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Predicting Crypto Trending by Using Markov Models

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ABSTRACT

Cryptocurrencies have become integral to daily transactions and income generation. It's a most important source to get income. This study is tried to predict the crypto trending movement based on the globally leading digital currencies value changes by employing Markov model. Thus, the primarily aim of this study is to determine the future tendency of increasing and decreasing the value of cryptocurrencies. The study focuses on Bitcoin, Ethereum and Monero, utilizing daily open values collected from Yahoo spanning 1795 days for Bitcoin and 1787 days for Ethereum and Monero. The dataset was Analyzed to determine the classification of price fluctuations across two successive days. Two Markov models were applied, one with two states (gain and loss) and the other with six states (large-gain, moderate-gain, small-gain, small-loss, moderate-loss, and large-loss). Acceptance of the Markov property and stationary assumption paves the way for further analysis. The steady state probability vector π is determined to understand the long-term patterns of crypto values, and expected recurrence times are computed. Finally, the future range for cryptocurrencies has been established using WMC (Weighted Markov Chain model). Cryptocurrency prices fluctuate daily, while the predicted values from the model fall within fixed intervals for several days. Therefore, accurately determining the exact value of a cryptocurrency on a specific day is very difficult. The findings offer valuable insights for investors and traders, aiding them in making informed decisions about future investments in the dynamic world of cryptocurrencies.

Keywords:Markov chain, Daily crypto value, Transition probabilities, Steady state probabilities, Mean recurrence time and Weighted Markov Chain.

1 Introduction

Cryptocurrency is a digital payment system that enables transactions to occur without the need for verification by traditional banks. It's designed to work as a medium of exchange through a computer network. It's not reliant on any central authority (Decentralized). Instead of existing as physical currency exchanged in person, cryptocurrency operates entirely in digital form, with transactions recorded as entries in an online database. When cryptocurrency is transferred, each transaction is documented in a public ledger known as the blockchain. These digital assets are stored in secure digital wallets, and the blockchain a decentralized and distributed system serves as the backbone of cryptocurrency technology. A record of all transactions updated and held by currency holders. Units of Cryptocurrency are created through a process called mining. It's not a fiat money. Currently, not only business even day-to-day transactions also people are using different cryptocurrencies because cryptocurrencies have some special features comparing with liquid money. People invest in cryptocurrencies for the same reason anyone can invest in anything. They hope its value will rise, getting them a profit. In this study we have considered the three popular cryptocurrencies. Such that Bitcoin (BTC), Ethereum (ETH) and Monero (XMR). The daily open values were obtained. In financial markets, including the cryptocurrency market trading days are typically divided into different intervals, such as minutes, hours or days, depending on the context. At the start of each trading day, the first recorded price of a cryptocurrency is considered its daily open value. This value is important because it provides a reference point for traders and investors. Changes in price throughout the trading day are often measured relative to this opening value.

P. Jasinthan et al (2015) have studied on analysis of market vegetable price using a Markov chain with two models. In first model has two states (loss, gain) and 2nd model has four states (large-gain, large-loss, small-gain, small-loss). A Similar study was conducted by R.Tharshan et al (2018), had analyzed A Markov model for investigating the stock market volume behavior. The successive price changes are independent and random, do not influence by past price movement. A study has been carried out by Kiki Ramadani et al (2020) for the prediction model for bitcoin price with fuzzy-time series Markov chain and Chen logical method. The study was built the forecasting model of the price movement of bitcoin. Kagame Richard et al (2022) have established the prediction Bitcoin prices using HMM (Hidden Markov Model). The project work focused on the optimal no of hidden states that provides the most accurate predictions. The stock's price of china sports industry was studied in 2015 by Qing-Xin-Zhou the theory of weighted Markov chain was applied to predict the stock's price a method which is used to forecast the future state.

Timur zekokh (2018) have studied at select the best model or set of models for modelling volatility of the four most popular cryptocurrencies, such that Bitcoin, Ethereum, Ripple and Litecoin. From this study results those could be improved by using instead the model specifications allowing for asymmetries and regime switching suggested by the analysis, from which both investors and regulators can get benefits. The same study was conducted by Tanya Araújo and Paulo Barbosa (2023), Reconstructing Cryptocurrency Processes via Markov Chain models. Similarly, Charfeddine et al (2020), show that BTC and Gold price series are cointegrated, implying that both series follow similar paths in the long term. They also show, through the Granger test, that gold prices affect BTC prices in a short time. In the same way, Corbet et al (2018) also study the stochastic properties of the leading cryptocurrencies and their linkages to standard stock market indices. The main findings show that the behavior of cryptocurrency markets is highly connected to each other but decoupled from the primary traditional stock indexes.

The primarily aim of the study is to determine the tendency of increasing and decreasing of crypto values. In this study, we analyze the behavior of daily trend of each three cryptocurrencies which are considered in study. The key goals of the current study are,

• To construct a Markov model that represents the daily movement of cryptocurrency values.

• To estimate the transition probabilities associated with daily price changes using the developed Markov model.

• To determine the stationary distribution of the cryptocurrency's daily price movements.

• To obtain the expected recurrence time of crypto price movement.

• To predict the future states and future price range of these currencies using Weighted Markov model based on this past value.

The remainder of the paper is structured as follows. Section 2 briefly reviews basic theoretical results related to the Markov chain, including transition probabilities, steady-state probabilities, mean recurrence time, and the Weighted markov chain. Section 3 describes the data used in the analysis and presents the results of the Markov chain model and weighted markov chain model. Finally, in section 4 summarize our study.

