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RESEARCH ARTICLE

Statistical Quality Control

Error detection through modified phase II process monitoring under different classical estimators

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Abstract: In real life, the distribution of the errors during any life testing of products or process does not meet the assumption of normality. Statistical process control (SPC) is defined as the use of statistical techniques to control a process or production method. SPC tools and procedures can help to monitor process behaviour, discover problems in internal systems, and find solutions for production issues. To identify and remove the variation in different reliability processes and to monitor the reliability of machines where the number of errors follows skewed distributions, we develop control charts to keep the process in control. For such situations, we have modified the existing control charts such as Shewhart control chart, exponentially weighted moving average (EWMA), hybrid exponentially weighted moving average (HEWMA) and extended exponentially weighted moving average (EEWMA) control charts. The current study introduced classical estimator based modified control charts for phase-II monitoring by assuming that the errors occur during the process follow skewed distribution called Beta Lehmann 2 Power function distribution (BL2PFD). The proposal for these control charts is based on the percentile estimator. We have compared all these control charts using Monte Carlo simulation studies and real-life applications to compare the proposed control charts. This study shows that an EEWMA control chart based on PE performs better than Shewhart, EWMA and HEWMA control charts, when the underlying distribution of the errors in process monitoring follows BL2PFD. These findings can be useful for researchers and practitioners in dealing with production errors and optimizing the output.

Keywords: Beta Lehmann 2 Power function distribution, exponentially weighted moving average, extended exponentially weighted moving averages, extended hybrid exponentially weighted moving averages, hybrid exponentially weighted moving average, percentile estimator.

INTRODUCTION

The Beta Lehmann 2 Power function distribution was generated by Zaka *et al*. (2020) is used in many real-life applications such as reliability and medical studies. It is a skewed distribution with the following probability density function.

$$f(x) = \frac{\left(1-\left(1-\left(\frac{x}{\beta}\right)^{\gamma}\right)^{\alpha}\right)^{a-1}\left(\left(1-\left(\frac{x}{\beta}\right)^{\gamma}\right)^{\alpha}\right)^{b-1}\alpha\left(1-\left(\frac{x}{\beta}\right)^{\gamma}\right)^{\alpha-1}\frac{\gamma x^{\gamma-1}}{\beta^{\gamma}}}{B(a,b)} \;,\quad 0 < x < \beta\,.$$

where γ, α, a and b are the shape, scale and tuning parameters.

The shape of the pdf of BL2PFD is approximately Normal for different values of the parameters.

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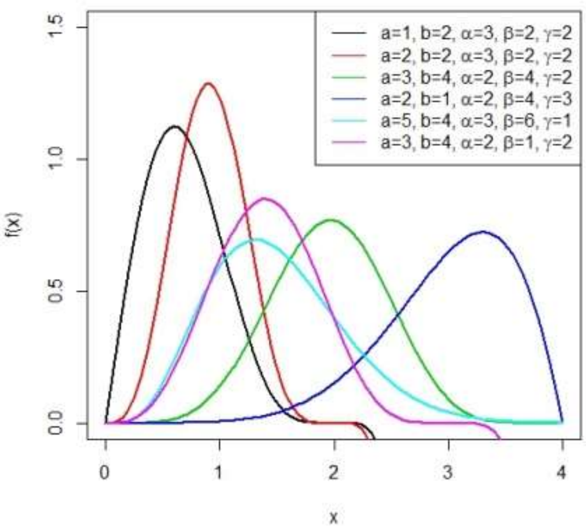

Figure 1: Shape of pdf for BL2PFD

The above plots show that the Beta Lehmann 2 power function distribution is sometimes negatively skewed. To control the variation in process for which the underlying distribution of process follows Beta Lehmann 2 power function distribution is discussed in this paper. We have proposed some modifications to the existing Shewhart, EWMA, HEWMA and EEWMA and compared the performance.

