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Development of Virtual Environment for Swing-Up Control of Rotary Double Inverted Pendulum for Engineering Education

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Abstract

The swing-up control of rotary double inverted pendulum is a benchmark problem discussed during advanced control systems engineering. It is a single input underactuated system which consists 4th order nonlinear kinematics. Coriolis forces of the pendulum links coupled with the gravitational forces makes it a challenging task to stabilize the total system. The fabrication of such a system under an acceptable level of accuracy requires precision machining and high-quality electronic components such as encoders and motors. Under the current economical fluctuations, affording a full product directly from the market is a financially challenging task for the Sri Lankan state universities. Considering current number of student enrolments, one product per an institute would not serve enough. In this study, an offline virtual model similar to the real-world conditions with encoder behaviour with sensor noise and actuator dead zone has been developed for the swing-up control of rotary double inverted pendulum. The mathematical model was developed using the Euler-Lagrangian method. The swing-up controller was designed as a muti-stage energy controller with a vertical stabilizing controller. The computer aided design shows the real time motion. Collectively, this virtual environment could be used by multiple users at once to engage in the teaching and learning process of advanced control systems engineering without depending on physical resources or the internet.

Keywords: Energy Control, Engineering Education, RDIP, Remote Learning, Swing-up Control

1. Introduction

Developing countries such as Sri Lanka has faced the prolonged challenge of teaching and learning advanced control systems engineering due the higher costs associated with it. The state universities are directly affected with the higher number of student enrolments under them. As per the accreditation standards, the relevant laboratory sessions should be conducted with prescribed number of students per group. Providing enough equipment for control systems laboratories to engage every student in the learning process has become a challenging situation with the current financial insecurity.

The rotary inverted pendulum systems are benchmark problems discussed in controls systems engineering. The single inverted pendulum contains simple control engineering principles and double inverted pendulum contains more advanced control principles with uncertainties. It teaches about controller design robust enough to withstand disturbances and moreover to handle non-linear control regime.

Rotary double inverted pendulum is a single input and multiple output underactuated system. The torque is applied as the input to drive the rotary arm at one end and the pendulum links are connected to the opposite end respectively. The non-linear kinematics equations can be derived using the Lagrangian method. The fundamental control strategy can be implemented with energy control and feedback control [1]. Later, different approaches with trajectory planning and predictive control has been implemented to control high end pendulum problems [2].

Several types of virtual learning platforms were designed for different control problems. A virtual model was developed by [3] for the linear inverted pendulum model which didn't require internet or payments. According to its authors, the requirement of online education would be risen by 8% by 2026 compared to 2021. A web browser based virtual laboratory was implemented by [4] for inverted pendulum and crane control applications. It requires a stable internet connection to utilize it. Another virtual platform was developed by Quanser for both single and double rotary pendulum upright stabilization. However it requires to purchase the full package with license. Although there are some developments which do not require an internet connection, they lack the option to interact with the system in real time [5]. Instead of the conventional pendulum systems, [6] developed a simulation and the control which used an inertia wheel to achieve balance through angular momentum exchange. It used a closed loop PID control to achieve balance with a noise filter. Another distance learning platform for control systems was developed by [7], which is a simulator for 2 DoF ball balancing platform to assist distance learning. It helps to gain an experience similar to the hands-on experience in laboratory using Matlab software. Another similar approach was taken by [8] for a 2 wheeled self-balancing robot simulator. The system allowed users to study about using LQR control, PID control and Kalman filtering as well. Although it was a 2 wheeled robot, it inherited single pendulum modelling procedure. The virtual model developed by [3], allowed students to change parameters and observe the system behavior. It also included building the mathematical model so students learn the fundamentals rather than mere visual observation. A similar approach could be ideal for the RDIP system as no studies have done for such a system with no internet connection required, real time user interaction and with a visual representation of system behavior.

The swing-up stage of the RDIP is inherently nonlinear. Therefore, linearized methods such as LQR or PID control is not suitable for this stage. The prominent method is the energy-based control where energy is injected to the system until desired level of energy is achieved. A structured energy controller that modulates the control torque for smooth energy transfer is a more refined method [9]. A more stable method was presented by [10] which includes energy-based control with linear stabilization. A switching mechanism was designed to smoothly switch between different states. Other advanced methods such as model predictive control (MPC) and partial feedback linearization have also been explored. Those methods provide improved performance but requires higher computational cost and complexity which can be challenging for undergraduate studies. The two stage energy control method was implemented for a linear double inverted pendulum cart by [11], using Lyapunov method and passivity based control. At first, the lower link is swung-up and upper link moves freely. Once the first link is stabilized, the energy control is again applied to second link for swing-up. It decouples the nonlinear dynamics and simplifies the problem. A similar approach was used by [12]

The PID control can be effective for single link systems, but for multi-link systems; it lacks robustness for handling nonlinear coupled forces [13]. The stabilization using PID control for multiple input output systems could be a decentralized system which is highly sensitive to parameter variations and unmodelled dynamics [14]. More robust optimal controls were used by [15] for the vertical upright stabilization of rotary double inverted pendulum. The LQR control method is the most prominent method due to its simplicity and robustness. The mathematical model of the system is linearized around the equilibrium point and aims to minimize the cost function for penalizing the control effort and deviation from the equilibrium point.

