\phi^3+\frac{16}{105}b_2\,\phi^{\frac{7}{2}}-\frac{1}{24}\phi^4+\ldots\ldots\ldots\ldots\right]$$

$$+\frac{1}{6}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\left[\begin{array}{c}\left\{\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2-2\phi\right\}^{\frac{3}{2}}-\left\{\left(V-u_{io}+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2-2\phi\right\}^{\frac{3}{2}}\\ +\left(V-u_{io}+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3-\left(V-u_{io}-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3\end{array}\right]$$

$$+\frac{1}{6}\sqrt{\frac{Q^3n_{io}^3}{3\sigma_j}}\left[\begin{array}{c}\left\{\left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2+\frac{2Z\phi}{Q}\right\}^{\frac{3}{2}}-\left\{\left(V-u_{jo}+\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2+\frac{2Z\phi}{Q}\right\}^{\frac{3}{2}}\\ +\left(V-u_{jo}+\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3-\left(V-u_{jo}-\sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3\end{array}\right]+\frac{\chi}{\sigma_p}\left(1-e^{-\sigma_p\phi}\right) \tag{15}$$

And for two-temperature non-isothermal electron plasma the expression for $\psi(\varphi)$ is

$$\psi(\varphi)=\left[-\varphi-\frac{1}{2}\phi^2+\frac{8}{15}\frac{\mu b_l+\nu\, b_h\beta_1^{\frac{3}{2}}}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\phi^{\frac{5}{2}}-\frac{1}{6}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2}\phi^3+\frac{16}{105}\frac{\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\phi^{\frac{7}{2}}\right.$$

$$\left.-\frac{1}{24}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3}\phi^4+\ldots\ldots\right]$$

$$+\frac{1}{6}\Sigma\sqrt{\frac{Q_\alpha^3\,n_{\alpha0}^3}{3\sigma_\alpha}}\left[\left\{\left(V-\ u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}-\left\{\left(V-\ u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}\right.$$
$$\left.+\,(V-\ u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha0}}})^3-\ (V-\ u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha\ n_{\alpha0}}})^3\right]$$
$$+\frac{\chi}{\sigma_p}\,(1-e^{-\sigma_p\phi}) \qquad (16)$$

In equation (14), the term $\frac{1}{2}(\frac{d\phi}{d\eta})^2$ may be assumed as the kinetic energy (K.E.) of a particle of unit mass at position ϕ and time η whereas the potential energy (P.E.) of the same particle at that instant is $\psi(\varphi)$.

For solitary wave solution the Sagdeev pseudopotential function $\boldsymbol{\psi}(\phi)$ satisfy the following conditions:

i) $\psi(\phi)|_{\phi=0} = 0$, $\left.\frac{\partial\psi}{\partial\phi}\right|_{\phi=0} = 0$

ii) $\left.\frac{\partial^2\psi}{\partial\phi^2}\right|_{\phi=0} < 0$

iii) $\psi(\phi)|_{\phi=\phi_m} = 0$ where ϕ_m is some max. value of ϕ

iv) $\psi(\phi) < 0$ in $0 < |\phi| < |\phi_m|$ (17)

The condition[1] for the existence of a solitary wave solution is

$$\frac{n_{i0}}{(V-u_{i0})^2-\frac{3\sigma_i}{n_{i0}}}+\frac{Z^2 n_{j0}}{Q(V-u_{j0})^2-\frac{3\sigma_j}{n_{j0}}} < 1+\chi\,\sigma_p \qquad (18)$$

For $\chi = 0$, inequality (18) supports Ref.[26] for warm ion ($\sigma_\alpha \neq 0$) plasma and also supports Ref.[26] for cold ion ($\sigma_\alpha = 0$) plasma .

In absence of positive and negative ion streams ($u_{i0} = 0$, $u_{j0} = 0$), we get the critical value of V (say V = V_c) from the inequation (18) as

$$\frac{n_{i0}}{V_c^2-\frac{3\sigma_i}{n_{i0}}}+\frac{Z^2 n_{j0}}{QV_c^2-\frac{3\sigma_j}{n_{j0}}} = 1+\chi\,\sigma_p \qquad (19)$$

After simplifying we get finally, $V_S^2 = V_c^2$ = Slow mode phase velocity.

$$=\frac{3}{2}\left[\frac{\left\{\left(\frac{\sigma_i}{n_{i0}}+\frac{\sigma_j}{Q\,n_{j0}}\right)+\frac{n_{i0}+\frac{Z^2 n_{j0}}{Q}}{3(1+\chi\sigma_p)}\right\}}{-\sqrt{\left\{\left(\frac{\sigma_i}{n_{i0}}+\frac{\sigma_j}{Q\,n_{j0}}\right)+\frac{n_{i0}+\frac{Z^2 n_{j0}}{Q}}{3(1+\chi\sigma_p)}\right\}^2-\frac{4}{Q}\left\{\frac{\sigma_i\sigma_j}{n_{i0}n_{j0}}+\frac{\frac{n_{i0}\sigma_j}{n_{j0}}+\frac{Z^2 n_{j0}\sigma_i}{n_{i0}}}{3(1+\chi\sigma_p)}\right\}}}\right] \qquad (20)$$

$V_F^2 = V_c^2$ = Fast mode phase velocity

$$= \frac{3}{2}\left[\begin{array}{c}\left\{\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Q\,n_{j0}}\right) + \frac{n_{i0}+\frac{Z^2 n_{j0}}{Q}}{3(1+\chi\sigma_p)}\right\} \\ \hline + \sqrt{\left\{\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Q\,n_{j0}}\right) + \frac{n_{i0}+\frac{Z^2 n_{j0}}{Q}}{3(1+\chi\sigma_p)}\right\}^2 - \frac{4}{Q}\left\{\frac{\sigma_i\sigma_j}{n_{i0}n_{j0}} + \frac{\frac{n_{i0}\sigma_j}{n_{j0}} + \frac{Z^2 n_{j0}\sigma_i}{n_{i0}}}{3(1+\chi\sigma_p)}\right\}}\end{array}\right] \tag{21}$$

Equations (20) and (21) are more general results than given in Ref.[29]. In absence of positron and negative ion, equations (20) and (21) reduce to the form $V_c = \sqrt{1 + 3\sigma_i}$ which is known as the normal ion-acoustic phase velocity when $n_{i0} \rightarrow 1$, but when only positive ion stream $(u_{i0} \neq 0)$is considered without taking positron and negative ion then we get $V_c = u_{i0} \pm \sqrt{1 + 3\sigma_i}$ which supports Ref.[18].

3. SOLITARY WAVE SOLUTIONS AND CORRESPONDING AMPLITUDES

In this section, we are trying to find out three types of solitary wave solutions in two-temperature non-isothermal and two-temperature isothermal electron plasma by the perturbative new approach "tanh-method" and their corresponding amplitudes are then compared under the variation of different plasma parameters. Now for non-isothermal two-temperature electron plasma we get finally from equation (13)

$$\frac{d^2\phi}{d\eta^2} = B_1\,\phi - B_2\phi^{\frac{3}{2}} + B_3\phi^2 - B_4\phi^{\frac{5}{2}} + B_5\,\phi^3 - \ldots\ldots\ldots\ldots \tag{22}$$

Where ,

$$B_1 = \left[1 - n_{i0}\left\{(V - u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}\right\}^{-1} - Z^2 n_{j0}\left\{Q(V - u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}\right\}^{-1} + \chi\sigma_p\right]$$

$$B_2 = \frac{4}{3}\,\frac{\left(\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{3}{2}}}$$

$$B_3 = \frac{1}{2}\left[\begin{array}{l}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V - u_{i0} - \sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3} - (V - u_{i0} + \sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3}\right\} \\ + \frac{Z^3 n_{j0}^{\frac{3}{2}}}{2Q\sqrt{3\sigma_{jQ}}}\left\{(V - u_{j0} - \sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3} - (V - u_{j0} + \sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3}\right\} - \chi\sigma_p^2\end{array}\right]$$

$$B_4 = \frac{8}{15}\,\frac{\left(\mu b_l^{(1)} + \nu b_h^{(1)} \beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}}$$

$$B_5 = \frac{1}{2}\left[\begin{array}{l} \frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5}\right\} \\ +\frac{Z^4 n_{j0}^{\frac{3}{2}}}{2Q^2\sqrt{3\sigma_{jQ}}}\left\{(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5} - (V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5}\right\} + \frac{\chi\sigma_p^3}{3} \end{array}\right] \quad (23)$$

For $\sigma_i = 0$ and $\sigma_j = 0$, the new form of the expression of B_1 , B_2 , B_3 , B_4 and B_5 in (23) are given below:

$$B_1 = \left[1 - \frac{n_{i0}}{(V-u_{i0})^2} - \frac{Z^2 n_{j0}}{Q(V-u_{j0})^2} + \chi\sigma_p\right]$$

$$B_2 = \frac{4}{3}\frac{\left(\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{3}{2}}}$$

$$B_3 = \frac{1}{2}\left[\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{3n_{i0}}{(V-u_{i0})^4} - \frac{3Z^3 n_{j0}}{Q^2(V-u_{j0})^4} - \chi\sigma_p^2\right]$$

$$B_4 = \frac{8}{15}\frac{\left(\mu b_l^{(1)} + \nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}}$$

$$B_5 = \frac{1}{2}\left[\frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{5n_{i0}}{(V-u_{i0})^6} - \frac{5Z^4 n_{j0}}{Q^3(V-u_{j0})^6} + \frac{\chi\sigma_p^3}{3}\right]$$

Equation (22) for two-temperature non-isothermal electron plasma reduces to two-temperature isothermal case when $\beta = 1$. Under this condition, we can easily find out the first and second order isothermal solitary wave solution and amplitude[1] . For applying the perturbative new approach "tanh-method" on equation (22), we use the substitution Z = tanh(η) and W(Z) = ϕ(η) from which we get a Fuchsian – like non-linear ordinary differential equation and thereafter a Frobeneous series solution is found. After returning to the original variable we get the first, second and third order solitary wave solutions with their corresponding amplitudes. As we have paid much attention on higher order amplitudes we can write straightway the solution of the differential equation (22), instead of showing the actual calculation.

We can now discuss separately two different cases of solutions namely non-isothermal and isothermal electron plasma.

