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The Lee-Carter approach for life expectancy analysis and mortality forecasting in Sri Lanka

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Abstract A person's life expectancy is one of the statistical measures used to determine how long they will live. However, in Sri Lanka, there is a lack of long-term, data-driven mortality forecasting, which limits the ability to make informed decisions in public health, insurance, and policy planning. This study addresses this gap by applying the Lee-Carter (LC) mortality estimation method to improve life expectancy analysis and mortality forecasting. In general, a life table that details the effects of mortality is used to calculate life expectancy. As a result, mortality predictions and life table analysis are important in the insurance, actuarial science, government, and pension planning sectors. The Registrar General's Office and the Department of Census and Statistics in Sri Lanka provided the mid-year population data and deaths as historical data for this study from 1992 to 2014. The Lee-Carter (LC) mortality estimation method is used to forecast mortality using actual death rates and mid-year populations from 1992 to 2014. a_x (the average age-specific log mortality rate over time), b_x (the sensitivity of mortality at each age to changes in the mortality index), and k_t (the overall mortality index over time) parameters are estimated using the Singular Value Decomposition (SVD) method in the Lee-Carter methodology between 1992 and 2014. k_t is forecasted for the years 2015–2035 using the traditional time series approach, and no specific time series model such as ARIMA or Exponential Smoothing was used. In 2035, the probability of dying as a 55-year-old woman and man are 0.023 and 0.051, respectively, according to the research. The following is a statement of the study's results: South Asia's average life expectancy is 69.41 years, while Sri Lanka's is 74.5 years. A country's future can be improved by thoroughly investigating demographics and life expectancy, as these factors directly affect financial stability

Keywords: Lee-Carter (LC) Approach, Life Expectancy, Singular Value Decomposition (SVD) Method

Introduction

In 2018, the approximate mid-year population of Sri Lanka was 21,670,000, with a growth rate of 1.1%. Sri Lanka has a male life expectancy of 72.1 years, a female life expectancy of 78.5 years, and a total life expectancy of 75.3 years, placing it 70th in the world. The country's approximate overall population is 20.3 million, according to the 14th Census of Population and Housing in 2012, and the gender ratio is 94% (Central Bank of Sri Lanka, 2018).

Demographic statistics are directly affected by fertility, mortality, and migration; Fertility has declined due to family planning programs and increased reproductive knowledge, particularly among older and more educated women (Silva, 1994). Malaria eradication is highlighted as a key milestone in mortality reduction, alongside other improvements such as enhanced healthcare infrastructure, food subsidies, and free education initiatives. The country's financial security is directly influenced by its demographics. The financial stability of a government is an immensely significant factor. As a result, the effects of factors on demography must be investigated. The three components that determine population statistics, fertility, mortality, and migration, must be examined and analyzed to develop policies and processes to improve Sri Lanka's financial security.

Furthermore, a life table illustrates the likelihood that a person of any age will die before their next birthday. Since life tables are typically generated separately for men and women, their mortality rates differ significantly. Life tables are used in biology, actuarial science, insurance, and other disciplines. Mortality forecasts are critical for predicting the future growth of the aging population and determining the sustainability of pension and social security systems. These are also useful for calculating life insurance rates and assisting governments in planning for their society' changing health-care and other service requirements. Because of the significant and growing importance of accurate mortality forecasting in society, the area of mortality forecasting is developing and progressing. This study creates life tables for Sri Lanka from 2012 to 2014 based on age and gender. Males and females have separate life tables due to differing mortality rates. In Sri Lanka, the Lee-Carter (1992) mortality modeling method is utilized for death forecasting by age and gender (Abeyrathna et al., 2009). The Lee-Carter age-based mortality model decomposes (recorded) age-specific mortality during a particular period for a single population based on the overall time trend, age-of-death model, and differences in extent by age of the overall temporal variation.

