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The Ant Colony Optimization For Solving Multi-Objective Assignment Problems Under Uncertainty

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Abstract

This study investigates the effectiveness of the Triangular Distribution (TD) and Ant Colony Optimization Algorithm (ACOA) in solving Multi-Objective Assignment Problems (MOAPs) under uncertainty. ACOA is a well-known biologically inspired metaheuristic algorithm which is widely used for solving large scale optimization problems. Its extension, the Multi-Objective Ant Colony Optimization Algorithm (MOACOA), provides a probabilistic framework for identifying near-optimal solutions in MOAPs. In real-world assignment problems, Assignment Costs (ACs) are often uncertain and cannot be precisely determined due to limited or incomplete information. To address the uncertainty, this study assumes that uncertain ACs follow a TD characterized by optimistic, pessimistic, and most probable values. Monte Carlo simulation is employed to estimate the expected assignment costs, allowing the uncertain problem to be deterministic equivalent. The resulting deterministic model is solved considering each objective separately using ACOA, yielding the positive ideal solution and the negative ideal solution for each objective. To handle multiple objectives simultaneously, a Fuzzy Exponential Membership Function (FEMF) is defined for each objective. The overall performance of each assignment is evaluated by aggregating the FEMF values across all objectives. The results demonstrate that variations in the shape parameter of the FEMF significantly influence the satisfaction levels of individual objectives. The obtained solutions consistently align with the preferences of the Decision Maker (DM). When a particular assignment does not meet expectations, alternative solutions can be generated by adjusting the shape parameters. The combined TD and ACOA framework effectively produce multiple reliable assignment alternatives and illustrates how different shape parameters impact objective trade-offs and convergence behavior. Overall, the proposed approach successfully identifies optimal assignments that reflect the DM's shape parameter under uncertainty. Without loss of generality, the algorithm is executed 100 times to compare the performance of the proposed ACOA with that of the Genetic Algorithm for a range of 1 to 10 shape parameter values. In general, the findings validate the capability of the proposed ACOA algorithm with triangular random parameters and exponential membership functions to reach near optimal solutions to multi-objective uncertain assignment problems. Different from prior studies that largely concentrate on either deterministic or fuzzy assignment problems, the proposed study focuses on uncertain assignment costs following TD, ACOA, and FEMFs to optimize multiple conflicting objectives, thereby addressing the limited research available with uncertain MOAPs along with decision preferences based on shape parameters. Future research can focus on metaheuristic hybrid approach to solve this problem to reach more stable solution with faster convergence.

Keywords: Ant colony optimization algorithm, Assignment problem, Monte Carlo simulation, Triangular distribution, Genetic algorithm.

1. Introduction

This study proposes a method for solving Multi-Objective Assignment Problems (MOAPs) under uncertainty. The classical assignment problem, a well-known combinatorial optimization problem in the field of Operations Research, seeks to optimally assign n tasks to n agents (e.g., workers, machines, etc.), such that one-to-one assignments between agents and tasks. However, real-life applications frequently involve multiple objectives and uncertain parameters, particularly in assignment costs. In many practical scenarios, uncertainty arises due to insufficient historical data, subjective judgment, or fluctuating operating conditions. To address this issue, the present study adopts the uncertainty theory proposed by Liu, which provides a structured framework for modeling uncertain variables [1, 2]. Lei Wang reported on the use of probabilistic methods to determine the feasibility of an optimal robust solution to uncertain problems [3]. In 2018, a comprehensive review was conducted on multi-objective approaches in Ant Colony Optimization (ACO) [4]. A comparison between different variants of multi-objective ACO was studied and observed that the strength of ACO lies in dealing with combinatorial discrete optimization problems due to probabilistic search strategy and pheromone learning. ACO has the potential to address the optimization problem even under the conditions where the uncertainties in the parameters exist in the systems. This was established by studying fuzzy stochastic optimization techniques using ACO [5]. Fuzzy and stochastic variables were incorporated in the model, which was developed under the framework of production distribution. In parallel to this study, an approach that gained particular relevance in uncertainty modeling was based on the use of triangular fuzzy numbers. In the same year, the study demonstrated that a general multi-objective optimization model under uncertainty, incorporating triangular fuzzy numbers to represent uncertain parameters, is highly effective for engineering and operational decision-making problems [6]. In 2019, a new pheromone update strategy was introduced that combined local pheromone communication with a global search mechanism to identify optimal solutions [7]. In align with this study, a fuzzy ant colony optimization algorithm was developed following the formulation of the mathematical model of the problem [8]. In this approach, heuristic information was generated for each candidate list based on the structure of the solution space to effectively guide the ants during the search process. Furthermore, a fuzzy local optimization procedure was incorporated to enhance the overall solution quality. Furthermore, several researchers have applied ant colony optimization techniques to determine and implement optimal routing paths between destinations [9]. In such dynamic environments, uncertainty associated with obstacles presents a significant challenge, highlighting the need for an intelligent and robust algorithm capable of producing efficient solutions with minimal computational overhead. Recent studies have further demonstrated the effectiveness of incorporating fuzzy uncertainty into multi-objective optimization problems. In 2024, a study employed triangular fuzzy numbers to model cost, supply, and demand parameters in a multi-objective transportation problem [10]. The authors showed that the fuzzy formulation was equivalent to the corresponding crisp model while remaining computationally efficient and capable of retaining uncertainty-related information. In the same year, a hybrid ant colony optimization approach was proposed to address optimization problems involving conflicting objectives with uncertainty [11]. Furthermore, the probabilistic search mechanism incorporated within the ACOA framework was shown to be highly compatible with fuzzy uncertainty environments, thereby enhancing its effectiveness in solving complex decision-making problems.

Also in the same year, broader methodological analyses of ACO emphasized the growing importance of developing approaches capable of addressing multi-criteria, stochastic, and uncertain optimization problems in modern applications [12]. Despite the growing body of research on ACO, studies specifically addressing multi-objective assignment problems remain limited. A recent study proposed the application of an ACO-based approach for solving multi-objective deterministic assignment problems and demonstrated that the probabilistic solution construction and pheromone updating mechanisms of ACO can effectively generate high-quality assignments [13]. However, the study focused solely on deterministic cost structures and did not consider uncertainty within the assignment process. In the year 2025, an analytical review identified five major research domains namely energy, sustainability, management, engineering, and related applied fields in which multi-objective stochastic optimization techniques are widely utilized [14]. The review highlighted the significance of these approaches in managing uncertainty and balancing conflicting objectives in complex real-world systems.

