

MAGNETIC PROPERTIES OF FERROMAGNETIC THIN FILMS WITH THREE SPIN LAYERS AS DESCRIBED BY FOURTH-ORDER PERTURBED HEISENBERG HAMILTONIAN

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ABSTRACT

The magnetic properties of ferromagnetic thin films with three spin layers were investigated for the first time using fourth-order perturbed Heisenberg Hamiltonian. Total magnetic energy was determined for ferromagnetic materials with simple cubic lattice. The simulation was carried out for Heisenberg Hamiltonian with different second-order magnetic anisotropy constants in different spin layers. The total magnetic energy periodically varies with spin-exchange interaction, azimuthal angle of spin and the second-order magnetic anisotropy constant in each spin layer. The peaks along the axis of the angle are closely packed except in the 3-D plot of energy versus angle and second-order magnetic anisotropy constant of the middle spin layer. 2-D plots were perceived by rotating 3-D plots in MATLAB. According to the 3-D plot, the angle between magnetic easy and hard directions is 90 degrees. The minimum energy range was found in the 3-D plot of energy versus angle and the second order anisotropy constant of the middle spin layer. The samples with the minimum magnetic anisotropy energies have potential application in transformer cores, magnetic switching and magnetic amplifiers.

Keywords: spin layers, Heisenberg Hamiltonian, perturbation, ferromagnetic films, magnetic easy direction

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INTRODUCTION

Ferromagnetic thin films are prime candidates in the applications of transformers, electromagnets, magnetic recording, hard drives, generators, telephones, loudspeakers and electric motors. The magnetic properties of ferromagnetic thin films depend on spin-exchange interaction, long-range dipole interaction, second-order magnetic anisotropy, fourth-order magnetic anisotropy, applied magnetic field, demagnetization factor and stress-induced anisotropy. The origin of both dipole interaction and demagnetization is the separation of magnetic dipoles in a material. Although magnetic dipole interaction is a microscopic property of material, the demagnetization factor depends on the geometry and dimension of the sample. Along the thick side of the sample, the demagnetization factor is small. Along the thin side of the sample, the demagnetization factor is large. Excess energy required to magnetize a specimen in a particular direction over that required to magnetize it along the easy direction is called the crystalline anisotropy energy. If there is a preferred direction of magnetization of a sample, the sample shows magnetic anisotropy. That preferred direction of magnetization is called magnetic easy direction. Magnetic easy axis oriented thin films provide the same magnetic energy density as the bulk material. As a result, magnetic easy axis oriented magnetic thin films are prime candidates in the applications of magnetic memory devices, microwave devices and monolithic microwave integrated circuits (MMIC) (Naoe *et al.*, 1981, Morisako *et al.*, 1988, Adam *et al.*, 1990, Hegde *et al.*, 1994). Ferromagnetic materials have spontaneous magnetization even in zero magnetic fields. Weiss introduced the domain hypothesis to explain this phenomenon. If the resultant magnetization of separate domains cancels out, demagnetization occurs. Ferromagnetic samples in a demagnetized state can be easily magnetized by applying a weak magnetic field. When an external magnetic field is applied to a sample, the

magnetization occurs due to the domain wall movement in the low field and magnetic moment rotation in domains in the high field.

Some theoretical research related to magnetic thin films can be summarized as follows. Heisenberg Hamiltonian has been employed to investigate the interfacial coupling dependence of the magnetic ordering in ferro-antiferromagnetic thin films (Tsai *et al.*, 2003). Spin exchange interaction, magnetic dipole interaction, applied magnetic field, and second and fourth-order magnetic anisotropy terms have been considered to study the ferromagnetic thin films (Hutch & Usadel, 1997, Hutch & Usadel, 1999, Usadel & Hutch, 2002). The domain structure and magnetization reversal in thin magnetic films have been determined using computer simulations (Nowak, 1995). Heisenberg Hamiltonian was employed to study in-plane dipole coupling anisotropy of a square ferromagnetic Heisenberg monolayer (Dantziger *et al.*, 2002). In addition, the step-induced magnetic-hysteresis anisotropy in ferromagnetic thin films has been theoretically studied using Monte Carlo simulations (Zhao *et al.*, 2002). First-principles band structure theory has been applied to study FeCo/W(110) surface alloys (Spisak & Hafner, 2005). Heisenberg Hamiltonian has been employed to study EuTe films with surface elastic stresses (Radomska & Balcerzak, 2003). De Vries theory was employed to calculate magnetic anisotropy energy of dc magnetron sputtered FeTaN thin films with (110) orientation (James & Chester, 1994). Layer-dependent magnetic moments in Ni films on Cu surface have been theoretically investigated using the Korringa-Kohn-Rostoker Green's function method (Ernst *et al.*, 2000). The affect of the surface, film thickness and temperature on magnetic properties in multiferroic thin films have been studied using modified Heisenberg model and transverse Ising model coupled with Green's function technique (Kovachev & Wesselinowa, 2009).