2 Methodology

This study analyzes three cryptocurrencies: Bitcoin, Ethereum, and Monero, using secondary data on their daily open values from Yahoo. Bitcoin data spans from February 2017 to December 2021 (1795 days), while Ethereum and Monero cover November 2017 to September 2022 (1787 days each). The data was converted to weekly and monthly averages. Two methodologies were used:

1. Markov Chain Analysis to study trends in each cryptocurrency.
2. Weighted Markov Chain (WMC) to predict future price range.

The study employs a Markov process, where the next state depends only on

the current state. It leads to equation (1),
$P(X_n = j/X_{n-1} = i, \cdots, X_1 = i_1, X_0 = i_0) = P(X_n = j/X_{n-1} = i)$
$$P(X_n = j/X_{n-1} = i) = P_{ij} \tag{1}$$

The equation (1) can be interpreted that, the next state of the system depends only on the present state, not on past states. In other word, equation (1) says Markov dependence.

Using equation (1) we define the transition probability from state i to j is called P_{ij} .The one-step transition probability matrix, denoted as P. It describes the probabilities of moving from one state to another in a single time step.

$$P = \begin{pmatrix} P_{11} & \cdots & P_{1m} \\ \vdots & \ddots & \vdots \\ P_{n1} & \cdots & P_{nm} \end{pmatrix} \tag{2}$$

In the matrix P, each entry P_{ij} represents the probability of transitioning from state i to state j in one step. $P^{(n)}$ represents the probabilities of transitioning from one state to another in n time steps in a Markov chain. In the matrix $P^{(n)}$, each entry $P_{ij}^{(n)}$ represents the probability of transitioning from state i to state j in n steps. It can be explicitly explained by the Chapman–Kolmogorov equation (3) as follows:

$$P_{ij}(m+n) = \sum_{k \in S} P_{ik}(m) P_{kj}(n) \tag{3}$$

In words, the probability of transitioning from state i to state j in $m+n$ steps is the sum over all k of the probability of going from i to k in m steps and then from k to j in n steps.
This is easy to see since, by repeated application of the Chapman–Kolmogorov equation, we have

$$P(n) = P(1)^n \tag{4}$$

But $P^{(1)}$ is the same as P, the one–step transition probability matrix. Therefore, $P^{(n)}$ can be determined by raising the transition matrix P to the nth power.

Intuitively, one of the most important quantities of interest in a Markov chain model is the **limiting probability**.

$$\lim_{n \to \infty} P_{ij}(n) = \lim_{n \to \infty} P(X_n = j \mid X_0 = i) \tag{5}$$

As an aside, we mention that this result follows from the Chapman–Kolmogorov equation.

There are two assumptions for the statistical test considered here. The first assumption pertains to the Markov property, and the second assumption pertains to stationarity. The likelihood ratio test statistic, denoted by Λ, is defined as the ratio of the likelihoods under the null and alternative hypotheses. Mathematically,

$$\Lambda = \frac{\text{Likelihood under } H_0}{\text{Likelihood under } H_1} \tag{a}$$

Transition probabilities are estimated using the formula (a) based on the maximum likelihood criterion, as formulated by Anderson and Goodman (1957). This approach assumes that the transition counts follow a multinomial distribution with probabilities P_{ij} and observed frequencies n_{ij}.

$$\hat{P}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}} \tag{b}$$

where n_{ij} is the number of observed transitions from state i to state j.
To verify the Markov property for our data set, a first order Markov chain and the homogeneity of the transition probabilities are tested using the likelihood ratio test proposed by Anderson and Goodman.

2.1 Test the Markov property

The successive events are from the first order Markov chain. To test this, the hypotheses are defined as:

$H_0 : P_{hij} = P_{ij}$ (The data satisfies the Markov property)
$H_1 : P_{hij} \neq P_{ij}$ (The data does not satisfy the Markov property)

The transition probabilities of the state h at time $n-2$, state i at time $n-1$, and state j at time n are considered. For this test, the likelihood ratio criterion is given by:

$$-2 \ln \Lambda = 2 \sum_{i=1}^{m} \sum_{j=1}^{m} n_{ij} \ln \left(\frac{\hat{P}_{ij}}{P_j} \right) \tag{*}$$

where,

n_{ij} : Total number of observed transitions from state i to j,
m : Total number of states,
P_j : Marginal probability of the j^{th} column.

The statistic $-2\ln\Lambda$ is asymptotically distributed as a chi-square distribution with $m(m-1)^2$ degrees of freedom.

2.2 Test the Stationary

Estimated transition probabilities are assumed to be constant throughout the study period. To test this, the hypotheses are defined as:

$H_0 : P_{ij} = P_{ij}(t)$ (The data satisfy stationarity)
$H_1 : P_{ij} \neq P_{ij}(t)$ (The data do not satisfy stationarity)

The likelihood ratio test statistic is given by:

$$-2\ln\Lambda = 2\sum_{t=1}^{T}\sum_{i,j=1}^{m} n_{ij}(t)\,\ln\left(\frac{P_{ij}(t)}{P_{ij}}\right) \qquad (**)$$

where,

T : Number of subintervals
$n_{ij}(t)$: Frequency count of transitions from state i to state j in the t^{th} subinterval

The statistic $-2\ln\Lambda$ follows a chi–square distribution with (T-1)(m(m-1)) degrees of freedom.

