

INSURANCE CONTRACT FOR EPIDEMIOLOGICAL DISEASES SPREAD

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Abstract

This paper presents the use of actuarial modeling for the spread of epidemiological diseases. Study is done based on a developed actuarial model of SIR infection which describes the transfer dynamics in an insurance contract in a given population. At the initial stage, we satisfied key assumptions and observed that the rate of infection is positively related and the rate of recovery is negatively related to the level premium payment. Hence, we developed a MATLAB program to calculate the minimum adjusted level premium for a hospitalization plan. Furthermore, this study expanded the basic model to eliminate some problems such as the Vector-Host relationship due to unsatisfied assumptions for the real data. It is reasonable to expand the SIR model by including Vector-Host transfer dynamics to find out an actuarial model for Dengue fever. Accordingly the length of an epidemic season for Dengue over the sample period can be estimated. Results demonstrate no impact from Vector-Host in determining the level premium payment and reveal the possibility of introducing an insurance policy for the spread of Dengue fever in Sri Lanka. Further, as a result, difficulties to clearly identify seasonal patterns of other diseases may also be overcome. We suggest the SIRS infection model with delayed differential equations as an appropriate solution to define an actuarial model for a wide range of diseases.

Key Words: *Benefit Payment, Benefit Reserve, Epidemiology, Transfer Dynamics, Vector-Host*

JEL Codes: C61; G22; I13

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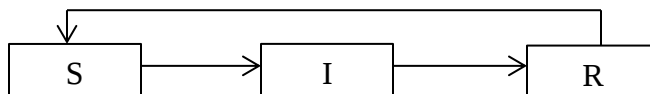
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INTRODUCTION

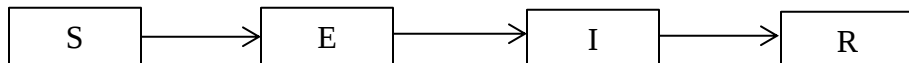
This study is focused on developing a more general actuarial model for epidemiological disease spread which can be used for an insurance contract. Epidemic disease is a kind of infectious disease which spreads rapidly in a particular population within a short time period. Dengue, Leptospirosis, Typhus fever and Chickenpox are some examples of epidemics in Sri Lanka. Those infectious diseases can spread in various ways, and pathogens such as Bacteria, Viruses, and Fungi cause infections through different modes of transmission (direct or indirect contact). Also transmission can take place through vectors such as mosquitos and rats. An insurance contract is an arrangement in which one party (the insurer) accepts significant insurance risk from another party (the policy holder) to compensate the policy holder if a specific uncertain future event impacts the policy holder. There are several insurance policies created for health care in various ways. As epidemics command much attention in public health it is reasonable to find out a specific actuarial model for epidemiological diseases spread. Feng (2005) and Feng and Garrido (2006) have developed an actuarial model based on SIR infection model. This study carried is out on the basis of the model so developed and expands it due to problems arising in relation to satisfying some assumptions made with Sri Lankan epidemiological data.

An epidemic model describes the transmission process of the population due to an infectious disease. Such a model can identify the number or proportion of the population that is susceptible, infected or recovered (Abramson, 2001). As usually described in theory, there are two typical schemes for disease transmission:

(a) SIRS infection



(b) SEIR infection



The population compartments are typically represented by letters such as S, E, I, and R denoting Susceptible, Exposed, Infectious and Recovered populations. Individuals who are vulnerable to infection are known as ‘Susceptible’ and belong to the S compartment. An individual who is already infected, but does not show symptoms or is unable to infect others belongs to the E (Exposed) compartment. Once an infected individual starts infecting others, that person is ‘Infectious’ and belongs to I compartment. Finally, when an individual is cured from the infection that individual belongs to the R (Recovered) compartment. Recovered individuals either remain

there if they achieve permanent recovery or may become susceptible again and move back into S compartment.

An extensive proportion of cohesive and diversified research was published in literature of epidemic models to explain interrelations among these compartments. The SIR model created by Kermack and McKendrick (1927) was set the mathematical and theoretical foundation for the concept. SIRS and SEIR were simply extended versions of SIR with inclusion of extra compartment to the basic model. Further comparisons and assessments were done by various mathematicians, researchers (Mollison, 1995; Allen & Burgin, 2000; Kaddar et al., 2011; Bhattacharya et al., 2015) to extended thresholds of these models using advanced statistical tools and analytical viewpoints. In addition to that, enormous numbers of applied possessions are also available in literature with some advanced developments due to heterogeneity of real data of epidemics, population characteristics and policies. HIV, Malaria and Dengue are some popular epidemics which were subjected to the development in models.

Feng and Hernandez (1997) devised a model to describe the population dynamics of dengue fever. They used a system of differential equations that model the population dynamics of dengue as an SIR vector transmitted disease with two pathogen states. Similarly, Chen and Hsieh (2012), Side and Noorani (2013) and Pandey et al. (2013) did extensive analysis and comparisons of epidemic models using real data from some East Asian countries. Work done by Side and Noorani (2013) was very much similar to Feng and Hernandez (1997) in methodology and studied re-breeding value based on the number of reported cases of dengue fever in South Sulawesi, Indonesia and Selangor, Malaysia. Application of the SIR model showed similarities between the countries and indicated that dengue fever has not become endemic in either country. Chen and Hsieh (2012) modeled transmission dynamics of dengue with implications from temperature effects. Their results show that the temperature climate factor was important and influenced the dynamic modeling of the Vector-Host interaction. Pandey et al. (2013) compared the impact of modeling assumptions on the dynamics of dengue fever in Thailand from simple Vector-Host and SIR models. Results concluded that the SIR model is considerably better than the vector-host model.

Apart from examining such dynamics of Dengue fever; Rahman (2016) used HIV as the subject to understand the spread, persistence and prevention mechanisms of infectious diseases by mathematical models. His attention focused on vaccination strategies and proposed a mathematical model to measure the outcome of vaccinations. Further, some other attempts were focused aggregated aspects of epidemic modeling towards global stability with disease-free equilibrium (Guo & Shuai, 2006; Huang & Takeuchi, 2011).