We have explained ferrite films using second-order and third-order perturbed Heisenberg Hamiltonian (Samarasekara, 2010, Samarasekara & Mendoza, 2011). In addition, Heisenberg Hamiltonian was employed to describe the magnetic easy axis orientation of experimentally deposited magnetic thin films at different temperatures (Samarasekara & Gunawardhane, 2011, Samarasekara & Saparamadu, 2012, Samarasekara & Saparamadu, 2013). According to our experimental studies, the orientation of the magnetic easy axis is highly sensitive to the deposition temperature of ferromagnetic thin films (Hegde *et al.*, 1994). We have applied second-order and third-order perturbed Heisenberg Hamiltonian to describe the ferromagnetic films (Samarasekara, 2006, Samarasekara & Mendoza, 2010). Unperturbed Heisenberg Hamiltonian was applied to describe ferrite thin films (Samarasekara, 2007). Magnetic properties of ferrite films have been elucidated using third-order perturbed Heisenberg Hamiltonian by us (Samarasekara, 2011). Magnetostatic energy of domains and their walls has been theoretically studied in different film thicknesses (Viot *et al.*, 2012). Magnetic thin films with thicknesses ranging from 2 to 4 layers have been modeled using anisotropic classical Heisenberg spins under the influence of mechanical uniaxial stresses (Laosiritaworn *et al.*, 2006). Monte Carlo simulation has been utilized to explore magnetic properties of very thin films with bcc structure (Santamaria & Diep, 2000). The properties of thin films made of stacked triangular layers of atoms bearing Heisenberg spins with an Ising-like interaction anisotropy have been investigated using extensive Monte Carlo simulations and analytical Green's function (Ngo & Diep, 2007). A Green's function technique has been implemented for the Heisenberg model to investigate the influence of the magnetic surface single-ion anisotropy on the spin-wave spectrum including damping effects in ferromagnetic thin films (Wesselinowa, 2006).

For this study, the fourth-order perturbation of the Heisenberg Hamiltonian was applied to describe the magnetic properties of ferromagnetic thin films with three spin layers. 3-D plots of total magnetic energy versus azimuthal angle of spin and spin-exchange interactions or second- order magnetic anisotropy constants of different spin layers were plotted using MATLAB. Spin- exchange interaction, long-range dipole interaction, second-order magnetic anisotropy, fourth- order magnetic anisotropy, applied magnetic field, demagnetization factor and stress-induced anisotropy were taken into account in our previous research work related to second and third- order perturbed Heisenberg Hamiltonian. To avoid tedious derivations, only spin exchange interaction, long range dipole interaction and second order magnetic anisotropy were considered in the fourth-order perturbed Heisenberg Hamiltonian.

MODEL

Classical Heisenberg Hamiltonian of ferromagnetic thin films with spin exchange interaction, long range dipole interaction and second order magnetic anisotropy can be conveyed as (Samarasekara, 2010, Samarasekara & Mendoza, 2011, Samarasekara & Gunawardhane, 2011, Samarasekara & Saparamadu, 2012).

$$H = -\frac{J}{2} \sum_{m,n} \vec{S}_m \cdot \vec{S}_n + \frac{\omega}{2} \sum_{m \neq n} \left(\frac{\vec{S}_m \cdot \vec{S}_n}{r_{mn}^3} - \frac{3(\vec{S}_m \cdot \vec{r}_{mn})(\vec{r}_{mn} \cdot \vec{S}_n)}{r_{mn}^5} \right) - \sum_m D_{\lambda_m}^{(2)} (S_m^z)^2 \quad (1)$$

After taking the dot products of spin vectors in equation (1), equation (1) can be deduced to the following form per unit spin with $\vec{S}_m = \vec{S}_n = 1$ (Samarasekara & Gunawardhane, 2011, Samarasekara & Saparamadu, 2012, Samarasekara & Saparamadu, 2013).

$$E(\theta) = -\frac{1}{2} \sum_{m,n=1}^N [(JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|}) \cos(\theta_m - \theta_n) - \frac{3\omega}{4} \Phi_{|m-n|} \cos(\theta_m + \theta_n)] - \sum_{m=1}^N D_m^{(2)} \cos^2 \theta_m \quad (2)$$

Where m and n indicate indices of two different spin layers, N exhibits the number of layers measured in the direction perpendicular to the film plane, J is the magnetic spin-exchange interaction, $Z_{|m-n|}$ stands for the number of nearest spin neighbors, ω represents the strength of long-range dipole interaction, $\Phi_{|m-n|}$ are constants for partial summation of dipole interaction, For non-oriented films, above angles θ_m and θ_n measured with film normal can be expressed in forms of $\theta_m = \theta + \varepsilon_m$ and $\theta_n = \theta + \varepsilon_n$, and cosine and sine terms can be expanded up to the fourth-order of ε as following. Here ε is the perturbation of the angle. For a ferromagnetic thin film with three spin layers, N varies from 1 to 3.

$$E(\theta) = E_0 + E(\varepsilon) + E(\varepsilon^2) + E(\varepsilon^3) + E(\varepsilon^4) \quad (3)$$

$$E_0 = -\frac{3}{2}(JZ_0 - \frac{\omega\phi_0}{4}) - 2(JZ_1 - \frac{\omega\phi_1}{4}) + \frac{3\omega}{8}(3\phi_0 + 4\phi_1)\cos 2\theta \\ - (D_1^{(2)} + D_2^{(2)} + D_3^{(2)})\cos^2 \theta \quad (4)$$

$$E(\varepsilon) = -\frac{3\omega}{4}[\phi_0(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \phi_1(\varepsilon_1 + 2\varepsilon_2 + \varepsilon_3)]\sin 2\theta \\ + \sin 2\theta(D_1^{(2)}\varepsilon_1 + D_2^{(2)}\varepsilon_2 + D_3^{(2)}\varepsilon_3) \quad (5)$$