For a homogeneous Markov chain, it is convenient to begin by classifying the states of the chain according to whether it is possible to move from one state to another. A state j is said to be an accessible state from state i if j can be reached from i in a finite number of steps, that is, if

$$P_{ij}(n) > 0 \quad \text{for some } n \geq 1.$$

If two states i and j are accessible to each other, then they are said to communicate. If a Markov chain has all its states belonging to one equivalence class, then it is said to be irreducible. That is, all states communicate with each other, or every state in this Markov chain can be reached from every other state in a finite number of steps. When the period of a state is one, the state is referred

to as aperiodic. If all states of a chain communicate and are aperiodic, then the chain is said to be ergodic. For an ergodic Markov chain, the limiting distribution exists.
An irreducible recurrent Markov chain is said to be positive recurrent if all states are positive recurrent and aperiodic. An irreducible, aperiodic, and positive recurrent Markov chain process has a unique long run (stationary) distribution. The asymptotic expression for the n^{th} step transition probability matrix is given by

$$\pi = \lim_{n\to\infty} P^{(n)} \tag{6}$$

A chain is said to have a stationary (or steady state) distribution if there exists a vector π such that, given a transition probability matrix P,

$$\pi = \pi P \tag{7}$$

The time taken to process, starting from state k and returning to the same state k, is called the mean recurrence time. Therefore, the mean recurrence time m_{kk} is given by

$$m_{kk} = \frac{1}{\pi_k} \tag{8}$$

The purpose of prediction by this study we are used the weighted Markov chain (WMC). A WMC is a variant of the traditional Markov Chain where additional weights or probabilities are assigned to the transitions between states. In a standard Markov Chain, each transition from one state to another is assumed to have an equal probability. However, in a Weighted Markov Chain, the transition probabilities are not uniform, and they can be adjusted to reflect the relative likelihood or importance of different state transitions.

Based on the discussion in this paper, the method of WMC prediction is expressed as follows:

1. Calculate the mean of value (x) and the standard deviation (S) of historical data :
   The mean and standard deviation respectively are given by the following equations (9) and (10),

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{9}$$

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \tag{10}$$

2. Classify m states based on the mean and standard deviation of this data.

3. Carrying out the Markov property inspection of the known closing price sequence.

   H_0 : The stochastic process has no Markov property.
   (Observations at successive time points are statistically independent.)

   H_1 : The stochastic process has Markov property.
   (Observations are from a 1^{st} order Markov chain)

   The statistical test,

$$\chi^2 = 2 \sum_{i,j=1}^{m} f_{ij} \ln \left(\frac{P_{ij}}{q_i} \right) \tag{11}$$

   P_{ij} : The probability of transitioning from state i to state j.
   f_{ij} : The total number of observed transitions from i to j.
   q_i : Marginal probabilities for i^{th} row.

If given the degree of freedom $(m-1)^2$ and the significance level α such that $\chi^2 > \chi^2_{(m-1)^2,\alpha}$, then H_0 is rejected. The stochastic process has a Markov property and we continue with the algorithm; otherwise, we cannot proceed.

4. Calculate the order of autocorrelation coefficient r_k and calculate the weight values w_k using equations (12) and (13).

$$r_k = \frac{\sum_{i=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{12}$$

$$w_k = \frac{|r_k|}{\sum_{k=1}^{K} r_k} \tag{13}$$

5. According to the statistical results, we can obtain various steps $(P[1], P[2], \ldots)$ of the Markov chain transition probability matrices.

6. Prediction of the future price range. The following equation indicates how to predict the future value of each of the three cryptocurrencies.

$$\tilde{P}_{ij} = \sum_{k=1}^{K} w_k P_{ij}^{(k)}, \qquad \text{where } w_k \geq 0 \text{ and } \sum_{k=1}^{K} w_k = 1 \quad (14)$$

The indicated maximum of the estimated values $\max\{\tilde{P}_{ij},\, j = 1, 2, \ldots, m\}$ represents the future state for each of the three cryptocurrencies. By using this method, we can calculate the future values of each of the three currencies. Here, $P_{ij}^{(k)}$ denotes the probability of transitioning from state i to state j in the k^{th} step.

2.3 THE MODEL CONSTRUCTION

The following assumptions were made before constructing model:

• "No after effect" that means that determine the future status of the process, know its state at the moment is enough, does not need to know its previous state.

• The process of crypto value trend is "time homogeneous" process.

We analyze the daily, weekly and monthly trend of crypto value by using Markov chain model and predict the future states and future values of each three currencies by using Weighted Markov Chain, which was considered in this study. The data was analyzed to classify the price movement change for two successive days. The data is transformed into weekly and monthly averages for analysis. The frequency approach was applied to estimate the transition probabilities. The transition probability p_{ij} can be calculated as

$$p_{ij} = \frac{N_{ij}}{N_i},$$

where N_i denotes the number of times the crypto value movement is in state i, and N_{ij} denotes the number of times a transition occurs from state i to state j.

The daily opening values of the cryptocurrency were not used directly; instead, the trend of the value was categorized as *increase*, *decrease*, or *no change*.

Based on the dataset and literature survey, two models were constructed:

• Model 1 : Gain or Loss

• Model 2 : Large-gain, Moderate-gain, Small-gain, Large-loss, Moderate-loss, Small-loss

Let X_n be the price of the cryptocurrency on the n^{th} day. Then we define Z_n as a random walk model:

$$Z_n = X_n - X_{n-1} \tag{15}$$

Here crypto value changes are taken as the first order of this chain since the nature of the crypto movement. Therefore the first order change is only appropriate for this study. So that, in this study, two models of markov chain analysis will be considered as follows.