The actuarial bases of epidemiological disease spread intend to address the financial and economic possessions of this venture. A book written by Slud (2001) provided imperative information on insurance and life annuity contracts. This was followed by the development of more applied mathematical models for insurance contracts relating to epidemic disease spread. Feng (2005) developed an actuarial model for epidemiology with the intention of building a bridge between epidemiology modeling and actuarial mathematics. In addition, his theory was utilized to design insurance contracts for the Great Plague in Eyam village, England and for the SARS epidemic in Hong Kong (Feng & Garrido, 2006). This was a significant contribution made to literature on epidemic modeling which has opened the doors to another testing ground for economic and financial analysts.

In the context of Sri Lankan epidemiology, Malaria was a disaster with its worst results taking place in the 1930s due to adverse climate conditions. There was a national attempt to eradicate Malaria fever, which succeed after large scale indoor residual spraying in 1947 (Fernando, 2013). As a result, in 2016, World Health Organization (WHO) officially certified the elimination of epidemics Malaria and Lymphatic Filariasis from Sri Lanka (WHO Annual Report, 2017). However, the regular occurrences of Dengue fever-related cases continue to trigger alarm and to threaten the success of Sri Lanka's anti-epidemic drive. According to Messer et al. (2002), Dengue transmission has grown since 1989 in Sri Lanka and is associated with large scale of urbanization and increasing human population. Unlike previous epidemics, Dengue vectors have been co-circulating in the environment with replacements of old genotypes with new (Kanakaratne et al., 2009; Sirisena and Noordeen, 2014).

Therefore, this has been a series issue where the increasing morbidity and mortality have had a significant economic impact over each season in Sri Lanka. For instance Sri Lanka reported 80,732 Dengue fever cases with 215 deaths within two quarters in January 2017. These records are 4.3 times larger than the average number of cases for the same period from 2010 onwards (WHO Official Web, 19 July 2017). As a result of epidemic waves, significant numbers of children and adults are getting infected and vulnerable since the majority are dependents (not in the labour force) in Sri Lankan society. Also such an issue requires vigilance in either preventing or curing epidemics. Public policy actors have difficulties in due time to care for them all as a result of continuous enhancements of economic burden over time. It requires the participation of investors from both public and private sectors to come up with a suitable solution where both get benefits.

As a solution, there is a possibility of having an insurance policy in action with a profit motive for investors and hospitalization benefits for people who get infected by epidemics. Also it requires proper research in the domestic context to observe the

applicability of the hypothesized solution. However, there were few studies recorded in Sri Lanka for modeling the spread of epidemic disease (Briet et al., 2008; Pathirana et al., 2009) and it is hard to discover an analysis based on SIR infection and actuarial based models. Hence, while putting the early steps for actuarial based modeling in Sri Lankan epidemic disease spread, this study intends to revisit the theory by Feng (2005) and to obtain an expanded version of it based on SIR infection which describes the transfer dynamics in an insurance contract considering highest total case recorded epidemics. The outcome of such an endeavor is intended to be utilized in developing an insurance policy to address concerns in practice.

METHOD

Mathematical Models in Epidemiology

Various models can be developed in terms of relevant compartments based on the nature of pathogens and diseases such as SIS, SIR, SEIS and etc. Without considering the demography of the host population, this simple model describes the conversion between susceptible, infectious and recovered sub-populations. If recovery is permanent and recovered individuals are no longer susceptible to that pathogen, then SIR model can be expressed as follows,



β is the infection rate for an individual per unit time and simultaneously γ is the recovery rate per unit time. According to this transformation dynamic of the population, the probabilities of corresponding compartments at time t denoted as $s(t)$, $i(t)$ and $r(t)$ respectively can be expressed as follows,

$$s'(t) = -\beta s(t)i(t), \quad t \geq 0 \quad (2.1)$$

$$i'(t) = \beta s(t)i(t) - \gamma i(t), \quad t \geq 0 \quad (2.2)$$

$$r(t) = 1 - s(t) - i(t), \quad t \geq 0 \quad (2.3)$$

With initial values $s(0) = s_0$, $i(0) = i_0$ and $s_0 + i_0 = 1$.

Assumptions:

1. The total population is constant for the considered time period.
2. β is the infectious rate per unit time and considered as a constant rate for a given disease.
3. γ is the recovery rate per unit time and considered as a constant rate for a given disease.
4. Natural births and deaths are not considered.

SIRS model is more general than SIR model. The only difference when compared with the SIR model is defining a new parameter f which represents the rate of recovered individuals transforming to susceptible again per unit time due to the temporary recovery from the infectious disease. Hence other assumptions and conditions remain same as previous model and the following equations can be obtained:

$$s'(t) = -\beta s(t)i(t) + fr(t), \quad t \geq 0 \quad (2.4)$$

$$i'(t) = \beta s(t)i(t) - \gamma i(t), \quad t \geq 0 \quad (2.5)$$

$$r'(t) = \gamma i(t) - fr(t), \quad t \geq 0 \quad (2.6)$$

Important formulas from Actuarial Mathematics

Actuarial mathematics concepts are emerged to describe the financial transactions between two parties called the insurer and the insured.