$$E(\varepsilon^2) = \left(JZ_1 - \frac{\omega\phi_1}{4}\right)\left(\frac{\varepsilon_1^2 + 2\varepsilon_2^2 + \varepsilon_3^2 - 2\varepsilon_1\varepsilon_2 - 2\varepsilon_2\varepsilon_3}{2}\right) \\ - \frac{3\omega}{8}\cos 2\theta(2\phi_0(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) + \phi_1(\varepsilon_1^2 + 2\varepsilon_2^2 + \varepsilon_3^2 + 2\varepsilon_1\varepsilon_2 + 2\varepsilon_2\varepsilon_3)) \\ + (D_1^{(2)}\varepsilon_1^2 + D_2^{(2)}\varepsilon_2^2 + D_3^{(2)}\varepsilon_3^2)\cos 2\theta \quad (6)$$

$$E(\varepsilon^3) = \frac{\omega}{8}\left(4\phi_0(\varepsilon_1^3 + \varepsilon_2^3 + \varepsilon_3^3) + \phi_1(\varepsilon_1^3 + 3\varepsilon_1\varepsilon_2^2 + 3\varepsilon_1^2\varepsilon_2 + 2\varepsilon_2^3) \right. \\ \left. + 3\varepsilon_2\varepsilon_3^2 + 3\varepsilon_2^2\varepsilon_3 + \varepsilon_3^3\right)\sin 2\theta \\ - \frac{4\cos\theta\sin\theta}{3}(D_1^{(2)}\varepsilon_1^3 + D_2^{(2)}\varepsilon_2^3 + D_3^{(2)}\varepsilon_3^3) \quad (7)$$

$$\begin{aligned}
 E(\varepsilon^4) = & \left(\frac{\omega}{4} \phi_1 - JZ_1 \right) \left(\frac{\varepsilon_1^4 + 2\varepsilon_2^4 + \varepsilon_3^4 + 6\varepsilon_1^2 \varepsilon_2^2}{-4\varepsilon_1^3 \varepsilon_2 - 4\varepsilon_1 \varepsilon_2^3 + 6\varepsilon_2^2 \varepsilon_3^2 - 4\varepsilon_2^3 \varepsilon_3 - 4\varepsilon_2 \varepsilon_3^3} \right) \\
 & + \frac{\omega}{8} \cos 2\theta \left[2\phi_0 (\varepsilon_1^4 + \varepsilon_2^4 + \varepsilon_3^4) \right] \\
 & + \frac{\phi_1}{4} \left(\varepsilon_1^4 + 2\varepsilon_2^4 + \varepsilon_3^4 + 6\varepsilon_1^2 \varepsilon_2^2 + 4\varepsilon_1^3 \varepsilon_2 + 4\varepsilon_1 \varepsilon_2^3 \right. \\
 & \quad \left. + 6\varepsilon_2^2 \varepsilon_3^2 + 4\varepsilon_2^3 \varepsilon_3 + 4\varepsilon_2 \varepsilon_3^3 \right) \\
 & - \frac{\cos 2\theta}{3} (D_1^{(2)} \varepsilon_1^4 + D_2^{(2)} \varepsilon_2^4 + D_3^{(2)} \varepsilon_3^4)
 \end{aligned} \tag{8}$$

Here $D_1^{(2)}$, $D_2^{(2)}$ and $D_3^{(2)}$ are the second-order anisotropy constants of bottom, middle and top spin layers, respectively.

Using $E(\varepsilon) = \vec{\alpha} \cdot \vec{\varepsilon}$, α_1 , α_2 and α_3 can be found.

$$\alpha_1 = -\frac{3\omega}{4} (\phi_0 + \phi_1) \sin 2\theta + D_1^{(2)} \sin 2\theta$$

$$\alpha_2 = -\frac{3\omega}{4} (\phi_0 + 2\phi_1) \sin 2\theta + D_2^{(2)} \sin 2\theta$$

$$\alpha_3 = -\frac{3\omega}{4} (\phi_0 + \phi_1) \sin 2\theta + D_3^{(2)} \sin 2\theta$$

Matrix elements of matrix C can be found using

$$E(\varepsilon^2) = \frac{1}{2} \vec{\varepsilon} \cdot \mathbf{C} \cdot \vec{\varepsilon}$$

$$C_{11} = JZ_1 - \frac{\omega \phi_1}{4} - \frac{3\omega}{4} (2\phi_0 + \phi_1) \cos 2\theta + 2D_1^{(2)} \cos 2\theta$$

$$C_{12} = C_{21} = C_{23} = C_{32} = -JZ_1 + \frac{\omega \phi_1}{4} - \frac{3\omega}{4} \phi_1 \cos 2\theta$$

$$C_{13} = C_{31} = 0$$

$$C_{22} = 2(JZ_1 - \frac{\omega\phi_1}{4}) - \frac{3\omega}{2}(\phi_0 + \phi_1)\cos 2\theta + 2D_2^{(2)}\cos 2\theta$$

$$C_{33} = JZ_1 - \frac{\omega\phi_1}{4} - \frac{3\omega}{4}(2\phi_0 + \phi_1)\cos 2\theta + 2D_3^{(2)}\cos 2\theta$$