Prior studies related to the control charts are mostly followed the normality assumption. The readers can study Shamma and Shamma (1992), Haq (2013) and Naveed *et al*. (2018). But what can organizations do if the normality assumption fails in the practical life. There are very few studies available in the literature that highlighted this issue, including Qiu and Li (2011), Noorossana *et al*., (2016), Zhang *et al*. (2017), Lin *et al*., (2017), Erto *et al*. (2018), Li *et al*. (2018), Liang *et al*., (2019). Further, Zaka *et al*. (2021a; 2021b), Jabeen *et al*. (2022, 2023) proposed some modified control charts for monitoring the shape parameter of Power function distribution (PFD), Reflected Power function distribution (RPFD) and Weighted Power function distribution (WPFD). As the many production processes are not necessarily follow the normality assumption, therefore, the aim of the study is to introduce the modified control charts, i.e. Shewhart, EWMA, HEWMA and EEWMA and assume that the distribution of underlying process follows a BL2PFD. Further, the proposed control charts are based on the different estimators of the shape parameter of BL2PFD.

This paper is about the use of the shape parameter to control the shape of the process when the distribution of process is BL2PFD. We use Percentile estimator (PE) discussed by Zaka *et al*. (2020) to propose the monitoring statistics under BL2PFD. We propose Monte Carlo simulation to estimate the shape parameter of the BL2PFD and the plotting statistics. It is observed that the proposed EEWMA control chart of the PE detects smaller shifts quicker than the HEWMA control charts.

MATERIALS AND METHODS

The BL2PFD introduced by Zaka *et al*. (2020) is used extensively to study the behavior of different phenomena in real life situations such as to test the life time of the subject of interest in medical and engineering sciences. The probability density function and cumulative density function are given as

$$f(x)=\frac{\left(1-\left(1-\left(\frac{x}{\beta}\right)^{\gamma}\right)^{\alpha}\right)^{a-1}\left(\left(1-\left(\frac{x}{\beta}\right)^{\gamma}\right)^{\alpha}\right)^{b-1}\alpha\left(1-\left(\frac{x}{\beta}\right)^{\gamma}\right)^{\alpha-1}\frac{\gamma x^{\gamma-1}}{\beta^{\gamma}}}{B(a,b)}\ ,\quad 0<x<\beta\,. \qquad \ldots(1)$$

and

$$F(x) = {B_{\left\{1-\left\{1-\left(\frac{x}{\beta}\right)^{\gamma}\right\}^{\alpha}\right\}}(a,b)} \Big/ {B(a,b)} \quad \ldots(2),$$

respectively.

Zaka *et al*. (2020) further illustrated the estimates of the shape and scale parameter. In the current study, we will use the shape parameter to provide its application in quality and reliability engineering. The estimate for shape parameter using method of common percentiles is given as

$$\hat{\gamma}_{PE} = \frac{\ln\left(\frac{1-(1-H)^{1/\alpha}}{1-(1-L)^{1/\alpha}}\right)}{\ln\left(\frac{P_H}{P_L}\right)} \quad \ldots(3),$$

where H= maximum percentile, L= minimum percentile and P = Percentile point.

Monitoring for the process following Beta Lehmann-2 Power function distribution.

We will discuss the monitoring for the shape parameter of the Beta Lehmann-2 Power function distribution using the following methods.

Modification of Shewhart control chart for shape parameter of Beta Lehmann-2 Power function distribution

In the following section, we have constructed a modified Shewhart control chart using PE for the shape parameter of a process following Zaka *et al.* (2021a; 2021b). The control limits are given as

$$LCL_{\hat{\gamma}_{PE}} = E(\hat{\gamma}_{PE}) - K * \sqrt{var(\hat{\gamma}_{PE})} \quad \ldots(4)$$

$$CL_{\hat{\gamma}_{PE}} = E(\hat{\gamma}_{PE}) \quad \ldots(5)$$

$$UCL_{\hat{\gamma}_{PE}} = E(\hat{\gamma}_{PE}) - K * \sqrt{var(\hat{\gamma}_{PE})}. \quad \ldots(6)$$

Modified EWMA control chart to monitor the shape parameter of BL2PFD

Following Zaka *et al*. (2021a; 2021b), the control limits of the modified EWMA are given as,

$$LCL_{EWPE_t} = E(\hat{\gamma}_{PE}) - K * \sqrt{var(\hat{\gamma}_{PE}) * \frac{\lambda}{(2-\lambda)}} \quad \ldots(7)$$

$$CL_{EWPE_t} = E(\hat{\gamma}_{PE}) \quad \ldots(8)$$

$$UCL_{EWPE_t} = E(\hat{\gamma}_{PE}) + K * \sqrt{var(\hat{\gamma}_{PE}) * \frac{\lambda}{(2-\lambda)}} \quad \ldots(9)$$

Modified HEWMA control chart to monitor the shape parameter of BL2PFD.

