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RESEARCH ARTICLE

Structural Engineering

Simplification of large-scale solid element model for seismic structural response analysis of buildings

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Abstract: The seismic response analysis of large-scale structural systems, including integrated earthquake simulations for urbanized cities, requires developing complex numerical models with over 10 million solid elements. However, such detailed models are often computationally intensive and time-consuming. Therefore, simplified models are necessary to efficiently assess the seismic response of structures in the time domain during the early stages of analysis. The authors are proposing a meta-modelling theory in which a simple mass-spring model (MSM) is constructed from a large-scale solid-element model. This paper presents examples of converting a three-dimensional solid element model to a one-dimensional MSM, which is denoted as the consistent mass-spring model (CMSM) in the viewpoint of structural mechanics. The performance of the proposed CMSM is compared with conventional MSM and the frequency-adaptive lumped mass-stick model (LMSM). For comparison, three-storey symmetrical and unsymmetrical (T-shaped and L-shaped) reinforced concrete (RC) buildings are selected. Numerical time history simulations are carried out to check the suitability of the proposed CMSM for three sets of ground motions. The proposed CMSMs solve the same physical problem as the solid element model, using suitable mathematical approximations, and show very good agreement with those of the solid models. The first three natural frequencies of the CMSM were found to match those of the solid element model for both symmetric and asymmetric RC buildings in the study, demonstrating the high accuracy of the developed MSM. It is also shown that such a simplified model is used as an alternative to a large-scale solid element model to estimate the overall responses of the structure at the initial stage of large-scale analysis.

Keywords: Continuum mechanics, mass-spring models, meta-modeling, seismic structural response analysis, structural mechanics.

INTRODUCTION

Modern modelling software enables us to automatically construct a solid element model using the data from Computer-Aided-Design (CAD) (Riaz *et al.*, 2021; Hori *et al.*, 2023). For a complicated structure, the constructed solid element model has a degree of freedom (DOF) of a few ten or hundred thousand (Kettil & Wiberg, 2002; Tuñón-Sanjur *et al.*, 2007; Quinay *et al.*, 2011; Tuñón-Sanjur *et al.*, 2013; Kusakabe *et al.*, 2022) but the model has high quality as it has high fidelity to the target structure. However, it is not an easy task to use such a large-scale model for seismic structural analysis that is carried out in the time domain. Large computer resources are needed for the computation of analyzing the model, especially when non-linear material properties or a state of finite or large deformation are considered.

For an efficient numerical analysis in the time domain, it is vital to have a simpler model such as a frame element model (Kettil & Wiberg, 2002). However, it is not certain that the frame model that is determined by the method is used as an alternative to a solid element model which is automatically constructed from CAD data. It is rare to

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have an identical natural frequency for the frame model and the solid element model even for linear seismic structural analysis (Jayasinghe *et al.*, 2016; 2017). A solid element model uses Young's modulus and Poisson's ratio for linear isotropic elasticity, but Poisson's ratio is fully ignored in a truss or beam theory which is the foundation of the frame model (Hori *et al.*, 2014).

To resolve the conflict between solid element model analysis and other analysis, the authors are proposing a new theory, called "meta-modelling" (Hori *et al.*, 2014; 2015a 2015b; Jayasinghe *et al.*, 2014; 2015a; 2015b; 2015c; 2015d; 2016), which regards structural mechanics (Hjelmstad, 2005; Zienkiewicz *et al.*, 2014) as a mathematical approximation of continuum mechanics (Gonzalez & Stuart, 2008; Reddy, 2013). The mathematical approximation is so smart that a spatially three-dimensional problem of continuum mechanics is reduced to a zero or one-dimensional problem, and it is shown that the accuracy loss due to the mathematical approximation is minimum when a structure or a structural component has a suitable configuration. There is a correlation between the accuracy of numerical analysis and the fidelity of the model (Jayasinghe *et al.*, 2015a; 2015b; 2015c).

This paper applies meta-modeling theory to develop a one-dimensional simplified model from a large-scale three-dimensional solid element model, enabling efficient seismic structural response analysis at the initial stages. A MSM is chosen as a simplified model, and a spatially three-dimensional problem of the seismic structural response is reduced to a spatially zero-dimensional problem. A key issue is the clarification of the advantages and disadvantages of using a simplified model as an alternative to a solid element model. Due to the rigorousness of the meta-modelling theory based on which it is constructed, the simplified model can provide accurate overall displacement behaviour.

The present paper is organized as follows. First, the significance of adopting a simpler numerical model compared to the complex solid element models for efficient numerical analysis is explained. In Section 2, the meta-modelling theory is explained using an energy theory or a Lagrangian of an MSM with a single mass point. It is shown that this Lagrangian is derived from that of continuum mechanics by making suitable mathematical approximations. The methodology for the development of the consistent mass-spring model (CMSM) using meta-modelling theory is explained in Section 3 along with the development of the conventional MSM and frequency adaptive lumped mass-stick model (LMSM). Section 4 describes a numerical example to evaluate the seismic performance of the proposed CMSM compared to the conventional MSM and frequency adaptive LMSM. Symmetric and asymmetric (T-shaped and L-shaped) three-storey RC building structures are considered as the numerical examples to evaluate the seismic performance of the MSMs. The seismic performance is evaluated by comparing the natural frequency, peak acceleration, the storey drifts and floor displacements of CMSM with the conventional MSM and frequency adaptive LMSM. The major conclusion of the present study is made in Section 5.

MATERIALS AND METHODS

Meta-modelling theory

A single mass point MSM which consists of one mass and one spring is considered to explain the meta-modelling theory (Hori *et al.*, 2014), for a structure as shown in Figure 1.