Equivalence Principle:

$$E[\text{Present Value of Benefits}] = E[\text{Present Value of Benefit Premiums}]$$

For a continuous whole life insurance policy with a unit benefit the level premium payment can be determined using equivalence principle as,

$$\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_x} \quad (2.7)$$

Where $\bar{A}_x$ is the actuarial present value of future benefit payments and $\bar{a}_x$ is the actuarial present value of future premium payments. The benefit reserve for the same policy can be defined as follows,

$$\text{Prospective method:} \quad {}_t\bar{V}(\bar{A}_x) = \bar{A}_{x+t} - \bar{P}(\bar{A}_x)\bar{a}_{x+t} \quad (2.8)$$

$$\text{Retrospective method:} \quad {}_t\bar{V}(\bar{A}_x) = \frac{\bar{P}(\bar{A}_x)\bar{a}_{x:t} - \bar{A}_{x:t}}{{}_tE_x} \quad (2.9)$$

Formulas of Actuarial Model based on SIR compartment model

An actuarial based model has developed for the epidemiological diseases and the following equations are given for the annuity for hospitalization plan which has defined by using the whole life insurance policy. When δ is the force of interest,

The total discounted future claim:

$$\bar{a}^i = \int_0^{\infty} e^{-\delta t} i(t) dt \quad (2.10)$$

The total discounted future premium:

$$\bar{a}^s = \int_0^{\infty} e^{-\delta t} s(t) dt \quad (2.11)$$

The force of infection:

$$\mu_t^s = \frac{-s'(t)}{s(t)} \quad (2.12)$$

The force of infection:

$$\mu_t^i = \frac{-i'(t)}{i(t)} \quad (2.13)$$

The level premium for the unit annuity for hospitalization plan:

$$\bar{P}(\bar{a}^i) = \frac{\bar{a}^i}{\bar{a}^s} = \frac{\delta \bar{a}^i}{1 - (\delta + \gamma) \bar{a}^i} \quad (2.14)$$

RESULTS AND DISCUSSION

Further Results on Actuarial model for SIR infection

As mentioned in the Section 2.3, there is an actuarial model which was developed by using the SIR infection model (Feng, 2005; Feng & Garrido, 2006) and this section (3.1) of the study consists of further findings which are obtained over the developments in previous model.

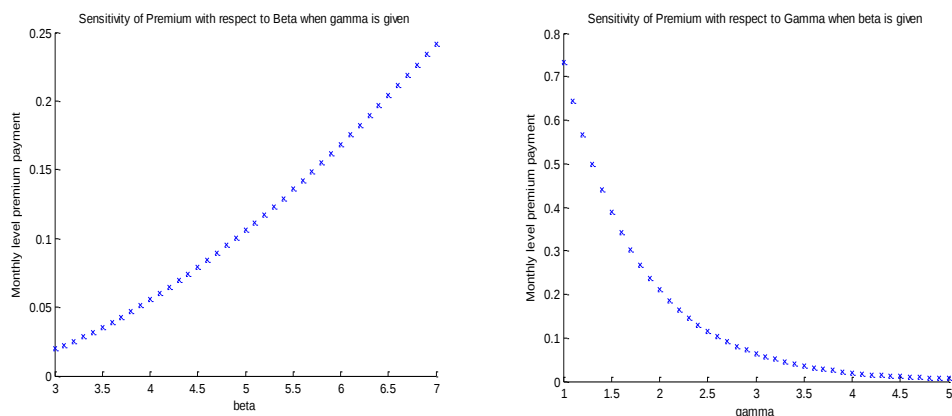
Sensitivity of Level Premium Payment with respect to the parameters

Determining the level premium payment with positive benefit reserve is mainly focused when the actuarial model is developed. According to the observation of this study, there is an effect from the parameters, α and β to determine the level premium payment.

- β , the rate of infection is positively related to the level premium payment.
- γ , the rate of recovery is negatively related to the level premium payment.

In general terms it is easy to explain, when infection rate increases the benefit payment also increases. Then according to the equivalence principle level premium must increase. When recovery rate increases benefit payment will decrease. Hence level premium should be increased due to the equivalence principle. Above results were obtained while developing MATLAB simulation [See APPENDIX 1] for the actuarial based model for SIR infectious disease developed by Feng (2005) for SARS epidemic.

Figure 1: Level Premium Payment with the Rate of Infection (β) and Rate of Recovery (γ)



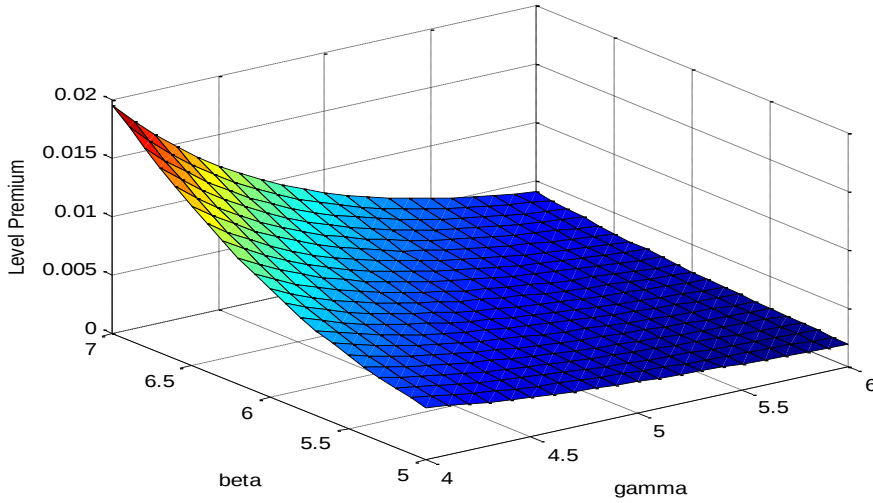
Source: Authors' Simulation Using MATLAB

The Figure 1 (left hand side) shows a positive relationship between level premium payment and rate of infection. It is easy to describe verbally that how it happens is as follows: when rate of infection increases the count of people who want medicine also increases. Thus the insurance company has to pay more medical expenses for covered people. According to the equivalence principle they increase the level premium payment to cover the cost. Hence the above graph is evidence of the trend applicable in real life situations.

Secondly we observed the sensitivity of level premium payment to recovery rate based on the same data. For the same data from SARS epidemic we can see negative relationship between level premium payment and recovery rate. In real terms, when recovery rate increases the length of hospitalization decreases and hospitalization costs decrease. Hence due to the equivalence principle the level premium payment decreases.

In addition, Figure 2 represents the sensitivity of level premium payment with simultaneous changes in the rates of infection (β) and recovery (γ). The rates given at the monthly basis vary from 5-7 for infection rate and 4-6 for recovery rate. Simulation shows that a simultaneous decline in the recovery rate and increase in infection rate leaning the level premium rates towards zero. Premium rate is rising to the highest possible level when an increase in recovery rate occurs simultaneous to an improvement in infection rate.