Following Zaka *et al*. (2021), Jabeen *et al*. (2021) and using Zaka *et al*. (2020), we get the following control limits of modified HEWMA.

$$LCL_{HEWPE_t} = E(\hat{\gamma}_{PE}) - K * \frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)}\sqrt{\left(\sum_{i=1}^{2}\frac{(1-\lambda_i)^2(1-(1-\lambda_i)^{2t})}{1-(1-\lambda_i)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)\{1-(1-\lambda_1)^t(1-\lambda_2)^t\}}{1-(1-\lambda_1)(1-\lambda_2)}\right)var(\hat{\gamma}_{PE})} \quad \ldots(10)$$

$$CL_{HEWPE_t} = E(\hat{\gamma}_{PE}) \quad \ldots(11)$$

$$UCL_{HEWPE_t} = E(\hat{\gamma}_{PE}) + K * \frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)}\sqrt{\left(\sum_{i=1}^{2}\frac{(1-\lambda_i)^2(1-(1-\lambda_i)^{2t})}{1-(1-\lambda_i)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)\{1-(1-\lambda_1)^t(1-\lambda_2)^t\}}{1-(1-\lambda_1)(1-\lambda_2)}\right)\mathrm{var}(\hat{\gamma}_{PE})} \quad \ldots(12)$$

Modified extended EWMA control charts for monitoring the shape parameter of BL2PFD

The EEWMA statistic using PE of the shape parameter of the RPFD was given by Zaka *et al*. (2021a; 2021b), we follow Zaka *et al*. (2021a; 2021b) and using Zaka *et al*. (2020), the control limits are given as

$$UCL_{EEWPE_t} = E(\hat{\gamma}_{PE}) + K\sqrt{V_{PE}\left\{({\lambda_1}^2+{\lambda_2}^2)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\}} \quad \ldots(13)$$

$$CL_{EEWPE_t} = E(\hat{\gamma}_{PE}) \quad \ldots(14)$$

$$LCL_{EEWPE_t} = E(\hat{\gamma}_{PE}) - K\sqrt{V_{PE}\left\{({\lambda_1}^2+{\lambda_2}^2)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\}} \quad \ldots(15)$$

RESULTS AND DISCUSSION

The following steps for simulation study assuming BL2 PFD as process distribution is executed.

Step 1: We generate a random sample of size n=100 from BL2PFD taking the $(\beta, \gamma) = (1.5, 2.5)$ and calculate the PE for the shape parameter from BL2PFD.

Step 2: We take 5000 repetitions of step 1 and compute $E(\hat{\gamma}_{PE})$ and $V(\hat{\gamma}_{PE})$. Compute the control limits to construct the proposed EWMA, HEWMA and EEWMA control limits using the statistics mention in equations (4) to (15).

Step 3: Compute ARL value for in control process and fix it at 500 by searching the suitable value of K.

Step 4: Now we take the shift of 0.05 in process parameter (γ). We calculate ARL and plot them against the shifted process. We observe the process and if it is in-control then we go to step 1. Otherwise, we record the Run Length and repeat the same process for 5000 times.