In this study, an offline virtual environment identical to the real world conditions was developed which could be redistributable among the students and teachers. The mathematical model was developed using the Lagrangian method. The swing-up was designed as a two stage energy controller. The vertical upright stabilization was developed using both partial state feedback and full state feedback Linear Quadratic Regulator (LQR). Moreover, the virtual environment was integrated with physical properties such as gravity and dampness. Furthermore, the measurements were taken using an encoder model under a low sample rate of 200Hz. The study also involves a novel design for the RDIP structure. In overall, this study aims to contribute control systems engineering education by providing a platform to validate complex control engineering approaches general hardware specifications under low sample rates.

2. Methods and Materials

The mathematical model of the system was developed using the Lagrangian method and linearized state space model. The swing-up control was developed using multi stage energy control with both partial and full state feedback control. The developed control system and the computer aided design (CAD) was validated using Simscape Multibody in Matlab Simulink.

2.1 The Mathematical Model

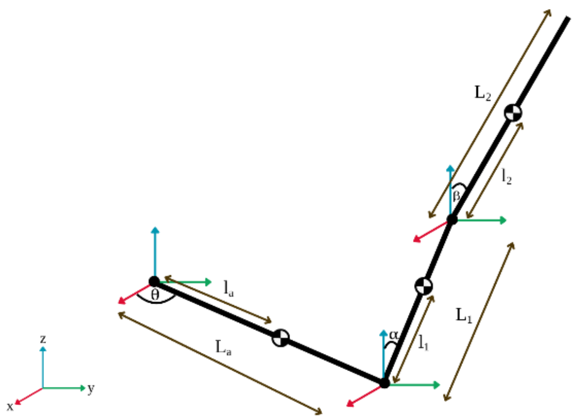

Fig. 1. Free body diagram of RDIP

The Lagrangian method was used to develop the non-linear equations of motion considering the three link coordinates of rotary arm, pendulum link 1 and link 2. Moreover, the system was linearized around the operating point to build the linearized state space model. Fig 1 shows the free body diagram of the systema and table 1 shows the parameter notations used in this study.

Table 1
Parameter Notations.

Notation	Description
$\mathbf{L_a}$	Length of Rotary Arm
$\mathbf{m_a}$	Mass of Rotary Arm
$\mathbf{I_a}$	Moment of Inertia of Rotary Arm
$\mathbf{L_1}$	Length of Link 1
$\mathbf{l_1}$	Distance to Center of Gravity of Link 1
$\mathbf{m_1}$	Mass of Link 1
$\mathbf{I_1}$	Moment of Inertia of Link 1

L_2	Length of Link 2
l_2	Distance to Center of Gravity of Link 2
m_2	Mass of Link 2
I_2	Moment of Inertia of Link 2
θ	Angle of Rotation of Rotary Arm
α	Angle of Rotation of Link 1
β	Angle of Rotation of Link 2
τ	Input Torque
b_a	Damping Coefficient of Rotary Arm
b_1	Damping Coefficient of Link 1
b_2	Damping Coefficient of Link 2
g	Gravitational Acceleration

Equation 1 shows the expression for the Lagrangian term, which is used in Euler-Lagrangian method in this study. L is the Lagrangian term, T is the total kinetic energy and V is the total potential energy of the system.

$$L = T - V \tag{1}$$

Where,

$$T = \frac{1}{2}I_a\dot{\theta}^2 + \frac{1}{2}I_1\dot{\alpha}^2 + + \frac{1}{2}I_2\dot{\beta}^2 + \frac{1}{2}m_1\left[\left(L_a\dot{\theta} + l_1\cos\alpha\,\dot{\alpha}\right)^2 + (-l_1\sin\alpha\,\dot{\alpha})^2\right] + \frac{1}{2}m_2\left[\left(L_a\dot{\theta} + L_1\cos\alpha\,\dot{\alpha} + l_2\cos\beta\,\dot{\beta}\right)^2 + \left(-L_1\sin\alpha\,\dot{\alpha} - l_2\sin\beta\,\dot{\beta}\right)^2\right]$$

$$V = m_1gl_1\cos\alpha + m_2g(L_1\cos\alpha + l_2\cos\beta)$$

Equation 2 shows the Euler-Lagrangian equation.

$$\frac{\partial^2 L}{\partial t\partial\dot{q_i}} - \frac{\partial L}{\partial q_i} = Q_i \tag{2}$$

Where,

$$q_i = [\theta\ \alpha\ \beta]^T$$

Once the Euler-Lagrangian equation is applied, following nonlinear kinematics equations were obtained as shown from equation 3-5 respectively.