However, abridged life tables for the period 2012-2014 were constructed as representative samples due to data availability in Sri Lanka has not been created in previous studies. As a result, the current study fills a gap in Sri Lankan mortality predictions until 2035. The rest of the paper is organized as follows: Section 2 Reviews Previous Studies, Section 3 Contains Data Details, hypotheses on mortality patterns in Sri Lanka, and the Life tables, Section 4 explores the findings of mortality forecasting in Sri Lanka, and Section 5 provides the conclusions.

Materials and methods

Data Description

Death rates are needed for the development of life tables. The mid-year population and number of deaths data are collected as age-specific and gender-specific from the Department of Census and Statistics, Sri Lanka, and the number of deaths registered from the Registrar General’s office. All data are collected for the period 1992-2014.

Statistical Analysis

Life Tables

Age-specific life tables are constructed for males and females separately. Life tables are constructed when basic inputs are added, which needed basic input is age-specific death rates. The age-specific death rate (m (x, n)) can be calculated as follows.

$$m(x,n) = \frac{number\ of\ deaths\ in\ the\ age\ gruop(x, x+n)}{population\ of\ the\ age\ group\ at\ mid\ interval}$$

Equation (1): Death rate

The average number of deaths in the period 2012-2014 are taken as a number of deaths in the age group (x, n). The population of the age group at mid-interval is obtained by using the 2012 census in Sri Lanka. Those data are used separately for males and females.

The notations used for different columns of a life table are given in Table 1.

Table 1: Notations used for different columns of a life table

Notation	Meaning of the notation
x	exact age
(x, n)	age group with initial age x with the length of interval n
m (x, n)	age specific death rate for age interval (x, x + n)
q (x, n)	probability of an individual age x dying before the end of the interval (x, x+n)
l (x)	number of survivors at age x in a life table with radix (l(0)) of 100,000 persons
d (x, n)	number of deaths in age interval (x, x + n)
L (x, n)	number of person – years lived in age interval (x, x + n)
s (x, n)	the proportion of the life table population in age group (x, x + n) who are alive n years later.
T (x)	number of person – years lived at ages x and older.
e (x)	expected future lifetime of individual age x
a (x, n)	average number of years lived in the age interval (x, x + n) by those dying during that age interval.

a(x,n) is used when converting death rates m(x,n) to probabilities of death q(x,n).

- For the first age group (infants), empirical values are used (commonly between 0.1 and 0.3, depending on mortality level).
- For most other age intervals, it's standard to assume a uniform distribution of deaths, so $a(x,n) \approx n/2$.

$$q(x,n) = \frac{n \cdot m(x,n)}{1 + \{n - a(x,n)\} \cdot m(x,n)}$$

Equation (2): Probability of Dying

Equation (2) helps adjust for the timing of deaths within each interval.

A complete life table consists of data for every single year of age. Nevertheless, an abridged life table consists of data for a 5-year age interval. In this study, an abridged life table was developed because of number of death rates was registered as age intervals (Abeyrathne et al., 2009).

Lee-Carter (LC) Approach

The Lee-Cater approach was introduced by R.D. Lee and L.R. Cater in a mortality study of United states (Lee and Carter, 1992). This method is used for long term mortality forecasting. The original Lee-Carter approach goes to forecast m (x, n) for given t, using the model given in equation 3.

$$\mathrm{Log}_e[m(x,t)] = a_x + b_x k_t + \varepsilon(x,t)$$

Equation (3): Lee-Carter approach model

Where; $\varepsilon\,(x,t)$ - Random error component (The error term reflects age-period effects not captured by the model.)

Final adjusted estimate of k_t using a standard time series approach.

a_x, b_x, k_t are estimated using a rank-1 approximation of the Singular Value Decomposition (SVD) of the matrix $\{log_e[m(x,t)] - a_x\}$.

Lee-Carter approach goes by six steps, that are,

1. Estimating model parameters using rank–1 approximation of the SVD of the matrix $\{log_e[m(x,t)] - a_x\}$. The success of long-term forecasting totally depends on the goodness-of-fit of the rank-1 approximation.

2. Re-scaling b_x and k_t so that b_x sum to unity and k_t sum to 0.

3. Adjusting k_t, the time-varying index, so that actual total number of deaths is equal to the fitted total number of deaths, for each t.