3.1.Solutions and amplitudes of two-temperature non-isothermal electron plasma

In case of two-temperature non-isothermal electron plasma, the first, second and third order solitary wave solutions and their corresponding amplitudes are shown in the following way:

Taking terms upto $\phi^{\frac{3}{2}}$ from equation (22) we get

$$\int \frac{d\phi}{\sqrt{B_1\phi^2 - \frac{4}{5}B_2\phi^{\frac{5}{2}}}} = \pm \int d\eta$$

Integrating with proper boundary conditions we get the first order[3,28] solitary wave solution (ϕ_1) as

$$\phi_1 = \left(\frac{5B_1}{4B_2}\right)^2 Sech^4\left(\sqrt{\frac{B_1}{16}}\,\eta\right) \tag{24a}$$

The first order amplitude (ϕ_{01}) of that solitary wave solution (24a) is

$$\phi_{01} = \left(\frac{5B_1}{4B_2}\right)^2 \tag{24b}$$

Similarly taking terms upto ϕ^2 from equation (22) and simplifying we get

$$\int \frac{d\phi}{\sqrt{B_1\phi^2 - \frac{4}{5}B_2\phi^{\frac{5}{2}} + \frac{2}{3}B_3\phi^3}} = \pm \int d\eta$$

Integrating the above relation with proper boundary conditions we get the second order[3,28] solitary wave solution (ϕ_2) as

$$\phi_2 = \left[\frac{3B_2}{5B_3} \pm \sqrt{\left\{\left(\frac{3B_2}{5B_3}\right)^2 - \frac{3B_1}{2B_3}\right\}}\, Sech\left(\frac{1}{2}\sqrt{B_1 - \frac{9B_2^2}{25B_3}}\,\eta\right)\right]^2 \tag{25a}$$

The second order amplitude (ϕ_{02}) of the solitary wave solution (25a) is

$$\phi_{02} = \left[\frac{3B_2}{5B_3} \pm \sqrt{\left\{\left(\frac{3B_2}{5B_3}\right)^2 - \left(\frac{3B_1}{2B_3}\right)\right\}}\right]^2 \tag{25b}$$

Again from equation (22) taking terms upto $\phi^{\frac{5}{2}}$ we get similarly

$$\int \frac{d\phi}{\sqrt{B_1\phi^2 - \frac{4}{5}B_2\phi^{\frac{5}{2}} + \frac{2}{3}B_3\phi^3 - \frac{4}{7}B_4\phi^{\frac{7}{2}}}} = \pm \int d\eta$$

Integrating similarly with proper boundary conditions, we get the third order[3,28] solitary wave solution (ϕ_3) as

$$\phi_3 = \phi_S = \phi_0^2 Sech^4\left[\left(\frac{\phi_0}{\phi_0 - \sqrt{\phi_S}}\right)^{\frac{1}{2}} \pm \frac{1}{2}\sqrt{\frac{B_4\phi_0^3}{7}}\,(\eta - \eta_0) + Sech^{-1}\left(\frac{\sqrt{\phi_m}}{\phi_0}\right)^{\frac{1}{2}} - \left(\frac{\phi_0}{\phi_0 - \sqrt{\phi_m}}\right)^{\frac{1}{2}}\right] \tag{26a}$$

In this case, ϕ_3 is an implicit function of η and is known as spiky solitary wave solution in $\sqrt{\phi_3} > 0$.

Similarly for $\sqrt{\phi_3}\ < 0$ the explosive (ϕ_3) solitary wave[28] solution in implicit form is

$$\phi_3=\phi_E=\phi_0^2 Cosech^4\left[\left(\frac{\phi_0}{\phi_0-\sqrt{\phi_E}}\right)^{\frac{1}{2}} \pm \frac{1}{2}\sqrt{\frac{B_4\phi_0^3}{7}}\ (\eta - \eta_0) + Cosech^{-1}\left(\frac{\sqrt{\phi_m}}{\phi_0}\right)^{\frac{1}{2}} - \left(\frac{\phi_0}{\phi_0-\sqrt{\phi_m}}\right)^{\frac{1}{2}}\right] \tag{26b}$$

Where $\phi_0 = \frac{7B_3}{18B_4}$ and ϕ_m is the optimal amplitude of the solitary wave represented by the profiles of ϕ_3 (= ϕ_S or ϕ_E)

The third order amplitude (ϕ_{03}) of the spiky and explosive solitary wave solution (26a) or (26b) is

$$\phi_{03} = \left[\left\{\frac{1}{2}(a_1+b_1)\right\}^{\frac{1}{3}} + \left\{\frac{1}{2}(a_1-b_1)\right\}^{\frac{1}{3}} + \frac{7B_3}{18B_4}\right]^2 \tag{27}$$

Where $a_1 = \left[\frac{1}{4}\left(\frac{7B_3}{9B_4}\right)^3 - \frac{49}{90}.\frac{B_2B_3}{B_4^2} + \frac{7B_1}{4B_4}\right]$ and $b_1^2 = \left|\left[a_1^2 + \frac{4}{27}\left\{\left(\frac{7B_2}{5B_4}\right) - \frac{1}{3}\left(\frac{7B_3}{6B_4}\right)^2\right\}^3\right]\right|$

Finally we can get easily the first, second and third order amplitudes for non-isothermal single temperature electron plasma from two-temperature non-isothermal electron plasma when μ = 1 and ν = 0.

3.2. Solutions and amplitudes of two-temperature isothermal electron plasma

For two-temperature isothermal electron plasma, equation (22) can be reduced to the following form in order to obtain the first, second and third order solitary wave solutions and their corresponding amplitudes:

$$\frac{d^2\phi}{d\eta^2} = B_1\,\phi + B_3\phi^2 + B_5\,\phi^3 + B_7\,\phi^4 + \ldots\ldots\ldots.. \tag{28}$$

Where $B_1 = \left[1 - n_{i0}\left\{(V-u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}\right\}^{-1} - Z^2 n_{j0}\left\{Q(V-u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}\right\}^{-1} + \chi\sigma_p\right]$

$$B_3 = \frac{1}{2}\left[\begin{array}{l}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-3}\right\}\\ +\frac{Z^3 n_{j0}^{\frac{3}{2}}}{2Q\sqrt{3\sigma_{jQ}}}\left\{(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3} - (V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-3}\right\} - \chi\sigma_p^2\end{array}\right]$$

$$B_5 = \frac{1}{2}\left[\begin{array}{l}\frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-5}\right\}\\ +\frac{Z^4 n_{j0}^{\frac{3}{2}}}{2Q^2\sqrt{3\sigma_{jQ}}}\left\{(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5} - (V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-5}\right\} + \frac{\chi\sigma_p^3}{3}\end{array}\right]$$

$$B_7 = \begin{bmatrix} \frac{1}{24}\frac{\mu+\nu\beta_1^4}{(\mu+\nu\beta_1)^4} - \frac{5n_{i0}^{\frac{3}{2}}}{16\sqrt{3\sigma_i}}\left\{(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-7} - (V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}})^{-7}\right\} \\ +\frac{5Z^5 n_{j0}^{\frac{3}{2}}}{16Q^3\sqrt{3\sigma_{jQ}}}\left\{(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-7} - (V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}})^{-7}\right\} - \frac{\chi\sigma_p^4}{24} \end{bmatrix} \quad (29)$$

In this situation we are not interested to find the actual calculation of first, second and third order solitary wave solution from equation (28). Instead, we only try to compare the first, second and third order amplitudes between non-isothermal and isothermal two-temperature electron plasma. For this reason, we write straightway the three kinds of solutions and their corresponding amplitudes by conventional way.

Taking terms up to ϕ^2 we get from equation (28)

$$\frac{d^2\phi}{d\eta^2} = B_1\,\phi + B_3\phi^2 \quad (30)$$

The corresponding first order solitary wave solution (ϕ_1) is

$$\phi_1 = \phi_{01}\, Sech^2\left(\sqrt{\frac{B_1}{4}}\,\eta\right) \quad (30a)$$

Where first order amplitude = $\phi_{01} = \left(-\frac{3B_1}{2B_3}\right)$ (30b)

Again taking terms up to ϕ^3 we get from equation (28)

$$\frac{d^2\phi}{d\eta^2} = B_1\,\phi + B_3\phi^2 + B_5\,\phi^3 \quad (31)$$

The second order solitary wave solution (ϕ_2) is

$$\phi_2 = \left[\left(-\frac{B_3}{3B_1}\right) + \sqrt{\frac{B_3^2}{9B_1^2} - \frac{B_5}{2B_1}}\;\; Cosh\left(\sqrt{\frac{B_1}{16}}\;\eta\right)\right]^{-1} \quad (32a)$$

The corresponding second order amplitude (ϕ_{02}) is

$$\phi_{02} = \frac{2B_3}{3B_5}\left[-1 + \sqrt{1 - \frac{9B_1B_5}{2B_3^2}}\right] \quad (32b)$$

For better approximation regarding the solution of the differential equation (28), taking terms upto ϕ^4 we get finally from equation (28)

$$\frac{d^2\phi}{d\eta^2} = B_1\,\phi + B_3\phi^2 + B_5\,\phi^3 + B_7\,\phi^4 \quad (33)$$

In the case of $\phi > 0$, the third order spiky solitary wave solution (ϕ_S) in explicit form is

$$\phi_3 = \phi_S = \phi_0 Sech^2\left[\left(\frac{\phi_0}{\phi_0-\phi_S}\right)^{\frac{1}{2}} \pm \frac{1}{2}\sqrt{\left(-\frac{2B_7}{5}\right)}\;\phi_0^{\frac{3}{2}}(\eta-\eta_0) + Sech^{-1}\left(\frac{\phi_m}{\phi_0}\right)^{\frac{1}{2}} - \left(\frac{\phi_0}{\phi_0-\phi_m}\right)^{\frac{1}{2}}\right]$$

(34)

Also the third order explosive solitary wave solution (ϕ_E) in explicit form for ϕ< 0 is

$$\phi_3=\phi_E=\phi_0 Cosech^2\left[\left(\frac{\phi_0}{\phi_0-\phi_E}\right)^{\frac{1}{2}} \pm \frac{1}{2}\sqrt{\left(-\frac{2B_7}{5}\right)}\ \phi_0^{\frac{3}{2}}(\eta-\eta_0)+Cosech^{-1}\left(\frac{\phi_m}{\phi_0}\right)^{\frac{1}{2}}-\left(\frac{\phi_0}{\phi_0-\phi_m}\right)^{\frac{1}{2}}\right] \tag{35}$$

Where $\phi_0 = \left(-\frac{5B_5}{12B_7}\right)$ and ϕ_m is the optimal amplitude of the solitary wave represented by the profiles of ϕ_3 (= ϕ_S or ϕ_E)

The third order amplitude(ϕ_{03}) is

$$\phi_{03} = \left[\left\{\frac{1}{2}(S_1+S_2)\right\}^{\frac{1}{3}}+\left\{\frac{1}{2}(S_1-S_2)\right\}^{\frac{1}{3}}-\frac{5B_5}{12B_7}\right] \tag{36}$$

Where $S_1 = -\frac{5}{12}\left[\frac{25}{72}\left(\frac{B_5}{B_7}\right)^3-\frac{5}{3}\left(\frac{B_3}{B_7}\right)\left(\frac{B_5}{B_7}\right)+6\left(\frac{B_1}{B_7}\right)\right]$

$$S_2^2 = \left|S_1^2+\ 4\left[\frac{5}{18}\left\{2\left(\frac{B_3}{B_7}\right)-\frac{5}{8}\left(\frac{B_5}{B_7}\right)^2\right\}\right]^3\right| \tag{37}$$

Similarly, the first, second and third order amplitudes for single temperature isothermal electron plasma will be obtained from two-temperature isothermal electron plasma when μ = 1 and ν = 0.