$$\tau - b_a\dot{\theta} = k_1\ddot{\theta} + k_2\cos\alpha\,\ddot{\alpha} + k_3\cos\beta\,\ddot{\beta} - k_2\sin\alpha\,\dot{\theta}^2 - k_3\sin\beta\,\dot{\beta}^2 \tag{3}$$

$$-b_1\dot{\alpha} = k_2\cos\alpha\,\ddot{\theta} + k_4\ddot{\alpha} + k_5\cos(\alpha-\beta)\,\ddot{\beta} + k_5\sin(\alpha-\beta)\,\dot{\beta}^2 - k_6\sin\alpha \tag{4}$$

$$-b_2\dot{\beta} = k_3\cos\beta\,\ddot{\theta} + k_7\ddot{\beta} + k_5\cos(\alpha-\beta)\,\ddot{\alpha} - k_5\sin(\alpha-\beta)\,\dot{\alpha}^2 - k_8\sin\beta \tag{5}$$

Where,

$$k_1 = I_a + L_a{}^2(m_1 + m_2)$$
$$k_2 = L_a(m_1l_1 + m_2L_1)$$
$$k_3 = m_2L_al_2$$
$$k_4 = I_1 + l_1{}^2m_1 + L_1{}^2m_2$$
$$k_5 = m_2l_2L_1$$
$$k_6 = g(m_1l_1 + m_2L_1)$$
$$k_7 = I_2 + m_2l_2{}^2$$
$$k_8 = m_2gl_2$$

This was further expressed as a Lagrangian dynamic model as below.

$$M\ddot{q} + C\dot{q} + G = \begin{bmatrix}\tau - b_a\dot{\theta}\\ -b_1\dot{\alpha}\\ -b_2\dot{\beta}\end{bmatrix} \tag{6}$$

Where,

$$M = \begin{bmatrix} k_1 & k_2\cos\alpha & k_3\cos\beta \\ k_2\cos\alpha & k_4 & k_5\cos(\alpha-\beta) \\ k_3\cos\beta & k_5\cos(\alpha-\beta) & k_7 \end{bmatrix}$$

$$C = \begin{bmatrix} -k_2\sin\alpha\,\dot{\alpha}^2 - k_3\sin\beta\,\dot{\beta}^2 \\ k_5\sin(\alpha-\beta)\,\dot{\beta}^2 \\ -k_5\sin(\alpha-\beta)\,\dot{\alpha}^2 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ -k_6\sin\alpha \\ -k_8\sin\beta \end{bmatrix}$$

The linearized state space model was developed by linearizing the nonlinear system around the vertical upright operating point. Equation 6 shows the developed state space model.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & r_{21}k_6 & r_{31}k_8 & -r_{11}b_a & -r_{21}b_1 & -r_{31}b_2 \\ 0 & r_{22}k_6 & r_{32}k_8 & r_{12}b_a & -r_{22}b_1 & -r_{32}b_2 \\ 0 & r_{23}k_6 & r_{33}k_8 & r_{13}b_a & -r_{23}b_1 & -r_{33}b_2 \end{bmatrix} \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix}$$

$$+\begin{bmatrix}0\\0\\0\\r_{11}\\r_{12}\\r_{13}\end{bmatrix}\tau \quad (7)$$

Where,

$$r_{11} = {}^{1}/_{P}\,(k_4k_7 - {k_5}^2)$$

$$r_{12} = {}^{1}/_{P}\,(k_2k_7 - k_3k_5)$$

$$r_{13} = {}^{1}/_{P}\,(k_2k_5 - k_3k_4)$$

$$r_{21} = {}^{1}/_{P}\,(k_2k_7 - k_5k_3)$$

$$r_{22} = {}^{1}/_{P}\,(k_1k_7 - {k_3}^2)$$

$$r_{23} = {}^{1}/_{P}\,(k_1k_5 - k_2k_3)$$

$$r_{31} = {}^{1}/_{P}\,(k_2k_7 - k_3k_5)$$

$$r_{32} = {}^{1}/_{P}\,(k_1k_5 - k_3k_2)$$

$$r_{33} = {}^{1}/_{P}\,(k_1k_4 - {k_2}^2)$$

$$P = k_1\left(k_4k_7 - {k_5}^2\right) - k_2(k_2k_7 - k_5k_3) + k_3(k_2k_5 - k_3k_4)$$

2.2 The Swing-Up Controller

The swing-up control includes swinging up the pendulum links through the nonlinear region and vertical upright stabilization in linearized region. In this study, above mentioned approaches were integrated together to work simultaneously rather than sequentially. Furthermore, the control approach was separated into two stages namely; swing-up of link 1 and link 2.