Matrix elements of matrix β can be found using

$$E(\varepsilon^3) = \varepsilon^2 \beta \cdot \vec{\varepsilon}$$

$$\beta_{11} = \frac{\omega}{8}(4\phi_0 + \phi_1)\sin 2\theta - \frac{4D_1^{(2)}\cos\theta\sin\theta}{3}$$

$$\beta_{12} = \beta_{21} = \beta_{23} = \beta_{32} = \frac{3\omega}{8}\phi_1\sin 2\theta$$

$$\beta_{31} = \beta_{13} = 0$$

$$\beta_{22} = \frac{\omega}{4}(2\phi_0 + \phi_1)\sin 2\theta - \frac{4D_2^{(2)}\cos\theta\sin\theta}{3}$$

$$\beta_{33} = \frac{\omega}{8}(4\phi_0 + \phi_1)\sin 2\theta - \frac{4D_3^{(2)}\cos\theta\sin\theta}{3}$$

Matrix elements of matrices F and G can be found using

$$E(\varepsilon^4) = \varepsilon^3 F \cdot \vec{\varepsilon} + \varepsilon^2 G \varepsilon^2$$

$$F_{11} = -\frac{1}{24}\left(JZ_1 - \frac{\omega\phi_1}{4}\right) + \frac{\omega}{8}\left(2\phi_0 + \frac{\phi_1}{4}\right)\cos 2\theta - \frac{D_1^{(2)}\cos 2\theta}{3}$$

$$F_{12} = F_{21} = F_{23} = F_{32} = \frac{1}{6}\left(JZ_1 - \frac{\omega\phi_1}{4}\right) + \frac{\omega}{8}\phi_1\cos 2\theta$$

$$F_{13} = F_{31} = 0$$

$$F_{22} = -\frac{1}{12}\left(JZ_1 - \frac{\omega\phi_1}{4}\right) + \frac{\omega}{8}\left(2\phi_0 + \frac{\phi_1}{2}\right)\cos 2\theta - \frac{D_2^{(2)}\cos 2\theta}{3}$$

$$F_{33} = -\frac{1}{24}\left(JZ_1 - \frac{\omega\phi_1}{4}\right) + \frac{\omega}{8}\left(2\phi_0 + \frac{\phi_1}{4}\right)\cos 2\theta - \frac{D_3^{(2)}\cos 2\theta}{3}$$

$$G_{11} = G_{22} = G_{33} = G_{13} = G_{31} = 0$$

$$G_{12} = G_{21} = G_{23} = G_{32} = -\frac{1}{8} \left(JZ_1 - \frac{\omega\phi_1}{4} \right) + \frac{3\omega}{32} \phi_1 \cos 2\theta$$

From equation 3, total energy can be expressed as

$$E(\theta) = E_0 + \vec{\alpha} \cdot \vec{\varepsilon} + \frac{1}{2} \vec{\varepsilon} \cdot C \cdot \vec{\varepsilon} + \varepsilon^2 \beta \cdot \vec{\varepsilon} + \varepsilon^3 F \cdot \vec{\varepsilon} + \varepsilon^2 G \varepsilon^2 \quad (9)$$

For the minimum energy of the second order-perturbed term (Hutch & Usadel, 1997, Hutch & Usadel, 1999, Usadel & Hutch, 2002),

$$\vec{\varepsilon} = -C^+ \cdot \vec{\alpha} \quad (10)$$

Here C^+ is the pseudo inverse of matrix C , and C^+ can be found using

$$C \cdot C^+ = 1 - \frac{E}{N} \quad (11)$$

Here E is the matrix with all elements given by $E_{mn}=1$.

RESULTS AND DISCUSSION

From equation (10),

$$\varepsilon_1 = -(C_{11}^+ \alpha_1 + C_{12}^+ \alpha_2 + C_{13}^+ \alpha_3)$$

$$\varepsilon_2 = -(C_{21}^+ \alpha_1 + C_{22}^+ \alpha_2 + C_{23}^+ \alpha_3)$$

$$\varepsilon_3 = -(C_{31}^+ \alpha_1 + C_{32}^+ \alpha_2 + C_{33}^+ \alpha_3)$$

From equation (11), matrix elements of C^+ can be found. After substituting elements of α and C^+ in the above equations, ε_1 , ε_2 and ε_3 can be determined. MATLAB computer software was applied to determine the elements of matrix C^+ . After substituting all parameters in equation (9), the total energy can be found as a function of spin exchange interaction (J), second order anisotropy constants ($D^{(2)}$), dipole interaction

(ω) and azimuthal angle (θ). All $\frac{D_1^{(2)}}{\omega}$, $\frac{D_2^{(2)}}{\omega}$, $\frac{D_3^{(2)}}{\omega}$ and $\frac{J}{\omega}$ are dimensionless physical quantities.

For s. c. (001) lattice, $Z_0=4$, $Z_1=1$, $\Phi_0=9.0336$ and $\Phi_f=-0.3275$ (Hutch & Usadel, 1997, Hutch & Usadel, 1999, Usadel & Hutch, 2002). Figure 1 shows the 3-D graph of total

energy versus angle and spin-exchange interaction for $\frac{D_1^{(2)}}{\omega}=10$, $\frac{D_2^{(2)}}{\omega}=10$ and

$\frac{D_3^{(2)}}{\omega}=5$. The peaks along the axis of angle are closely packed in the fourth-order

perturbed case compared to the second-order and third-order perturbed cases (Samarasekara, 2006, Samarasekara & Mendoza, 2010, Samarasekara & Mendoza, 2011).