4. Modeling adjusted k_t using standard time series approach.

5. Forecasting k_t, and hence m (x, t), for future time periods.

6. Constructing life tables and forecasting life expectancy at birth (or at specific ages).

1992-2014 death rate data were used for this forecast. Therefore, the number of deaths and mid-year population data were collected for 1992-2014 separately for males and females.

Results and Discussion

Mortality Pattern

According to Figure 1, there are two peak points in 1995 and 2004 because of the rise of civil war and tsunami cases. Both males and females have a similar pattern of several deaths. Factors affecting several deaths appear to be affecting males and females alike.

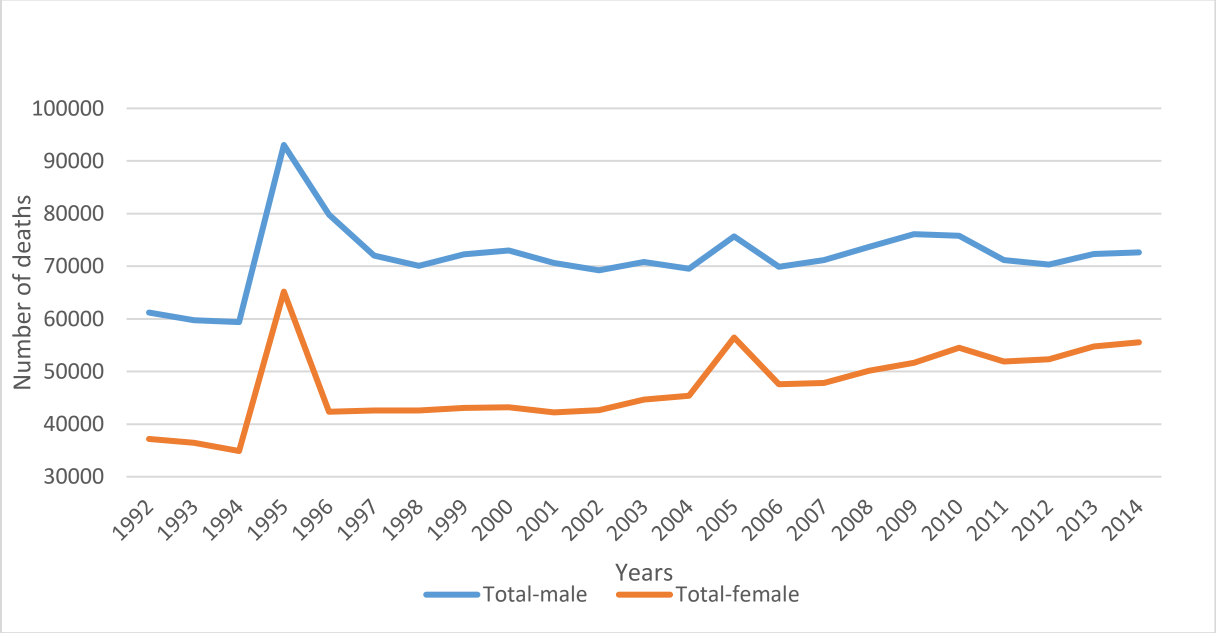

Figure 1. Number of deaths variation with years in Sri Lanka

Life Tables

Sri Lankan life tables are generated for males and females separately for the 2012-2014 period because of mortality rates are significantly difference of male and female. Table 2 and Table 4 illustrate those life tables. Those life tables are called abridged life tables. Abridge table contains the data by intervals of five or ten years (in here five), except in the initial years. This type of table is constructed due to basic number of deaths data registered by five-year age interval. Although annual death data was available, abridged intervals were used for increased reliability and consistency, as death rates can fluctuate significantly year-to-year, especially for smaller age groups or regions.