The swing-up of pendulum links were achieved using energy control. Using the expressions for kinetic energy and potential energy of the system, target energy levels for both links were derived separately as shown in equations 7 and 8.

$$E_{1\,ref} = m_1gl_1 \quad (8)$$

$$E_{2\,ref} = m_2g(L_1 + l_2) \quad (9)$$

Equations 9 and 10 shows the general expressions for the energies of the pendulum links.

$$E_1 = 0.5I_1\dot{\alpha}^2 + m_1gl_1\cos\alpha \quad (10)$$

$$E_2 = 0.5I_1\dot{\beta}^2 + m_2g(L_1\cos\alpha + l_2\cos\beta) \quad (11)$$

The RDIP is a nonlinear system 3-dimensional movement. Instead of calculating the torque required for 3D motion, the system was reduced to a linear 2D motion equivalent to a linear double pendulum system. The linear force (u) required was calculated using the rate of change of energy ($\dot{E}$) and the energy deference (ΔE) between the target energy and the current energy of respective links. Equations 11 - 14 show the respective mathematical expressions.

$$\Delta E = E_i - E_{i,ref} \quad (12)$$

Where $i = 1,2$

$$\dot{E}_1 = \dot{\alpha}(m_1l_1 + m_2(L_1 + l_2)) \quad (13)$$

$$\dot{E}_2 = m_2\dot{\beta}(L_1 + l_2)\cos\beta \quad (14)$$

$$u = -u_{max}\,sign(\dot{E}_i)\,sign(\Delta E_i) \quad (15)$$

The calculated linear force was then mapped into 3D rotational torque using the equation 16.

$$\tau = \frac{u - L_af}{L_az} \quad (16)$$

where,

$$f = -M^{-1}(C + G)$$

$$z = [M^{-1}]_{1,1}$$

Fig 2 and 3 show the control architecture of the energy based swing-up controller.