Therefore, independent as well as simultaneous changes in the rates of infection and recovery specify the characteristics of the level of premium payment to be considered for a hospitalization plan.

Figure 2: Sensitivity of Premiums with Parameters

Source: Authors' Simulation using MATLAB

Adjusting Level Premium Payment

According to the retrospective approach the individual benefit reserve at time t for the annuity for hospitalization plan with unit benefit can be formulated as follows:

$$\bar{V}_t(\bar{a}^i) = \int_0^t [\bar{P}(\bar{a}^i)s(t)e^{\delta t} - i(t)e^{\delta t}] dt, \quad t > 0$$

However, to satisfy the requirement of positivity of benefit reserve curve, for all $t > 0$

$$\bar{P}(\bar{a}^i) \geq \frac{\int_0^t i(t)e^{\delta t} dt}{\int_0^t s(t)e^{\delta t} dt}$$

Feng (2005) has carried out some tests by setting up $\delta = 0$ and those results do not make sense of the time value of money. Given the complexity of solving equations without neglecting the force of interest, an algorithm is defined and a MATLAB program developed through this study to calculate the minimum adjusted level premium for the hospitalization plan to satisfy the above condition. [See APPENDIX 2]. This program could be used to calculate the level premium of diseases which have permanent immunity with the absence of Vector-Host transfer dynamics. Otherwise it will not be adequate to obtain 100 percent accuracy in results. Therefore, it is important to identify nature and characteristics of Sri Lankan epidemic diseases to recognize the applicability of the program developed.

Brief Analysis of Epidemics in Sri Lanka

This sub section provides an understanding on the epidemics in Sri Lanka based on the data for a 40-week period beginning from 26th December 2015 to 30th September 2016. The data were collected from the official website of the Epidemiology Unit of Sri Lanka. According to available data there are 18 diseases identified as epidemics in Sri Lanka. However some types of diseases are rarely recorded relative to other diseases. This analysis carried out based on only the top 10 epidemics which have highest total cases recorded for the above mentioned period.

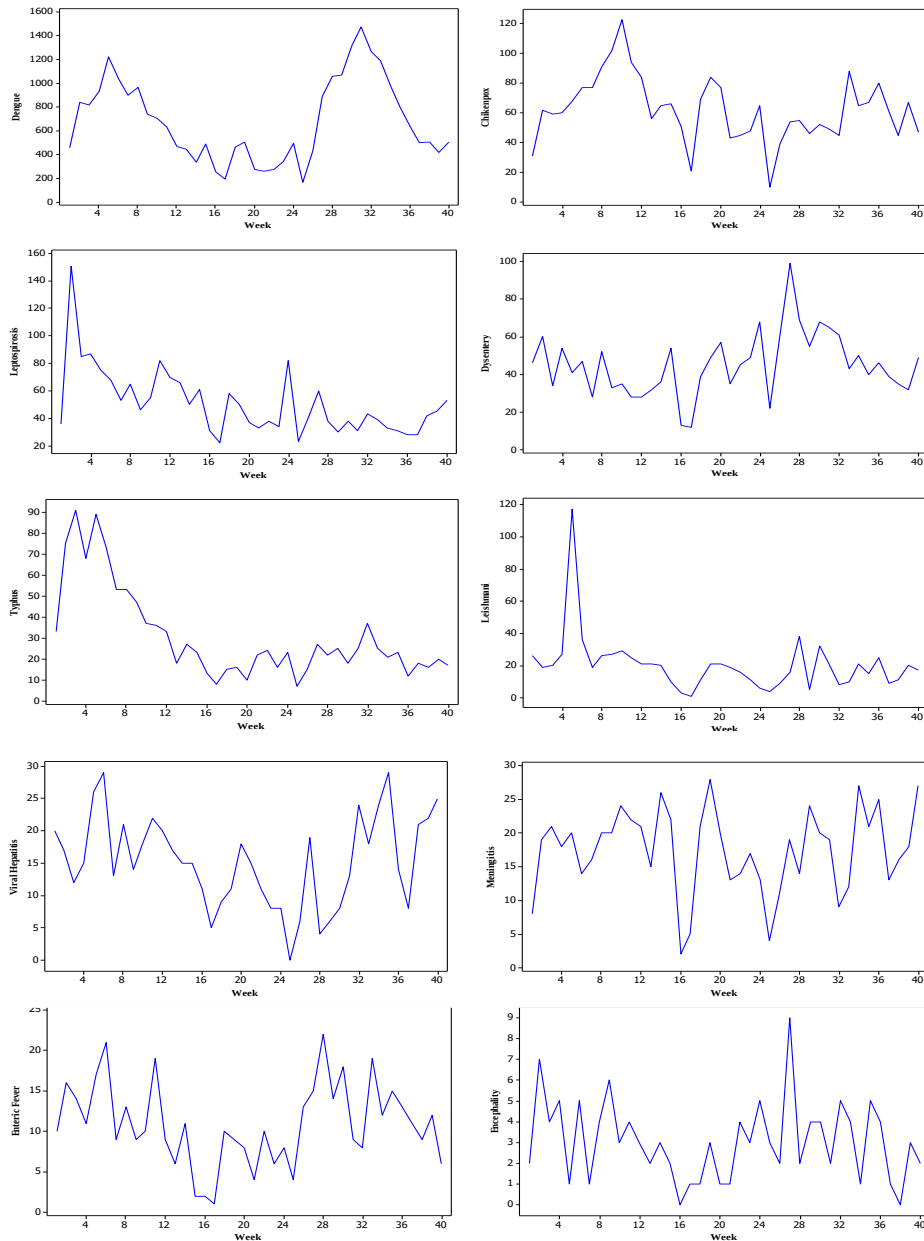
Table 1: Total Cases of Epidemic Diseases (26th December 2015 to 30th September 2016)