Table 2. Male life table for 2012-2014 period in Sri Lanka

Male life table 2012-2014 Sri Lanka									
Age	m (x,n) (x1000)	q (x,n) (x1000)	l (x)	d (x,n)	L (x,n)	S (x,n)	T (x)	e (x)	a (x,n)
0	9.439	9.356	100000	936	99129	0.989971	7183615	71.836	0.069
1	0.427	1.705	99064	169	395856	0.998347	7084485	71.514	1.625
5	0.251	1.254	98895	124	494167	0.998605	6688629	67.633	2.500
10	0.308	1.537	98771	152	493478	0.997324	6194462	62.715	2.500
15	0.870	4.342	98620	428	492157	0.994392	5700984	57.808	2.802
20	1.322	6.589	98191	647	489397	0.993262	5208827	53.048	2.589
25	1.349	6.725	97544	656	486100	0.992948	4719430	48.382	2.526
30	1.518	7.564	96889	733	482671	0.991272	4233331	43.693	2.583
35	2.045	10.174	96156	978	478459	0.988026	3750659	39.006	2.629
40	2.878	14.296	95177	1361	472729	0.981044	3272201	34.380	2.680
45	4.981	24.623	93817	2310	463768	0.968712	2799471	29.840	2.699
50	7.866	38.620	91507	3534	449258	0.953209	2335703	25.525	2.658
55	11.523	56.090	87973	4934	428237	0.931086	1886445	21.444	2.644
60	17.601	84.515	83038	7018	398725	0.892889	1458209	17.561	2.654
65	28.736	134.578	76020	10231	356017	0.829443	1059484	13.937	2.646
70	47.258	212.117	65790	13955	295296	0.750396	703467	10.693	2.589
75	70.505	301.403	51835	15623	221589	0.457118	408171	7.875	2.594
80	194.078	...	36211	36211	186582	...	186582	5.153	5.153

First, second and last entry of s (x, n) as follows:

- First entry of s (x, n) is for survivorship of 5 cohorts of birth in the interval from age 0 to 5 = L (0,5) /500000
- Second entry of s (x, n) is for s (0,5) = L (5,5) / L (0,5)
- Last entry of s (x, n) is s (75,5) = T (80) / T (75)

The Table 3 demonstrates a summary of a previous life table. It contains some age intervals and probability of dying.

Table 3. Summary table of the male life table

Age interval	Probability of dying
q(0,5)	0.011046
q(15,20)	0.024985
q(15,35)	0.072126
q(15,45)	0.157995
q(30,40)	0.320976
q(60,20)	0.563919

Table 4. Female life table for 2012-2014 period in Sri Lanka

Female life table 2012-2014 Sri Lanka									
Age	m (x,n) (x1000)	q (x,n) (x1000)	l (x)	d (x,n)	L (x,n)	S (x,n)	T (x)	e (x)	a (x,n)
0	7.450	7.399	100000	740	99313	0.992051	7770682	77.7068	0.0722
1	0.333	1.329	99260	132	396712	0.998736	7671368	77.2855	1.5120
5	0.196	0.978	99128	97	495398	0.998918	7274656	73.3864	2.5000
10	0.238	1.187	99031	118	494862	0.998444	6779258	68.4557	2.5000
15	0.412	2.056	98914	203	494093	0.997639	6284395	63.5341	2.6604
20	0.515	2.573	98710	254	492926	0.997474	5790303	58.6596	2.5377
25	0.496	2.476	98456	244	491681	0.997276	5297377	53.8043	2.5356
30	0.614	3.068	98212	301	490341	0.996448	4805696	48.9316	2.6058
35	0.829	4.136	97911	405	488599	0.995039	4315355	44.0742	2.6375
40	1.199	5.978	97506	583	486176	0.992383	3826756	39.2463	2.6744
45	1.938	9.645	96923	935	482473	0.987246	3340580	34.4662	2.7062
50	3.290	16.324	95989	1567	476319	0.979964	2858108	29.7755	2.6874
55	4.922	24.333	94422	2298	466776	0.968723	2381788	25.2250	2.6791
60	8.163	40.066	92124	3691	452176	0.946042	1915013	20.7873	2.7123
65	14.799	71.588	88433	6331	427778	0.900608	1462836	16.5417	2.7274
70	28.193	132.293	82102	10862	385260	0.837240	1035059	12.6069	2.6752
75	45.478	205.912	71241	14669	322555	0.503607	649798	9.1212	2.7062
80	172.873	...	56571	56571	327243	...	327243	5.7846	5.7846

First, second and last entry of S (x, n) as follows:

- First entry of S (x, n) is for survivorship of 5 cohorts of birth in the interval from age 0 to 5 = L (0,5) /500000
- Second entry of S (x, n) is for S (0, 5) = L (5, 5) / L (0, 5)
- Last entry of S (x, n) is S (75+,5) = T (80) / T (75)

Following Table 5 shows a summary of a previous life table. It contains some age intervals and probability of dying.