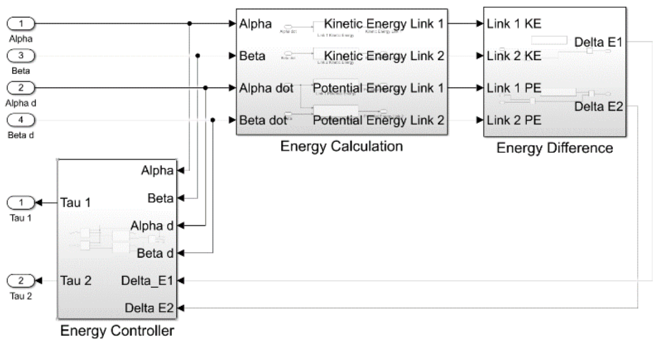

Fig. 2. Energy-based swing-up control architecture

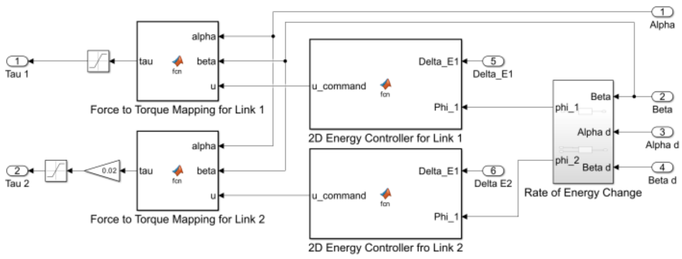

Fig. 3. Inside the energy controller

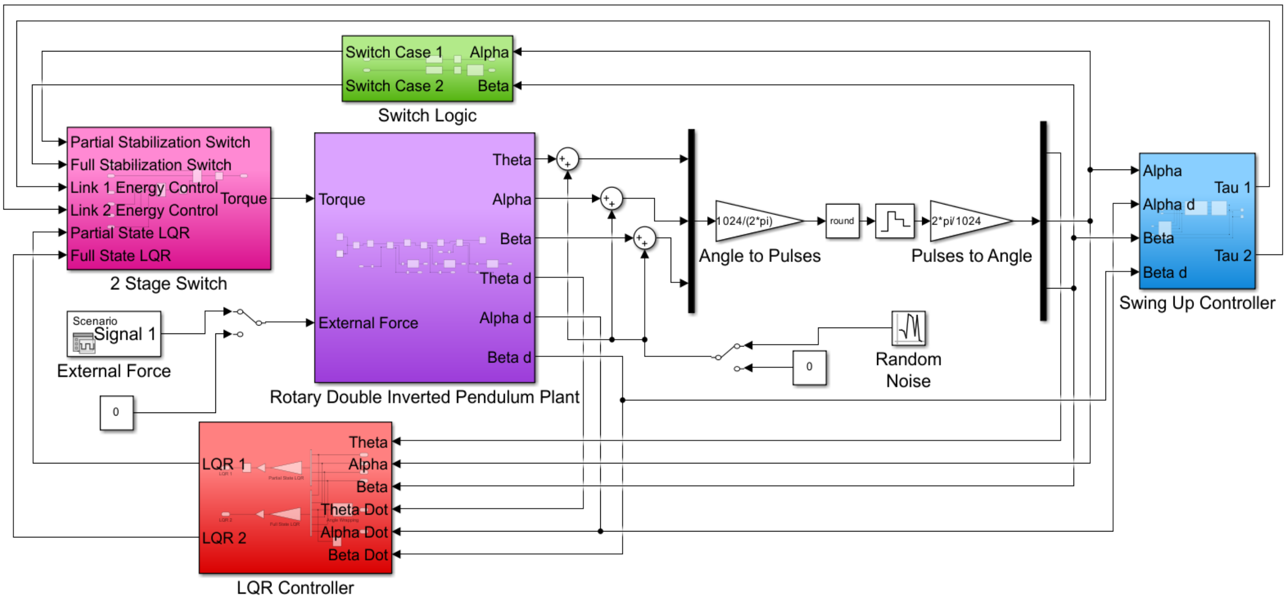

Fig. 4. Total system with the controllers

The upright stabilization was achieved using Linear Quadratic Regulator (LQR) under both partial state feedback and full state feedback. During the first swing-up stage (swing-up of link 1), energy was pumped only to the link 1. Once the link 1 reached to the vicinity of vertical upright position, partial LQR controller (LQR 1) stabilizes the link 1. It is designed for the stability of a rotary single inverted pendulum. The Coriolis forces acting on link 1 due to the random movement of link 2 was considered as a disturbance to the link 1 stability. Once the link 1 was stabilized, energy was pumped to the link 2 while maintaining the link 1's stability. When the link 2 reaches to vertical upright position, full state LQR controller (LQR 2) activates. Fig 4 shows the control architecture of the two LQR controllers.

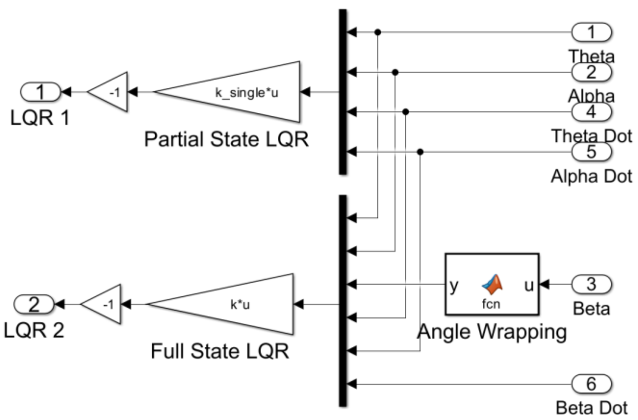

Fig. 5. Vertical stability control architecture

Furthermore, switching logic was developed to correctly switch between different control schemes. Table 2 shows how the controllers were switched based on the state values.

As Fig 6 shows, how the controllers are integrated with the plant model. Moreover, an encoder model of 1024 resolution was modelled with the plant. It made sure that the controller is developed to cooperate with low end dc motors found in the market instead of expensive motors with higher resolutions.

Table 2
Two stage switching logic.

State value	Control scheme
$\alpha > 10°$	Link 1 swing-up
$\alpha < 10°$ **AND** $\beta > 3°$	Link 2 swing-up while LQR 1 On
$\alpha < 10°$ **AND** $\beta < 3°$	LQR 2 On

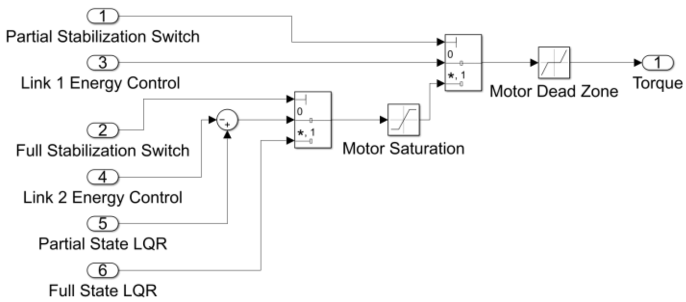

Fig. 6. Controller switch architecture

2.3 The Computer Aided Mechanical Design

Fig 7 shows the CAD model of the system. The mechanical design addressed a critical issue regarding the center of gravity of the link 2. The centers of gravity of the two pendulum links should be on one plane to consider a constant rotary arm length for both links. The mentioned criteria could not be maintained in conventional RDIP models due to the nature of the revolute joint between link 1 and link 2. The model used in this study eliminated the issue with a modified revolute joint.