Assignment costs are rarely deterministic in real-world applications. To capture this variability, this study introduces Uncertain Assignment Costs (UACs) for each objective using the Triangular Distribution (TD), characterized by optimistic, pessimistic, and most probable values. The UACs are estimated by performing Monte Carlo simulation to transform the uncertain MOAPs into an equivalent deterministic model. The resulting deterministic model is then solved using the ACOA, a biologically inspired metaheuristic known for its effectiveness in solving complex and large-scale combinatorial optimization problems. For each objective, positive ideal and negative ideal solutions are obtained. In MOAPs, a Fuzzy Exponential Membership Function (FEMF) is obtained for each objective, according to the different shape parameters (SPs) of the Decision Maker (DM), and subsequently sum of the FEMF values for each path is found. This study shows that the change in the SPs of the FEMF influences the level of satisfaction for each objective function, and all the obtained solutions are reliable with the preference of the DM.

Furthermore, the relationship between optimal solution of single objective and compromise solution of each objective was studied. This demonstrates that the compromise solution represents a balanced allocation that slightly sacrifices optimality of individual objective in order to overall gain of all objectives. Subsequently, to verify the effectiveness of the proposed methodology, an analysis is performed between the Genetic Algorithm (GA) approach and the proposed methodology. GA, one of the most prominent metaheuristic optimization techniques and was chosen due to its robustness and adaptability, has been successfully used to solve NP hard combinatorial discrete optimization problems. Recent studies have demonstrated the effectiveness of Genetic Algorithms (GA) in solving multi-objective optimization problems under uncertain environments. Dhodiya and Tailor developed a hybrid algorithm using GA to solve fuzzy multi-objective assignment problems by adopting exponential membership functions with triangular possibility distribution, proving the capability of GA to tackle uncertain environments effectively [15]. Subsequent research further extended the application of GA to fuzzy multi-objective optimization and uncertainty-driven optimization models [16]. In 2019, it was shown that GA has the potential to solve MOAP that the overall operating cost, labor time, and quality level of the tasks are nearly optimized simultaneously [17]. In addition, GA has been successfully applied to various multi-objective assignment and allocation problems [18]. Collectively, these studies indicate that GA provides a reliable and effective benchmark for evaluating the performance of the proposed ACOA within a triangular uncertain environment.

2. Methodology

2.1 Materials and Methods

A method of solution is proposed to solve a Multi-Objective Uncertain Assignment Problem (MOUAP) subject to set of operational constraints. Here, each problem involves a one-to-one assignment between n tasks and n agents with the objective of achieving the optimal assignments.
The MOUAP is formulated under the following conditions:

- Each task is assigned to exactly one agent.
- Each agent is assigned to exactly one task.

The MOUAC matrix can be represented as below:

$$\zeta^k = \begin{pmatrix} \zeta_{11}^k & \zeta_{12}^k & \cdots & \zeta_{1n}^k \\ \zeta_{21}^k & \zeta_{22}^k & \cdots & \zeta_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{n1}^k & \zeta_{n2}^k & \cdots & \zeta_{nn}^k \end{pmatrix}_{n\times n}, \quad k = 1,2,\dots,m,$$

where parameters are defined as;

m number of objectives
n number of agents/number of tasks
ζ_{ij}^k uncertain assignment cost of assigning j^{th} task to i^{th} agent in the k^{th} objective.

The general formulation of the mathematical model of the MOUAP

$$\text{Min } Z^k(x) = \sum_{i=1}^{n}\sum_{j=1}^{n}\zeta_{ij}^k x_{ij}, \; k = 1,2,\dots,m, \qquad (1)$$
$$x = [x_{ij}]_{nxn}$$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = 1, \; i = 1,2,\dots,n$ –only one agent is assigned to a task

$\sum_{i=1}^{n} x_{ij} = 1, \; j = 1,2,\dots,n$ –only one task is assigned to an agent,

where x_{ij} is defined as

$$x_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{agent is assigned to } j^{th} \text{task} \\ 0, & \text{if } i^{th} \text{ agent is not assigned to } j^{th} \text{ task.} \end{cases}$$

In this study, ζ_{ij}^k is assumed to be Triangular distributed. The TD is widely used in simulation, optimization, and decision analysis when only limited information is available. A typical application in TD is cost estimation, where only three estimates: the smallest possible value, the most likely value and the largest possible value, are needed. Also, TD is computational efficient in modeling the uncertainties of MOAPs. Unlike the normal distribution, which requires large historical datasets and has unbounded values, the TD uses only three parameters. Compared with the uniform distribution, it better represents practical situations because it includes a most probable value rather than assuming all values are equally likely. Furthermore, the TD is easy to integrate with Monte Carlo simulation and ACO, reducing computational complexity while effectively representing uncertain assignment costs. As mentioned earlier, the three parameters of TD are: the optimistic (minimum) value a, the pessimistic (maximum) value b, and the most probable value c. That is,

$$\zeta_{ij} \sim Tr(a,c,b), \qquad \text{where } a < c < b.$$

A TD is a continuous distribution with a probability density function has a triangular shape in appearance. Its cumulative distribution function can be defined as,

$$F(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)}, & a \le x < c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)}, & c \le x \le b \end{cases}. \qquad (2)$$

The inverse cumulative distribution function (CDF) of the TD was utilized in this study to efficiently generate random samples for the Monte Carlo simulation process. Since Monte Carlo simulation requires repeated random realizations of uncertain parameters, uniformly distributed random numbers in the interval between 0 and 1 were transformed into TD random variables using the inverse CDF method. Thus, the inverse CDF can be derived as,

$$f^{-1}(u) = \begin{cases} a + \sqrt{(b-a)(c-a)u}, & 0 < u < \frac{(c-a)}{(b-a)} \\ b - \sqrt{(b-a)(b-c)(1-u)}, & \frac{(c-a)}{(b-a)} \le u < 1 \end{cases}, \qquad (3)$$

where $0 < u < 1$.