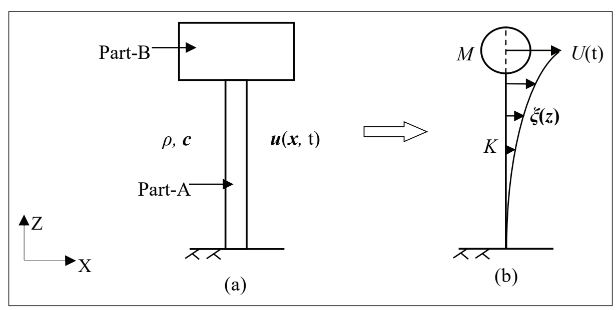

Figure 1: Highly mass concentrated structure

The structure is modeled as a linear MSM when the mass of part-*A* is negligible compared to that of part-*B* of the structure. A Lagrangian of this MSM is;

$$\mathcal{L}_s[U,V] = \mathcal{K}_s[V] - \mathcal{P}_s[U]. \qquad ...(1)$$

Here, U and V are displacement and velocity of the mass in a particular direction, and $\mathcal{K}_s$ and $\mathcal{P}_s$ are the kinematic and strain energy defined as;

$$\mathcal{K}_s[V] = \frac{1}{2}MV^2,$$

$$\mathcal{P}_s[U] = \frac{1}{2}KU^2, \qquad ...(2)$$

where M is lumped mass and K is the spring constant for the lateral stiffness, and $V = \dot{U}$.

Since this structure is made of a certain material (steel, Reinforced Concrete (RC), or wood), the structure can be modeled according to the continuum mechanics (Gonzalez & Stuart, 2008; Reddy, 2013). That is, we consider a three-dimensional displacement vector function, $\boldsymbol{u}$. Assuming linear elasticity, we have the following Lagrangian;

$$\mathcal{L}_{\mathrm{c}}[\boldsymbol{v}, \boldsymbol{\epsilon}] = \mathcal{K}_{\mathrm{c}}[\boldsymbol{v}] - \mathcal{P}_{\mathrm{c}}[\boldsymbol{\epsilon}], \qquad ...(3)$$

where $\boldsymbol{v} = \dot{\boldsymbol{u}}$ is velocity and $\boldsymbol{\epsilon} = sym\boldsymbol{\nabla u}$ is strain with dot and $\boldsymbol{\nabla}$ being the temporal and spatial derivatives, and with *sym* meaning the symmetric part, and $\mathcal{K}_{\mathrm{c}}$ and $\mathcal{P}_{\mathrm{c}}$ are;

$$\mathcal{K}_{\mathrm{c}}[\boldsymbol{v}] = \int_V \frac{1}{2}\rho \boldsymbol{v} \cdot \boldsymbol{v} \, \mathrm{dv},$$

$$\mathcal{P}_{\mathrm{c}}[\boldsymbol{\epsilon}] = \int_V \frac{1}{2}\boldsymbol{\epsilon}:\boldsymbol{c}:\boldsymbol{\epsilon} \, \mathrm{dv}, \qquad ...(4)$$

where ρ and $\boldsymbol{c}$ are density and elasticity, with $\cdot$ and $:$ standing for the inner product and second-order contraction, respectively. The form of $\mathcal{L}_{\mathrm{s}}$ and $\mathcal{L}_{\mathrm{c}}$ given by Equations (1) and (3) are similar to each other, even though the functions and functionals are given by Equations (2) and (4) are utterly different.

The meta-modeling theory takes advantage of the similarity in the form of $\mathcal{L}_{\mathrm{s}}$ and $\mathcal{L}_{\mathrm{c}}$, and derives $\mathcal{L}_{\mathrm{s}}$ from $\mathcal{L}_{\mathrm{c}}$. The procedure of deriving $\mathcal{L}_{\mathrm{s}}$ from $\mathcal{L}_{\mathrm{c}}$ is simple, as we have to approximate the three-dimensional vector-valued function, $\boldsymbol{u}$. Indeed, if we approximate it as the product of an unknown temporal function, $U(\mathrm{t})$, and a known three-dimensional vector-valued function, $\boldsymbol{\xi}(\boldsymbol{x})$, as;

$$\boldsymbol{u}(\boldsymbol{x}, t) = U(\mathrm{t})\boldsymbol{\xi}(\boldsymbol{x}), \qquad ...(5)$$

then, substituting Equation (5) into $\mathcal{L}_{\mathrm{c}}$, we obtain $\mathcal{L}_{\mathrm{s}}$. Here, the mass and spring constants, M and K, are explicitly computed in terms of ρ and $\boldsymbol{c}$ together with the assumed $\boldsymbol{\xi}$. That is,

$$M = \int \rho\, \boldsymbol{\xi}.\boldsymbol{\xi} \, \mathrm{dv},$$

$$K = \int \boldsymbol{\nabla\xi}:\boldsymbol{c}:\boldsymbol{\nabla\xi} \, \mathrm{dv}, \qquad ...(6)$$

The derivation of $\mathcal{L}_{\mathrm{s}}$ from $\mathcal{L}_{\mathrm{c}}$ is made mathematically, without making any physical assumptions. The Lagrangian problem of a continuum model with ρ and $\boldsymbol{c}$ ($\mathcal{L}_{\mathrm{c}}$) is reduced to another Lagrangian problem of an MSM of M and K ($\mathcal{L}_{\mathrm{s}}$), by using an approximated function of the displacement. Therefore, the solution of $\mathcal{L}_{\mathrm{s}}$ is regarded as an approximate solution of $\mathcal{L}_{\mathrm{c}}$.

The meta-modelling theory is able to derive a bar model, a beam model or a plate model from a continuum model in the same manner as shown in the above, *i.e.*, substituting a suitably approximated displacement function into $\mathcal{L}_{\mathrm{c}}$ (Hjelmstad, 2005; Hori *et al.*, 2014; Zienkiewicz *et al.*, 2014). The key concept of this theory is that all the modeling solves the same physical problem of continuum mechanics but uses their own distinct mathematical approximations. According to this theory, therefore, it is guaranteed that when a simplified model is constructed by making use of natural frequencies and mode shapes of the continuum modelling, it will share the same fundamental dynamic characteristics as the continuum mechanics' model.