Table 5. Summary table of the female life table

Age interval	Probability of dying
q(0,5)	0.008718
q(15,20)	0.010135
q(15,35)	0.029573
q(15,45)	0.068642
q(30,40)	0.164034
q(60,20)	0.385921

The main outcome of the life tables is life expectancy. The life expectancy of newborn male and female are 71.836 and 77.7068.

Comparisons Using Life Tables

Following figures which are Figure 2, Figure 3 and Figure 4 show comparison between male and female according to death rates, life expectation and probability of dying.

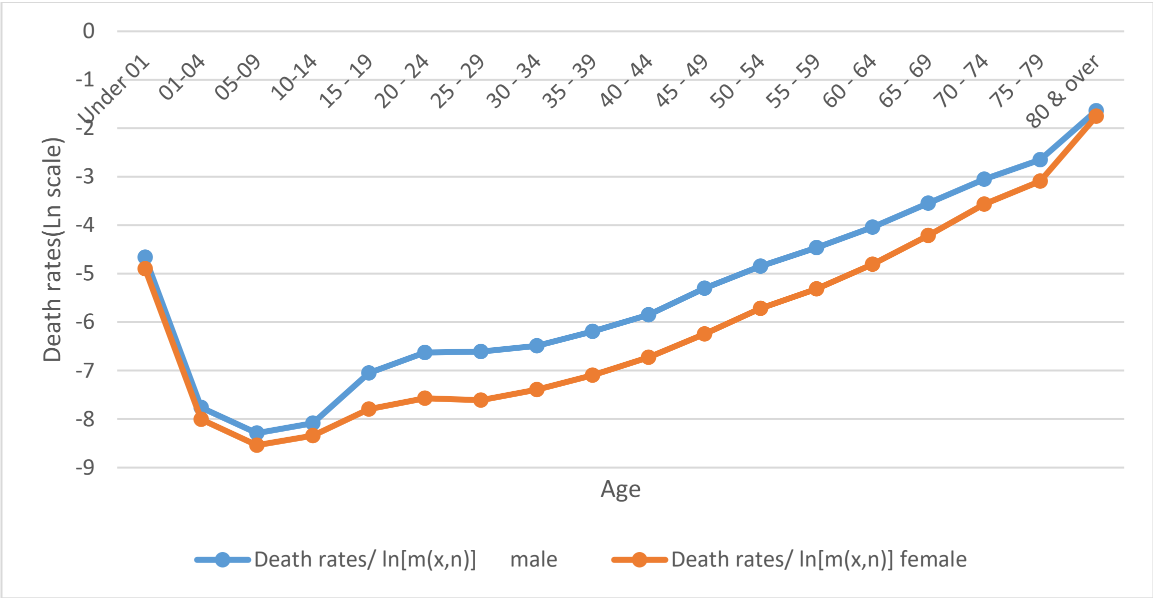

Figure 2. Death rate versus age in 2012-2014 period of male and female

The Figure 2 shows death rates (Ln scale) for different age groups for both males and females. There is a similar pattern between males and females. It appears that factors affecting the death rate equally affect both males and females.