Here, each UAP is reformulated as a deterministic assignment model and solved it using the ACOA. In the proposed method, the procedure begins with the first row of the selected deterministic Assignment Cost Matrix (ACM). Initially, transition probabilities (p_{ij}^q) of all the paths in the first row of the ACM are calculated. Based on these probabilities, the corresponding cumulative probability (CP) for that row will also be calculated. For the random selection of the new path, a random number (RNs) between 0 and 1 is generated uniformly for each selected path. Comparing the CP and the generated random number, the new path is determined. If the comparison between the CP and the generated random number across all candidate paths does not yield a unique path, that is, if multiple paths become candidates, the pheromone levels are updated and the transition probabilities are recomputed accordingly, and subsequently the process described earlier is repeated for the corresponding row of the ACM until the comparison converges to a unique shortest path. After selecting the shortest path for the corresponding row, this process is repeated sequentially for the remaining rows of the ACM as well. This whole process will run through all the deterministic ACMs.

In ACOA, the transition probability of the q^{th} ant moving from agent j to task i is as follows [19, 20]:

$$P_{ij}^{q} = \frac{\tau_{ij}^{\alpha}(t).\eta_{ij}^{\beta}(t)}{\sum_{ij}\tau_{ij}^{\alpha}(t).\eta_{ij}^{\beta}(t)} \quad , \tag{4}$$

where τ_{ij} is the pheromone trails for arc (i,j) for the objective and $\eta_{ij} = 1/c_{ij}$, where c_{ij} is the assignment cost between j^{th} task and i^{th} agent of the deterministic model. Also, η_{ij} is the corresponding heuristic value, α and β are constants that determine the relative importance of τ_{ij} and η_{ij} respectively.

Pheromone evaporation, and the complete update rule thus takes the form

$$\tau_{ij}(t+1) \leftarrow (1-\rho)\tau_{ij}(t) + \sum_{a}^{r} \Delta\tau_{ij}^{a}, \tag{5}$$

where ρ is the evaporation rate of pheromone, $\rho \in [0,1]$, r is the number of paths of ants, $\tau_{ij}(t+1)$ is the total pheromone deposited by ants on path (i,j) and $\Delta\tau_{ij}^{a}$is the quantity of pheromone laid on edge (i,j) by ant a. In ant colonies, pheromone intensity decreases over time because of evaporation. In ACOA, evaporation is simulated by applying an appropriately defined pheromone evaporation rule,

$$\Delta\tau_{ij}^{a} = \begin{cases} \frac{1}{c_a}, & a^{th}\text{ant's assignment cost on the edge } (i,j), \\ 0, & \text{otherwise.} \end{cases}$$

Subsequently, the positive and negative ideal solutions of each objective in the above deterministic model are determined, and hence, the values of FEMF are determined [21]. Here, the FEMF for the minimization case is defined as,

$$\mu_k(Z^k(x)) = \begin{cases} 1, & if\ Z^k \le L_t \\ \frac{e^{-s\psi_r^k(x)} - e^{-s}}{1-e^{-s}}, & if\ L_t < Z^k < U_t, \\ 0, & if\ Z^k \ge U_t, \end{cases} \tag{6}$$

$\forall\ k$ and $\quad r = 1, 2, \dots, n,$

where, $\psi_r^k(x) = \frac{Z_r^k - L_t}{U_t - L_t}$, Z_r^k is objective value, and L_t is Positive Ideal Solution (PIS), U_t is Negative Ideal Solution (NIS) of the objective function, n is alternative path and s are non-zero SPs decided by the decision maker [7]. In this study, to transform the deterministic multi-objective assignment problem into a single-objective problem, the additive operator is applied to the membership value. In this approach, the overall measure is obtained as the sum of the individual membership values.

Accordingly, to make a decision, the additive operator (Fitness function) is applied to the member value as follows:

$$V_r = \sum_{k=1}^{m} \mu_r(Z^k(x)) , \quad r = 1, 2, \dots, n. \tag{7}$$

$$V = Max\{V_1, V_2, V_{3,} \ \dots\ , \ V_n\}. \tag{8}$$

This study proposes a novel method for multi-objective optimization under uncertainty.

The step-wise form of the proposed method to reach the optimal assignment for the MOUAP is given below:

Step 1: Perform Monte Carlo simulation by randomly generating values of u between 0 and 1, and hence, estimating the values of $\zeta_{ij}^{k} \sim Tr(a,c,b)$ for each objective.
Step 2: Using the values of ζ_{ij}^{k} generated in Step 1, estimate the cost coefficients for each objective.
Step 3: Using the estimated cost coefficients determined in Step 2, formulate the deterministic ACM.
Step 4: Define the values of α, β, ρ and initial pheromone level.
Step 5: Determine the transition probability values (p_{ij}^{q}) for the first row of the ACM.
Step 6: Compute the CP and generate the RNs for the corresponding row.
Step 7: Find the shortest path for the row of the ACM comparing the CP and RNs.
Step 8: Exclude the row and column of the ACM corresponding to the selected shortest path.
Step 9: If the unique shortest path cannot be found, update the pheromones and go to Step 5, otherwise go to Step 10.
Step 10: If the unique shortest path is found, repeat the process for the remaining rows of ACM and find the optimal solution of objective ($Z^k(x)$).
Step 11: Find the PIS (L_t) and NIS (U_t) of the corresponding objective.
Step 12: Find FEMF value of the objective using exponential membership function.
Step 13: If all objectives have not been selected, proceed to Step 5, otherwise go to Step 14.
Step 14: Determine the values of V_r, $r = 1,2, \dots, n.$
Step 15: Find the value of V.
Step 16: Solve the problem formulated in Step 15 to obtain the optimal compromised solution to MOUAP.

The initial pheromone value was set to 1, while the parameters α and β were both assigned a value of 1. In addition, the pheromone evaporation rate ρ was fixed at 0.5. A flowchart of the proposed algorithm is shown in the Fig. 1 given below and the pseudocode is given in the annexure.