4. DETERMINATION OF ENERGY IN NON-ISOTHERMAL PLASMA

Energy is a very sensitive entity in relation to the solitary wave solutions. In two-temperature non-isothermal electron plasma, we are trying to calculate the required energy from the difference of higher order amplitude to its next lower order amplitude. Once we find the successive orders of solitary wave solutions, we can then obtain different orders of amplitudes. Actually the amplitude of vibration is not always constant but it decreases rapidly. This decrease is due to the loss of energy. The loss of energy (ΔE_r)for first and second order amplitudes can be expressed as

$$\Delta E_r = \phi_{02} - \phi_{01} = \left[\frac{3B_2}{5B_3} \pm \sqrt{\left\{\left(\frac{3B_2}{5B_3}\right)^2-\left(\frac{3B_1}{2B_3}\right)\right\}}\right]^2 - \left(\frac{5B_1}{4B_2}\right)^2 \tag{38}$$

And the loss of energy (ΔE_S) for second and third order amplitudes can be similarly expressed as

$$\Delta E_S = \phi_{03} - \phi_{02} = \left[\left\{\frac{1}{2}(a_1 + b_1)\right\}^{\frac{1}{3}} + \left\{\frac{1}{2}(a_1 - b_1)\right\}^{\frac{1}{3}} + \frac{7B_3}{18B_4}\right]^2 - \left[\frac{3B_2}{5B_3} \pm \sqrt{\left\{\left(\frac{3B_2}{5B_3}\right)^2 - \left(\frac{3B_1}{2B_3}\right)\right\}}\right]^2 \tag{39}$$

5. DISCUSSION

In this paper, we studied the first (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of ion-acoustic solitary waves in presence of positron for non-isothermal two-temperature electrons and compared those amplitudes between two-temperature isothermal and non-isothermal electron plasma under the variation of different plasma parameters. These are represented by the Figs. 1 – 5.

Fig.1 shows the comparison of first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes between two-temperature non-isothermal and isothermal electron plasma against negative ion concentration (n_{j0}) with the variation of the concentration (μ, ν) of two-temperature electron represented by a_1, a_2 , a_3 and b_1, b_2, b_3 respectively when μ = 0.15, ν = 0.85 shown in Fig.1(a) and those by a_4 , a_5 , a_6 and b_4 , b_5 , b_6 respectively when μ= 1, ν = 0 shown in Fig.1(b). It is found that these amplitudes are decreasing for both two-temperature non-isothermal and two-temperature isothermal electron plasma except at n_{j0} = 0.02 but these three types of amplitudes are larger for two- temperature isothermal electron than those of two-temperature non-isothermal electron plasma.

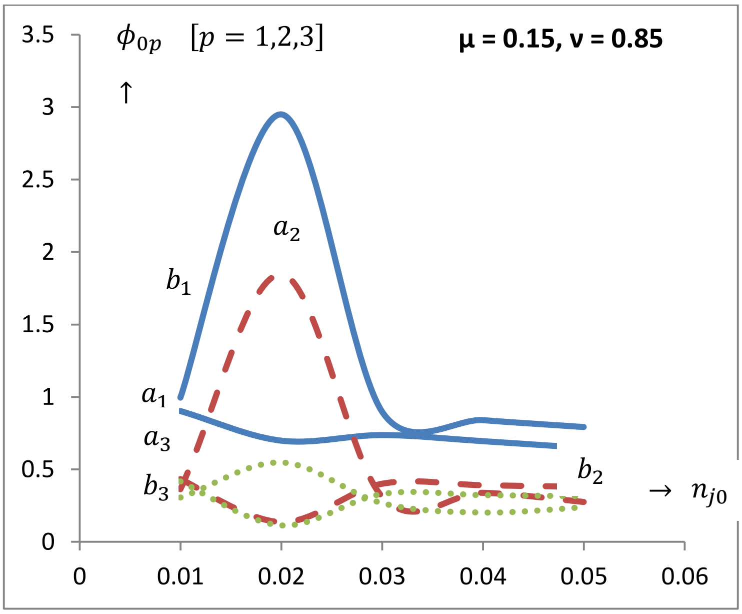

Fig.1(a) Comparison of first, second and third order amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma verses negative ion concentration with the variation of two-temperature electron concentration (μ, ν) for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17, σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, **μ = 0.15, ν =0.85.**

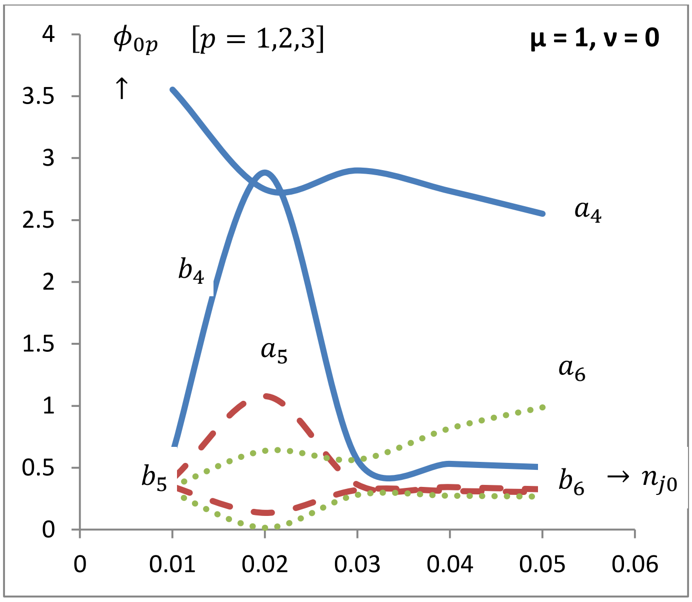

Fig.1(b) Comparison of first, second and third order amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma verses negative ion concentration with the variation of two-temperature electron concentration (μ, ν) for V = 1.6, u_{i0} = 0.5, u_{j0}= 0.3, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17 , σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, **μ = 1 , ν = 0**.

Fig.2 represents the first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of two-temperature non-isothermal and isothermal electron plasma against negative ion concentration (n_{j0}) for the variation of the temperature of positive (σ_i) and negative (σ_j) ions shown by a_1 ,a_2 , a_3 and b_1 , b_2 , b_3 shown in Fig.1(a) when $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ and also by d_1 , d_2 , d_3 and k_1 , k_2 , k_3 when $\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{30}$ shown in Fig.2. When $\sigma_i = \frac{1}{30} < \sigma_j = \frac{1}{25}$, amplitudes are decreasing for both non-isothermal and isothewrmal electron plasma with increasing negative ion concentration except at n_{j0}= 0.02 but when $\sigma_i = \frac{1}{25} > \sigma_j = \frac{1}{30}$ the amplitudes are similarly increasing for both non-isothermal and isothermal electron plasma with increasing negative ion concentration.

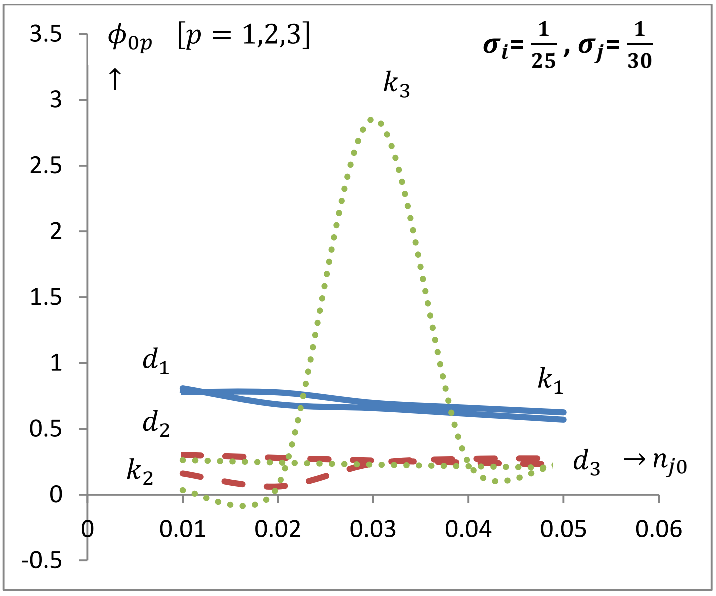

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Fig.2 Comparison of first, second and third order amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma verses negative ion concentration with the variation of the temperature of positive and negative ions for V = 1.6, u_{i0}= 0.5, u_{j0}= 0.3,$\boldsymbol{\sigma_i} = \frac{\mathbf{1}}{\mathbf{25}}$, $\boldsymbol{\sigma_j} = \frac{\mathbf{1}}{\mathbf{30}}$, χ = 0.17 , σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$= 0.51, μ = 0.15, ν = 0.85.

In **Fig.3**, the first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of two-temperature non-isothermal and isothermal electron plasma against negative ion concentration (n_{j0}) for the variation of the ratios (σ_p) of the temperature of electron (T_e)and positron (T_p) represented by a_1, a_2, a_3 and b_1 , b_2 , b_3 for both non-isothermal and isothewrmal two – temperature electron plasma shown in Fig.1(a) when σ_p= 0.41 and those by c_1 , c_2, c_3 and f_1 , f_2, f_3 when σ_p= 0.90 shown in Fig.3. It is evident from the figure that the first, second and third order amplitudes are decreasing except at some points of n_{j0} when σ_p increases for increasing negative ion concentration (n_{j0}).