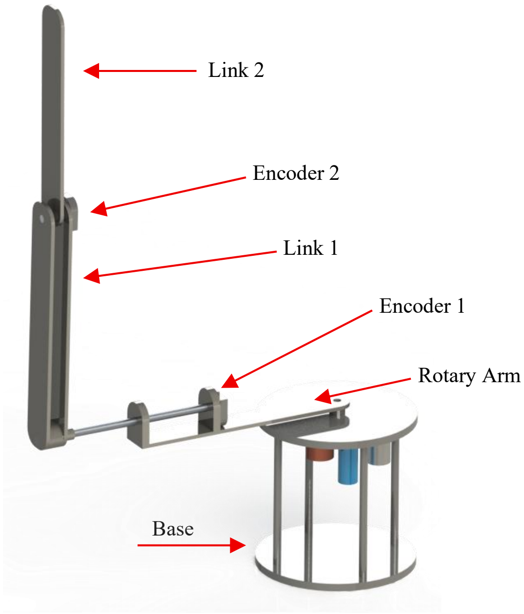

Fig. 7. CAD model of the RDIP

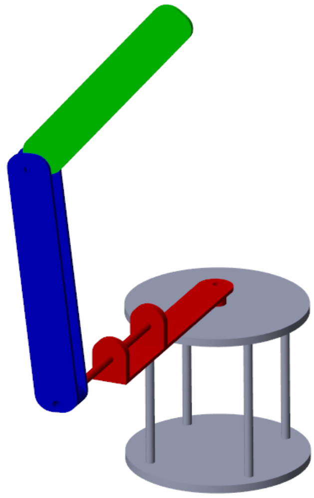

Fig. 9. Swing-up of link 2 while link 1 vertically upright

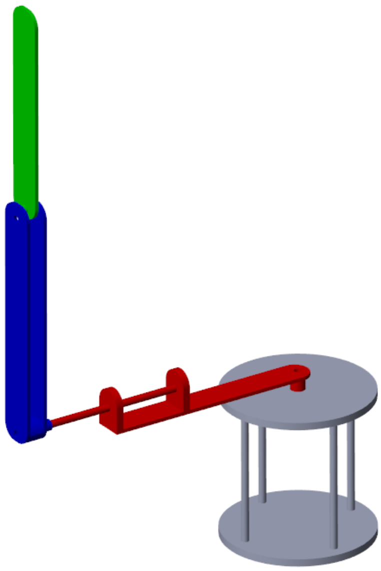

Fig. 10. Both links vertically upright

2.4 Simplified Single Inverted Pendulum Model

A similar virtual environment was developed for the rotary single inverted pendulum. It was implemented among students with introductory knowledge on control systems engineering. The solid model was developed using CAD and the control system was developed using the Matlab Simulink. The students had the opportunity to change the system parameters by changing the datafile of the Simscape Multibody model. A Matlab script file was used to initialize the control gains. The same script file had the provision of changing the datafile of the model to customize the system parameters such as mass, lengths and centre of gravity.

3. Results and Discussion

Once the controller design was completed, the developed virtual model was validated inside the Matlab Simulink environment under a low sample rate of 200Hz. The obtained results verified the successful multistage swing-up control of the RDIP model with the encoder model under 200Hz. Fig 8-10 show the three stages of the swing-up of RDIP.

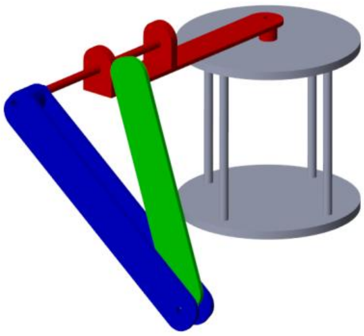

Fig. 8. Swing-up of the link 1

Fig 11-13 show the response of the three joint angles when Gaussian sensor noise and actuator dead zone simulated. The system was simulated with sensor noise of variance 1×10^{-4} rad^2 and actuator dead zone of 1% of maximum system torque. As the figures show, the variation can be segmented into 3 regions with respect to the three controller stages. The stage 1 has taken a long time period to achieve stabilization of link 1. The main reason is the disturbance forces from the arbitrary movements of the link 2. It is similar to a rotary single pendulum trying to achieve swing-up control under external disturbances. Furthermore, the results show the robustness of the LQR 1 controller to withstand the disturbances. The intermediate stage is a hybrid stage where, both the energy control and LQR control acts simultaneously. During the second stage, energy was pumped only considering the swing-up of link 2. Once the system reaches third stage, all the three parameters have reached steady state under LQR 2. Moreover, joint angle readings were wrapped between 180° and -180°. Due to the sensor noise, vibrations were observed even after reaching steady state.