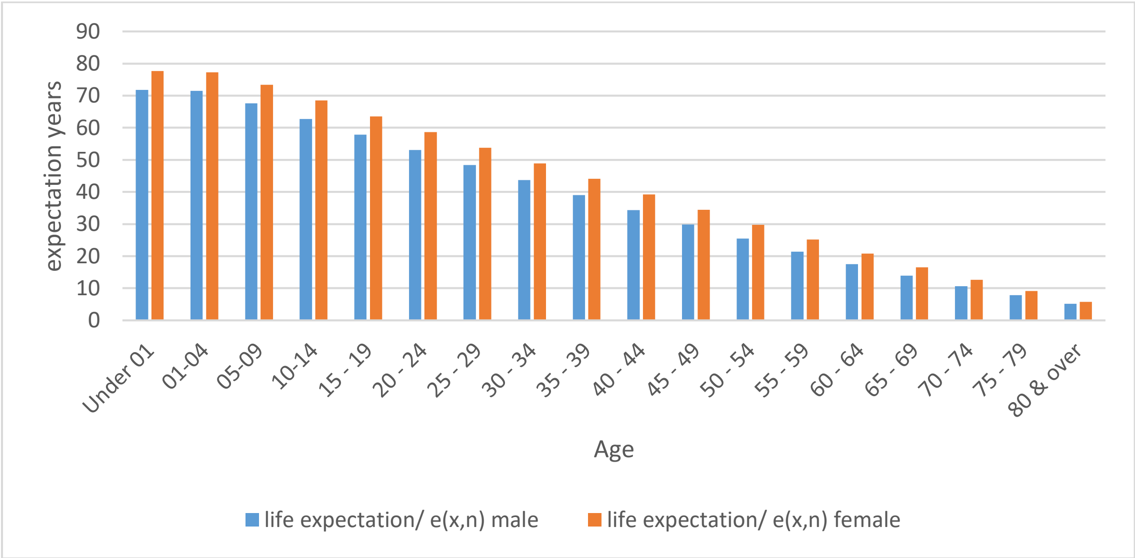

Figure 3. Life expectation versus in 2012-2014 period of male and female

Figure 3 shows life expectancy for different age groups for both males and females. It appears when people are aging the life expectancy decreases. The life expectancy of the males is always less than females.

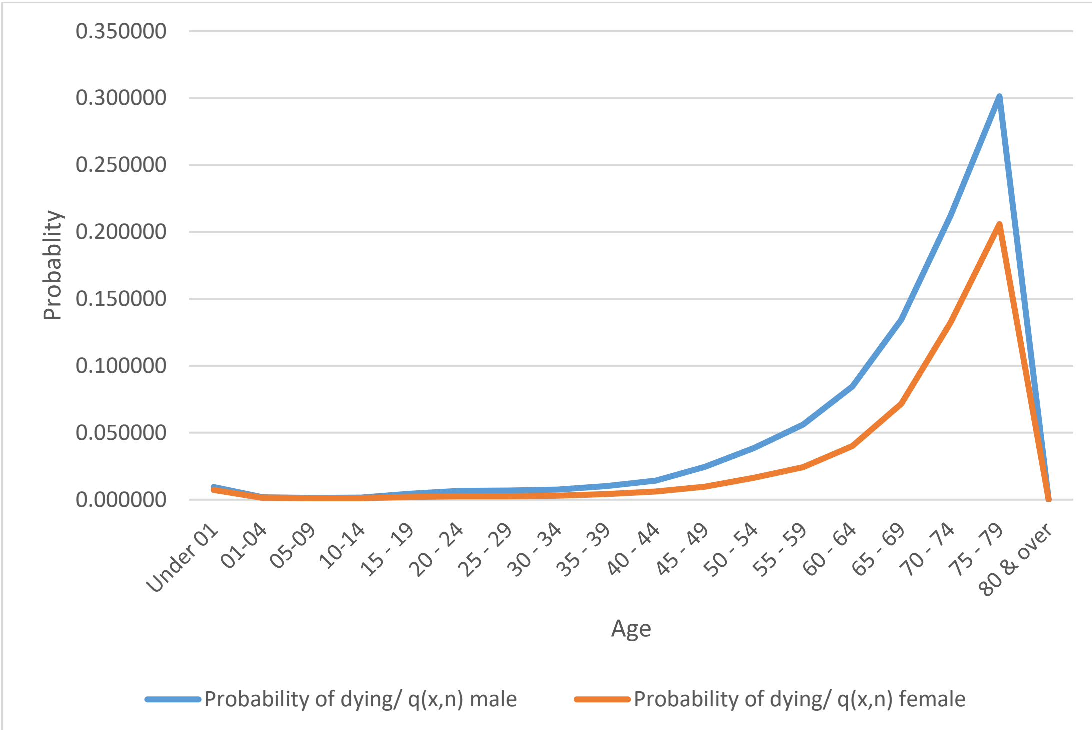

Figure 4. Probability of dying versus ages in 2012-2014 of male and female.