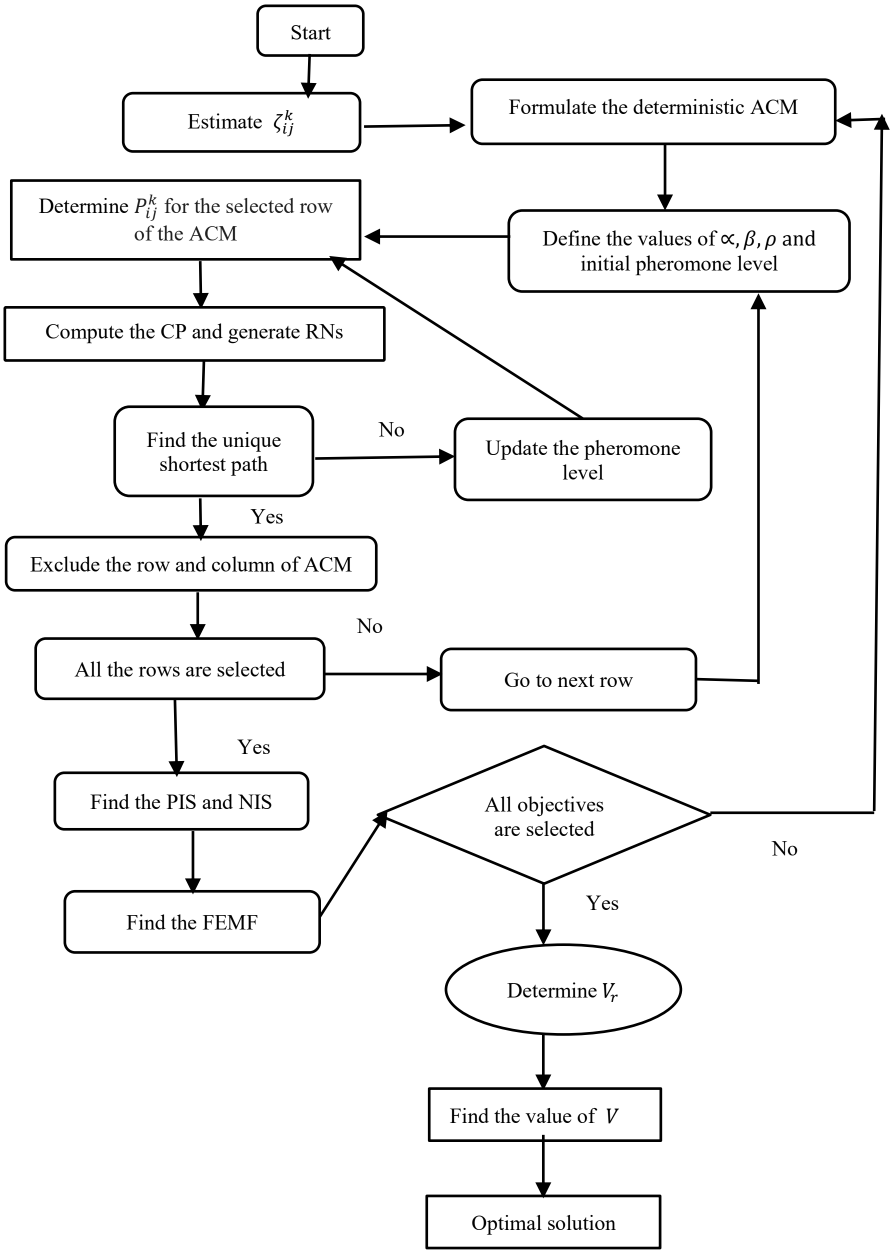

Fig.1. Flowchart of the proposed algorithm

2.2 Illustration of the proposed method

In this illustration, the uncertain assignment cost coefficients ζ_{ij}^{k} corresponding to the three objectives, were represented in the form of uncertain assignment cost matrices (UACMs), denoted by ζ^1, ζ^2, and ζ^3. These coefficients were randomly generated following the TD. Each uncertain assignment cost coefficient was represented by three parameters, namely the minimum value, the most likely value and the maximum value, as defined earlier.

$$\zeta^1 = \begin{pmatrix} (2.51, 5.92, 9.20) & (1.52, 6.59, 7.52) & (3.35, 5.82, 9.88) & (3.57, 6.08, 10.23) \\ (0.75, 6.84, 7.97) & (3.24, 5.62, 7.04) & (0.87, 5.40, 7.05) & (2.95, 4.89, 8.99) \\ (2.03, 6.98, 10.49) & (2.91, 6.55, 9.86) & (9.98, 5.62, 10.14) & (3.37, 4.97, 8.93) \\ (3.65, 4.62, 7.29) & (1.29, 6.85, 9.53) & (1.42, 5.96, 8.62) & (3.42, 6.82, 10.14) \end{pmatrix}_{4\times4}$$

$$\zeta^2 = \begin{pmatrix} (1.64, 5.37, 8.82) & (3.79, 5.67, 10.64) & (3.94, 4.41, 8.22) & (3.52, 5.38, 8.42) \\ (2.32, 6.19, 8.05) & (2.68, 4.54, 7.63) & (2.40, 4.77, 8.27) & (3.21, 6.99, 9.33) \\ (1.43, 5.38, 9.68) & (1.45, 4.26, 9.25) & (0.86, 6.47, 8.35) & (2.60, 6.44, 7.47) \\ (3.49, 4.24, 9.94) & (2.27, 4.21, 10.85) & (2.34, 5.09, 8.61) & (3.15, 5.11, 10.76) \end{pmatrix}_{4\times4}$$

$$\zeta^3 = \begin{pmatrix} (3.43, 5.08, 10.87) & (2.44, 6.22, 8.05) & (0.96, 5.70, 8.30) & (3.45, 4.63, 8.91) \\ (2.67, 4.38, 8.16) & (1.78, 4.19, 9.94) & (2.32, 5.35, 7.30) & (3.45, 6.63, 7.17) \\ (3.99, 4.95, 10.97) & (0.07, 5.21, 9.39) & (1.35, 6.86, 9.21) & (0.54, 6.47, 10.22) \\ (2.56, 5.08, 8.08) & (2.98, 4.29, 7.96) & (3.36, 5.51, 8.38) & (2.60, 4.04, 10.42) \end{pmatrix}_{4\times4}$$