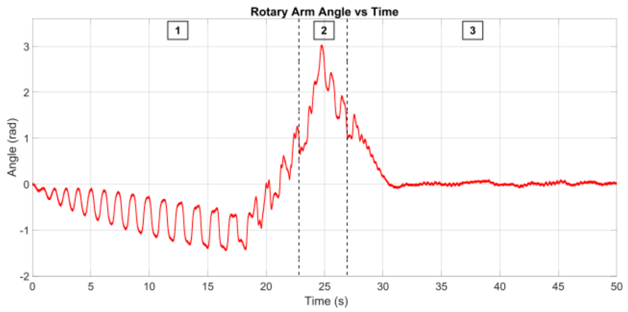

Fig. 11. Rotary arm angle vs. time

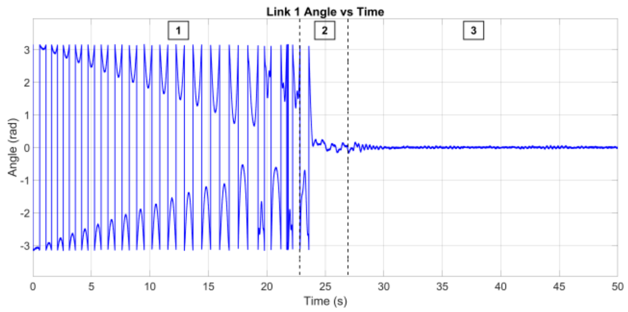

Fig. 12. Link 1 angle vs. time

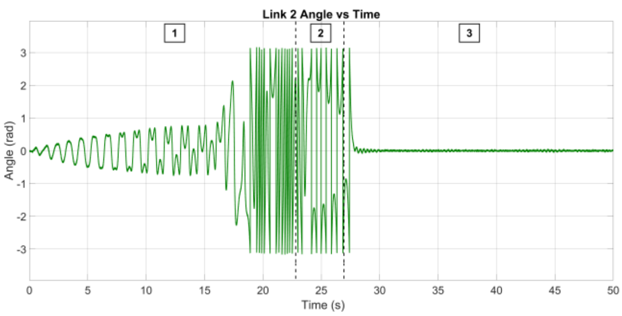

Fig 13. Link 2 angle vs. time

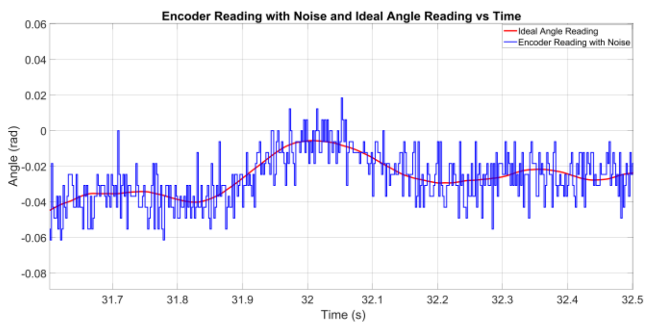

Fig 14. Ideal angle and encoder reading vs. time

Moreover, the research team has integrated the low resolution (1024) encoder model under a low sample rate 200 Hz for the system. Fig 14 shows difference between ideal joint angle readings and the quantized encoder angle readings. It shows the capabilities of the controller to handle complex control algorithms using mediocre components found in the market and with less computing power.

Fig 15 shows the variation of the total energies of the pendulum links. Both the pendulum links have reached the required target energy level. Initially both links have negative potential energy and zero kinetic energy. During the swing-up, negative potential energy is converted to target potential energy level through kinetic energy.

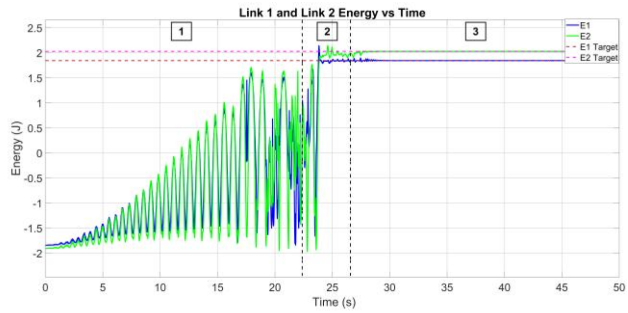

Fig.15. Energy variation vs. time

Fig 16 shows the variation of the input torque. The variation is not smoother during steady state due to the low sample rate of 200Hz and encoder resolution of 1024. The switching between control schemes results the sudden peak. However the system was designed with a safety limit, such that input torque does not exceed 20 Nm and with an actuator dead zone of 1% of maximum system torque.

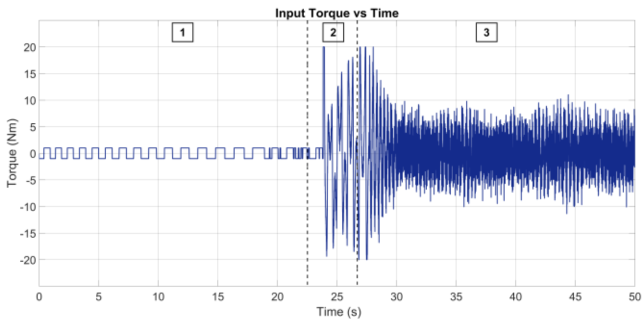

Fig. 16. Input torque vs. time

Furthermore, above results were again validated considering the energy provided by the motor during the swing-up and the total energy obtained by the pendulum links. As Fig 17 shows, the total energy provided by the motor is less than the energy change of pendulum links verifying that performance of the virtual system could be achieved by a real DC motor available in the market.

The required amount of maximum torque is 20 Nm at the axis of rotary arm with a peak angular velocity of 4.5 rad/s. The torque can be provided using a 1:14 gear ratio gear box which are already available in the market. Therefore, the maximum required power of the motor is 6.4 W. Furthermore, the mentioned system requirement could be achieved using available products in the market also with a safety margin (Maxon DCX 19S 11 W, 12 V).