A cross sectional view of Figure 1 is given in Figure 2. The graph of Figure 2 can be obtained by rotating 3-D plot in Figure 1 using MATLAB. The shape of the graph is also entirely different from the graphs obtained using the second and third order perturbed Heisenberg Hamiltonian (Samarasekara, 2006, Samarasekara & Mendoza, 2010, Samarasekara & Mendoza, 2011). Major energy maximums can be observed at $J/\omega=27$, 52, 77, etc. Major energy minimums can be discerned at $J/\omega=3$, 28, 53, etc. Peaks are periodically distributed. In addition to the major maxima and minima, minor maxima and minima can be observed. The total energy is in the range of 10^{13} . The energy found using third order perturbed Heisenberg Hamiltonian was in the range from 10^{16} to 10^{19} (Samarasekara & Mendoza, 2010). The energy slightly reduces due to the fourth order perturbation.

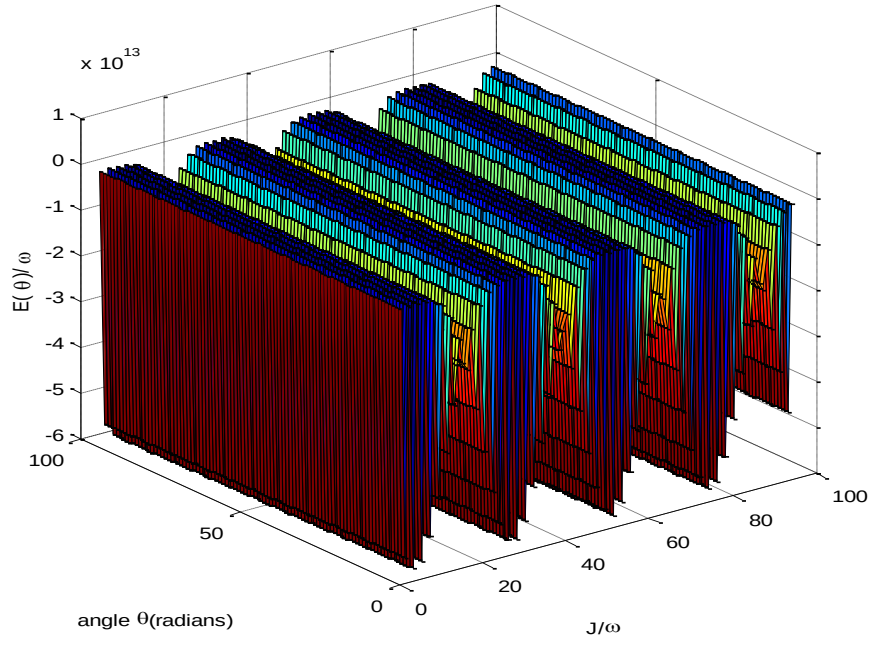


Figure 1: 3-D plot of $\frac{E(\theta)}{\omega}$ versus $\frac{J}{\omega}$ and angle at $\frac{D_1^{(2)}}{\omega} = 10$, $\frac{D_2^{(2)}}{\omega} = 10$ and

$$\frac{D_3^{(2)}}{\omega} = 5$$

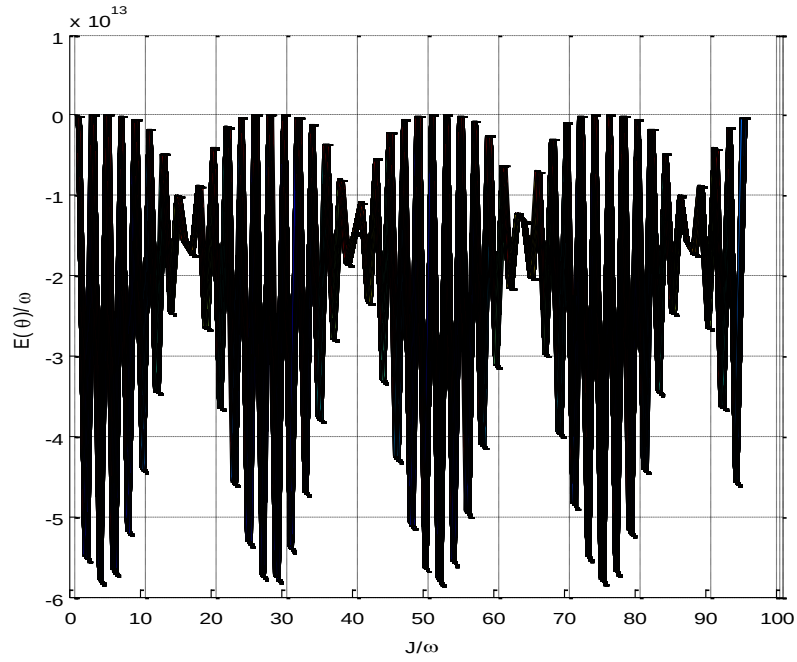


Figure 2: Graph of $\frac{E(\theta)}{\omega}$ versus $\frac{J}{\omega}$

3-D plot of total energy versus angle and spin exchange interaction for $\frac{D_1^{(2)}}{\omega} = 5$,

$\frac{D_2^{(2)}}{\omega} = 10$ and $\frac{D_3^{(2)}}{\omega} = 10$ is similar to the graph in Figure 1. This implies that the

switching the values of the second order anisotropy constant in the bottom and top spin layers does not change the total magnetic energy. Figure 3 represents the 3-D plot of total

energy versus angle and spin exchange interaction for $\frac{D_1^{(2)}}{\omega} = 10$, $\frac{D_2^{(2)}}{\omega} = 5$ and

$\frac{D_3^{(2)}}{\omega} = 10$. The total energy is in the order of 10^{10} in this case. Therefore, the total

energy was considerably reduced compared to the energy found using third order perturbed Heisenberg Hamiltonian (Samarasekara & Mendoza, 2010). When the second order anisotropy constant in the middle spin layer is less than those of the top and bottom spin layers, the total magnetic energy reduces according to the Figures 1 and 3. This implies that the film can be easily magnetized when the second order anisotropy constant in the middle spin layer is less. The graph of energy versus angle in this case is similar to Figure

2. Major energy minima can be observed at $\frac{J}{\omega} = 25, 50, 75$, etc in this case. Major energy

maxima appeared at $\frac{J}{\omega} = 24, 49, 74$, etc.