Model 1: Two State Model

In the first model, there are two states defined for daily data as follows:

• State 1 (gain) = Today cryptocurrency price is greater than or equal to the previous day price.
• State 2 (loss) = Today cryptocurrency price is less than the previous day price.

The state of this system can be represented by a binary random variable Y_n:

$$Y_n = \begin{cases} 1, & \text{if } Z_n \geq 0, \\ 2, & \text{if } Z_n < 0, \end{cases}$$

where $Z_n = X_n - X_{n-1}$ denotes the daily change in price.
For this two-state Markov chain, the (2×2) transition probability matrix can be obtained. Furthermore, the steady-state probabilities and the mean recurrence times were also calculated.

Model 2: Six State Model

In second model, there are six states defined for daily data as follows:

• State 1 (Large Gain): Today's cryptocurrency price gain is greater than $2w$ compared to the previous day.
• State 2 (Moderate Gain): Today's cryptocurrency price gain is between w and $2w$ from the previous day.
• State 3 (Small Gain): Today's cryptocurrency price gain is less than w from the previous day.
• State 4 (Small Loss): Today's cryptocurrency price loss is less than w from the previous day.
• State 5 (Moderate Loss): Today's cryptocurrency price loss is between w and $2w$ from the previous day.

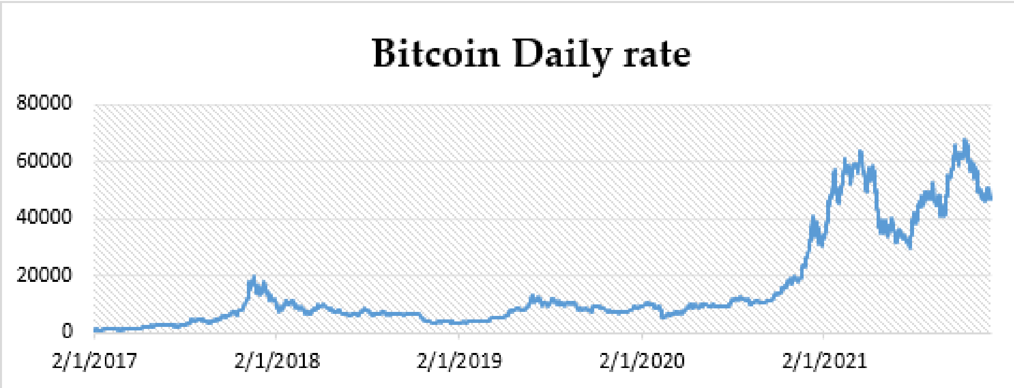

Fig. 1: plot of daily open value of bitcoin

• State 6 (Large Loss): Today's cryptocurrency price loss is greater than $2w$ from the previous day.

A sequence of daily observations on the state of the system can be represented by a random variable Y_n:

$$Y_n = \begin{cases} 1, & \text{if } |Z_n| > 2w \text{ (gain)}, \\ 2, & \text{if } w < |Z_n| \leq 2w \text{ (gain)}, \\ 3, & \text{if } |Z_n| \leq w \text{ (gain)}, \\ 4, & \text{if } |Z_n| \leq w \text{ (loss)}, \\ 5, & \text{if } w < |Z_n| \leq 2w \text{ (loss)}, \\ 6, & \text{if } |Z_n| > 2w \text{ (loss)}, \end{cases}$$

where w is the absolute average daily change in price, $Z_n = X_n - X_{n-1}$. For this six-state Markov chain, the (6×6) transition probability matrix can be obtained. Furthermore, the steady-state probabilities and mean recurrence times were also calculated.

Model Identification of Bitcoin

Bitcoin value is a frequently changing variable in the trading market.

Plot of Bitcoin Rate

The following plot indicates how the value changes on a daily basis.

From this plot we can able to check the randomness of the data set. Here, there is no specific pattern, therefore we can make an assumption of random-

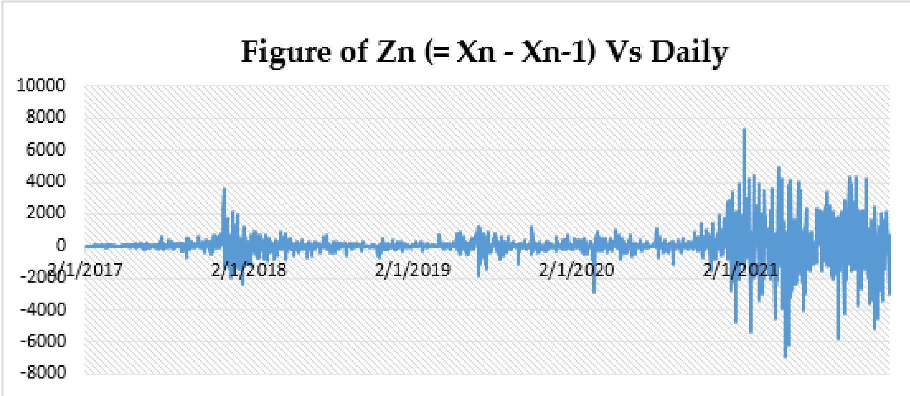

Fig. 2: Movement of daily rate behavior for Bitcoin

ness from this graph. But there is a trend here. This means the dataset does not satisfy the stationary assumption. That's why we changed the dataset into the random walk model. Here, we take the first differences. Figure 2 describes the relationship between Z_n and the daily analysis of Bitcoin rates.

Markov Property Test Results

H_0 : The first difference of the crypto value exhibits the Markov property.
H_1 : The first difference of the crypto value does not exhibit the Markov property.