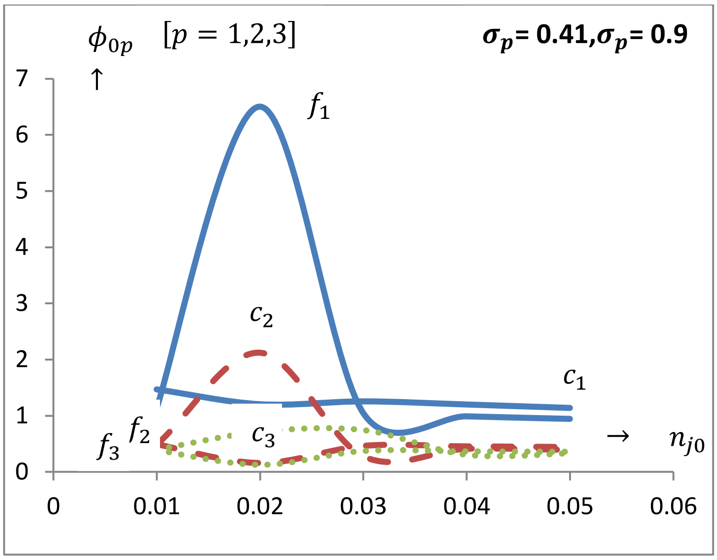


Fig.3 Comparison of first, second and third order amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma verses negative ion concentration with the variation of the temperature ratios (σ_p) of electron (T_e) and positron (T_p) for V = 1.6, u_{i0}= 0.5, u_{j0}= 0.3,$\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17 , $\boldsymbol{\sigma_p}$ **= 0.41,** $\boldsymbol{\sigma_p}$ **= 0.9**, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$= 0.51, μ = 0.15, ν = 0.85.

Fig.4 shows the first (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes of two-temperature non-isothermal and isothermal electron plasma against negative ion concentration (n_{j0}) for the variation of the ratios of negative to positive ion masses (Q) represented by a_1 , a_2, a_3 and b_1 , b_2 , b_3 shown in Fig.1(a) when Q = 4 and those by g_1 , g_2, g_3 and l_1 , l_2 , l_3 shown in Fig.4 when Q = 16. When the mass ratios (Q) are increasing then the amplitudes are decreasing except at some values of n_{j0} for increasing negative ion concentration.

Fig.5 shows the first (Φ_{01}), second (Φ_{02}) and third (Φ_{03}) order amplitudes of two-temperature non-isothermal and isothermal electron plasma against negative ion concentration (n_{j0}) for the variation of positron density (χ) represented by a_1 , a_2, a_3 and b_1 , b_2 , b_3 when χ = 0.17 shown in Fig.1(a) and those by h_1 , h_2 , h_3 and r_1 , r_2 , r_3 shown in Fig. 5 when χ = 0 . As positron density (χ)increases, the first (Φ_{01}) , second (Φ_{02}) and third (Φ_{03}) order amplitudes are increasing except at some values of n_{j0} for increasing negative ion concentration.

We are now investigating the change of energy by which the amplitudes are progressively decreasing and for this reason, the graphical approach for the change of energy is inevitable under the variation of different plasma parameters.

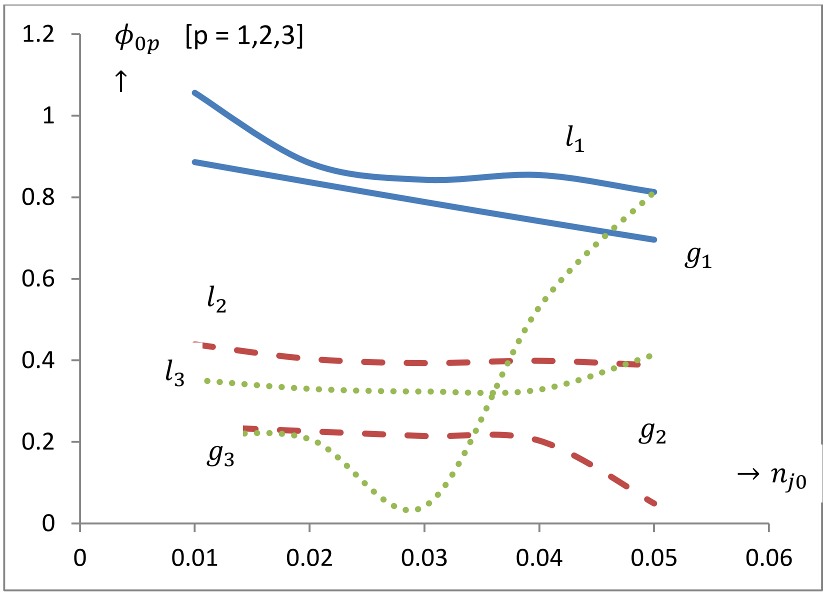


Fig.4 Comparison of first, second and third order amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma verses negative ion concentration with the variation of the mass ratios (Q) of negative to positive ions for V = 1.6, u_{i0}= 0.5, u_{j0}= 0.3,σ_i = $\frac{1}{30}$, σ_j = $\frac{1}{25}$, χ = 0.17 , σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$= 0.51, μ = 0.15, ν =0.85, Q = 16.

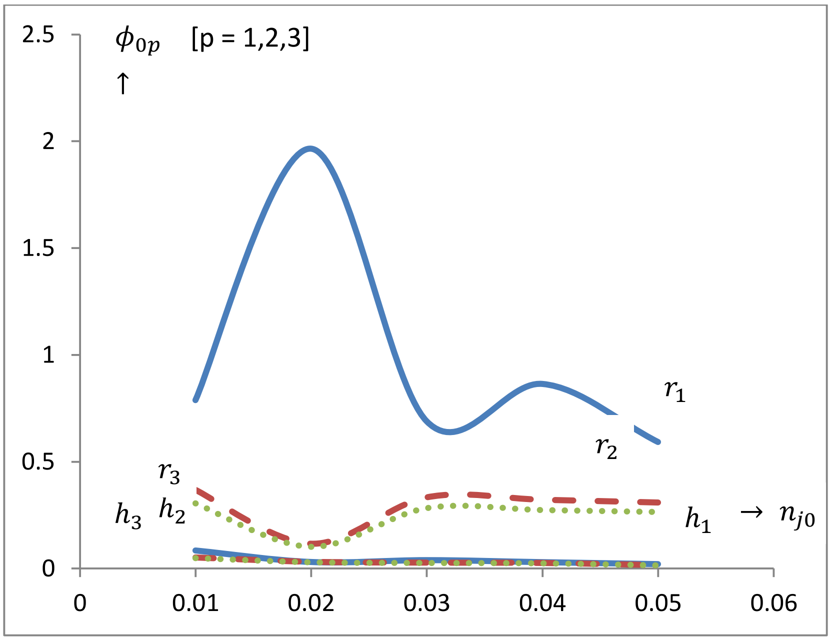


Fig.5 Comparison of first, second and third order amplitudes between two-temperature non-isothermal and two-temperature isothermal electron plasma verses negative ion concentration with the variation of positron density (χ) for V = 1.6, u_{i0}= 0.5, u_{j0}= 0.3,σ_i = $\frac{1}{30}$, σ_j = $\frac{1}{25}$, χ = 0 , σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$= 0.51, μ = 0.15, ν =0.85, Q = 4, Z = 1.

The loss or change of energy is presently shown by the Figs.6 – 10 for the variation of the concentration (μ, ν) of two-temperature electron , temperature of positive (σ_i) and negative (σ_j) ions, temperature ratios (σ_p) of electron (T_e) and positron (T_p), mass ratios (Q) of negative to positive ions and density of positron (χ) .

In **Fig.6**, the change of energy (ΔE_r) generated by the decrease of amplitude from second order to first order denoted by a_4 and the change of energy (ΔE_S)due to the decrease of amplitude from third order to second order denoted by a_5 are shown against negative ion concentration (n_{j0}) for the variation of the concentration (μ,ν) of two-temperature non-isothermal electron when μ = 0.15, ν = 0.85 and those by a_6 , a_7 when μ = 1, ν = 0. It is found that the numerical values of the change of energy $(\Delta E_r \ or \ \Delta E_S)$ for μ = 1, ν = 0 are larger than those values of ΔE_r or ΔE_S at μ = 0.15, ν = 0.85.

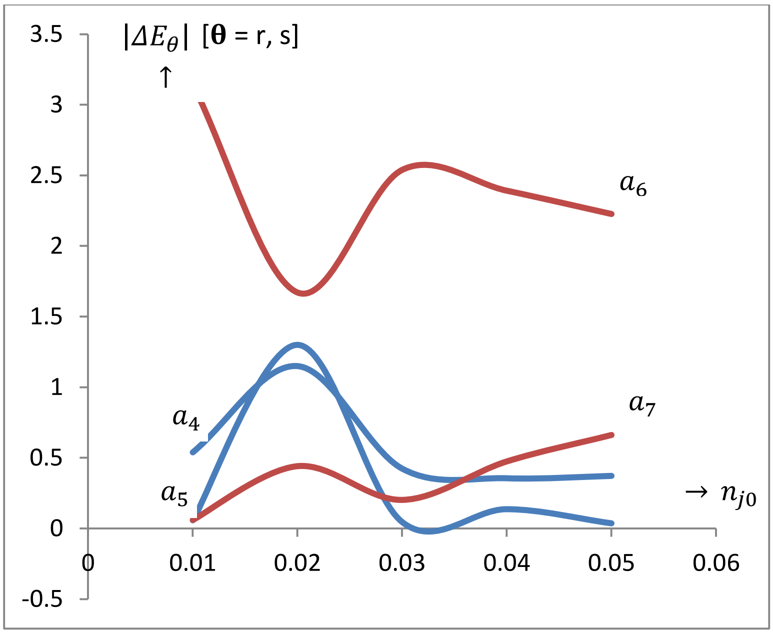


Fig.6 Change of energy (ΔE_r or ΔE_S) verses negative ion concentration (n_{j0}) with the variation of the concentration (μ, ν) of two-temperature non-isothermal electron for V = 1.6, u_{i0}= 0.5, u_{j0}= 0.3,$\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17 , σ_p = 0.41, β_1 = 0.25, b_l = 0.15, b_h = 0.4, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$= 0.51, μ = 0.15, ν =0.85, μ = 1, ν = 0, Q = 4, Z = 1.

Fig.7 shows the change of energy (ΔE_r) due to the decrease of amplitude from second ordder to first order denoted by a_4 and the change of energy (ΔE_S)due to the decrease of amplitude from third order to second order denoted by a_5 against negative ion concentration (n_{j0}) for the variation of the temperature of positive (σ_i) and negative (σ_j) ions when $\sigma_i = \frac{1}{30}$, σ_j =

$\frac{1}{25}$ and those by b_4 , b_5 when $\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{30}$. The following diagram shows that the numerical values of the change of energy (ΔE_r *or* ΔE_S) for $\sigma_i = \frac{1}{30} < \sigma_j = \frac{1}{25}$ are greater than the numerical values of ΔE_r or ΔE_S for $\sigma_i = \frac{1}{25} > \sigma_j = \frac{1}{30}$.