Disease	No. of cases	Percentage from total cases
Dengue	27,209	72.65%
Chickenpox	2,488	6.64%
Leptospirosis	2,038	5.44%
Dysentery	1,809	4.83%
Typhus	1,231	3.29%
Leishmani	813	2.17%
Meningitis	698	1.86%
Viral Hepatitis	611	1.63%
Enteric Fever	435	1.16%
Encephalitis	122	0.33%
Total	37,454	100%

Source: Epidemiological Unit, Sri Lanka

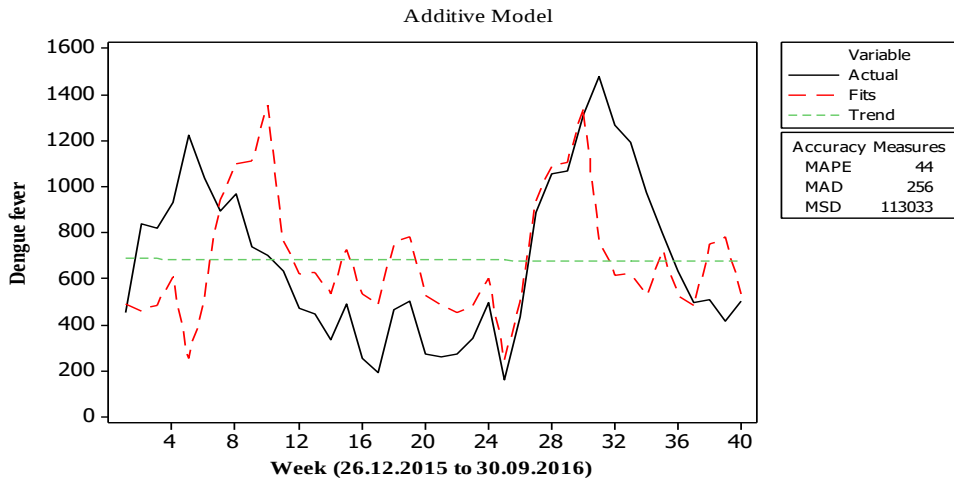
According to the above table, the highest recorded number of cases arise in relation to Dengue which accounts for 72.65% of the total top 10 epidemic cases. This implies that the probability of being infected with Dengue for a person in Sri Lanka is much higher than in other diseases. However there are considerable percentages for the diseases Chickenpox (6.64%), Leptospirosis (5.44%), Dysentery (4.83%) and Typhus (3.29%).

There are several patterns which can be seen when constructing time plots for the above diseases. Some have clear seasonal patterns (Dengue fever). Also some have very short term fluctuations and it is difficult to determine the length of a season (Dysentery, Meningitis and etc.).

Figure 3: Time Series Plots for Epidemiological Data

Source: Authors' Preparation

Additionally some diseases have declining patterns (Leptospirosis, Typhus and Leishmaniasis). However it is important to study the reasons behind those patterns. Thus this study is focused on developing an actuarial model for the spread of epidemiological diseases which can be used more generally to reduce the impact of several patterns

Figure 4: Time Series Decomposition Plot for Dengue Fever

Source: Authors' Preparation

Dengue fever only contains a clear seasonal pattern based on the data for 40 weeks. The above figure provides further evidence on the seasonal behaviour of dengue epidemic through a comparison of actual data with estimated measures for a given optimal lag length of 20 weeks for each season. Therefore, dengue fever has the expected seasonal features and appears to be a testing ground for the feasibility of the insurance contract. However, there are some questions to be addressed through further advancements of actuarial model in light of long term effects such as Vector-Host transfer dynamics embedded with epidemic disease spread.

According to the above data the length of an epidemic season for some diseases such as dengue can be estimated. But the SIR model is defined by neglecting the type of disease which can be transferred by a vector. Dengue fever is the major epidemic disease in Sri Lanka which is generally spread by mosquitoes. Hence it is reasonable to expand the SIR model by including Vector-Host transfer dynamics to find out an actuarial model for diseases such as Dengue fever.

Actuarial Based Model for Dengue Fever Spread using SIR (Vector-Host) Model

The simplified SIR model for Dengue fever can be formulated as follows,

$$\begin{aligned}
 s'_h(t) &= \tau - n\beta_h b s_h(t) i_v(t) - \mu_h s_h(t), & t > 0 \\
 i'_h(t) &= n\beta_h b s_h(t) i_v(t) - (\gamma + \mu_h) i_h(t), & t > 0 \\
 i'_v(t) &= \beta_v b (1 - i_v(t)) i_h(t) - \mu_v i_v(t), & t > 0
 \end{aligned}$$

Where $s_h(t)$ – probability of being susceptible host at time t , $i_h(t)$ – probability of being infected host at time t , $i_v(t)$ – probability of being infected vector at time t , τ - human birth rate, μ_h - mortality rate of human, γ - recovery rate of humans, b - daily biting rate, β_h - probability of transmission of dengue virus from infected mosquitoes to human per bite, β_v - probability of infection from human to mosquito per unit time and n is equal to (vector population/host population). Same as the assumptions made for previous models when assuming that birth rate and mortality rates are negligible for the short term period, above system of ODEs can be simplified by setting $\tau = 0$ and $\mu_h = 0$.

$$s'_h(t) = -n\beta_h b s_h(t) i_v(t), \quad t > 0$$

$$i'_h(t) = n\beta_h b s_h(t) i_v(t) - \gamma i_h(t), \quad t > 0$$

$$i'_v(t) = \beta_v b (1 - i_v(t) i_h(t)) - \mu_v i_v(t), \quad t > 0$$

Using the same procedure carried out to obtain Result 2 it can be easily shown that,

$$\bar{a}^{s_h} = \frac{1 - (\delta + \gamma) \bar{a}^{i_h}}{(\delta)}$$

And it yields to the level premium payment which formulated for the SIR infection model is same here. Hence further results can be obtained using the same procedure until calculating adjusted level premium payment.