Cryptocurrency	χ^2	P-value	Decision of H_0
Bitcoin Daily Data	5.1124	0.0776	Do not reject H_0
Ethereum Daily Data	5.8918	0.05255	Do not reject H_0
Monero Daily Data	4.0362	0.13291	Do not reject H_0

Table 1: Test results of Markov property assumption

Table 1 displays the test results concerning the Markov property. Using equation (*), calculate the test statistic value for the corresponding p-value. We have set alpha to 0.05, which is a typical value in this scenario. Given that all the p-values exceed alpha, it implies that the results of the Markov property test leads to the decision of not rejecting the null hypothesis. This implies that the first difference of the crypto value exhibits a Markov property.

Stationary Assumption Test Results

H_0 : The first difference of the crypto value exhibits stationarity.
H_1 : The first difference of the crypto value does not exhibit stationarity.

Cryptocurrency	χ^2	P-value	Decision of H_0
Bitcoin Daily Data	12.3572	0.05446	Do not reject H_0
Ethereum Daily Data	10.34548	0.1108	Do not reject H_0
Monero Daily Data	12.1208	0.0593	Do not reject H_0

Table 2: Test results of Stationary assumption

Table 2 displays the test results concerning the stationary assumption. Using equation (**), calculate the test statistic value for the corresponding p-value. Given that all the p-values exceed alpha, it implies that the results of the Stationary assumption test leads to the decision of not rejecting the null hypothesis. This implies that the data set satisfy the stationary assumption. We have already assumed that the Random walk model satisfies the Markov property and stationarity. These results confirm the accuracy of our assumption.

3 RESULTS AND DISCUSSION

The data was analyzed to classify the open value movement of change on two successive days. Transition probability matrix was determined for finding the trend of the Bitcoin value movement. By calculating the number of days that both first day and second day were gain, and first day was gain and second day was loss, we could find the probability to make a transition from gain to gain and gain to loss. The transition frequencies can be represented in the Table 3 and Table 4 below, which takes into account the data for bitcoin for two models separately:

State	Gain(1)	Loss(2)	Sum
Gain(1)	501	469	970
Loss(2)	469	354	823

Table 3: The frequency of Bitcoin's price movement for Model 1

State	Large Gain (1)	Moderate Gain (2)	Small Gain (3)	Small Loss (4)	Moderate Loss (5)	Large Loss (6)	Sum
Large Gain (1)	292	35	35	39	31	287	719
Moderate Gain (2)	31	11	16	14	11	25	108
Small Gain (3)	29	14	39	23	10	28	143
Small Loss (4)	42	10	24	13	05	33	127
Moderate Loss (5)	33	06	06	06	07	23	081
Large Loss (6)	293	32	22	32	17	219	615

Table 4: The frequency of bitcoin's price movement for model 2

Consequently, from the transition frequencies, the following are estimated as follows: the first step transition probability matrix (P), steady state vector (π) and mean recurrence times (M) of the Markov chain.

$$P^{(1)} = \begin{bmatrix} 0.5165 & 0.4835 \\ 0.5699 & 0.4301 \end{bmatrix}$$

According to the one step transition probability matrix we can conclude that the Markov chain is aperiodic. It is also irreducible and positive recurrent. Therefore, it is an ergodic Markov chain and the limiting distribution exists.

$$\lim_{n\to\infty} P^n = \begin{bmatrix} 0.541 & 0.45899 \\ 0.541 & 0.45899 \end{bmatrix}$$

So the steady state vector is

$$\pi = [0.541 \quad 0.45899]$$

And the mean recurrence time vector is

$$M = [1.8484 \quad 2.1787]$$

In daily data, if the process is currently in state 1 (gain), the probability of remaining in the gain state on the next day is 0.5165, while the probability of moving to state 2 (loss) is 0.4835. Similarly, if the process is in state 2 (loss), the probability of moving to the gain state is 0.5699, while the probability of remaining in the loss state is 0.4301. These transition probabilities provide short-term probabilistic predictions of future price states.The limiting distribution of the Markov chain indicates that, in the long run, the process spends about 54.1% of the time in the gain state and 45.9% in the loss state. This steady-state distribution describes the long-term behavior of the system and should not be interpreted as the probability of the next price movement.The mean recurrence times for the gain and loss states are 1.8484 and 2.1787, respectively, indicating that the process returns to the gain state slightly faster than to the loss state.

To improve the first model and calculate the correct and accurate prediction we are doing six state model. In this model first we calculated average absolute value ofZ_n.

Here,

$$w = 25.75163328$$

$$2w = 51.50326657$$

For the second model, the transition matrix was found to be:

$$P^{(1)} = \begin{bmatrix} 0.4161 & 0.0487 & 0.0487 & 0.0542 & 0.0431 & 0.3992 \\ 0.2870 & 0.1019 & 0.1481 & 0.1296 & 0.1019 & 0.2315 \\ 0.2028 & 0.0979 & 0.2727 & 0.1608 & 0.0699 & 0.1958 \\ 0.3307 & 0.0787 & 0.1890 & 0.1024 & 0.0394 & 0.2598 \\ 0.4074 & 0.0741 & 0.0741 & 0.0741 & 0.0864 & 0.2840 \\ 0.4764 & 0.0520 & 0.0358 & 0.0520 & 0.0276 & 0.3561 \end{bmatrix}$$