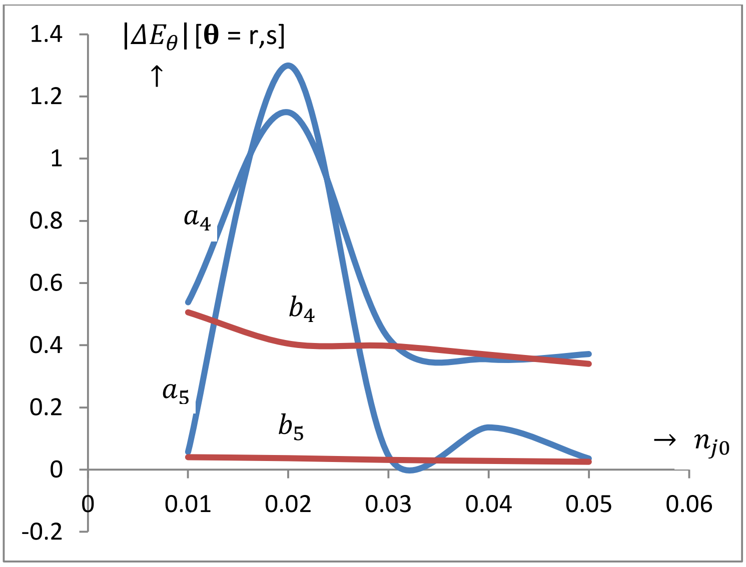


Fig.7 Change of energy (ΔE_r or ΔE_S) verses negative ion concentration (n_{j0}) with the variation of the temperature of positive (σ_i) and negative (σ_j) ions for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, Q =4, $\boldsymbol{\sigma_i = \frac{1}{30}}$, $\boldsymbol{\sigma_j = \frac{1}{25}}$, $\boldsymbol{\sigma_i = \frac{1}{25}}$, $\boldsymbol{\sigma_j = \frac{1}{30}}$, $\chi = 0.17$, $\sigma_p = 0.41$, $b_l = 0.15$, $b_h = 0.4$, $\beta_1 = 0.25$, $b_l^{(1)} = 0.25$, $b_h^{(1)} = 0.51$, μ = 0.15, ν = 0.85, Z = 1.

Fig.8 represents the change of energy (ΔE_r) due to the decrease of amplitude from second order to first order denoted by a_4 and the change of energy (ΔE_S)due to the decrease of amplitude from third order to second order denoted by a_5 against negative ion concentration (n_{j0}) for the variation of the ratios (σ_p) of the temperature of electron (T_e) and positron (T_p) when σ_p =0.41 and those by b_6 , b_7 when σ_p =0.9. The change of energy for σ_p = 0.41 is numerically less than the change of energy (ΔE_r *or* ΔE_S) for σ_p = 0.90.

In **Fig.9,** the change of energy (ΔE_ror ΔE_S) against negative ion concentration (n_{j0}) for the variation of the mass ratios (Q) of negative to positive ions are shown. The curves a_4 , a_5 due to the decrease of amplitude from second order to first order and from third order to second order are formed for Q = 4 whereas curves b_8 , b_9 are similarly formed for Q = 16. For larger Q i.e. at Q = 16, the value of ΔE_r is numerically greater than the value of ΔE_r at smaller mass ratio i.e. at Q = 4 but for larger Q i.e. at Q = 16 , at first ΔE_S is numerically less than the value of ΔE_S at Q = 4 for some increasing values of n_{j0} and after that ΔE_S are gradually increasing for increasing values of n_{j0} .

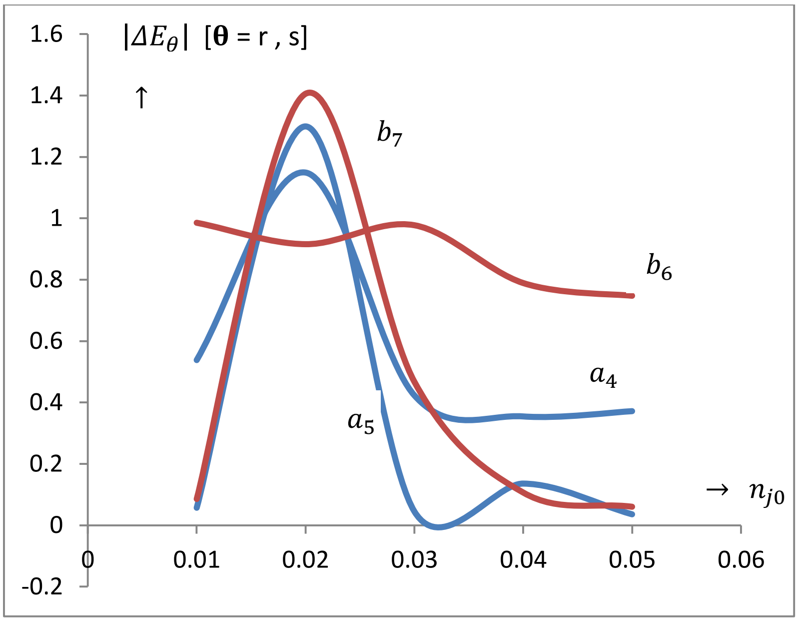


Fig.8 Change of energy (ΔE_r or ΔE_S) verses negative ion concentration (n_{j0}) with the variation of the ratios (σ_p) of the temperature of electron (T_e) and positron (T_p) for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, Q =4, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17, $\boldsymbol{\sigma_p}$ **= 0.41,** $\boldsymbol{\sigma_p}$ **= 0.9**, b_l = 0.15, b_h = 0.4, β_1 = 0.25, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, μ = 0.15, ν = 0.85, Z = 1.

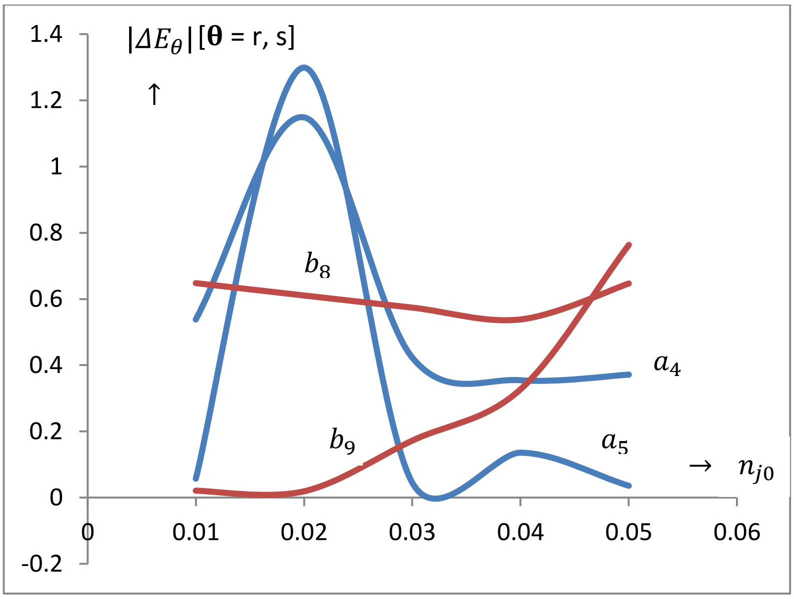


Fig.9 Change of energy (ΔE_r or ΔE_S) verses negative ion concentration (n_{j0}) with the variation of the mass ratios (Q) of negative to positive ions for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, Q =4, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17, σ_p = 0.41, b_l = 0.15, b_h = 0.4, β_1 = 0.25, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, μ = 0.15, ν = 0.85, **Q =4, Q = 16**, Z = 1.

Fig.10 shows the change of energy (ΔE_r or ΔE_S) against negative ion concentration (n_{j0}) for the variation of the density of positron (χ). The previously formed curves a_4 , a_5 due to the decrease of amplitude from second order to first order and from third order to second order are formed for $\chi = 0.17$ whereas b_{10} , b_{11} curves are similarly formed for $\chi = 0$. The change of energy ΔE_r or ΔE_s for $\chi = 0.17$ are larger than the change of energy ΔE_r or ΔE_s for $\chi = 0$ which are clearly visible in this figure.

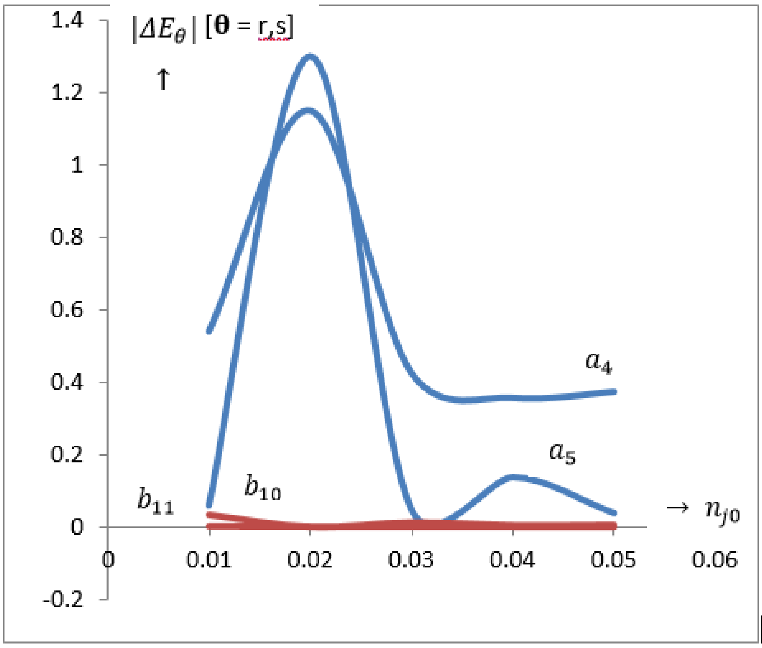


Fig.10 Change of energy (ΔE_r or ΔE_S) verses negative ion concentration (n_{j0}) with the variation of the density of positron (χ) for V = 1.6, $u_{i0} = 0.5$, $u_{j0} = 0.3$, Q =4, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, $\boldsymbol{\chi} = \mathbf{0}$, $\boldsymbol{\chi} = \mathbf{0.17}$, $\sigma_p = 0.41$, $b_l = 0.15$, $b_h = 0.4$, $\beta_1 = 0.25$, $b_l^{(1)} = 0.25$, $b_h^{(1)} = 0.51$, μ = 0.15, ν = 0.85, Z =1.