Table 1: Shewhart control chart based on PE for BL2PFD

Estimation method		Shift 0	0.01	0.03	0.120	0.180	0.60	1.00	1.5
PE	ARL	500.112	419.257	315.537	120.781	67.769	4.435	1.531	1.244
	SDRL	494.452	400.9711	323.4627	123.102	62.744	3.888	0.9294	0.210
	P10	51.0	44.9	33.9	14.9	7.90	1	1	1
	P25	145	111.0	94.0	39.0	20	2	1	1
	P50	348	262.5	216.0	81.5	52	3	1	1
	P75	699	535.5	422.5	161.0	97.25	6	2	1
	P90	1179.5	919.4	752.0	277.1	151.2	10	3	1

Table 2: EWMA control chart based on PE for BL2PFD

Estimation method		Shift 0	0.01	0.03	0.120	0.180	0.60	1.00	1.5
PE L=4.20 $\lambda = 0.10$	ARL	500.748	290.138	183.057	60.016	30.803	2.036	1.03	1
	SDRL	478.0	336.079	170.97	21.079	9.248	1.223	0.662	0.3776
	P10	64.0	48.00	32.00	12	2	1	1	1
	P25	162.0	110.00	62.75	37	11	2	1	1
	P50	368.0	264.00	165.00	46	28	2.3	1	1
	P75	677.0	493.25	247.00	64	32	4	1	1
	P90	1107.1	786.30	410.6	90	39	5	2	1

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	ARL	500.137	330.241	203.07	80.6	50.03	2.12	1.07	1
	SDRL	492.2264	422.12	328.01	85.615	58.602	2.015	0.674	0.269
	P10	54.90	51.00	34.90	10.0	8.0	1	1	1
PE	P25	141.00	118.75	92.75	25	19	2	1	1
L=4.20	P50	341.00	302.5	225.00	61	46	3	1.03	1
$\lambda = 0.20$	P75	726.75	623	427.25	118	86	4	2	1
	P90	1211.20	1019.5	736.10	195.1	135.1	6	2	1
	ARL	500.663	370.1	198.07	92.01	60.03	2.32	1.13	1
	SDRL	484.7689	432.3127	319.7975	104.374	51.22	2.71	0.753	0.248
PE	P10	55.90	51.9	38.0	11.9	6.00	1	1	1
L=4.15	P25	141.75	127.0	95.0	30	17	2	1	1
$\lambda = 0.30$	P50	341.50	312.5	231.0	75	42	3	1	1
	P75	707.50	627.0	459.0	147	76.25	5	2	1
	P90	1184.10	1023.4	752.2	235	115.2	7	2	1
	ARL	500.112	392.257	250.537	105.781	63.769	3	1.2	1
	SDRL	494.452	400.9711	323.4627	123.102	62.744	3.888	0.9294	0.210
PE	P10	51.0	44.9	33.9	14.9	7.90	1	1	1
L=4.175	P25	145	111.0	94.0	39.0	20	2	1	1
$\lambda = 0.75$	P50	348	262.5	216.0	81.5	52	3	1.1	1
	P75	699	535.5	422.5	161.0	97.25	6	2	1
	P90	1179.5	919.4	752.0	277.1	151.2	10	3	1