According to the one-step transition probability matrix we can conclude that the Markov chain is aperiodic. It is also irreducible and positive recurrent. Therefore, it is an ergodic Markov chain and the limiting distribution exists. The limiting distribution is,

$$\lim_{n\to\infty} P^n = \begin{bmatrix} 0.4017 & 0.0602 & 0.0790 & 0.0707 & 0.0451 & 0.3432 \\ 0.4017 & 0.0602 & 0.0790 & 0.0707 & 0.0451 & 0.3432 \\ 0.4017 & 0.0602 & 0.0790 & 0.0707 & 0.0451 & 0.3432 \\ 0.4017 & 0.0602 & 0.0790 & 0.0707 & 0.0451 & 0.3432 \\ 0.4017 & 0.0602 & 0.0790 & 0.0707 & 0.0451 & 0.3432 \\ 0.4017 & 0.0602 & 0.0790 & 0.0707 & 0.0451 & 0.3432 \end{bmatrix}$$

Thus the long-run descriptive behavior (steady-state vector) is

$$\pi = [0.4017 \quad 0.0602 \quad 0.0790 \quad 0.0707 \quad 0.0451 \quad 0.3432]$$

And the mean recurrence time vector is

$$M = [2.4894 \quad 16.6113 \quad 12.6582 \quad 14.1443 \quad 22.1729 \quad 2.9138]$$

Each row of the transition probability matrix $P^{(1)}$ is a probability vector describing the likelihood of changes in Bitcoin daily price states between two successive days. The transition probabilities were estimated from the observed counts in the daily data. Based on the one-step transition matrix, if the process is currently in state 1 (large gain), the probability of remaining in the large gain state on the next day is 0.4161, while the probabilities of moving to moderate gain, small gain, small loss, moderate loss, and large loss are 0.0487, 0.0487, 0.0542, 0.0431, and 0.3992, respectively. These probabilities provide short-term probabilistic predictions of the next-day price state. The limiting (steady-state) distribution indicates that, in the long run, the process spends about 40.17% of the time in the large gain state, 6.02% in moderate gain, 7.90% in small gain, 7.07% in small loss, 4.51% in moderate loss, and 34.32% in large loss. These values describe the long-term behavior of the process rather than the probability of the next-day price movement. The mean recurrence times for the states show that the largest recurrence time occurs for state 5 (moderate loss), with an average return time of about 22 days, while the smallest recurrence time occurs for state 1 (large gain), with an average return time of about 2 days.

In a similar manner, the results for the remaining cryptocurrencies, Ethereum and Monero, are estimated and displayed in Table 3 and Table 4. From the transition probability matrix, the long-run descriptive behavior and mean recurrence times are obtained for each model separately.

	Model 1	Model 2
$P^{(1)}$	$\begin{bmatrix} 0.4739 & 0.5261 \\ 0.5559 & 0.4441 \end{bmatrix}$	$\begin{bmatrix} 0.4358 & 0.0177 & 0.0130 & 0.0236 & 0.0212 & 0.4888 \\ 0.5161 & 0.0323 & 0 & 0 & 0.0323 & 0.4194 \\ 0.5405 & 0.0270 & 0.0270 & 0.1081 & 0.0270 & 0.2703 \\ 0.3590 & 0.0256 & 0.0256 & 0.0513 & 0.0769 & 0.4603 \\ 0.4722 & 0 & 0.1111 & 0.0278 & 0 & 0.3889 \\ 0.5183 & 0.0164 & 0.0252 & 0.0151 & 0.0164 & 0.4086 \end{bmatrix}$
π	$[0.5138 \quad 0.4862]$	$[0.4751 \quad 0.0174 \quad 0.0207 \quad 0.0218 \quad 0.0202 \quad 0.4448]$
M_{kk}	$[1.9463 \quad 2.0568]$	$[2.1048 \quad 57.4713 \quad 48.3092 \quad 45.8716 \quad 49.5049 \quad 2.2482]$

Table 5: Results of the estimated model parameters for Ethereum

	Model 1	Model 2
$P^{(1)}$	$\begin{bmatrix} 0.4649 & 0.5351 \\ 0.5781 & 0.4219 \end{bmatrix}$	$\begin{bmatrix} 0.1395 & 0.0930 & 0.1628 & 0.2093 & 0.1163 & 0.2791 \\ 0.0625 & 0.0500 & 0.3000 & 0.3500 & 0.1375 & 0.1000 \\ 0.0137 & 0.0348 & 0.4279 & 0.4739 & 0.0386 & 0.0112 \\ 0.0124 & 0.0289 & 0.5254 & 0.3810 & 0.0371 & 0.0151 \\ 0.0602 & 0.1928 & 0.3976 & 0.2410 & 0.0723 & 0.0361 \\ 0.1458 & 0.1458 & 0.2917 & 0.2500 & 0.0625 & 0.1042 \end{bmatrix}$
π	$[0.5193 \quad 0.4807]$	$[0.0241 \quad 0.0448 \quad 0.4504 \quad 0.4073 \quad 0.0465 \quad 0.0269]$
M_{kk}	$[1.9257 \quad 2.0803]$	$[41.4938 \quad 22.3214 \quad 2.2202 \quad 2.4552 \quad 21.5054 \quad 37.1747]$

Table 6: Results of the estimated model parameters for Monero

For Model 1, the long-run behavior of the Markov chain indicates a higher proportion of time spent in the gain state compared to the loss state. This suggests that, in the long run, Bitcoin price movements are more frequently associated with gains than losses. However, these results describe the overall tendency of the system rather than predicting the exact price of Bitcoin on a specific day. The findings imply that investors may benefit from understanding the probabilistic patterns of price movements, such as buying when prices decrease and selling when prices increase. Therefore, the results of this study may help investors better understand market behavior and support more informed investment decisions.