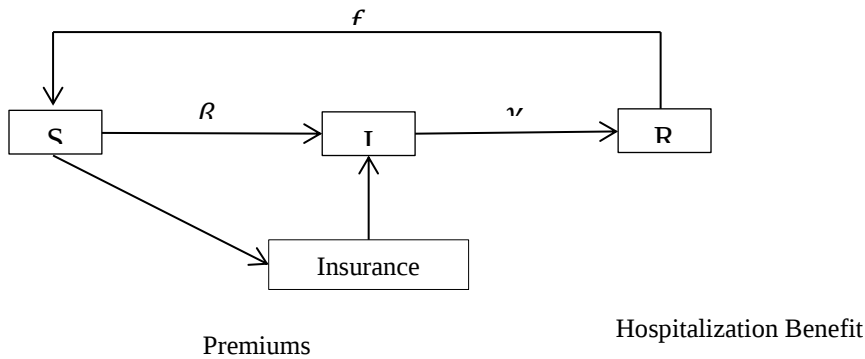
Actuarial Model using SIRS Model

According to the above analysis of epidemics in Sri Lanka, it can be observed that some diseases do not show clear seasonal patterns. Moreover some diseases consist of short term fluctuations, and it is difficult to determine the length of epidemic period. Also some persons can be infected by the same disease more than once for the time period considered.

On the other hand, an insurance contract is usually for an annum or six month period and is rarely adjusted with the epidemic season. Hence the transformation within compartments for a long term can be described more generally using SIRS model than SIR model. Moreover when the parameter f (the rate of transferring recovered person to a susceptible) set as zero, it implies the SIR infection model. Hence using SIRS infection model, it is possible to define an actuarial model for a wide range of diseases.

The transfer dynamic among S, I, R compartments with insurance can be displayed as in the Figure 5 (below).

Figure 5: Transfer dynamics between S, I, R compartments and insurance



Source: Authors' Preparation

According to the above transfer dynamics, similar to SIR model to obtain a general equation for the level premium payment for the annuity for hospitalization plan based on Whole Life Insurance policy following results are obtained:

Result 1:

$$\int_0^{\infty} e^{-\delta t} \int_0^t f(r) dr dt = \frac{1}{\delta} \int_0^{\infty} e^{-\delta r} f(r) dr$$

Proof: Considering the derivative of $\exp(-\delta t)$ with respect to t ,

$$\int_0^{\infty} e^{-\delta t} \int_0^t f(r) dr dt = \frac{-1}{\delta} \int_0^{\infty} \int_0^t f(r) dr d(e^{-\delta t}) \quad \text{as } d(e^{-\delta t}) = \frac{-1}{\delta} e^{-\delta t} dt$$

Then by integrating by parts,

$$\int_0^{\infty} e^{-\delta t} \int_0^t f(r) dr dt = \frac{1}{\delta} \int_0^{\infty} e^{-\delta r} f(r) dr$$

Result 2: In the SIRS model of (1.4) to (1.6)

$$\bar{a}^s = \frac{1 - (\delta + \gamma + f)\bar{a}^i}{(\delta + f)} \quad (3.1)$$

Proof: Consider the addition of (1.4) and (1.6),

$$s'(t) + i'(t) = fr(t) - \gamma i(t), \quad t > 0$$

Thus,

$$s'(t) + i'(t) = 1 - fs(t) - (\gamma + f)i(t), \quad t > 0 \quad \text{as } r(t) = 1 - s(t) - i(t)$$

Then by integrating both sides from 0 to a fixed T,

$$s(t) + i(t) - 1 = -f \int_0^T s(r) dr - (\gamma + f) \int_0^T i(r) dr, \quad T > 0$$

Multiplying both sides by $\exp(-\delta t)$ and integrates with respect to T from 0 to ∞ and also considering Result 1,

$$\bar{a}^s + \bar{a}^i - \frac{1}{\delta} = -\frac{f}{\delta} \bar{a}^s - \frac{(\gamma + f)}{\delta} \bar{a}^i, \quad T > 0$$

Yields to the Result 2.

Hence the level premium for the annuity for hospitalization plan for SIRS infection can be formulated as,

$$\bar{P}(\bar{a}^i) = \frac{\bar{a}^i}{\bar{a}^s} = \frac{(\delta + f)\bar{a}^i}{1 - (\delta + \gamma + f)\bar{a}^i} \quad (3.2)$$

CONCLUSION AND POLICY RECOMMENDATIONS

This study based on a developed actuarial model of SIR infection which describes transfer dynamics in an insurance contract in a given population. At the initial stage, we satisfied key assumptions and observed that the rate of infection is positively related and the rate of recovery is negatively related to the level premium payment. Therefore, when infection rate increases the benefit payment also increases. Then according to the equivalence principle level premium must increase. When recovery rate increases the benefit payment will decrease. Hence level premium should be increased due to the equivalence principle.

Further, we developed a MATLAB program to calculate the minimum adjusted level premium for a hospitalization plan. However if those diseases are transferred by vectors the model should be expanded by including impact of vector for disease spread. But the equation for level premium payment is not changed due to the impact of vector. As a result of these verifications, there is a possibility of introducing an insurance policy for the spread of Dengue in Sri Lanka. The developed MATLAB program can be used for the premium calculation purposes for a given hospitalization plan.

The study also focused on obtaining a more general actuarial based model for epidemiological diseases spread in Sri Lanka. When a disease has permanent immunity, the simple SIR model can be used to develop an actuarial model. However it is difficult to identify seasonal patterns clearly for diseases which confer temporary immunity. An insurance contract is determined usually for an annum or six month period. The appropriate epidemic period may have the same length as the insurance

contract. According to the definition of an epidemic, epidemic period is considered as a short time period such as two-three months. Hence it is reasonable to study the long term behavior of epidemic data using SIRS model to obtain a more accurate Actuarial Based Model for the spread of epidemiological diseases. But SIRS model is expressed using Delay Differential Equations and this study was not focused on simulating that model. Finally, as a result of difficulties in identifying seasonal patterns clearly for other diseases except Dengue fever, we recommend SIRS infection model with delayed differential equations as an appropriate solution for further research in the Sri Lankan context due to its possibility to define an actuarial model for a wide range of diseases.