The MOUAP can be formulated as below according to the notations defined earlier (eq^n. 1):

$$Min\ Z^k(x) = \sum_{i=1}^{4}\sum_{j=1}^{4}\zeta_{ij}^k x_{ij}, \qquad k=1, 2, 3$$

subject to the constraints

$$\sum_{j=1}^{4} x_{ij} = 1, \qquad i = 1, 2, 3, 4$$

$$\sum_{i=1}^{4} x_{ij} = 1, \qquad j = 1, 2, 3, 4$$

$$x_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{agent assigned to } j^{th} \text{task} \\ 0, & \text{if } i^{th} \text{ agent not assigned to } j^{th} \text{ task.} \end{cases}$$

First, using TD, the data obtained from UACMs were converted into deterministic ACMs by performing a Monte Carlo simulation with 100 replications. Subsequently, each objective with deterministic coefficients is taken separately keeping the rest of the objectives relaxed and solved it using ACOA. After repeating the process for the three objectives, PIS and NIS are recorded for each objective to find the FEMF values. Subsequently, the optimal compromise solution to the MOUAP is obtained by applying additive operator (fitness function) on the FEMF values.

3. Results and Discussion

Monte Carlo simulation was performed to estimate the uncertain assignment cost coefficients using the inverse TD. Based on these estimates, deterministic assignment cost matrices (DACMs) were constructed for each objective. The resulting matrices were subsequently employed as inputs to the proposed ACOA in order to determine the optimal consensus assignment solution under uncertainty.
The deterministic assignment cost matrices, denoted as DACM1, DACM2, and DACM3, derived from the uncertain assignment cost matrices ζ^1, ζ^2 and ζ^3, are shown below:

$$\text{DACM1} = \begin{pmatrix} 5.8912 & 5.2407 & 6.3558 & 6.6324 \\ 5.2235 & 5.3131 & 4.4668 & 5.6125 \\ 6.5255 & 6.4562 & 6.5801 & 5.7576 \\ 5.1868 & 5.9221 & 5.3585 & 6.8077 \end{pmatrix}_{4\times4}$$

$$\text{DACM2} = \begin{pmatrix} 5.4676 & 6.8679 & 5.6257 & 5.8968 \\ 5.6832 & 5.0746 & 5.2957 & 6.6807 \\ 5.7119 & 5.1821 & 5.4466 & 5.6464 \\ 6.0443 & 5.9850 & 5.5077 & 6.5258 \end{pmatrix}_{4\times4}$$

$$\text{DACM3} = \begin{pmatrix} 6.4116 & 5.5577 & 4.9688 & 5.6276 \\ 5.0384 & 5.2552 & 4.9760 & 5.7458 \\ 6.5866 & 4.8582 & 5.7913 & 5.7166 \\ 5.2161 & 5.0459 & 5.7269 & 5.6334 \end{pmatrix}_{4\times4}$$

The next step is to solve the MOUAP, considering each objective alone, using ACOA, thereby each problem becomes a single objective assignment problem.

Consider the first objective relaxing the last two objectives

$$\begin{aligned} \text{Min } Z^1(x) = {} & 5.8912\, x_{11} + 5.2407\, x_{12} + 6.3558\, x_{13} \\ & +6.6324\, x_{14} + 5.2235\, x_{21} + 5.3131 x_{22} \\ & +4.4668\, x_{23} + 5.6125\, x_{24} + 6.5255\, x_{31} \\ & + 6.4562\, x_{32} + 6.5801\, x_{33} + 5.7576 x_{34} \\ & +5.1868\, x_{41} + 5.9221\, x_{42} + 5.3585\, x_{43} \\ & +6.8077 x_{44} \end{aligned}$$

Solving it using ACOA, the optimal solution obtained for the $Z^1(x)$ is,
$x^{(1)} = \{\{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}\}$. .

$Z^1\left(x^{(1)}\right) = 20.6519.$

Consider the second objective relaxing the first and the last objectives

$$\begin{aligned} \text{Min } Z^2(x) = {} & 5.4676\, x_{11} + 6.8679\, x_{12} + 5.6257\, x_{13} \\ & +5.8968\, x_{14} + 5.6832\, x_{21} + 5.0746\, x_{22} \\ & +5.2957\, x_{23} + 6.6807\, x_{24} + .7119\, x_{31} \\ & +5.1821\, x_{32} + 5.4466\, x_{33} + 5.6464\, x_{34} \\ & +6.0443\, x_{41} + 5.9850\, x_{42} + .5077\, x_{43} \\ & + 6.5258\, x_{44} \end{aligned}$$

Similarly, the optimal solution obtained for the $Z^2(x)$ is,
$x^{(2)} = \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}$.

$Z^2\left(x^{(2)}\right) = 21.6963.$

Consider the third objective relaxing the first two objectives

$$\begin{aligned} \text{Min } Z^3(x) = {} & 6.4116\, x_{11} + 5.5577\, x_{12} + 4.9688\, x_{13} \\ & +5.6276 x_{14} + 5.0384\, x_{21} + 5.2252\, x_{22} \\ & +4.9760\, x_{23} + 5.7458\, x_{24} + .5866\, x_{31} \\ & +4.8582\, x_{32} + 5.7913\, x_{33} + 5.7166\, x_{34} \\ & +5.2161\, x_{41} + 5.0459\, x_{42} + 5.7269\, x_{43} \\ & +5.6334\, x_{44} \end{aligned}$$

Finally, solving the third objective alone, the optimal solution obtained for $Z^3(x)$ is,
$x^{(3)} = \{\{0, 0, 1, 0\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 1\}\}$.
$Z^3(x^{(3)}) = 20.4988$.

The PIS and NIS values determined by the above process are summarized in the Table 1 given below:

Table 1
PIS and NIS values for each objective

Value \ Objective	$Z^1(x)$	$Z^2(x)$	$Z^3(x)$
PIS (L_t)	20.6519	21.6963	20.4988
NIS (U_t)	25.0021	25.0395	23.1963

Finding the fuzzy exponential membership function value for $Z^1(x)$:

$$\mu_1(Z^1(x)) = \begin{cases} 1, & if\ Z^1(x) \le L_1 \\ \frac{e^{-5\psi_r^1(x)} - e^{-5}}{1-e^{-5}}, & if\ L_1 < Z^1(x) < U_1 \\ 0, & if\ Z^1(x) \ge U_1 \end{cases},$$

$$r = 1, 2, \dots, n,$$

where for the n alternative path, $\psi_r^1(x) = \frac{Z_r^1(x) - L_1}{U_1 - L_1}$.