Table 3: HEWMA control chart based on PE for BL2PFD

Estimation Methods		Shift							
		0	0.01	0.03	0.120	0.180	0.60	1.00	1.5
PE	ARL	500.748	205.13	123.07	20.01	5.803	1.03	1	1
L=4.065	SDRL	478.0	336.079	170.97	21.079	9.248	1.223	0.662	0.3776
$\lambda_1 = 0.20$,	P10	64.0	48.00	32.00	12	2	1	1	1
$\lambda_2 = 0.10$	P25	162.0	110.00	62.75	17	3	1	1	1
	P50	368.0	254.00	125.00	26	6	1.01	1	1
	P75	677.0	493.25	247.00	41	22	3	1	1
	P90	1107.1	786.30	410.6	59	31	5	1.02	1
	ARL	500.137	220.138	144.05	40.016	20.03	1.152	1	1
	SDRL	492.2264	422.12	328.01	85.615	58.602	2.015	0.674	0.269
PE	P10	54.90	51.00	34.90	10.0	8.0	1	1	1
L=3.82	P25	141.00	118.75	92.75	25	19	1.08	1	1
$\lambda_1 = 0.20$,	P50	341.00	230.5	162.01	51	32	1.1	1	1
$\lambda_2 = 0.60$	P75	726.75	623	427.25	118	86	3	1	1
	P90	1211.20	1019.5	736.10	195.1	135.1	5	1.07	1
	ARL	500.663	260.13	168.07	51.01	27.20	1.372	1	1
	SDRL	484.7689	432.3127	319.7975	104.374	51.22	2.71	0.753	0.248
PE	P10	55.90	51.9	38.0	11.9	6.00	1	1	1
L=3.885	P25	141.75	127.0	95.0	30	17	2	1	1
$\lambda_1 = 0.20$,	P50	341.50	252.5	161.0	48	21.2	1.2	1	1
$\lambda_2 = 0.75$	P75	707.50	627.0	459.0	147	76.25	4	1	1
	P90	1184.10	1023.4	752.2	235	115.2	7	1.02	1
	ARL	500.748	215.12	133.03	25.01	6.203	1.046	1	1
	SDRL	922.781	638.93	298.782	28.718	12.675	1.3705	0.6443	0.478
	P10	55.90	50.1	38.0	11.9	2	1	1	1
PE	P25	140.5	128.01	95.0	28	4.1	1.01	1	1
L=4.065	P50	345.01	212.5	130.0	38	5.8	1.03	1	1
$\lambda_1 = 0.30$,	P75	654.5	415	226.25	36	17	3	1	1
$\lambda_2 = 0.10$	P90	1785.8	1039.5	574.5	61.1	27	6	1.001	1

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	ARL	500.137	226.2	154.057	47.016	24.203	1.252	1	1
	SDRL	491.464	373.008	234.365	37.25	15.797	1.2435	0.593	0.253
PE	P10	58.00	45.90	27.00	8.0	5	1	1	1
L=3.82	P25	152.5	113.75	71	15	8	1	1	1
$\lambda_1 = 0.30$,	P50	365.5	221.50	141	41	19	1.1	1	1
$\lambda_2 = 0.60$	P75	694.75	528.50	337.25	56	26	3	1	1
	P90	1137.10	869.10	543.30	92.1	39	4	1.1	1
	ARL	500.663	270.1	178	61.01	32.03	1.472	1.02	1
	SDRL	476.8266	365.166	239.856	39.8348	16.93	1.289	0.5901	0.243
PE	P10	56.90	50.0	27.00	9.0	5.0	1	1	1
L=3.885	P25	150.0	115	72	16	9	1	1	1
$\lambda_1 = 0.30$,	P50	368.0	260.5	162	51.5	30.1	1.5	1	1
$\lambda_2 = 0.75$	P75	695.75	544.0	334.25	58	27	3	1.08	1
	P90	1130.0	887.5	572.30	97.1	42.1	4	1.1	1

Table 4: ARL$_S$ for PE Based EEWMA control charts.