REFERENCES

- Abramson, G. (2001). Mathematical modeling of the spread of infectious diseases. Retrieved from <https://fisica.cab.cnea.gov.ar/estadistica/abramson/notes/Epidemics-Lectures-PANDA.pdf>
- Allen, L. J., & Burgin, A. M. (2000). Comparison of deterministic and stochastic SIS and SIR models in discrete time. *Mathematical biosciences*, 163(1), 1-33.
- Bhattacharya, P., Paul, S., & Choudhury, K. S. (2015). Different Types of Epidemic Models and their Characteristic Behaviour by using Matlab. *Journal of Interdisciplinary Mathematics*, 18(5), 569-592.
- Briet, O. J., Vounatsou, P., Gunawardena, D. M., Galappaththy, G. N., & Amerasinghe, P. H. (2008). Models for short term malaria prediction in Sri Lanka. *Malaria Journal*, 7(1), 76.
- Chen, S. C., & Hsieh, M. H. (2012). Modeling the transmission dynamics of dengue fever: implications of temperature effects. *Science of the total environment*, 431, 385-391.
- Feng, R. H. (2005). Epidemiological models in actuarial mathematics (Master thesis, Concordia University). Retrieved from <http://spectrum.library.concordia.ca/8666/1/MR16244.pdf>
- Feng, R., & Garrido, J. (2006). Application of Epidemiological Models in Actuarial Mathematics. Retrieved from <https://pdfs.semanticscholar.org/e034/72ea8f54.pdf>
- Feng, Z., & Velasco-Hernández, J. X. (1997). Competitive exclusion in a vector-host model for the dengue fever. *Journal of mathematical biology*, 35(5), 523-544.
- Fernando, P. (2013). Past Malaria Epidemics in Sri Lanka-An Analysis. *Journal of the College of Community Physicians of Sri Lanka*. 19(1). 27-41.

- Guo, H., Li, M. Y., & Shuai, Z. (2006). Global stability of the endemic equilibrium of multigroup SIR epidemic models. *Canadian Applied Mathematics Quarterly*, 14(3), 259-284.
- Huang, G., & Takeuchi, Y. (2011). Global analysis on delay epidemiological dynamic models with nonlinear incidence. *Journal of mathematical Biology*, 63(1), 125-139.
- Kaddar, A., Abta, A., & Alaoui, H. T. (2011). A comparison of delayed SIR and SEIR epidemic models. *Nonlinear Analysis: Modelling and Control*, 16(2), 181-190.
- Kanakarathne, N., Wahala, W. M. P. B., Messer, W. B., Tissera, H. A., Shahani, A., Abeyasinghe, N., De Silva, A. M. and Gunasekera, M. (2009). Severe Dengue Epidemics in Sri Lanka, 2003–2006 . *Emerging Infectious Diseases*, 15(2), 192–199.
- Kermack, W. O., & McKendrick, A. G. (1927). A contribution to the mathematical theory of epidemics. In *Proceedings of the Royal Society of London A: mathematical, physical and engineering sciences*. 115(772), 700-721. Retrieved from <http://rspa.royalsocietypublishing.org/content/royprsa/115/772/700.full.pdf>
- Messer, W. B., Vitarana, U. T., Sivananthan, K., Elvtigala, J., Preethimala, L. D., Ramesh, R., Withana, N., Gubler, D. J. and De Silva, A. M. (2002). Epidemiology of Dengue in Sri Lanka Before and After the Emergence of Epidemic Dengue Hemorrhagic Fever. *The American journal of tropical medicine and hygiene*, 66(6), 765-773.
- Mollison, D. (1995). *Epidemic models: their structure and relation to data*, 5. Cambridge University Press.
- Pandey, A., Mubayi, A., & Medlock, J. (2013). Comparing vector–host and SIR models for dengue transmission. *Mathematical biosciences*, 246(2), 252-259.
- Pathirana, S., Kawabata, M., & Goonatilake, R. (2009). Study of potential risk of dengue disease outbreak in Sri Lanka using GIS and statistical modelling. *Journal of Rural and Tropical Public Health*, 8, 8. Retrieved from http://epubs.scu.edu.au/esm_pubs/865/
- Rahman, S. A. (2016). *Study of Infectious Diseases by Mathematical Models: Predictions and Controls* (Doctoral dissertation, The University of Western Ontario). Retrieved from <http://ir.lib.uwo.ca/etd/3487/>
- Rihan, F. A., & Anwar, M. N. (2012). Qualitative analysis of delayed SIR epidemic model with a saturated incidence rate. *International Journal of Differential Equations*. Retrieved from <https://www.hindawi.com/journals/ijde/2012/408637/abs/>

- Side, S., & Noorani, M. S. M. (2013). A SIR model for spread of dengue fever disease (simulation for South Sulawesi, Indonesia and Selangor, Malaysia). *World Journal of Modelling and Simulation*, 9(2), 96-105.
- Sirisena, P. D. N. N. and Noordeen, F. (2014). Evolution of dengue in Sri Lanka- changes in the virus, vector, and climate. *International Journal of Infectious Diseases*, 19, 6-12.
- Slud, E. V. (2012). *Actuarial mathematics and life-table statistics*. Chapman & Hall/CRC.
- WHO (2017). *Communicable Diseases. Annual Report 2016: A Journey of Commitment, Passion and Dedication*. Published by World Health Organization (WHO) Country Office for Sri Lanka. 17-23.