Subsequently, 24 alternative paths are identified, and the corresponding cost associated with each path is computed and shown in Table 2 below:

Table 2
Twenty-four alternative paths and their corresponding costs

Path No:	Alternative paths	Cost of $Z^1(x)$	Cost of $Z^2(x)$	Cost of $Z^3(x)$
1	{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}	24.5921	22.5146	23.0915
2	{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}	22.3204	21.6963	23.1103
3	{1, 0, 0,0}, {0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}	23.6219	22.4712	21.8792
4	{1, 0, 0,0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {0, 1, 0, 0}	22.0377	22.3947	22.1501
5	{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 1, 0, 0}, {0, 0, 1, 0}	23.3184	22.8381	22.7425
6	{1, 0, 0,0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}	24.0059	23.5799	22.9946
7	{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}	23.8520	24.5235	22.0208
8	{0, 1, 0,0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}	21.5803	23.7052	22.0396
9	{0, 1, 0,0}, {0, 0, 1, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}	23.0407	24.4013	22.7537
10	{0, 1, 0,0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}	20.6519	22.8543	21.4664
11	{0, 1, 0,0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 0, 1, 0}	22.7372	24.7682	23.1591
12	{0, 1, 0,0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {1, 0, 0, 0}	22.6201	25.0395	22.3109
13	{0, 0, 1, 0}, {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}	24.8432	23.0168	20.4988
14	{0, 0, 1, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 1, 0, 0}	23.2590	22.9403	20.7697
15	{0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}	25.0021	22.9380	22.4440
16	{0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}	22.6133	22.3910	21.1567
17	{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}	24.4159	24.0033	22.3471
18	{0, 0, 1, 0}, {0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}	23.6113	23.5328	20.7889
19	{0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}	23.6706	22.2698	21.2511
20	{0, 0, 0, 1}, {1, 0, 0, 0}, {0, 0, 1, 0}, {0, 1, 0, 0}	24.3581	23.0116	21.5032
21	{0, 0, 0, 1}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 1, 0}	23.8295	22.1910	23.1963
22	{0, 0, 0, 1}, {0, 1, 0, 0}, {0, 0, 1, 0}, {1, 0, 0, 0}	23.7124	22.4623	21.8902
23	{0, 0, 0, 1}, {0, 0, 1, 0}, {1, 0, 0, 0}, {0, 1, 0, 0}	23.5468	22.8894	22.2361
24	{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}	22.7422	22.4189	20.6779

Furthermore, the values of the sum of the membership functions (V_r) associated with the various SPs for the corresponding paths are given in Table 3 below.
The following graph (Fig. 2) illustrates the relationship between the alternative path and V corresponding to positive values of the SPs.

The summary of the results of the corresponding V values for different SPs with respect to each objective are presented in Table 4 below.

As illustrated in Figure 1(a), when the SPs vary from 1 to 5, the highest values of V are obtained as 2.0600, 1.8299, 1.6263, 1.6002, and 1.5824, respectively. Accordingly, path 10 is identified as the optimal assignment, {0,1,0,0}, {0,0,1,0}, {0,0,0,1}, {1,0,0,0}}, with corresponding objective values $Z^1(x)$ = 20.6519, $Z^2(x)$ = 22.8543, and $Z^3(x)$ = 21.4664.

Furthermore, as shown in Figure 1(b), path 13 produces the highest V values for SPs ranging from 6 to 10. The associated optimal assignment is {{0,0,1,0}, {1,0,0,0}, {0,1,0,0}, {0,0,0,1}}, with corresponding objective values $Z^1(x)$ = 24.8432, $Z^2(x)$ = 23.0168, and $Z^3(x)$ = 20.4988.

Table 3
Values of the V_r corresponding to the different SPs for each path

Path No.	$V_r = \sum_{k=1}^{3} \mu_r(Z^k(x))$									
	$s = 1$	$s = 2$	$s = 3$	$s = 4$	$s = 5$	$s = 6$	$s = 7$	$s = 8$	$s = 9$	$s = 10$
1	0.7371	0.5974	0.4762	0.3757	0.2949	0.2309	0.1806	0.1413	0.1106	0.0865
2	1.5149	1.3908	1.2859	1.2035	1.1423	1.0984	1.0676	1.0463	1.0316	1.0216
3	1.2564	0.9687	0.7303	0.5450	0.4067	0.3053	0.2311	0.1765	0.1360	0.1054
4	1.5459	1.2436	0.9776	0.7586	0.5858	0.4525	0.3506	0.2728	0.2131	0.1672
5	0.9240	0.6731	0.4747	0.3282	0.2249	0.1539	0.1056	0.0728	0.0505	0.0352
6	0.5135	0.3344	0.2067	0.1228	0.0710	0.0404	0.0227	0.0127	0.0071	0.0040
7	0.5911	0.3833	0.2355	0.1390	0.0797	0.0449	0.0250	0.0139	0.0077	0.0043
8	1.2930	1.0019	0.7608	0.5736	0.4338	0.3309	0.2552	0.1989	0.1565	0.1240
9	0.5577	0.3627	0.2241	0.1333	0.0773	0.0442	0.0250	0.0142	0.0080	0.0045
10	**2.0600**	**1.8299**	**1.6263**	**1.6002**	**1.5824**	1.2370	1.1680	1.1187	1.0837	1.0589
11	0.4548	0.3188	0.2141	0.1393	0.0887	0.0558	0.0348	0.0216	0.0134	0.0083
12	0.6504	0.4566	0.3063	0.1988	0.1263	0.0792	0.0494	0.0308	0.0192	0.0120
13	1.5054	1.3802	1.2755	1.1941	1.1343	**1.5621**	**1.5110**	**1.4644**	**1.4220**	**1.3834**
14	1.6441	1.3748	1.1404	0.9483	0.7959	0.6763	0.5819	0.5062	0.4444	0.3929
15	0.6964	0.5106	0.3615	0.2502	0.1710	0.1162	0.0790	0.0537	0.0366	0.0250
16	1.7866	1.4732	1.1854	0.9395	0.7389	0.5797	0.4551	0.3580	0.2824	0.2233
17	0.5107	0.3201	0.1888	0.1062	0.0576	0.0305	0.0159	0.0081	0.0042	0.0021
18	1.3893	1.1453	0.9442	0.7868	0.6658	0.5724	0.4987	0.4388	0.3889	0.3463
19	1.5740	1.3019	1.0589	0.8542	0.6877	0.5544	0.4484	0.3639	0.2964	0.2422
20	1.0865	0.8166	0.5922	0.4186	0.2911	0.2005	0.1374	0.0939	0.0641	0.0438
21	0.9625	0.8156	0.6880	0.5811	0.4930	0.4201	0.3595	0.3085	0.2652	0.2283
22	1.2394	0.9572	0.7235	0.5419	0.4061	0.3062	0.2329	0.1787	0.1382	0.1076
23	1.0052	0.7215	0.4989	0.3370	0.2250	0.1499	0.1001	0.0672	0.0455	0.0309
24	1.9873	1.7361	1.5044	1.3032	1.1348	0.9958	0.8812	0.7860	0.7061	0.6381