		Shift							
		0	0.01	0.03	0.120	0.180	0.60	1.0	1.5
	ARL	500.836	114.863	73.211	11.464	1.067	1.003	1.001	1
PE	SDRL	492.8587	9.447194	1.4527	0.5558	0.2501	0.0547	0.0316	0
$\lambda_1 = 0.20$,	P10	56.80	25	20	4	1	1	1	1
$\lambda_2 = 0.10$	P25	153.75	88	31	7	1	1	1	1
L= 7.25	P50	358.50	113	65	10.3	1.03	1	1.0001	1
	P75	707.00	189	99	42	1.1	1.08	1.0012	1
	P90	1124.10	270	200	82	1.8	1.3	1.01	1
	ARL	500.748	132.614	83.661	15.453	1.259	1.1	1	1
PE	SDRL	496.0317	34.778	2.1005	0.5693	0.2357	0.04469	0.0316	0
$\lambda_1 = 0.20$,	P10	53.90	6.0	11	1	1	1	1	1
$\lambda_2 = 0.60$	P25	146.75	11.0	21	5	1.1	1.08	1	1
L=7.59	P50	337.50	125.0	76	14	1.19	1.17	1	1
	P75	708.75	152.0	95	29	1.8	1.7	1	1
	P90	1156.20	198.1	181	62	2.1	1.5	1.05	1
	ARL	500.222	166.909	101.239	17.526	1.377	1.2	1.02	1
PE	SDRL	495.0974	45.73822	2.846	0.6047	0.26672	0.07056	1.001	0
$\lambda_1 = 0.20$,	P10	60.0	16	11	4	1	1	1	1
$\lambda_2 = 0.75$,	P25	147.0	114	102	9	1.1	1.1	1	1
L=7.66	P50	337.5	164	114	16	1.3	1.17	1.01	1
	P75	711.0	196	175	22	2.1	2	1.7	1
	P90	1217.7	207	188	42	2.12	2.01	1.8	1
	ARL	500.836	138.863	94.211	14.464	1.167	1.1	1.002	1
PE	SDRL	515.024	17.76437	1.6695	0.5572	0.2241	0.0446	0.0316	0
$\lambda_1 = 0.30$,	P10	46.00	35	21	1	1	1	1	1
$\lambda_2 = 0.10$	P25	142.00	109	89	6	1.09	1.08	1.001	1
L=7.00	P50	332.00	136	93	13	1.13	1.1	1.002	1
	P75	712.25	192	164	29	2.1	2	1.1	1
	P90	1160.10	246	206	62	5	4	1.2	1
	ARL	500.748	142.614	98.661	17.453	1.3	1.2	1.01	1
PE	SDRL	486.214	66.673	4.607	0.7368	0.3281	0.0772	0.03162	0
$\lambda_1 = 0.30$,	P10	64.9	29.0	25	3	1	1	1	1
$\lambda_2 = 0.60$	P25	154.0	82.0	71	9	1.1	1.06	1.002	1
L=7.35	P50	343.5	133.0	120	16	1.2	1.1	1.01	1
	P75	688.0	201.0	188	40	3	2	1.9	1
	P90	1174.4	260.1	240	99	7	5	4	1
	ARL	500.222	178.909	106.239	20.26	1.577	1.3	1.1	1.03
PE	SDRL	490.6586	86.438	7.653	0.9214	0.3561	0.08341	0.0316	0
$\lambda_1 = 0.30$,	P10	63.9	111.00	22	3	1.1	1	1	1
$\lambda_2 = 0.75$	P25	155.0	169.00	78	9	1.2	1.2	1.08	1.02
L=7.44	P50	346.0	266.00	102	18	1.4	1.28	1.1	1.03
	P75	693.0	292.25	211	32	2	2	2	2
	P90	1178.7	306.00	298.10	63	4	3.9	3.5	3.2

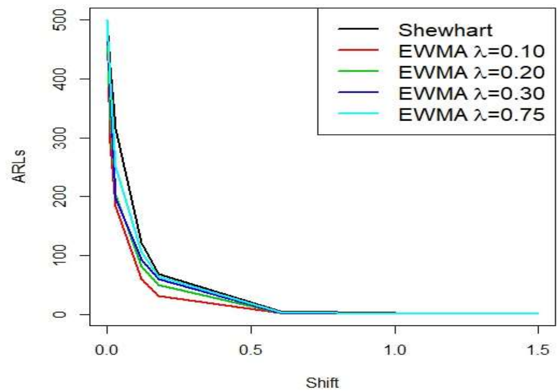

Figure 2: ARLs for the shape parameter of BL2PFD under Shewhart and EWMA control charts

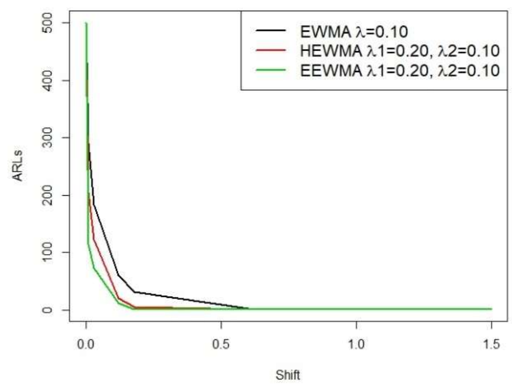

Figure 3: ARLs for the shape parameter of BL2PFD under EWMA (λ=0.10), HEWMA and EEWMA control charts at λ_1=0.20, λ_2=0.10