BIBLIOGRAPHY

- Epidemiological Unit - Sri Lanka, <http://www.epid.gov.lk/web/index.php?lang=en>
- WHO Official Web (19 July 2017). Dengue fever – Sri Lanka. Disease Outbreak News. World Health Organization, <http://www.who.int/csr/don/19-july-2017-dengue-sri-lanka/en/>

APPENDIX - A

MATLAB Code for simulate level premium payment with β and γ

1) M-file: sirmodelID

```
function f=sirmodelD(t,Y,P)

S=Y(1); % the number of susceptible individuals
I=Y(2); % the number of infected individuals
R=Y(3); % the number of recovered individuals

%parameter values; [ s ->(beta)-> i ->(gamma)-> r ]
beta=P(1);
gamma=P(2); %per day
N=261;

%System of ODE
f(1,1)=-beta*S*I/N;
f(2,1)=beta*S*I/N-gamma*I;
f(3,1)=gamma*I;
```

2) M-file: beta_PAH.m

```
figure;
hold on
for i=3:0.1:7
P=[i/30 2.73/30];

S0=254;
I0=7;
R0=0;
Y0=[S0 I0 R0]; %initial values for S'(t), I'(t) and R'(t)
tspan=[0 365]; % time range in days
sol=ode45(@sirmodelD,tspan,Y0,[],P);

x = linspace(0,365,365);
y = deval(sol,x);

N=S0+I0;
delta=0.2/30;

ai0=0;
as0=0;
for t=1:365
    it=y(2,t)/N;
    st=y(1,t)/N;
    ai0=ai0+it*(exp(-delta*(t-1))-exp(-delta*t))/delta;
    as0=as0+st*(exp(-delta*(t-1))-exp(-delta*t))/delta;
end
Ai0=(delta+P(1))*ai0-I0/N;
PAH=ai0/as0;
plot(i,PAH,'x');
```

```
title('Sensitivity of Premium with respect to Beta when gamma is given');
xlabel('beta');
ylabel('Monthly level premium payment');
end
hold off
```

3) M-file: gamma_PAH.m

```
figure;
hold on
for i=1:0.1:5
P=[4.68/30 i/30];

S0=254;
I0=7;
R0=0;
Y0=[S0 I0 R0]; %initial values for S'(t), I'(t) and R'(t)
tspan=[0 365]; % time range in days
sol=ode45(@sirmode1D,tspan,Y0,[],P);

x = linspace(0,365,365);
y = deval(sol,x);

N=S0+I0;
delta=0.2/30;

ai0=0;
as0=0;
for t=1:365
    it=y(2,t)/N;
    st=y(1,t)/N;
    ai0=ai0+it*(exp(-delta*(t-1))-exp(-delta*t))/delta;
    as0=as0+st*(exp(-delta*(t-1))-exp(-delta*t))/delta;
end
Ai0=(delta+P(1))*ai0-I0/N;
PAH=ai0/as0;
plot(i,PAH,'x');
title('Sensitivity of Premium with respect to Gamma when beta is given');
xlabel('gamma');
ylabel('Monthly level premium payment');
end
hold off
```

4) beta_gamma_PAH.m

```
1) beta_gamma_PAH.m
A=[];
B=[];
C=[];
l=1;
for j=0:0.01:1 % define the range for gamma(monthly rate)
    k=1;
    for i=0:0.01:1 % define the range for beta(monthly rate)

        P=[i/30 j/30];
```

```

S0=254;
I0=7;
R0=0;
Y0=[S0 I0 R0]; %initial values for S(t), I(t) and R(t)
tspan=[0 150]; % time range in days
sol=ode45(@sirmodelD,tspan,Y0,[],P);

x = linspace(0,150,150);
y = deval(sol,x);

N=S0+I0;
delta=0.2/30; %force of interest per day

ai0=0;
as0=0;
for t=1:150
    it=y(2,t)/N;
    st=y(1,t)/N;
    ai0=ai0+it*(exp(-delta*(t-1))-exp(-delta*t))/delta;
    as0=as0+st*(exp(-delta*(t-1))-exp(-delta*t))/delta;
end

PAH=ai0/30/as0;
C(k,1)=PAH;
A(k)=P(1);
k=k+1;
end
B(1)=P(2);
l=l+1;
end
figure;
hold on
surf(B,A,C);
xlabel('gamma');
ylabel('beta');
zlabel('Level Premium Payment');
title('Sensitivity of Level Premium Payment with parameters');
hold off

```

APPENDIX - B

MATLAB Code for Simple SIR Model

```

adjusted_BR_AH.m
N=261; % total population
S0=254; % initial probability of susceptible
I0=7; % initial probability of infected
R0=0; % initial probability of recovered
initial=[S0 I0 R0];
time=[0 150]; % length of epidemic period in days
P=[4.33/30 2.73/30]; % assigning monthly rates

sol=ode45(@sirmodelD,time,initial,[],P);
x = linspace(0,150,150);
y = deval(sol,x);

```

```

delta=0.2/30; % force of interest (daily)
beta=P(1);
gamma=P(2);
%%
%following section calculates the level premium payment PAH
ai0=0;
as0=0;
for t=1:150
    it=y(2,t)/N;
    st=y(1,t)/N;
    ai0=ai0+it*(exp(-delta*(t-1))-exp(-delta*t))/delta;
    as0=as0+st*(exp(-delta*(t-1))-exp(-delta*t))/delta;
end
PAH=ai0/30/as0;
disp('The Level Premium Payment before the adjustment');
disp(PAH);
%%
%following section calculates the adjusted Level Premium Payment
ai0=0;
as0=0;
for t=1:150
    it=y(2,t)/N;
    st=y(1,t)/N;
    ai0=ai0+it*(exp(-delta*(t-1))-exp(-delta*t))/delta;
    as0=as0+st*(exp(-delta*(t-1))-exp(-delta*t))/delta;

    while PAH<ai0/as0/30
        PAH=PAH+0.0000001;
    end

end

disp('The Adjusted Level Premium');
disp(PAH);
%%
%Plotting the benefit reserve curve for the adjusted level premium
figure;
hold on;
for time=1:150
    ai0=0;
    as0=0;
    for t=1:time
        it=y(2,t)/N;
        st=y(1,t)/N;
        ai0=ai0+it*(exp(-delta*(t-1))-exp(-delta*t))/delta;
        as0=as0+st*(exp(-delta*(t-1))-exp(-delta*t))/delta;
    end
    pt=as0*PAH;
    bt=ai0/30;
    vt=(pt-bt);
    plot(t,vt,'x');
end
xlabel('Number of days');
title('Plot of Adjusted Benefit Reserve for Plan AH');
hold off

```