Results and Discussion for Weighted Markov Chain

In this study, a **weighted Markov chain model** was applied to forecast the future values of Bitcoin. The model incorporates both the transition probabilities between states and the relative weights of past observations, allowing for a more nuanced prediction of price movements compared to a simple Markov chain.

Using the data and the model, we computed the **mean** and **standard deviation** of Bitcoin prices according to equations (9) and (10):

$$\text{Mean } (\bar{x}) = 15732.29145$$

$$\text{Standard Deviation } (S) = 17020.32117$$

In this model were partitioned into eight states based on mean and standard deviation. States are defined in mathematical notation,

$$y_n = \begin{cases} 1, & \text{if } x < 1288.02972 \\ 2, & \text{if } 1288.02972 \leq x < 7222.130865 \\ 3, & \text{if } 7222.130865 \leq x < 15732.29145 \\ 4, & \text{if } 15732.29145 \leq x < 24242.45204 \\ 5, & \text{if } 24242.45204 \leq x < 32752.61262 \\ 6, & \text{if } 32752.61262 \leq x < 41262.77321 \\ 7, & \text{if } 41262.77321 \leq x < 49772.93379 \\ 8, & \text{if } x \geq 49772.93379 \end{cases}$$

The number of transitions between states were obtained and converted into the one-step transition probability matrix.

$$P^{(1)} = \begin{bmatrix} 0.9884 & 0.0116 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9674 & 0.0326 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0259 & 0.9669 & 0.0072 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0656 & 0.9180 & 0.0164 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7407 & 0.2593 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0619 & 0.8866 & 0.0515 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0400 & 0.8600 & 0.1000 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0685 & 0.9315 \end{bmatrix}$$

Check the Markov property using the Chi-square test (equation (11)). The Chi-square test is used to test the independence of observations.

$$H_0 : \text{The stochastic process has no Markov property}$$

(Observations at successive time points are statistically independent.)

$$H_1 : \text{The stochastic process has the Markov property}$$

(Observations follow a first-order Markov chain.)

The results derived from equation (11) are:

$$\text{Calculated value} = 468.350676$$

$$\text{Table value} = \chi^2_{((m-1)^2,\alpha)}$$

$$55.7885 < \chi^2 < 67.5048$$

Since,

$$\text{Calculated value} > \text{Table value},$$

we reject H_0 at the 5% significance level.
From these results, we conclude that the stochastic process follows the Markov property. Therefore, we can proceed with the Weighted Markov Chain model.

This task involves applying equations (12) and (13) to obtain the autocorrelation coefficients and Weight values for different orders within the Markov chain and then organizing these results into a table labeled as Table 7,

k	1	2	3	4	5	6
r_k	0.9973	0.9948	0.9923	0.9895	0.9866	0.9836
w_k	0.1678	0.1674	0.1669	0.1665	0.1660	0.1655

Table 7: Results of Autocorrelation coefficient and Weight values for Bitcoin.

By using equation (14), we predict the state of the open value in the future with the values on 1790$^{\text{th}}$ day, 1791$^{\text{th}}$ day, 1792$^{\text{th}}$ day, 1793$^{\text{th}}$ day, 1794$^{\text{th}}$ day and 1795$^{\text{th}}$ day as the initial states, and the corresponding state one step transition probability matrix to forecast the open value on 1796$^{\text{th}}$ day (01$^{\text{th}}$ of January 2022)

Table 8 illustrates the forecasted value for Bitcoin on the 1796$^{\text{th}}$ day;

Initial Day	State	k	w_k	1	2	3	4	5	6	7	8
1795	7	1	0.1678	0	0	0	0	0	0.04	0.86	0.10
1794	7	2	0.1674	0	0	0	0	0.0025	0.0699	0.7485	0.1792
1793	7	3	0.1669	0	0	0	0	0.0062	0.0925	0.6596	0.2417
1792	8	4	0.1665	0	0	0	0	0.0006	0.0132	0.1994	0.7868
1791	8	5	0.1660	0	0	0	0	0.0012	0.0198	0.2261	0.7529
1790	8	6	0.1655	0	0	0	0	0.0021	0.0269	0.2470	0.7239
			$\hat{p}_{ij}$	0	0	0	0	0	0	0.0012	0.0438

Table 8: Forecasted value of Bitcoin on 1796$^{\text{th}}$ day

From Table 8, the one-step transition probability matrix $P^{(1)}$ was first obtained using the observed number of transitions between states. Using this matrix, the multi-step transition probability matrices for two-step $P^{(2)}$, three-step $P^{(3)}$, four-step $P^{(4)}$, five-step $P^{(5)}$, and six-step $P^{(6)}$ transitions were calculated. The rows in Table 8 correspond to the transition probabilities derived from successive step matrices based on the initial day's state. The weights w_k used in the weighted Markov model were obtained from Table 7.

The weighted transition probabilities $\hat{p}_{ij}$ were calculated using Equation (14), and the state with the maximum probability was identified. For example, the highest probability value was 0.4912, corresponding to state 7, indicating that the most probable state for the 1796^{th} day was state 7 with a probability of 49.12%, which corresponds to the price range $41262.77 \leq x < 49772.93$. The observed (actual) Bitcoin price of 46311.75 falls within this range. Similarly, the predicted state for the 1797^{th} day was state 6, and the observed value also lies within that state range. These results indicate that the weighted Markov model can estimate the most probable future price range of a cryptocurrency. However, it should be noted that the model provides probabilistic predictions of price ranges rather than exact cryptocurrency values. The same procedure was applied to Ethereum and Monero to estimate their probable future price ranges.