Formulation of CMSM meta-modelling theory

We seek to construct a rigorous simplified model which shares the identical fundamental dynamic characteristics as those of the target solid element model; this simplified model is called Consistent Mass-Spring Model (CMSM).

This study is an extension of previous work done by Jayasinghe *et al.* (2015). The previous CMSM formulation is limited to only two mass points, but the current formulation is extended to *n* mass points, where *n* is a positive real integer. This improvement of CMSM will increase the accuracy of response. The previous formulation of CMSM is restructured by using a linear transformation matrix which transfers dynamic modes to approximate displacement functions to incorporate the effect of higher modes.

According to the meta-modelling theory, we consider an approximate displacement function of the following form;

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{\alpha=1}^{n} U^{\alpha}(t)\boldsymbol{\Phi}^{\alpha}(\boldsymbol{x}), \qquad ...(7)$$

where U^{α} is a certain direction displacement component of the α-th mass point and $\boldsymbol{\Phi}^{\alpha}$ is an approximate displacement mode. By definition, a component of $\boldsymbol{\Phi}^{\alpha}(\boldsymbol{x}^{\alpha})$ corresponding to U^{α} takes on a value of 1, with $\boldsymbol{x}^{\alpha}$ being the location of the α-th mass point, and $\boldsymbol{\Phi}^{\alpha}(\boldsymbol{x}^{\beta}) = 0$ for $\alpha \neq \beta$ which can be called requirement 1.

For simplicity, we substitute the approximate displacement ($\boldsymbol{u}$) of Equation (7) into $\mathcal{L}$ of Equation (3). Then, we obtain

$$\mathcal{L} = \sum_{\alpha,\beta=1}^{n} \frac{1}{2} m^{*\alpha\beta} \dot{U}^{\alpha} \dot{U}^{\beta} - \frac{1}{2} k^{*\alpha\beta} U^{\alpha} U^{\beta}, \quad ...(8)$$

where,

$$m^{*\alpha\beta} = \int_V \rho \boldsymbol{\Phi}^{\alpha} \boldsymbol{\Phi}^{\beta} \, \mathrm{d}v \text{ and } k^{*\alpha\beta} = \int_V \boldsymbol{\nabla\Phi}^{\alpha} : \boldsymbol{c} : \boldsymbol{\nabla\Phi}^{\beta} \, \mathrm{d}v. \quad ...(9)$$

The right side of Equation (8) is rewritten as

$$\sum_{\alpha=1}^{n} \frac{1}{2} M^{\alpha} \left(\dot{U}^{\alpha}\right)^2 + \sum_{\alpha,\beta=1\,(\alpha\neq\beta)}^{n} M^{\alpha\beta} \dot{U}^{\alpha} \dot{U}^{\beta} - \sum_{\alpha,\beta=1\,(\alpha\neq\beta)}^{n} \frac{1}{2} K^{\alpha\beta} \left(U^{\alpha} - U^{\beta}\right)^2 - \sum_{\alpha=1}^{n} \frac{1}{2} K^{\alpha} (U^{\alpha})^2,$$

where

$M^{\alpha} = m^{*\alpha\alpha}$, $M^{\alpha\beta} = m^{*\alpha\beta}$, $K^{\alpha\beta} = -k^{*\alpha\beta}$ and

$$K^{\alpha} = \sum_{\beta=1}^{n} k^{*\alpha\beta}.$$

According to Equation (8), it is straightforward to understand the necessity of a third spring for the MSM which consists of two mass points; see Figure 2 for a two point MSM. If we can choose a $\boldsymbol{\Phi}^{\alpha}$ so that $M^{\alpha\beta}$ and K^{α} vanish (requirement 2), then the above $\mathcal{L}$ of Equation (8) becomes $\mathcal{L}$ of the typical MSM which includes an equal number of linear springs and mass points.

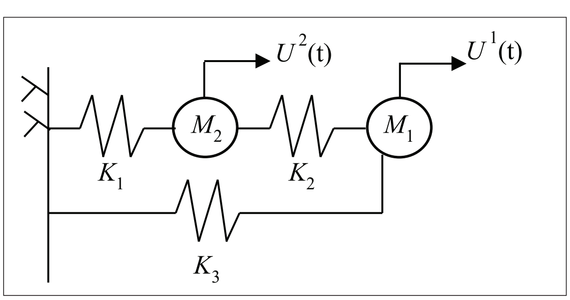

Figure 2: Schematic view of a CMSM consisting of two mass points

Introduction of dynamic modes with linear transformation matrix

Dynamic mode shapes of a target problem need to be incorporated in the construction of a MSM, so that it shares the same dynamic fundamental characteristics with a continuum model. We suppose that an *n* number of dynamic modes $\{\boldsymbol{\Psi}^{\alpha}, \omega^{\alpha}\}$ $(\alpha = 1,2,3,\ldots,n)$ are given; $\boldsymbol{\Psi}^{\alpha}$ and ω^{α} are mode shape and natural frequency of the α-th mode respectively. By definition, $\boldsymbol{\Psi}^{\alpha}$ and ω^{α} satisfy;

$$\int_V \rho \boldsymbol{\Psi}^{\alpha} \cdot \boldsymbol{\Psi}^{\beta} \, \mathrm{d}v = 0, \quad \int_V \boldsymbol{\nabla\Psi}^{\alpha} : \boldsymbol{c} : \boldsymbol{\nabla\Psi}^{\beta} \, \mathrm{d}v = 0, \quad ...(10)$$

and,

$$\rho(\omega^{\alpha})^2 \boldsymbol{\Psi}^{\alpha} + \boldsymbol{\nabla} \cdot (\boldsymbol{c} : \nabla \boldsymbol{\Psi}^{\alpha}) = 0, \quad ...(11)$$

for $\alpha \neq \beta$.