Fig.11 shows fast (V_F) and slow (V_S) ion-acoustic phase velocity against negative ion concentration (n_{j0}) for the variation of the ratios (σ_p) of electron (T_e)and positron (T_p) temperature. As negative ion concentration (n_{j0}) increases the fast ion-acoustic phase velocity (V_F) increases whereas slow ion-acoustic phase velocity (V_S) decreases and also for increasing values of σ_p, the fast(V_F) and slow (V_S) ion-acoustic phase velocities are gradually decreasing i.e. $V_1 > V_2 > V_3$ and $v_1 > v_2 > v_3$where V_1, V_2, V_3 and v_1, v_2, v_3 are respectively the fast and slow ion-acoustic phase velocities at σ_p = 0.41, 0.7 and 0.9.

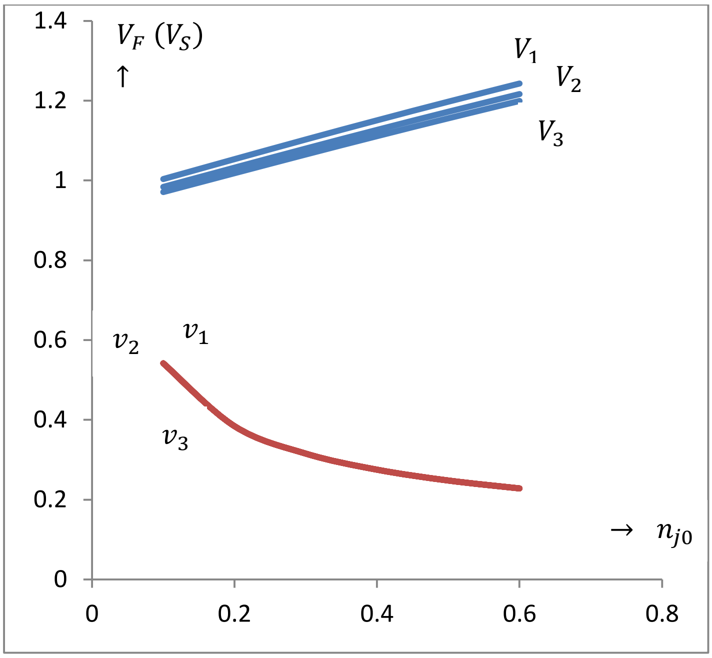


Fig.11 Fast (V_F) and Slow (V_S) mode phase velocity verses negative ion concentration (n_{j0}) with the variation of the temperature ratios (σ_p) of electron $(T_e)$and positron (T_p) for $\boldsymbol{\sigma_p}$ **= 0.41, 0.7 , 0.9,** $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, $\chi =$ 0.17, Q = 4 , Z = 1.

In **Fig.12**, the fast ion-acoustic phase velocity (V_F) and slow ion-acoustic phase velocity (V_S) against negative ion concentration (n_{j0}) for the variation of positive(σ_i) and negative(σ_j) ion temperatures are shown. When temperatures of positive (σ_i) and negative(σ_j) ions change, fast mode phase velocity (V_F) or slow mode (V_S) phase velocity increases for increasing values of negative ion concentration (n_{j0}) at a particular temperature of σ_p when Q = 4, χ = 0.17 and Z = 1. V_F or V_S will be largest for $\sigma_i = \frac{1}{25} = \sigma_j$ $(\sigma_i = \sigma_j)$ and they will be least for $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ $(\sigma_i < \sigma_j)$ when all other parameters are same as before. In this case, V_1 (v_1) is the Fast (Slow) mode phase velocity at $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, V_4 (v_4) is the Fast (Slow) mode phase velocity at $\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{30}$ and V_5 (v_5) is the Fast (Slow) mode phase velocity at $\sigma_i = \frac{1}{25}$, $\sigma_j = \frac{1}{25}$.

Fig.13 shows the fast (V_F) and slow (V_S) ion-acoustic phase velocity against negative ion concentration (n_{j0}) for the variation of the ratios of negative to positive ion masses (Q). When the mass ratios (Q) of negative to positive ions are increasing, the fast mode phase velocity (V_F) is increasing for increasing values of n_{j0} whereas slow mode(V_S) is gradually decreasing. Here V_1, V_6, V_7 and v_1, v_6, v_7 are respectively the Fast and Slow mode phase velocities at Q = 4, 8.875 and 35.5.

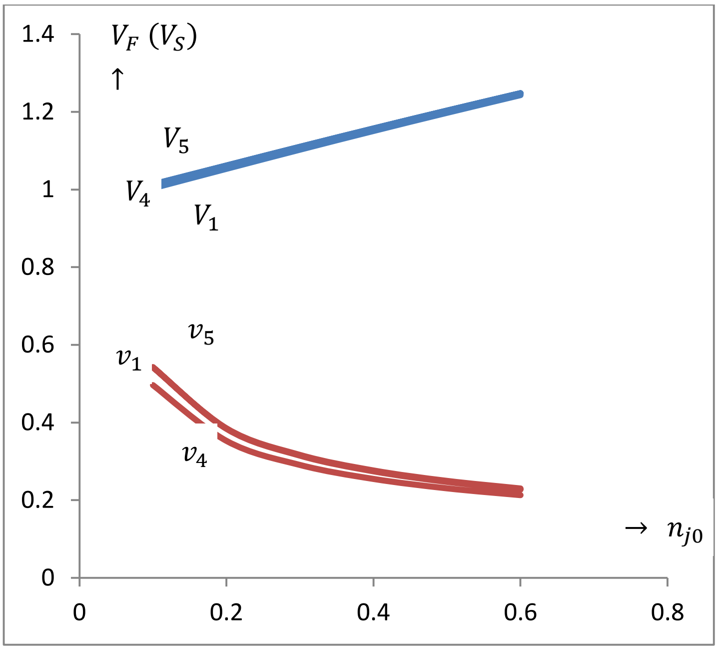


Fig.12 Fast (V_F) and Slow (V_S)mode phase velocity verses negative ion concentration (n_{j0}) with the variation of the temperature of positive $(\sigma_i)$and negative (σ_j) ion temperature for σ_p = 0.41, $\boldsymbol{\sigma_i = \frac{1}{30}}$, $\boldsymbol{\sigma_j = \frac{1}{25}}$, $\boldsymbol{\sigma_i = \frac{1}{25}}$, $\boldsymbol{\sigma_j = \frac{1}{30}}$, $\boldsymbol{\sigma_i = \frac{1}{25}}$, $\boldsymbol{\sigma_j = \frac{1}{25}}$, χ = 0.17, Q = 4 , Z = 1.

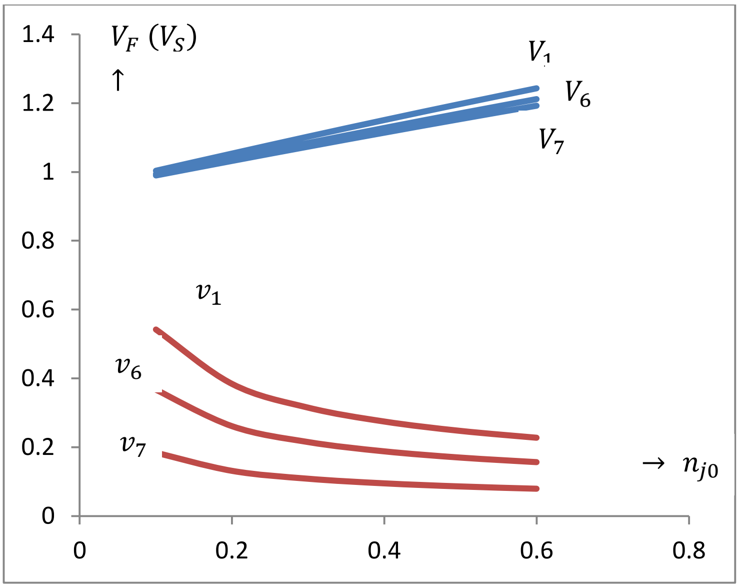


Fig.13 Fast(V_F) and Slow (V_S) mode phase velocity verses negative ion concentration (n_{j0}) with the variation of the mass ratios $(Q)$of negative to positive ions for σ_p = 0.41, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ = 0.17, **Q = 4 , Q = 8.875 , Q = 35.5** , Z = 1.

In **Fig.14**, the fast (V_F) and slow (V_S) ion-acoustic phase velocity against negative ion concentration (n_{j0}) for the variation of positron density (χ) are shown. The fast mode phase velocity (V_F) increases for increasing negative ion (n_{j0}) concentration at a particular χ

whereas slow mode phase velocity (V_S) decreases for increasing negative ion concentration(n_{j0}). Here V_8 (v_8) is the Fast (Slow) mode phase velocity at $\chi = 0$, V_1 (v_1) is the Fast (Slow) mode phase velocity at $\chi = 0.17$ and V_9 (v_9) is the Fast (Slow) mode phase velocity at $\chi = 0.22$.