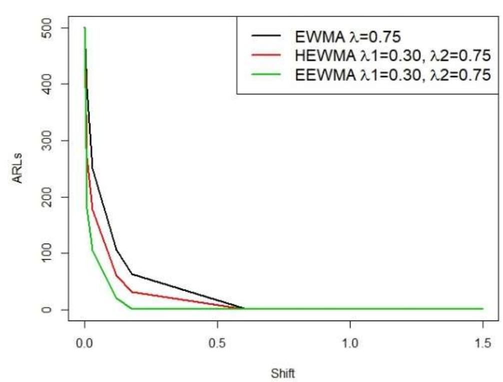

Figure 4: ARLs for the shape parameter of BL2PFD under EWMA (λ=0.75), HEWMA and EEWMA control charts at λ_1=0.20, λ_2=0.75

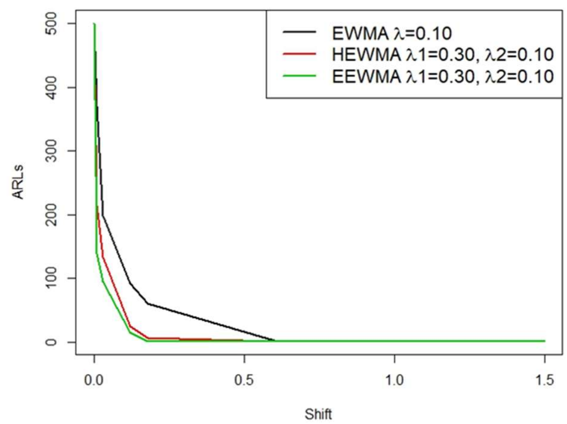

Figure 5: ARLs for the shape parameter of BL2PFD under EWMA (λ=0.10), HEWMA and EEWMA control charts at λ_1=0.30, λ_2=0.10

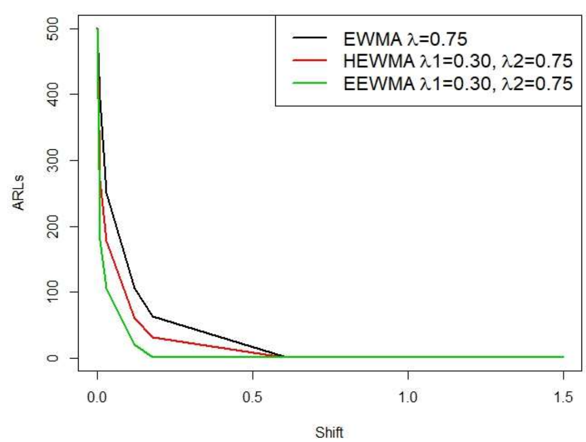

Figure 6: ARLs for the shape parameter of BL2PFD under EWMA (λ=0.75), HEWMA and EEWMA control charts at λ_1=0.30, λ_2=0.75

We observe from Tables 1-4 and Figures 2-6, that EEWMA control chart under PE ($\lambda_1 = 0.20, \lambda_2 = 0.10$) performs better as compared to Shewhart, EWMA and HEWMA control charts under PE. We take a shift of 0.01 in the shape parameter of BL2PFD and see that Shewhart control chart under PE provide ARL_1=419.257, EWMA control chart under PE ($\lambda = 0.60$) gives ARL_1=290.138, HEWMA control chart under PE ($\lambda_1 = 0.20, \lambda_2 = 0.10$) gives ARL_1=205.13 and EEWMA control chart under PE gives ARL_1=114.863. So, we see that a control chart under EEWMA is better than Shewhart, EWMA and HEWMA control charts under PE. We take different shifts and observe the same behavior. We observe that the ARL_1 approaches to 1 more efficiently when we use EEWMA control chart under PE as compared to the other control charts under PE.

We also observe from that EEWMA control chart (for all λ_1 and λ_2) performs better as compared to Shewhart, EWMA and HEWMA control charts under PE by giving less ARL_1 value which means an early detection of any shift in the process.