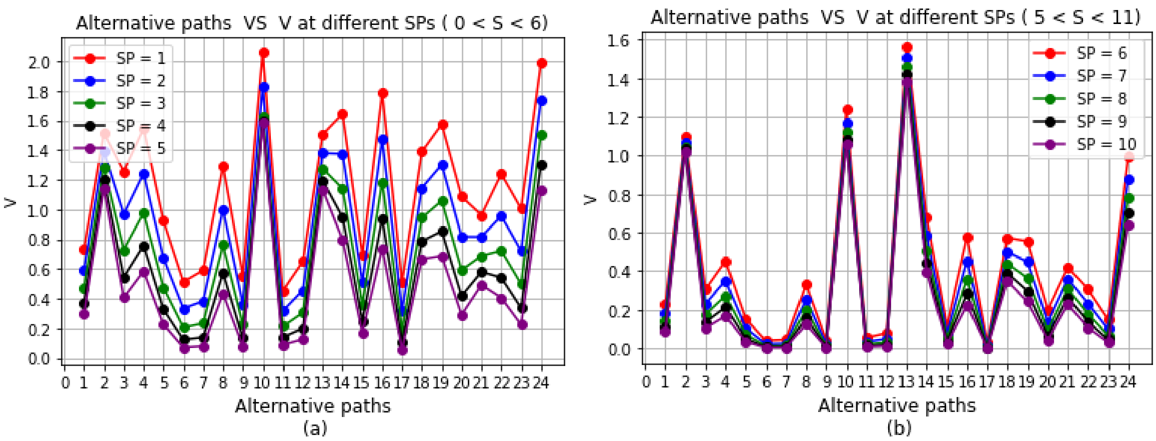

Fig. 2. The relationship between alternative paths and the V values at different SPs

Table 4
Summary of the results of the corresponding V values of different SPs for each objective

Shape parameters	V	Optimal value $Z^1(x)$	Optimal value $Z^2(x)$	Optimal value $Z^3(x)$	Optimal path	Path No.
(1, 1, 1)	2.0600					
(2, 2, 2)	1.8299					
(3, 3, 3)	1.6263	20.6519	22.8543	21.4664	{0, 1, 0,0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}	10
(4, 4, 4)	1.6002					
(5, 5, 5)	1.5824					
(6, 6, 6)	1.5621					
(7, 7, 7)	1.5110					
(8, 8, 8)	1.4644					
(9, 9, 9)	1.4220	24.8432	23.0168	20.4988	{0, 0, 1, 0},{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}	13
(10, 10, 10)	1.3834					

3.1 Relationship between single-objective optimal solutions and multi-objective compromise solutions

In multi-objective optimization, each objective typically yields its own optimal solution when considered independently. However, these single-objective optimal solutions often conflict with each other, making it impossible to satisfy all objectives simultaneously in a multi-objective context. As a result, decision makers must search for compromise solutions that balance the competing goals rather than optimizing any single objective in isolation. This relationship is especially important in multi-objective assignment problems (MOAPs), where different objectives such as cost, time, reliability, or resource utilization may lead to entirely different assignment structures. Studying how far a compromise solution deviates from each single-objective optimum solution provides insights into the degree of conflict among objectives, sensitivity of assignments to preference changes, and robustness of the chosen solution. In this study, the relationship between the optimal solution of a single objective and the compromise solution of each objective was studied. It is shown in Table 5 below:

Table 5
Optimal Single-Objective Solutions and Compromised Multi-Objective Solutions

Objective	Optimal solution of Single-objective	Compromised solution	
		$1 \le s \le 5$	$6 \le s \le 10$
$Z^1(x)$	20.6519	20.6519	24.8432
$Z^2(x)$	21.6963	22.8543	23.0168
$Z^3(x)$	20.4988	21.4664	20.4988

The results (Table 5) indicate that the optimal solution obtained for each single-objective assignment problem is better than the corresponding objective value of the compromise solution in the multi-objective assignment problem. This outcome confirms the inherent trade-off nature of multi-objective optimization, where achieving optimality for all objectives simultaneously is not feasible. The compromise solution therefore represents a balanced allocation that slightly sacrifices individual objective's optimality in order to reach a trade-off optimal solution considering all three objectives together.