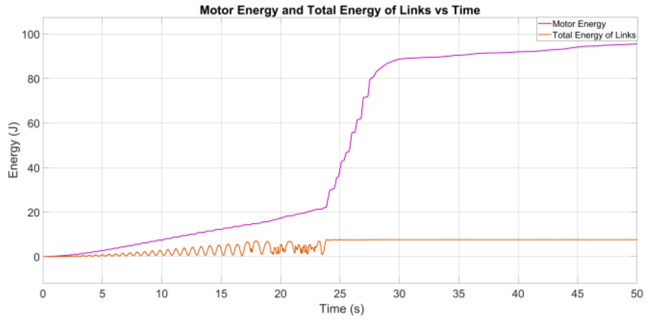

Fig. 17. Motor input energy and pendulum link energy variation vs time

The system response and the ability to withstand external disturbances were validated by providing an external force of 0.3N to second pendulum link. Fig 18-20 shows how three joint angles varied under the external impulse force and recovered back to its vertical upright position. The highlighted region between 35 s - 40 s represents the response to the external impulse force.

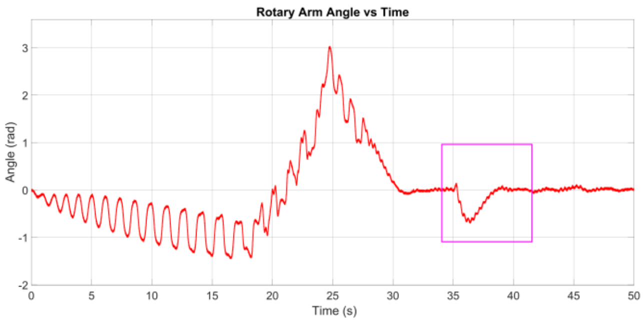

Fig.18. Rotary arm angle under external disturbance vs time

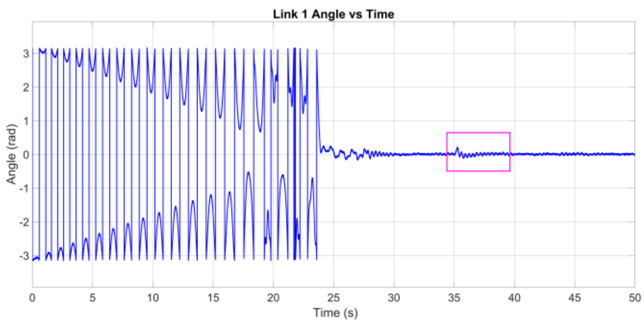

Fig.19. Link 1 angle under external disturbance vs time

Moreover, the outcomes of the implemented virtual rotary single inverted pendulum system shows that students have gained better understanding on control principles than with the physical setup. Rather than observing a system, the students got the chance to differentiate system behaviour with different gain values as well as different system parameters. The safety concerns of hardware components were eliminated. Furthermore, the students engaged more with the teacher regarding the concerns they faced during their work.

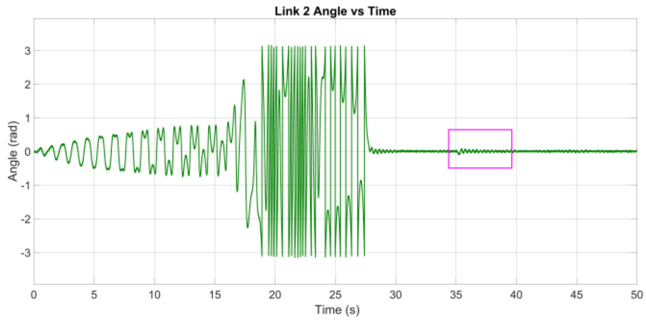

Fig. 20. Link 2 angle under external disturbance vs time

4. Conclusion

The study has successfully been able to develop a virtual environment for the swing-up control of rotary double inverted pendulum under low sample rate. Followed by the system design process, the system performance was validated in Matlab Simulink environment. The results show achieving multi-stage energy control and vertical upright stabilization under a sample rate of 200Hz and low-resolution encoders with a noise variance of $1x10^{-4}$ rad^2. The developed control system operates under conditions identical to real world. Hence it fulfils the need of a physical apparatus. This can be utilized by multiple users at once as well, engaging more participants in the learning and teaching process. Apart from the technical differences, students experienced using a similar virtual platform for rotary single inverted pendulum. The outcomes showed that the higher number of students got more familiarized on the system behaviour and the control algorithms than with the physical setup. Furthermore, the study shows how to implement advanced control algorithms with limited hardware resources and less computing power. Overall, this study provides a virtual environment for swing-up of a rotary double inverted pendulum identical to real world conditions aiding the controls system engineering education.

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Competing Interests

The authors have no competing interests to declare

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