Figure 4 shows that the probability of dying increases with getting older. According to this graph probability of dying of females is less than the probability of dying of males in the same age group. Therefore, the risk of dying of females is less than males in the same age group. The seemingly low death probability among those over 80 may reflect survivorship bias and the limitations of abridged life table estimation at older ages, where the population base is much smaller and data may be underreported.

Mortality Forecasting with Lee-Carter Approach

Estimated a_x, b_x and k_t values used for LC approach to get forecasted death rates for future periods. Table 6 includes the estimated a_x, b_x values for both males and females. 1992-2014 death rates are used for this purpose of applying LC approach.

Table 6 and Table 7 show the forested age-specific death rates (mortality rate) for the period 2015-2035 separately for males and female, which are calculated by using LC estimate model. This is the final outcome of mortality forecasting analysis. Variation of mortality in future (2015-2035) years in Sri Lanka as follows.

Table 6. Estimated a_x and b_x values

Age group	male		female	
	a_x	b_x	a_x	b_x
0 – 4	-4.201361	0.060464	-4.723154	0.061345
5 – 9	-6.325484	0.0771275	-6.470146	0.077015
10 – 14	-6.99148	0.0868905	-7.250741	0.084374
15 – 19	-6.602354	0.0781635	-6.720453	0.079008
20 – 24	-6.187015	0.0753565	-6.504583	0.078887
25 – 29	-6.002785	0.0733905	-6.400008	0.074800
30 – 34	-5.866912	0.0720915	-6.098021	0.073911
35 – 39	-5.614803	0.0680255	-5.872015	0.069860
40 – 44	-5.348162	0.0666175	-5.69871	0.063609
45 – 49	-5.062184	0.0616185	-5.358014	0.059619
50 – 54	-4.785215	0.0534005	-5.200147	0.058370
55 – 59	-4.025874	0.0494995	-4.875412	0.052743
60 – 64	-3.802574	0.0477515	-4.278451	0.048280
65 – 69	-3.504891	0.0384635	-3.901423	0.040009
70 – 74	-3.000012	0.0358325	-3.354308	0.035610
75–79	-2.284864	0.0256779	-2.305477	0.026321
80 +	-1.45248	0.0147655	-1.482372	0.015862

Table 7. Forecasted values of mortality index k_t

Period	Male	Female
2015	-10.359236	-8.538726
2020	-11.864382	-9.423456
2025	-13.024536	-11.9832
2030	-14.389563	-12.392715
2035	-15.86234	-14.745321

Table 8 and Table 9 show the forecasted death rates by years 2015-2035, by using those details it can produce life tables for future years. Those life tables are used for predicting number of deaths, number of survivors (specially 55 aged people), probability of dying in future years. Those matters are very useful for actuarial science, modelling pension plans, modelling job creation, improve educational facilities, other government plans, medical aspects and many studies.