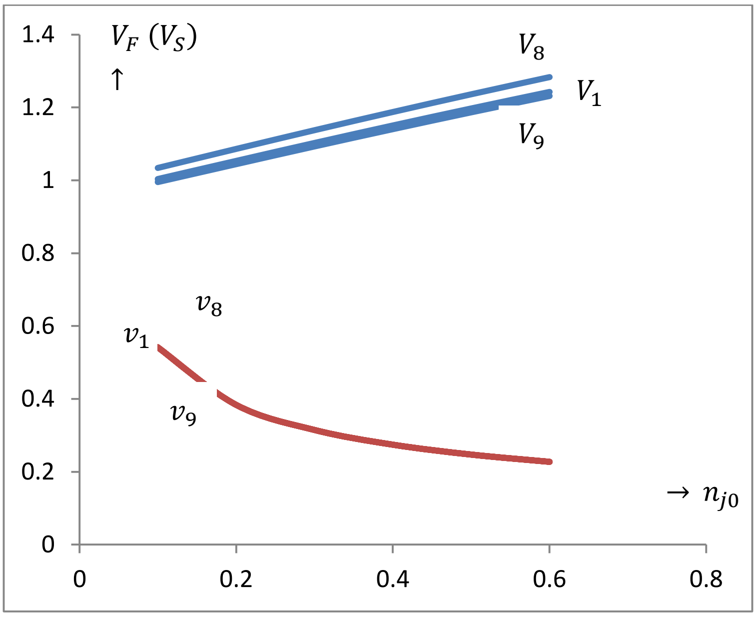
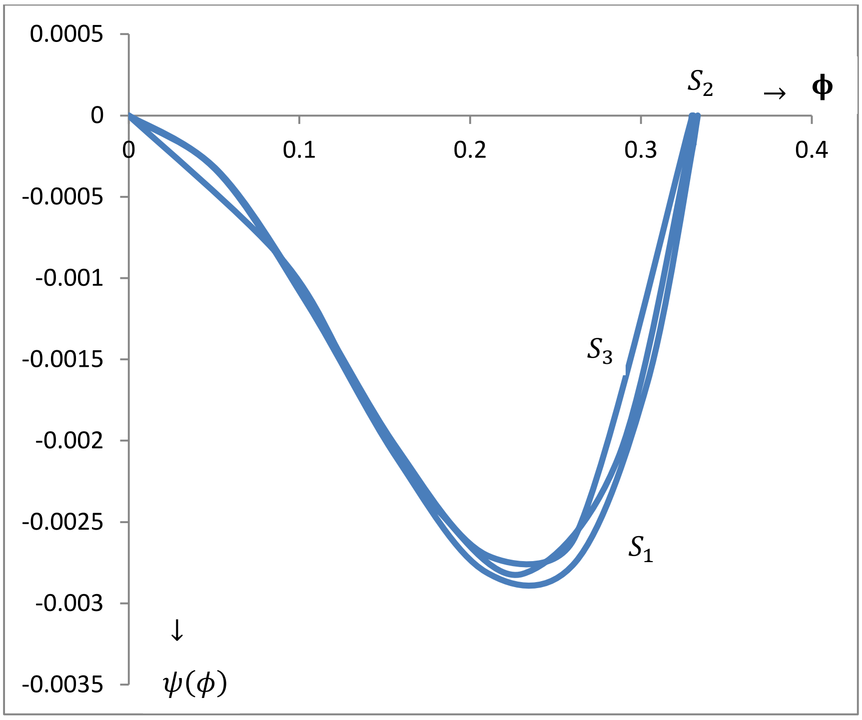


Fig.14 Fast (V_F) and Slow (V_S) mode phase velocity verses negative ion concentration (n_{j0}) with the variation of positron density (χ) for $\sigma_p = 0.41$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, χ **= 0 ,** χ **= 0.17,** χ **= 0.22** , Q = 4 , Z = 1.

Fig.15 shows the profiles of Sagdeev pseudopotential curves $\psi(\phi)$ against the electrostatic potential ϕ with the variation of the increasing values of the temperature of negative ions (σ_j). The curve $\psi(\phi)$ cuts ϕ axis at $\phi = \phi_m = 0.333322$ for $\sigma_i = 0$, $\sigma_j = 0$ represented by S_1 , at $\phi = \phi_m = 0.33081$ for $\sigma_i = 0$, $\sigma_j = \frac{1}{25}$ represented by S_2 and at $\phi = \phi_m = 0.329854$ for $\sigma_i = 0$, $\sigma_j = \frac{1}{20}$ represented by S_3 . This shows that the amplitudes are decreasing for increasing temperature of negative ions (σ_j) when cold positive ions are taken and effect of negative ions on Sagdeev pseudopotential is clearly visible from this figure which is a very interesting situation.

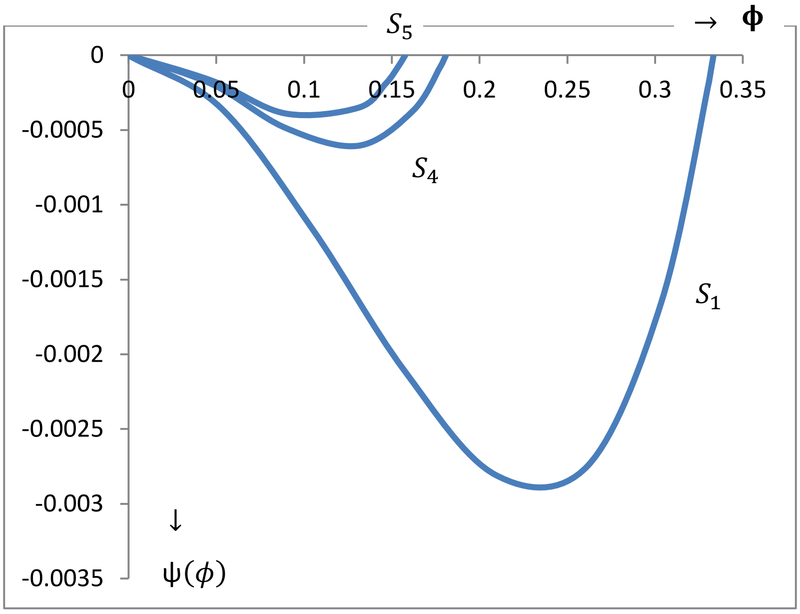


S_1 : $\boldsymbol{\sigma_i}$ **= 0,** $\boldsymbol{\sigma_j}$ **= 0**; S_2 : $\boldsymbol{\sigma_i}$ **= 0,** $\boldsymbol{\sigma_j}$ **=** $\boldsymbol{\frac{1}{25}}$; S_3 : $\boldsymbol{\sigma_i}$ **= 0,** $\boldsymbol{\sigma_j}$ **=** $\boldsymbol{\frac{1}{20}}$

Fig.15 Profiles of **S**agdeev pseudopotential $\boldsymbol{\psi(\phi)}$ verses electrostatic potential $\boldsymbol{\phi}$ with the variation of the increasing values of the negative ion temperature (σ_j) for V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3, Q =4, $\boldsymbol{\sigma_i = 0}$, $\boldsymbol{\sigma_j}$ **=0** ; $\boldsymbol{\sigma_i = 0}$, $\boldsymbol{\sigma_j = \frac{1}{25}}$; $\boldsymbol{\sigma_i = 0}$, $\boldsymbol{\sigma_j = \frac{1}{20}}$, χ = 0.17, σ_p = 0.41, b_l = 0.15, b_h = 0.4, β_1 = 0.25, $b_l^{(1)}$ = 0.25, $b_h^{(1)}$ = 0.51, μ = 0.15, ν = 0.85.

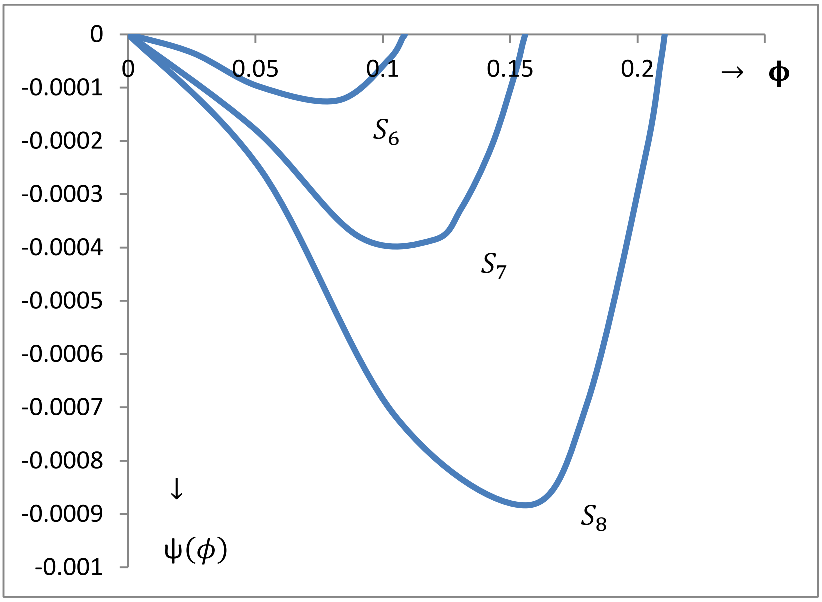
In **Fig.16,** the Sagdeev pseudopotential curves ψ(ϕ) against the electrostatic potential ϕ are drawn for the variation of the increasing values of the temperature of positive ions. As positive ion temperature increases from 0 to $\frac{1}{25}$ while negative ion temperature (σ_j) is zero, then the corresponding amplitudes are also decreasing but take smaller values than before. It shows the effect of the temperature of positive ion (σ_i) on Sagdeev pseudopotential ψ(ϕ). The curves so formed are represented by S_1 for σ_i = 0, σ_j = 0 when ϕ = ϕ_m = 0.333322, by S_4 for $\sigma_i = \frac{1}{30}$, σ_j = 0 when ϕ = ϕ_m = 0.180873 and by S_5 for $\sigma_i = \frac{1}{25}$, σ_j = 0 when ϕ = ϕ_m = 0.157691.

Fig.17 shows the Sagdeev pseudopotential curves ψ(ϕ) against the electrostatic potential ϕ with the variation of the temperature of positive (σ_i) and negative (σ_j) ions. Three different temperatures of positive (σ_i) and negative (σ_j) ions are considered in this situation. The Sagdeev pseudopotential curve ψ(ϕ) for $\sigma_i = \frac{1}{20} < \sigma_j = \frac{1}{10}$ represented by S_6 cuts the ϕ axis at ϕ = ϕ_m = 0.108655 , curve ψ(ϕ) for $\sigma_i = \frac{1}{25} > \sigma_j = \frac{1}{30}$ represented by S_6 cuts the ϕ axis



S_1 : $\boldsymbol{\sigma_i = 0, \sigma_j = 0}$; S_4 : $\boldsymbol{\sigma_i = \frac{1}{30}, \sigma_j = 0}$; S_5 : $\boldsymbol{\sigma_i = \frac{1}{25}, \sigma_j = 0}$

Fig.16 Profiles of **S**agdeev pseudopotential $\boldsymbol{\psi(\phi)}$verses electrostatic potential $\boldsymbol{\phi}$ with the variation of the increasing values of the positive ion temperature (σ_i) for $V = 1.6, u_{i0} = 0.5, u_{j0} = 0.3$, Q =4, $\boldsymbol{\sigma_i = 0}$ **,** $\boldsymbol{\sigma_j = 0}$ **;** $\boldsymbol{\sigma_i = \frac{1}{30}, \sigma_j = 0}$ **;** $\boldsymbol{\sigma_i = \frac{1}{25}, \sigma_j = 0}$, $\chi = 0.17$, $\sigma_p = 0.41$, $b_l = 0.15$, $b_h = 0.4$, $\beta_1 = 0.25$, $b_l^{(1)} = 0.25$, $b_h^{(1)} = 0.51$, μ = 0.15, ν = 0.85.