Employing $\mathcal{L}$ of Equation (3) and substituting the approximate displacement function ($\boldsymbol{u} = \sum U^{\alpha} \boldsymbol{\Psi}^{\alpha}$) into it, we can obtain,

$$\mathcal{L} = \sum_{\alpha,\beta=1}^{2} \frac{1}{2} m^{\alpha} (\dot{u}^{\alpha})^2 - \frac{1}{2} k^{\alpha} (u^{\alpha})^2, \quad ...(12)$$

where,

$$m^{\alpha} = \int_V \rho \boldsymbol{\Psi}^{\alpha} \boldsymbol{\Psi}^{\alpha} \, \mathrm{d}v \text{ and } k^{\alpha} = \int_V \boldsymbol{\nabla\Psi}^{\alpha} : \boldsymbol{c} : \boldsymbol{\nabla\Psi}^{\alpha} \, \mathrm{d}v. \quad ...(13)$$

As a result of the orthogonality property of dynamic mode shapes, Equation (10), dynamic modes do not produce coupling terms in Equation (12). Additionally, due to Equation (11), it is readily seen that m^{α} and k^{α} of Equation (13) satisfy;

$$(\omega^{\alpha})^2 m^{\alpha} = k^{\alpha}, \quad ...(14)$$

for $\alpha = 1, 2, 3, \ldots, n$. Note that, if $\boldsymbol{\Psi}^{\alpha}$ is replaced by $\boldsymbol{\Psi}'^{\alpha} = a\boldsymbol{\Psi}^{\alpha}$, we have

$$m'^{\alpha} = c^2 m^{\alpha} \text{ and } k'^{\alpha} = c^2 k^{\alpha}, \quad ...(15)$$

which still satisfies

$$(\omega^\alpha)^2 m'^\alpha = k'^\alpha. \quad ...(16)$$

Furthermore, note that;

$$\boldsymbol{u}(\boldsymbol{x},t) = \sum_{\alpha=1}^{n} u'^\alpha(t)\boldsymbol{\Psi}'^\alpha(\boldsymbol{x}) = \sum_{\alpha=1}^{n} u^\alpha(t)\boldsymbol{\Psi}^\alpha(\boldsymbol{x}). \quad ...(17)$$

Pure dynamic modes can satisfy neither requirement 1 nor requirement 2 which are mentioned above. In general, it is not possible to find $\{\boldsymbol{\Phi}^\alpha\}$ which satisfies both requirements; thus, we need to find a suitable linear combination of $\{\boldsymbol{\Psi}^\alpha\}$ that satisfies, at least, requirement 1. To this end, we consider the following combination.

$$U^\alpha = \sum_{\beta=1}^{n} t^{\alpha\beta} u^\alpha \text{ and } \boldsymbol{\Phi}^\alpha = \sum_{\beta=1}^{n} \left(t^{\alpha\beta}\right)^{-1} \boldsymbol{\Psi}^\alpha, \quad ...(18)$$

where $\left[t^{\alpha\beta}\right]$ is a linear transformation matrix with $\left[t^{\alpha\beta}\right]^{-1}$ being the inverse of $\left[t^{\alpha\beta}\right]$.

Then, Equation (9) is rewritten as.

$$m^{*\alpha\beta} = \sum_{\gamma,\beta,\alpha=1}^{n} m^\gamma (t^{\alpha\gamma})^{-1} \left(t^{\beta\gamma}\right)^{-1} \text{ and}$$

$$k^{*\alpha\beta} = \sum_{\gamma,\beta,\alpha=1}^{n} k^\gamma (t^{\alpha\gamma})^{-1} \left(t^{\beta\gamma}\right)^{-1} \quad ...(19)$$

To determine the components of $\left[t^{\alpha\beta}\right]$, we introduce a vector $[\Xi^\alpha]$, which consists of zeros and ones, zero for mass points not on $z = z^\alpha$ and one for the mass point on $z = z^\alpha$. Here, z^α is the location of α-th mass point along the axial direction of a target problem, and then we can compute,

$$(t^{\alpha\gamma})^{-1} = [\Xi^\alpha][\boldsymbol{\Psi}^\gamma(z^\alpha)]^{-1} \quad ...(20)$$

Here, $[\boldsymbol{\Psi}^\gamma(z^\alpha)]$ is a regular square matrix with $[\boldsymbol{\Psi}^\gamma(z^\alpha)]^{-1}$ being the inverse of $[\boldsymbol{\Psi}^\gamma(z^\alpha)]$. $\boldsymbol{\Psi}^\gamma(z^\alpha)$ is a displacement component along a fixed direction for the MSM of the γ-th dynamic mode at the α-th mass point location. It is clear that coupling mass and stiffness terms of mass and stiffness matrices should be there to assure the consistency of the MSM.

Application of CMSM for building structures and comparative study

Problem setting for building structures

A symmetric and asymmetric three-storey RC building (Figure 3 and Figure 5) is considered in the present study to develop the Mass-spring models described in Section 3. The selected three-storey RC building consists of square columns having cross-section dimensions of 0.35 m x 0.35 m and a uniform slab thickness of 0.15 m. The elastic modulus, Poisson's ratio, and density of the concrete are 24 GPa, 0.2, and 2000 kg m^{-3}, respectively. Three different earthquake ground motions are selected to perform the time history analysis. Figure 4 shows the acceleration time histories of the selected earthquake ground motions. All the ground motions are applied in the X-direction of the symmetric and asymmetric three-storey RC buildings in the time history analysis.