Table 8: Forecasted age specific death rates 2015-2035 for male

Age group	Period				
	2015	2020	2025	2030	2035
0 – 4	0.008005	0.007308	0.006813	0.006274	0.005739
5 – 9	0.000805	0.000717	0.000656	0.00059	0.000527
10 – 14	0.000374	0.000328	0.000297	0.000263	0.000232
15 – 19	0.000604	0.000537	0.00049	0.000441	0.000393
20 – 24	0.000942	0.000841	0.00077	0.000695	0.000622
25 – 29	0.001156	0.001035	0.00095	0.00086	0.000772
30 – 34	0.001342	0.001204	0.001107	0.001003	0.000902
35 – 39	0.001801	0.001626	0.001502	0.001369	0.001239
40 – 44	0.002386	0.002158	0.001998	0.001824	0.001653
45 – 49	0.003344	0.003048	0.002838	0.002609	0.002383
50 – 54	0.004804	0.004433	0.004166	0.003873	0.00358
55 – 59	0.010688	0.00992	0.009367	0.008755	0.008139
60 – 64	0.013606	0.012662	0.01198	0.011224	0.010462
65 – 69	0.020174	0.01904	0.018209	0.017277	0.016326
70 – 74	0.034348	0.032545	0.031219	0.029729	0.028201
75–79	0.078014	0.075056	0.072853	0.070344	0.067733
80 +	0.200801	0.196388	0.193052	0.1892	0.18513

Table 9. Forecasted age specific death rates 2015-2035 female

Age group	Period				
	2015	2020	2025	2030	2035
0 – 4	0.005263	0.004985	0.004261	0.004155	0.003597
5 – 9	0.000803	0.00075	0.000616	0.000596	0.000498
10 – 14	0.000345	0.00032	0.000258	0.000249	0.000205
15 – 19	0.000614	0.000573	0.000468	0.000453	0.000376
20 – 24	0.000763	0.000712	0.000581	0.000563	0.000468
25 – 29	0.000877	0.000821	0.000678	0.000658	0.000551
30 – 34	0.001196	0.00112	0.000927	0.000899	0.000756
35 – 39	0.001552	0.001459	0.00122	0.001185	0.001006
40 – 44	0.001946	0.00184	0.001563	0.001523	0.001311
45 – 49	0.002831	0.002686	0.002306	0.00225	0.001956
50 – 54	0.003351	0.003182	0.002741	0.002676	0.002332
55 – 59	0.004865	0.004643	0.004056	0.00397	0.003507
60 – 64	0.00918	0.008796	0.007774	0.007622	0.006803
65 – 69	0.014364	0.013864	0.012515	0.012311	0.011205
70 – 74	0.025774	0.024975	0.022799	0.022469	0.020663
75–79	0.079641	0.077808	0.072738	0.071959	0.067638
80 +	0.198332	0.195568	0.187787	0.186571	0.179737

Conclusion

In the 2012-2014 period, based on our results life expectancy is respectively male and female 71.8 and 77.7 for newborns. According to our study, People 55 aged life expectation is respectively male and female another 21.4 and 25.2 years. 55 aged person dying probability for males and females estimate as 0.056 and 0.024, respectively. There are two peak points for Sri Lankan deaths in 1995 and 2005 because of the civil war and the tsunami (Abeyrathna et al., 2009). These two points appear in both male and female death graphs.

The limitation of this study is the reliability concerns of mortality and population data from the Northern Province and Eastern Province, particularly during the civil war period (1983–2009). Conflict likely disrupted data collection, death registration, and population census activities, which may have introduced bias or underreporting. Thus, the national life expectancy and mortality forecasts could slightly underestimate regional disparities.

From 2015 to 2035, mortality rates are forecasted by using estimated a_x, b_x and k_t. According to those results, in 2035 probability of dying aged 55 people identified as 0.051 and 0.023 respectively male and female. Sri Lankan life expectation is higher than compared of the South Asian region (World Health Rankings, 2019). It indicates that Sri Lanka's health sector has more potential than the South Asian region (Central Bank of Sri Lanka, 2018). However, the other side of having high life expectancy is Sri Lankan elderly population is increasing compared with the South Asian region (World Health Rankings, 2019).

Conflicts of interest

The authors declare that they have no conflict of interest.

Author Contributions

N.A.D.N. Napagoda led the analytical work, including statistical analyses, life table development, and implementation of the Lee-Carter model. She authored the analysis and results section, contributed to research design and methodology, and provided academic supervision throughout.

P.B.W.S.R. Kumarasinghe drafted the abstract, introduction, literature review, methodology, and conclusion. She developed tables and figures, structured the manuscript, and ensured proper formatting and coherence based on journal guidelines.

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