The energy of ultra-thin film with three spin layers varies with the angle as shown in these graphs. The change of energy with angle represents the energy required to rotate the magnetic moment from one direction to another direction. In some cases as this manuscript describes, the energy required to rotate the magnetic moments is higher. In the magnetization process of a sample, magnetic moments rotate from various directions to the applied magnetic field direction. If the energy required to rotate magnetic moments in a

sample is higher, it is difficult to magnetize that sample. When the anisotropy energy is less, the field required to magnetize a sample is also less. When the anisotropy constant in the middle spin layer is lower compared to those in the top and bottom spin layers, it is easy to magnetize the sample. For hard (permanent) magnetic materials, the magnetic field required to magnetize the sample is higher. For soft (permeable) magnetic materials, the magnetic field required to magnetize the sample is less. When the anisotropy constant in the middle spin layer is lower compared to those in the top and bottom spin layers, the ultra-thin film becomes a soft magnetic material. Soft magnetic materials are the prime candidates in the application of transformer cores, magnetic switching and magnetic amplifiers.

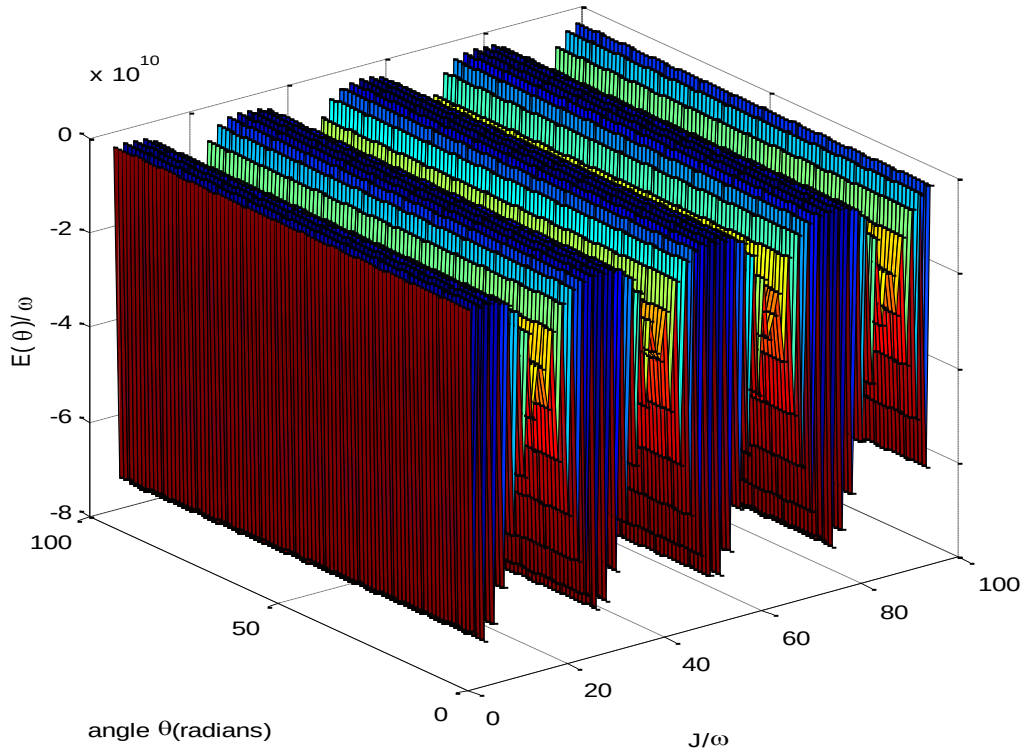


Figure 3: 3-D plot of $\frac{E(\theta)}{\omega}$ versus $\frac{J}{\omega}$ and angle at $\frac{D_1^{(2)}}{\omega} = 10$, $\frac{D_2^{(2)}}{\omega} = 5$ and

$$\frac{D_3^{(2)}}{\omega} = 10$$

3-D plot of energy versus angle and the second order magnetic anisotropy constant of the bottom spin layer for $\frac{J}{\omega} = 10$, $\frac{D_2^{(2)}}{\omega} = 10$ and $\frac{D_3^{(2)}}{\omega} = 10$ is shown in Figure 4. Energy varies in the order of 10^{14} in this case. The range of the energy in this case is slightly less than the range of the energy found using third order perturbed Heisenberg Hamiltonian (Samarasekara & Mendoza, 2010). The graph of energy versus angle in this case has resemblance to Figure 2. The major maximums of energy can be observed at $\frac{D_1^{(2)}}{\omega} = 20$,

45, 70, etc. The major energy minimums can be seen at $\frac{D_1^{(2)}}{\omega} = 23, 48, 73$, etc. In addition to major minima and maxima, minor minima and maxima can be observed in this case similar to Figure 2.