S_6 : $\boldsymbol{\sigma_i = \frac{1}{20}, \sigma_j = \frac{1}{10}}$; S_7 : $\boldsymbol{\sigma_i = \frac{1}{25}, \sigma_j = \frac{1}{30}}$; S_8 :$\boldsymbol{\sigma_i = \frac{1}{40}, \sigma_j = \frac{1}{30}}$

Fig.17 Profiles of **S**agdeev pseudopotential $\boldsymbol{\psi(\phi)}$verses electrostatic potential $\boldsymbol{\phi}$ with the variation of the different values of the temperatures of positive (σ_i) and negative (σ_j) ions for $V = 1.6$, $u_{i0} = 0.5$, $u_{j0} = 0.3$, Q =4, $\boldsymbol{\sigma_i = \frac{1}{20}, \sigma_j = \frac{1}{10}}$ **;** $\boldsymbol{\sigma_i = \frac{1}{25}, \sigma_j = \frac{1}{30}}$ **;** $\boldsymbol{\sigma_i = \frac{1}{40}, \sigma_j = \frac{1}{30}}$, $\chi = 0.17$, $\sigma_p = 0.41$, $b_l = 0.15$, $b_h = 0.4$, $\beta_1 = 0.25$, $b_l^{(1)} = 0.25$, $b_h^{(1)} = 0.51$, μ = 0.15, ν = 0.85.

at ϕ = ϕ_m = 0.156017 and curve ψ(ϕ) for $\sigma_i = \frac{1}{40} < \sigma_j = \frac{1}{30}$ represented by S_8 cuts the ϕ axis at ϕ = ϕ_m = 0.210734. It is also evident from this figure that the amplitude will be maximum for $\sigma_i = \frac{1}{40} < \sigma_j = \frac{1}{30}$ and minimum for $\sigma_i = \frac{1}{20} < \sigma_j = \frac{1}{10}$ which shows the effect of both positive (σ_i) and negative (σ_j) ion temperatures on Sagdeev pseudopotential curve ψ(ϕ).

5. CONCLUSION:

In this paper, the author investigated the first, second and third order amplitudes compared between non-isothermal and isothermal electron plasma, slow and fast mode phase velocities, loss of energy generated by the decrease of amplitude and profiles of Sagdeev potential for solitary wave solutions.

In the presence of positron, the contribution of third order amplitudes and the comparison of amplitudes for first, second and third order between non-isothermal and isothermal two-temperature electron plasma and loss of energy due to decrease of amplitude are most probably a new finding that previous researchers did not consider.

The comparison of first, second and third order amplitudes between two-temperature isothermal and non-isothermal electron plasma and the loss of energy due to decrease in amplitude are one of our important motivations and new idea associated with the variation of two-temperature electron concentration (μ, ν), temperature of both ions (σ_i, σ_j), ratio (σ_p) of electron (T_e) and positron temperature (T_p), ratio of negative to positive ion masses (Q) and positron density (χ). Fast (V_F) and slow (V_S) ion-acoustic phase velocity are also considered and analysed properly with the variation of different plasma parameters.

The temperature of positive and negative ions help us to understand the nature and behaviour of Sagdeev pseudopotential curves. We have shown separately the effect of the temperature of positive ion, negative ion and also the effect of the temperatures of both ions through the various profiles of the Sagdeev pseudopotential for two-temperature non-isothermal electron plasma.

In space, the plasma acoustic modes are produced by the non-isothermality of electrons. The higher order spiky and explosive solitary wave solutions are the new findings in space plasma due to trapping of electrons that Freja Scientific Satellite observations was unable to find out.

Our future plan is to solve the two-temperature non-isothermal electron plasma with positron along with relativistic cold positive and cold negative ion.

ACKNOWLEDGEMENT

The present author would like to thank Dr. S.N. Paul for his valuable suggestions and discussions in the preparation of this paper to its present form**.**

References

[1]S.Chattopadhyay, *Effect of ionic temperatures on ion-acoustic solitary waves in a drift negative ion plasma with single temperature electron*,Fizika A (Zagreb) **19**,1(2010) 31 – 46.

[2] S.Chattopadhyay, *Contribution of non-thermal electrons to ion-acoustic soliton formation in warm positive and negative ion plasma*,The African Review of Physics , **9**,0041 (2014) 317 – 331.

[3] R.Roychoudhury,G.C.Das and J.Sarma, *Quasipotential analysis for deriving the multidimensional Sagdeev potential equation in multicomponent plasma*,Phys. of Plasmas,Vol. **6**, No.7, July (1999)2721 – 2726; https://doi.org/10.1063/1.873228.

[4] S.N.Paul, C. Das, I. Paul, B. Bandhopadhyay, S .Chattopadhyay and S. S. De ,*Ion-acoustic solitary waves in an electron-ion-positron plasma*, Indian J. Phys.,vol-**86**,No.6(2012) 545 – 553; https://doi.org/10.1007/s12648-012-0080-8.

[5] A.Jeffrey and T.Kakutani, *Propagation of solitary waves in relativistic plasmas*,SIAM Rev., Vol.**14** (1972) 582.

[6] H.Ikezi, *Experiments on ion-acoustic solitary waves*,Phys. Fluids, Vol. **16** (1973) 1668;https://doi.org/10.1063/1.1694194.

[7] H.Ikezi, R.J.Taylor and D.K.Baker,*Formation and interaction of ion-acoustic solitons*, Phys.Rev.Letts.,Vol.**25**(1970)11;https:doi.org/10.1103/physRevLett.25.11.

[8] G.C.Das and S.G.Tagare,*Propagation of ion-acoustic waves in a multicomponent plasma*, Plasma Phys., Vol. **17**,No.-12(1975) 1025;https://doi.org/10.1088/0032-1028/17/12/002.

[9] M.Q.Tran and P.J.Hirt, *The Kortweg-de Vries equation for a two component plasma*,Plasma Phys., Vol. **16** (1974) 617.

[10] S.Watanabe, *Ion-acoustic soliton in plasma with negative ion,* J. Phys. Soc. Japan, Vol.**53**,No.-3 (1984)950– 956.

[11] M.Q.Tran, *Ion-acoustic solitons in a plasma: A review of their experimental properties and related theories*,Phys.Scripta, Vol. **20**, No.3&4(1979)317;https://doi.org/10.1088/0031-8949/20/3-4/004.

[12] K. E.Lonngren, *Soliton experiments in plasmas*, Plasma Phys. Vol. **25** (1983) 943.

[13] Y.Nakamura and I. Tsukabayashi, *Observation of modified Kortweg-de Vries solitons in a multicomponent plasma with negative ions*, Phys. Rev. Lett., Vol**. 52**, No.26(1984) 2356.

[14] G.O.Ludwig, J.L.Ferreira and Y.Nakamura,*Observation of ion-acoustic rarefaction solitons in a multicomponent plasma with negative ions*, Phys. Rev. Lett.,Vol.- **52**,No.4 (1984) 257.

[15] H.Schamel, *Stationary solitary, snoidal and sinusoidal ion acoustic waves*, Plasma Phys. **14** (1972) 905.

[16] H. Schamel, *A modified Kortweg-de Vries equation for ion-acoustic waves due to resonant electrons*, J. Plasma Phys.,Vol.- **9**,No.3(1973) 377.

[17] M. K.Kalita and S. Bujarbarua, *Higher order contributions to ion acoustic solitary waves in a plasma consisting of warm ions and nonisothermal electrons*, Canad. J. Phys.Vol.- **60**, No.4(1982) 392-396; https://doi.org/10.1139/p82-057.

[18] S.S.Ghosh and A.N.S. Iyengar, *Forbidden regions in ion temperature for ion acoustic compressive solitary waves for a plasma with two electron temperatures*, Phys. Plasmas **4**, 6 (1997) 2139;https://doi.org/10.1063/1.872378.

[19] S.S.Ghosh, K.K.Ghosh and A.N.S. Iyengar, *Large Mach number ion acoustic rarefactive solitary waves for a two electron temperature warm ion plasma*, Phys. Plasmas **3**,10 (1996) 3939; https://doi.org/10.1063/1.871567.

[20] S.G. Tagare and R.V.Reddy, *Effect of higher order nonlinearity on propagation of nonlinear ion-acoustic waves in a collisionless plasma consisting of negative ions*,J. Plasma Phys. **35** (1986) 219.

[21] K.P.Das, S.R. Majumdar and S.N.Paul,*Higher order contributions to ion-acoustic solitary waves in a multicomponent plasma consisting of warm ions and two-component nonisothermal electrons*, Phys. Rev. **E 51** (1995) 4796.

[22] S. Bhattacharaya and R.K.Roychowdhury, *Effect of ion temperature on ion-acoustic solitary waves in a two-ion plasma*,Canad. J. Phys. **66**,6 (1988) 467-470;https://doi.org/10.1139/p88-076.

[23] K.P.Das and S.R.Majumdar, *Higher order nonlinear and dispersive effects on ion-acoustic solitary waves*, Canad. J. Phys. **69** (1991) 822.

[24] A.Roychowdhury, G. Pakira, S.N.Paul and K.Roychowdhury, *Higher order corrections to solitary waves in a non-isothermal relativistic plasma with cold ions and two-temperature electrons*, J. Plasma Phys. **44**, 2 (1990)253 – 263.

[25] S.R.Majumdar, S.N.Paul and K.P. Das, *Third-order contributions to ion-acoustic solitons in a plasma with two types of cold positive ions and two-temperature non-isothermal electrons,* J. Plasma Phys. **64**,3 (2000)297 – 308.

[26] S.Chattopadhyay&S.N.Paul,*Compressive and rarefactive solitary waves in plasma with cold drifting positive and negative ions*, The African Review of Physics, **7**, 0033 (2012) 289 – 299.

[27] S. Chattopadhyay, S.N.Paul and D. Ray, *Influence of negative ions on ion-acoustic solitary waves in a two-electron-temperature plasma*, Fizika A (Zagreb) **18**,3 (2009) 89 – 106.

[28] G.C.Das, S.G.Tagare and J.Sarma, *Quasipotential analysis for ion-acoustic solitary waves and double layers in plasmas*, Planet Space Science, Vol.**46**,No.4 (1998)417-424.

[29] R.Z.Sagdeev, *In Reviews of Plasma Physics, edited by M.A.Leontovich*, consultants Bureau, New York, Vol.-**4**(1966) 23.

[30] S.N.Paul, S.Chattopadhyay and K.K.Ghosh,*Higher-order contribution to the ion-acoustic soliton in a drift negative-ion plasma*, Fizika A (Zagreb)**6**, 3(1997)139 – 148.