Summary of Prediction Range

The following table summarizes the future price range of Bitcoin values from the 1796^{th} day to the 1801^{th} day using the Weighted Markov Chain (WMC) method.

Day	Date	Future Price Range	Actual Value
1796	01$^{\text{st}}$ January 2022	$41262.77321 \leq x < 49772.93379$	46311.75
1797	02$^{\text{nd}}$ January 2022	$41262.77321 \leq x < 49772.93379$	47680.93
1798	03$^{\text{rd}}$ January 2022	$41262.77321 \leq x < 49772.93379$	47343.54
1799	04$^{\text{th}}$ January 2022	$41262.77321 \leq x < 49772.93379$	46458.85
1800	05$^{\text{th}}$ January 2022	$41262.77321 \leq x < 49772.93379$	45899.36
1801	06$^{\text{th}}$ January 2022	$41262.77321 \leq x < 49772.93379$	43565.51

Table 9: Future price range of Bitcoin on 1796^{th} day

The following table summarizes the future price range of Ethereum value from 1788^{th} day to 1801^{th} day with the WMC method.
The following table summarizes the future price range of Monero value from 1788^{th} day to 1801^{th} day with the WMC method.
From these results, it is evident that cryptocurrency values change from day to day. However, the observed values obtained from the model fall within a fixed interval (state) for a few days. Therefore, obtaining the exact value of a cryptocurrency on a particular day is very difficult. Using this method, we can estimate the most probable price interval (range) for a particular cryp-

Day	Date	Future Price Range	Actual Value
1788	01st October 2022	$1120.77154 \leq x < 1729.666866$	1328.194
1789	02nd October 2022	$1120.77154 \leq x < 1729.666866$	1311.753
1790	03rd October 2022	$1120.77154 \leq x < 1729.666866$	1276.163
1791	04th October 2022	$1120.77154 \leq x < 1729.666866$	1323.278
1792	05th October 2022	$1120.77154 \leq x < 1729.666866$	1361.973
1793	06th October 2022	$1120.77154 \leq x < 1729.666866$	1352.807

Table 10: Future price range of Ethereum on 1788th day

Day	Date	Future Price Range	Actual Value
1788	01st October 2022	$104.7961527 \leq x < 149.6207562$	147.3167
1789	02nd October 2022	$104.7961527 \leq x < 149.6207562$	140.8834
1790	03rd October 2022	$104.7961527 \leq x < 149.6207562$	136.9156
1791	04th October 2022	$104.7961527 \leq x < 149.6207562$	141.7044
1792	05th October 2022	$104.7961527 \leq x < 149.6207562$	146.8237
1793	06th October 2022	$104.7961527 \leq x < 149.6207562$	146.2632

Table 11: Future price range of Monero on 1788st day

tocurrency. However, the model does not provide the exact numerical value of the cryptocurrency price.

4 Conclusion

In this study, the Markov Chain Model (MCM) and Weighted Markov Chain Model (WMCM) were applied to analyze the price movements of three leading cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), and Monero (XMR). These models provide a probabilistic framework for studying the dynamics of cryptocurrency prices, where the predicted outcomes reflect the likelihood of transitioning between different price states rather than exact numerical values. The transition probability matrices were estimated from historical daily data and used to analyze both short term transitions and long term steady state behavior.

The analysis confirms that all identified states in the Markov chains are communicating, aperiodic, and ergodic, ensuring the existence of a stable limiting distribution. In the two state model, the probability of remaining in a gain state is higher than that of staying in a loss state across all three cryptocurrencies, suggesting a general tendency toward upward movements. Mean recurrence times indicate that gain states are typically revisited more frequently than loss states, highlighting the dynamic nature of price changes. Ethereum exhibits behavior similar to Bitcoin, while Monero shows a higher probability of small gains compared with other states.

For the six state weighted Markov model, Bitcoin shows the highest likelihood of remaining in the large gain state, with a steady state probability of 0.4017, indicating a long term tendency toward higher price states. Ethereum's analysis reveals a higher probability of large gains, while Monero displays a higher probability of small gains (0.4504). These results describe the probabilistic behavior of the cryptocurrencies and provide insights into the potential distribution of future price states. Importantly, these outcomes reflect ranges or intervals of likely prices, not exact predictions for a given day, in line with the theoretical properties of Markov models.

The weighted Markov model allows for estimating the most probable price intervals for future periods. For instance, the calculated probability for Bitcoin being in a particular state on a given day indicates the most likely price range, even though the exact price cannot be determined. Similar probabilistic forecasts were made for Ethereum and Monero, demonstrating that WMCM effectively captures the likely ranges of cryptocurrency movements. This provides valuable guidance for investors and traders seeking to understand market behavior and make informed decisions based on probable trends rather than exact price forecasts.

Overall, the results indicate that gain states are generally more probable than loss states across all three cryptocurrencies. While precise daily price prediction remains challenging due to inherent market volatility, the WMCM provides reliable estimates of likely future price ranges. These findings can help investors identify opportunities for buying during price declines and selling when prices are expected to rise, thereby supporting more effective investment strategies. Future work will explore advanced approaches, such as fuzzy Markov chain models, to further improve the estimation of cryptocurrency prices on specific days, potentially enhancing profitability while managing risk.

The Weighted Markov Chain Model effectively estimates the most probable future price ranges of Bitcoin, Ethereum, and Monero, reflecting the probabilistic behavior of cryptocurrency markets. Gain states are generally more likely than loss states, providing insights into potential upward trends. While exact daily prices cannot be predicted, the model helps investors make informed decisions based on likely price intervals.

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