Construction of MSMs

First, 3D finite element models of the symmetric and asymmetric three-storey RC buildings are developed using FE software. Modal analysis is performed to

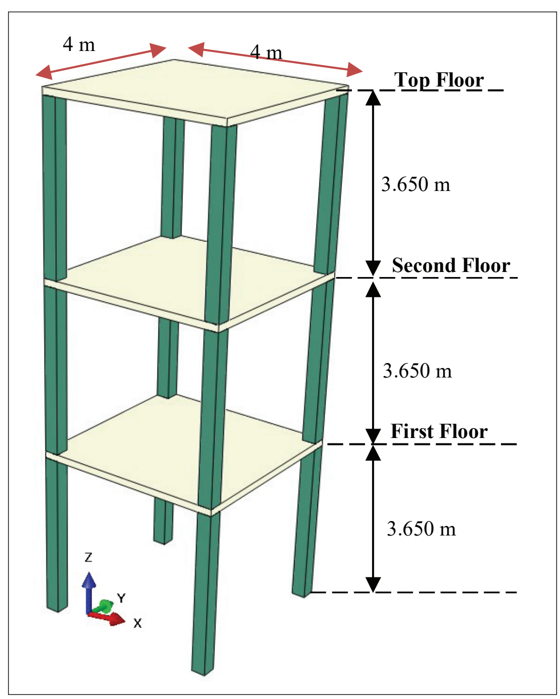

Figure 3: Selected symmetric building

obtain the necessary undamped natural frequencies and the mode shapes of the symmetric and asymmetric three-storey RC buildings. Figure 6, depict the mode shapes of the symmetric three-storey RC buildings. The obtained mode shapes are then used to develop the low-fidelity MSMs considered in this study. Time history analysis is performed to determine the displacement and acceleration response of the developed low-fidelity MSMs. The spring stiffness is determined using the static pushover analysis of the 3D finite element (FE) model.

The mass and stiffness matrices of the CMSM, as well as the mass matrix of the frequency adaptive LMSM, are then determined using *Mathematica* code. The tributary area consideration method is used to compute the mass matrix of the conventional MSM. Then, *Mathematica* coding is developed based on the Newmarks' method to determine the displacement and acceleration of the low-fidelity MSMs. Finally, the drift ratio and peak acceleration of the MSMs are compared with the 3D solid model to investigate the utilization of the low-fidelity models of the structures, compared to the 3D solid models.