3.2 Comparision of Efficency Between ACOA and GA in Solving the MOUAP

Due to significant computational run time, Hungarian algorithm is not the most desirable method in solving large scale MOUAP, besides the fact that it has the capability of finding the exact optimal solution. Therefore, in this study, another prominent metaheuristic algorithm known as GA is considered to compare the performance of the proposed ACOA method, where both algorithms were executed on the same determinant matrices generated from the triangular distribution and produced results were statistically compared. Here, 100 iterations were used for the ACOA implementation and 100 generations for the GA. The above illustrative MOUAP was solved using ACOA and GA coded in Python programming language**.**

The process of the comparison in brief is shown in the Fig. 3 given below:

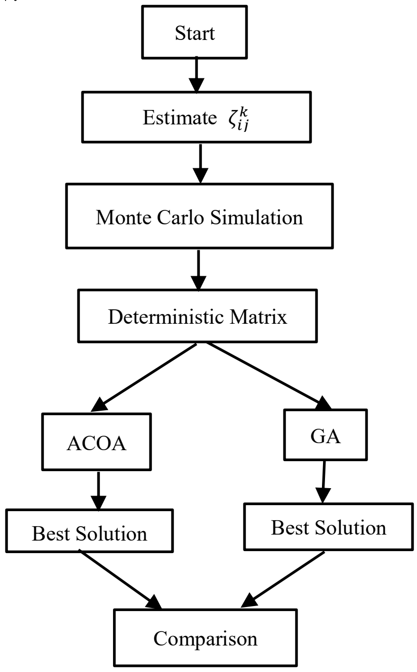

Fig. 3. Process of the comparison between ACOA and GA

The proposed ACOA and GA were tested on multi-objective uncertain assignment problems with triangular distribution of parameters and fuzzy exponential membership function with the same population size, number of iterations, order of matrices and termination criteria. The computational tests were performed for various values of shape parameter s =1, 2, …, 10, and each test was executed independently 100 times. In this study, the performance of the ACOA and GA is measured by varying the shape parameter from 1 to 10 in terms of the fitness value and standard deviation required to obtain an optimal solution for MOUAP. Table 6 below displays the results of the analysis. According to the Table 6 given below, from the comparison analysis, it can be seen that for each shape parameter value considered for the analysis, the mean fitness value obtained from ACOA is greater than to one attained from GA. Since the fitness value represents the average satisfaction with respect to all objectives, ACOA is able to generate solutions with the efficiency of the pheromone-guided search mechanism, which provides a better balance between all objectives than GA. Furthermore, for all shape parameters, the standard deviations of the fitness value for ACOA are relatively low compared to those of GA. Standard deviation is an indication of how stable and consistent the algorithm is, and lower values show greater stability. This is vital in stochastic problems since they involve uncertainties in the parameters.

In addition, it is seen from the comparision study that the exponential membership function is able to reflect the level of satisfaction of the decision maker depending on various values of the shape parameter. That is, ACOA manages to retain superior average fitness values with lower variance in comparison with GA.

Table 6
Performance comparison of ACOA and GA in terms of Mean and Standard Deviation of the Fitness (V) for solving the MOUAP under shape parameters ranging from 1 to 10

Shape parameter (s)	Fitness value (V)				Average computational time (seconds)	
	Mean		Stdndard Deviation			
	ACOA	GA	ACOA	GA	ACOA	GA
1	2.1348	2.1331	0.2425	0.2428	0.2348	0.0293
2	2.0042	2.0009	0.3117	0.3163	0.1933	0.0242
3	1.8284	1.8166	0.3162	0.3228	0.1917	0.0243
4	1.6850	1.6793	0.3413	0.3461	0.1805	0.0234
5	1.5797	1.5758	0.3246	0.3260	0.1841	0.0238
6	1.5548	1.5452	0.3709	0.3733	0.1924	0.0251
7	1.4304	1.4282	0.3683	0.3685	0.2309	0.0294
8	1.4647	1.4566	0.4195	0.4218	0.1862	0.0240
9	1.3392	1.3338	0.3306	0.3321	0.1884	0.0245
10	1.3032	1.3025	0.3412	0.3437	0.1827	0.0240

Also, Table 6 shows that both the proposed ACOA and the GA require less than one second of computational time to obtain the assignment solutions. In multi-objective optimization problems that are not well-defined, the main objective of this study is to find robust solutions rather than giving priority to reducing the computational time. Thus, the better mean fitness values and standard deviations that are obtained by ACOA show the effciency and reliability of the ACOA besides marginally higher computaional time over to GA in reaching a near optimal solution to the MOUAP problem.

Overall, the numerical findings clearly validate the ability of the suggested ACOA algorithm to offer a robust optimization method for solving multi-objective uncertain assignment problems, where assignment costs are assumed to follow a Triangular distrbution and conflicting objectives are compromised by the exponential membership functions.

4. Conclusions

The results demonstrate that variations in the SPs significantly influence the optimal assignment and the corresponding satisfaction levels of the objective functions. The proposed approach effectively captures the dynamic preference structure of the DM, ensuring that the obtained solutions remain consistent with the desired level of satisfaction. Moreover, the flexibility of the model allows for the generation of alternative assignment plans by adjusting the SPs values within the adopted membership functions, thereby enabling a broader exploration of feasible solutions.

From the computational results, it can be noted that the proposed ACOA approach was able to produce higher mean fitness values compared to GA at all levels of the considered shape parameter values. The above result shows that the proposed algorithm is able to produce more promising solutions for the assignment problem considered in this study. This conclusion is clearly evidenced by the fact that standard deviations obtained by ACOA which are lower than those of GA.

Overall, the proposed method successfully addresses the inherent conflicts among multiple objectives and yields robust and acceptable compromise solutions. This highlights its effectiveness as a decision-support tool for solving multi-objective assignment problems under uncertainty, while maintaining alignment with the fundamental principles of multi-objective decision making.

Here, the proposed method can be extended to large-scale assignment problems involving more agents, jobs, and objectives. Then, as the complexity of the search space grows rapidly, the advanced pheromone updating strategies of ACOA can be further improved. Furthermore, future studies can investigate hybrid optimization approaches by combining ACOA with other metaheuristic techniques such as genetic algorithms, particle swarm optimization, or differential evolution, which can improve the convergence stability and global search performance. Furthermore, different forms of uncertainty can be incorporated into the assignment problem model. In addition to triangular distributions, uncertainty representations such as trapezoidal fuzzy numbers, interval uncertainty, and stochastic distributions may be considered to better model realistic uncertain decision-making problems.

Moreover, the effectiveness of the proposed methodology can be further validated through applications to real-world industrial datasets, including supply chain management, manufacturing systems, healthcare scheduling, and transportation networks. Such practical implementations would further demonstrate the robustness and effectiveness of the proposed approach in complex uncertain optimization environments.

Competing Interests

The authors have no competing interests to declare

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