Figure 5 represents the 3-D plot of energy versus angle and second order magnetic anisotropy constant in the middle spin layer for $\frac{J}{\omega} = 10$, $\frac{D_1^{(2)}}{\omega} = 10$ and $\frac{D_3^{(2)}}{\omega} = 10$.

This 3-D plot is different from the other 3-D plots given in this manuscript. The energy range in this case is close to the energy found using the third order perturbed Heisenberg Hamiltonian (Samarasekara & Mendoza, 2010). Peaks along the axis of the angle are slightly separated in this case. Energy maximums can be observed (in radians) at $\theta = 1, 4.14, 7.28$, etc. Energy minimums can be found at $\theta = 2.57, 5.71, 8.85$, etc in radians. Energy maximums and minimums provide the magnetic hard and easy directions, respectively. The angle between easy and hard directions is experimentally 1.57 radians or 90 degrees (Hegde *et al.*, 1994). Therefore, our theoretical data agree with the experimental data.

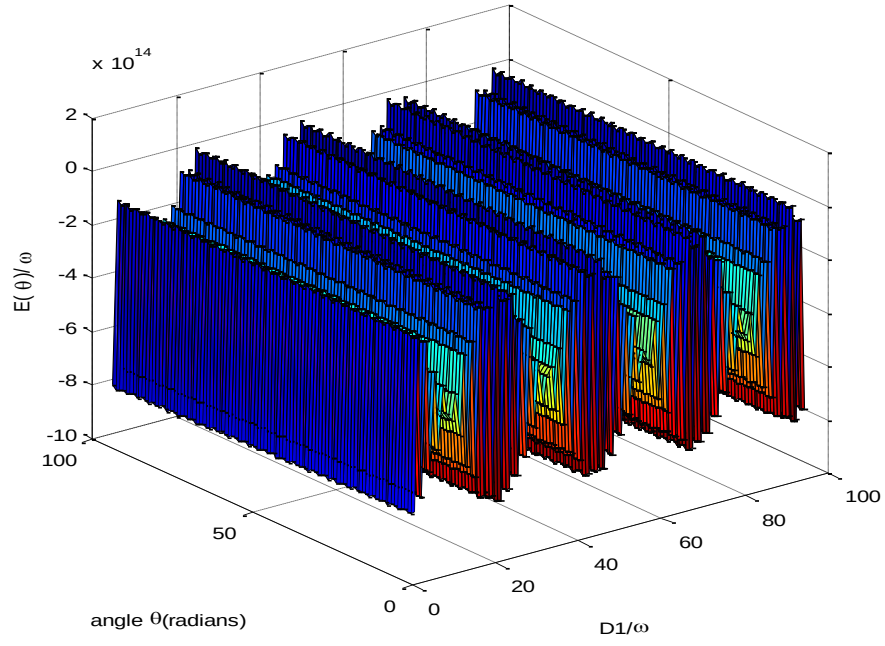


Figure 4: 3-D plot of $\frac{E(\theta)}{\omega}$ versus $\frac{D_1^{(2)}}{\omega}$ and angle at $\frac{J}{\omega} = 10$, $\frac{D_2^{(2)}}{\omega} = \frac{D_3^{(2)}}{\omega} = 10$

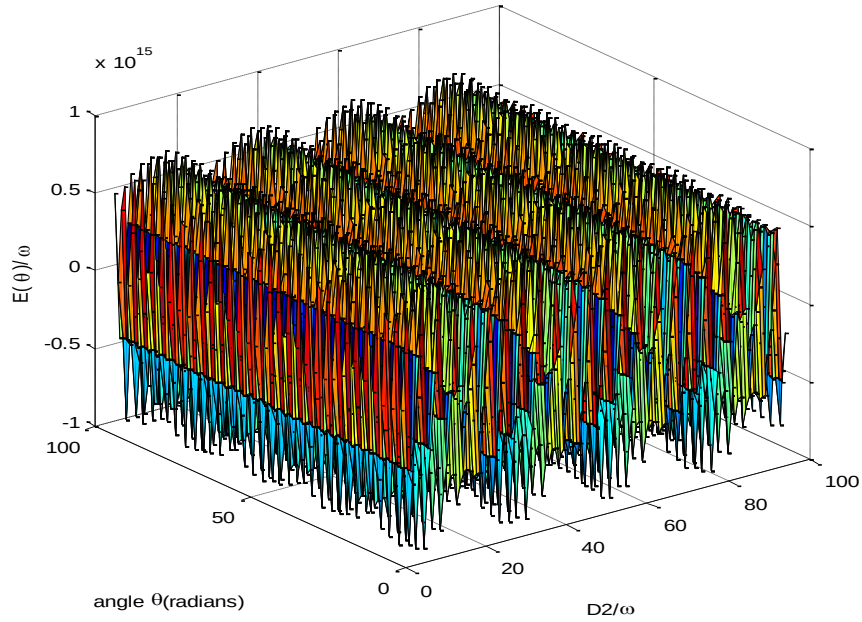


Figure 5: 3-D plot of $\frac{E(\theta)}{\omega}$ versus $\frac{D_2^{(2)}}{\omega}$ and angle at $\frac{J}{\omega} = 10$, $\frac{D_1^{(2)}}{\omega} = \frac{D_3^{(2)}}{\omega} = 10$ nd

The 3-D plot of energy versus second order magnetic anisotropy constant at the top spin

layer and angle is given in Figure 6 for $\frac{J}{\omega}=10$, $\frac{D_1^{(2)}}{\omega}=10$ and $\frac{D_2^{(2)}}{\omega}=10$. Energy

range in this case is 10^{10} , which is less than the energy ranges in graphs in Figures 4 and 5.