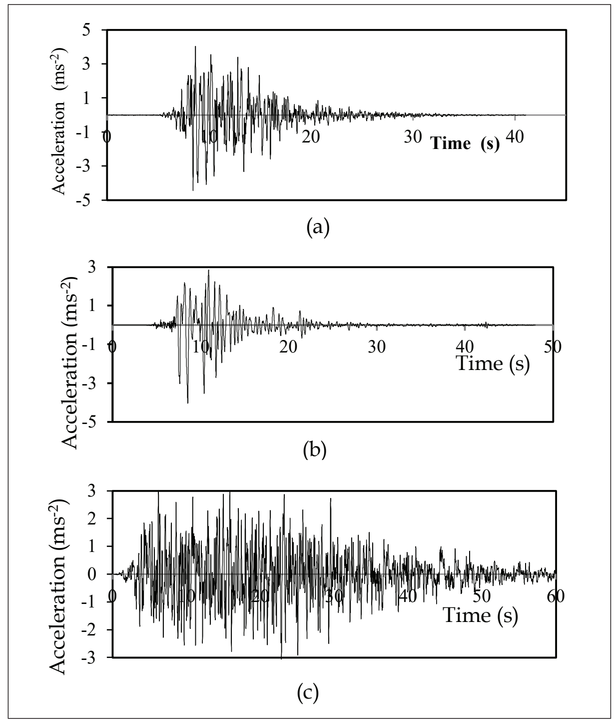

Figure 4: Selected Ground Motions (a) 1st (b) 2nd (c) 3rd

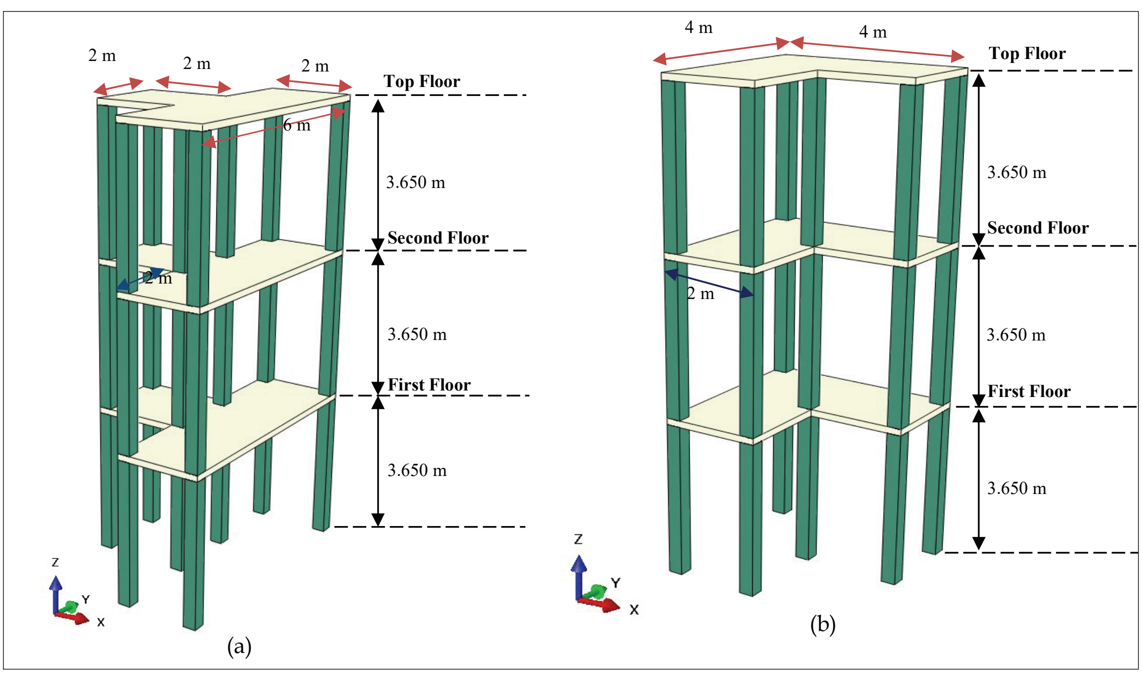

Figure 5: Selected asymmetrical buildings: (a) T-Shape (b) L-Shape

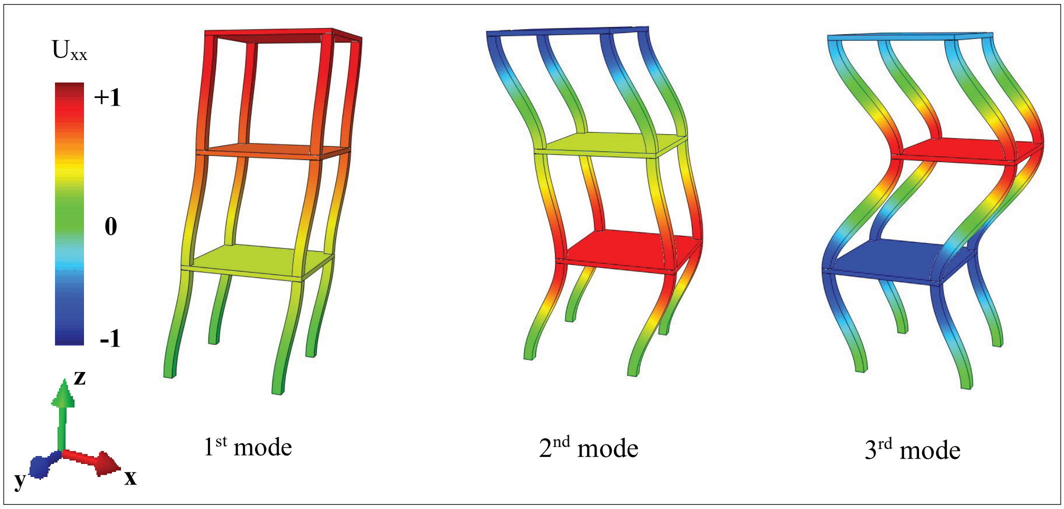

Figure 6: Mode shapes in x-direction of the symmetric three-storey RC building

RESULTS AND DISCUSSION

Comparison of the natural frequency of MSMs

The natural frequency, drift ratio, and peak acceleration of the mass point in the low-fidelity MSMs are compared with those of the solid model to evaluate the significance and potential advantages of using low-fidelity models over solid models. First, to evaluate the consistency of the developed MSMs for symmetric and asymmetric three-storey RC buildings, their natural frequencies are compared with those of the solid model (Table 1).

The natural frequencies of the conventional MSM and the frequency adaptive LMSM show a marginal error compared to the natural frequencies of the solid element model. Moreover, the frequency-adaptive LMSM outperforms the conventional MSM in terms of natural frequencies. However, the first three natural frequencies of the CMSM coincide with the corresponding natural frequencies of the solid element model for symmetric and asymmetric RC buildings considered in the study. These findings confirm the consistency of the developed CMSM with the solid model.

Table 1: Natural frequencies comparison in x-direction between the CMSM and other MSMs

Building type		Symmetric building			Asymmetric building L-shaped			Asymmetric building T - shaped		
Mode Number (x-direction)		1	2	3	1	2	3	1	2	3
	Solid model / (Hz)	2.33	6.60	9.80	3.03	8.71	13.02	3.02	8.69	13.20
Conventional MSM	Frequency (Hz)	2.39	6.66	9.53	3.20	8.87	12.60	3.16	8.75	12.43
	Error (%)	2.58	0.91	2.76	5.61	1.84	3.23	4.64	0.69	5.83
Frequency adaptive LMSM	Frequency (Hz)	2.36	6.62	9.61	3.11	8.72	12.67	3.11	8.73	12.72
	Error (%)	1.29	0.30	1.94	2.64	0.11	2.69	2.98	0.46	3.64
CMSM	Frequency (Hz)	2.33	6.60	9.80	3.03	8.71	13.02	3.02	8.69	13.20
	Error (%)	0	0	0	0	0	0	0	0	0

Comparison of the peak acceleration and storey drifts

The damages to the building structures can be estimated based on the peak displacement, peak velocity, peak acceleration response, and maximum drift ratio under seismic loading. The present study considered the peak acceleration response and drift ratio of the symmetric and asymmetric three-storey RC buildings to estimate the damage under seismic loading. Figure 7 shows the response of the symmetrical three-storey RC building from the time history analysis using the second and third earthquake ground motions. It can be identified that the conventional MSM has more or less the same response as the frequency-adaptive LMSM with regard to peak acceleration and drift ratio. Moreover, the conventional MSM and the frequency-adaptive LMSM depict a significant variation between their seismic responses compared to the solid element model. Therefore, adopting the conventional MSMs and the frequency-adaptive LMSM to determine the seismic response of the solid element model yields less accurate results. However, the peak acceleration and the drift ratio response of the CMSM model due to the seismic loading have coincided with the solid element model. Therefore, the developed CMSM for the symmetric three-storey RC building can be used to determine the seismic response of the symmetrical three-storey RC building instead of adopting the solid model.

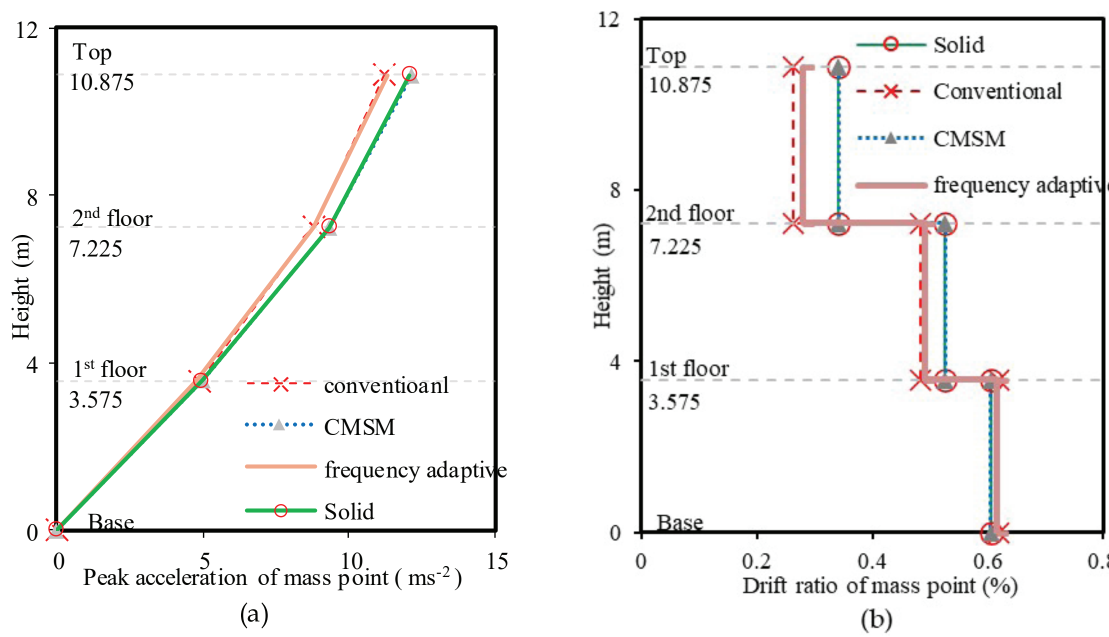

Figure 7: Response of symmetrical building: (a) peak acceleration for second ground motion and (b) drift ratio for third ground motion

Figure 8 and Figure 9 show the peak acceleration and drift ratio responses of the L-shape and T-shape three-storey RC buildings for the 2nd and 3rd earthquake ground motions.

According to Figures 8 and 9, the peak acceleration and drift ratio responses of the conventional MSM and the frequency adaptive LMSM differ significantly from the peak acceleration and drift ratio responses of the solid element model for both L-shaped and T-shaped asymmetrical buildings. The frequency-adaptive LMSM produces more accurate results compared to the conventional MSM. However, the peak acceleration and drift ratio response of the CMSM almost coincides with the peak acceleration and drift ratio of the solid model. Therefore, the developed CMSM can be used to determine the seismic response of the L-shaped and T-shaped asymmetrical buildings instead of the solid element model.

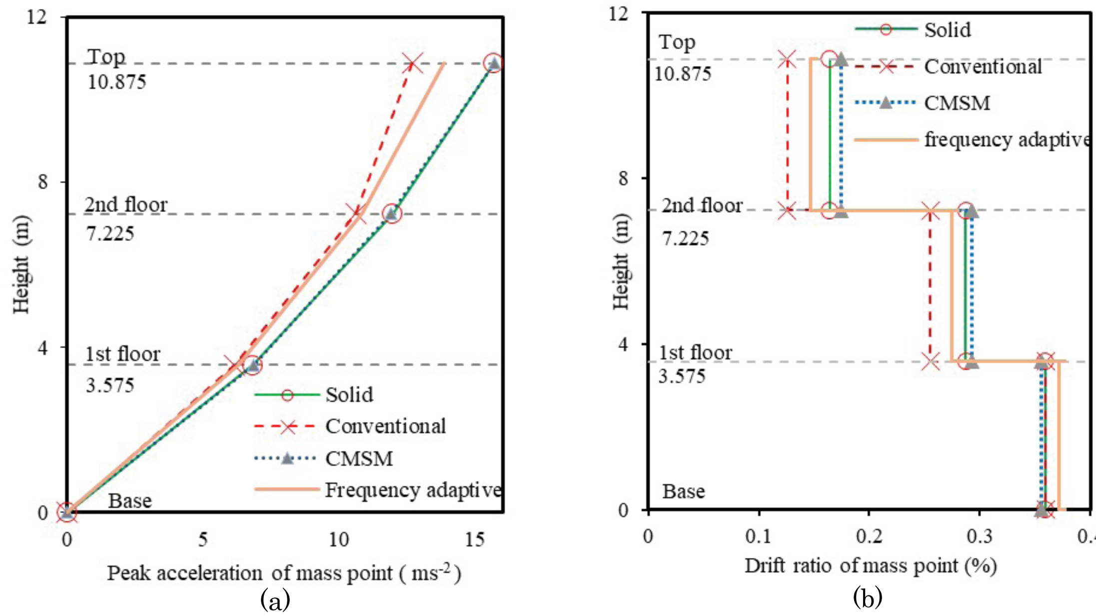

Figure 8: Response of L-shaped asymmetrical building: (a) peak acceleration for second ground motion and (b) drift ratio for third ground motion