This implies that the films can be easily magnetized by varying the second order magnetic

anisotropy constant in the top spin layer. Energy maximums can be observed at $\frac{D_3^{(2)}}{\omega} = 6,$

10, 14, 18, etc. Energy minimums can be perceived at $\frac{D_3^{(2)}}{\omega} = 7, 4, 11, 15,$ etc.

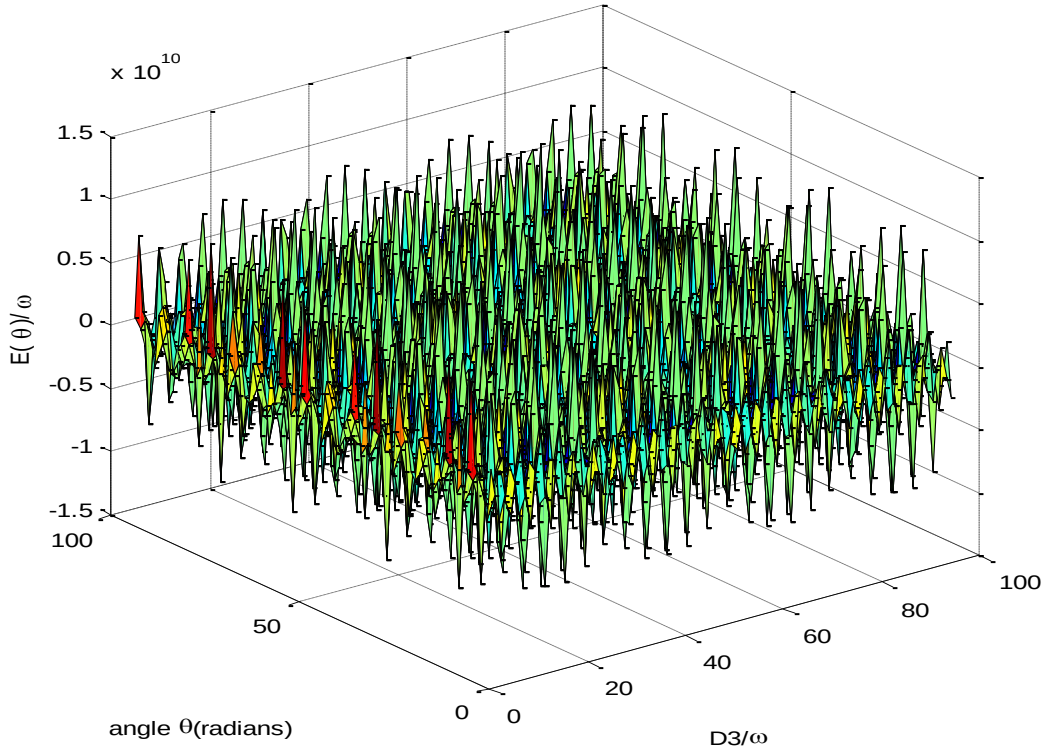


Figure 6: 3-D plot of $\frac{E(\theta)}{\omega}$ versus $\frac{D_3^{(2)}}{\omega}$ and angle at $\frac{J}{\omega}=10$, $\frac{D_1^{(2)}}{\omega}=10$ and

$$\frac{D_2^{(2)}}{\omega}=10$$

CONCLUSIONS

When the second order magnetic anisotropy constant in the middle spin layer is higher than those of top and bottom spin layers, the total magnetic energy is in the order of 10^{13} . On the other hand, the total magnetic energy is in the order of 10^{10} , when the second-order magnetic anisotropy constant in the middle layer is less than those of top and bottom spin layers. Therefore, when the second order magnetic anisotropy constant in the middle spin layer is less than those of top and bottom spin layers, the total magnetic energy becomes smaller. If the energy required for rotating the magnetic moments from the magnetic easy direction to any particular direction is smaller, the magnetic anisotropy energy become smaller. Magnetic hard direction can be observed along the directions given by $\theta = 1, 4.14, 7.28$, etc in radians. Here θ is the angle between the spin and the normal drawn to film plane. Magnetic easy directions can be found along the directions given by $\theta = 2.57, 5.71, 8.85$, etc in radians. The magnetic easy direction is perpendicular to the magnetic hard direction as expected. According to the 3-D plot of energy versus angle and the second order magnetic anisotropy constant of the bottom spin layer for other values fixed at

$\frac{J}{\omega} = 10$, $\frac{D_2^{(2)}}{\omega} = 10$ and $\frac{D_3^{(2)}}{\omega} = 10$, the major maximums of energy can be observed at

$\frac{D_1^{(2)}}{\omega} = 20, 45, 70$, etc, and the major energy minimums can be seen at $\frac{D_1^{(2)}}{\omega} = 23, 48, 73$,

etc. In addition to major minima and maxima, minor minima and maxima can be perceived in all graphs. According to the 3-D plot of total energy versus angle and spin exchange

interaction for other values fixed at $\frac{D_1^{(2)}}{\omega} = 10$, $\frac{D_2^{(2)}}{\omega} = 5$ and $\frac{D_3^{(2)}}{\omega} = 10$, major

energy minima can be noticed at $\frac{J}{\omega} = 25, 50, 75$, etc and major energy maxima can be

observed at $\frac{J}{\omega} = 24, 49, 74$, etc.

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