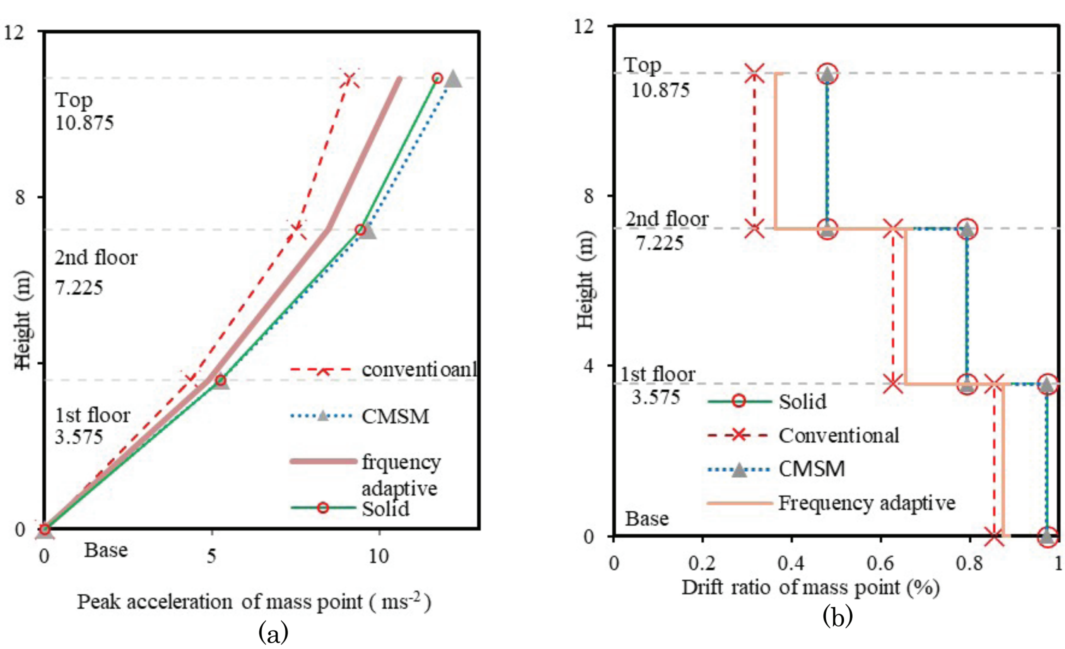

Figure 9: Response of T-shaped asymmetrical building: (a) peak acceleration for second ground motion and (b) drift ratio for third ground motion

As per the significant comparison, it is evident that conventional MSMs and frequency-adaptive LMSM yield less accurate results compared to the solid element model. However, the developed CMSM in the present study produces almost the same results as the solid element model for both symmetrical and asymmetrical three-storey RC buildings. Therefore, the developed CMSM can be used to evaluate the seismic response of both symmetric and asymmetric three-storey RC buildings considered in the present study instead of using the solid element model. Table 2 illustrates the percentage of error in the peak acceleration response and drift ratio from the developed CMSMs for each floor of the symmetric and asymmetric three-storey RC buildings considered in the study.

Table 2: Percentage of error in peak acceleration and maximum drift ratio of CMSM with the solid model

Symmetric building						
Ground motions	1st		2nd		3rd	
Peak Values	Acc. / ms^{-2}	Drift ratio (%)	Acc. / ms^{-2}	Drift ratio (%)	Acc. / ms^{-2}	Drift ratio (%)
3rd floor/ (10.95m)	0.11	0.20	0.21	0.07	0.48	0.20
2nd floor/ (7.30m)	0.37	0.01	0.13	0.03	0.03	0.10
1st floor/ (3.65m)	0.21	0.07	0.14	0.05	0.19	0.03
L-shaped building						
Ground motions	1st		2nd		3rd	
Peak Values	Acc. / ms^{-2}	Drift ratio (%)	Acc. / ms^{-2}	Drift ratio (%)	Acc. / ms^{-2}	Drift ratio (%)
3rd floor/ (10.95m)	2.35	8.87	3.85	8.95	0.76	6.12
2nd floor/ (7.30m)	1.54	3.75	2.36	4.67	1.10	1.89
1st floor/ (3.65m)	1.82	1.02	0.15	1.61	0.40	1.03
T-shaped building						
Ground motions	1st		2nd		3rd	
Peak Values	Acc. / ms^{-2}	Drift ratio (%)	Acc. / ms^{-2}	Drift ratio (%)	Acc. / ms^{-2}	Drift ratio (%)
3rd floor/ (10.95m)	0.24	0.10	0.13	0.23	0.29	0.06
2nd floor/ (7.30m)	0.38	0.03	0.12	0.14	0.01	0.11
1st floor/ (3.65m)	0.48	0.11	0.07	0.03	0.87	0.00

When compared to the symmetric building and T-shaped asymmetrical building, the L-shaped asymmetrical building has the largest error percentages for both peak acceleration and maximum drift ratio across all levels. This is due to the torsional irregularity of the L-shaped building which caused the torsional mode which should be considered in the development of the CMSM. However, T-shaped buildings are not subject to any torsional response due to the application of the earthquake ground motion through the centre of the stiffness. Therefore, it is evident that the developed CMSM can be used to determine the seismic response of the symmetric and asymmetric three-storey RC buildings considered in the study instead of using the solid model.

CONCLUSION

In this paper, the authors propose a systematic method for the rigorous development of a simplified 1D model based on the solid element model. This model is entitled CMSM. The consistency of CMSM is assured by implementing the meta-modelling theory in the construction process of CMSM. According to the meta-modelling theory, additional springs are incorporated in CMSM to guarantee consistency with that of the solid element model. The proposed CMSM can incorporate any number of dynamic mode shapes of a target structure to improve the accuracy of the response.

The performance of the proposed MSM is compared with the available MSMs, such as the conventional MSM and the frequency-adaptive LMSM. The target building types are three-storey symmetrical and L-shaped and T- shaped asymmetrical RC building structures. The seismic response is evaluated by comparing the peak displacement, the peak acceleration, and the maximum drift ratio of each floor level. The study found that the first three natural frequencies of the CMSM align closely with those of the solid element model for both symmetric and asymmetric RC buildings. A comparative analysis further demonstrates that the CMSM shows excellent agreement with the solid element model for these structures. Additionally, the peak acceleration and drift ratio responses of the CMSM show good agreement with those of the solid element model. Therefore, it can be concluded that the proposed CMSM can solve the same physical problem of the solid element model using suitable mathematical approximations. However, an L-shaped asymmetric RC building has a small deviated response compared to the solid element model. This is due to the neglect of the torsional modes of vibrations in the construction of all MSMs. The T-shaped asymmetrical building provides a better response to the application of the ground motions along the symmetric axis of the building in all MSMs.

The proposed CMSM needs to be extended to non-linear analysis, which will result in more benefits by saving computational resources at the initial stage of design. At least, it is straightforward to apply the meta-modelling to obtain an incremental response of a non-linear elastoplastic structure. Furthermore, by incorporating the torsional mode into the MSM formulation process, the selected MSMs can be improved for seismic response analysis of asymmetric building structures.

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