Table 5: Simulated data

HEWMA $\lambda_1 = 0.20$, $\lambda_2 = 0.10$, L= 4.065			EEWMA $\lambda_1 = 0.20$, $\lambda_2 = 0.10$,L= 7.25		
$HEWPE_t$	LCL	UCL	$EEWPE_t$	LCL	UCL
2.003035	2.001297	2.038704	1.999922	2.001297	2.038704
2.015575	1.984710	2.055290	2.009740	1.984710	2.055290
2.035488	1.969518	2.070482	2.013428	1.969518	2.070482
2.037292	1.956267	2.083733	2.009999	1.956267	2.083733
2.032129	1.945017	2.094983	2.007248	1.945017	2.094983
2.030353	1.935621	2.104379	2.013095	1.935621	2.104379
2.034773	1.927854	2.112146	2.020617	1.927854	2.112146
2.033791	1.921479	2.118521	2.022756	1.921479	2.118521
2.032368	1.916270	2.123730	2.026333	1.916270	2.123730
2.034350	1.912028	2.127972	2.024896	1.912028	2.127972
2.037700	1.908582	2.131418	2.019637	1.908582	2.131418
2.044160	1.905786	2.134214	2.011215	1.905786	2.134214
2.043985	1.903520	2.136480	1.995523	1.903520	2.136480
2.043668	1.901684	2.138316	1.992991	1.901684	2.138316
2.053700	1.900198	2.139802	1.987555	1.900198	2.139802
2.058514	1.898995	2.141005	1.988376	1.898995	2.141005
2.052912	1.898022	2.141978	1.981422	1.898022	2.141978
2.055571	1.897234	2.142766	1.983265	1.897234	2.142766
2.048172	1.896597	2.143403	1.983712	1.896597	2.143403
2.046945	1.896081	2.143919	1.989081	1.896081	2.143919
2.040349	1.895664	2.144336	1.990513	1.895664	2.144336
2.031455	1.895326	2.144674	2.008520	1.895326	2.144674
2.025016	1.895052	2.144948	2.006625	1.895052	2.144948
2.014828	1.894831	2.145169	2.010102	1.894831	2.145169
2.004909	1.894652	2.145348	2.017664	1.894652	2.145348
2.010919	2.001297	2.038704	2.027329	1.894507	2.145493
2.025538	1.984710	2.055290	2.048474	1.894389	2.145611
2.038631	1.969518	2.070482	2.063326	1.894294	2.145706
2.045303	1.956267	2.083733	2.070312	1.894217	2.145783
2.054403	1.945017	2.094983	2.077926	1.894155	2.145845
2.078672	1.935621	2.104379	2.094428	1.894104	2.145896
2.089144	1.927854	2.112146	2.111844	1.894063	2.145937
2.087476	1.921479	2.118521	2.122475	1.894030	2.145970
2.075752	1.916270	2.123730	2.134089	1.894003	2.145997
2.073981	1.912028	2.127972	2.139766	1.893981	2.146019
2.073262	1.908582	2.131418	2.140586	1.893964	2.146036
2.073125	1.905786	2.134214	2.137590	1.893949	2.146051
2.078779	1.903520	2.136480	2.125775	1.893938	2.146062
2.078492	1.901684	2.138316	2.128186	1.893929	2.146071
2.072080	1.900198	2.139802	2.126809	1.893921	2.146079
2.056733	1.898995	2.141005	2.131641	1.893915	2.146085
2.039243	1.898022	2.141978	2.127647	1.893910	2.146090
2.026515	1.897234	2.142766	2.132809	1.893906	2.146094
2.030019	1.896597	2.143403	2.136058	1.893903	2.146097
2.028108	1.896081	2.143919	2.144269	1.893900	2.146100
2.024789	1.895664	2.144336	2.147808	1.893898	2.146102
2.020850	1.895326	2.144674	2.169608	1.893896	2.146104
2.018151	1.895052	2.144948	2.169412	1.893895	2.146105
2.013678	1.894831	2.145169	2.174811	1.893894	2.146106
2.021826	1.894652	2.145348	2.184422	1.893893	2.146107

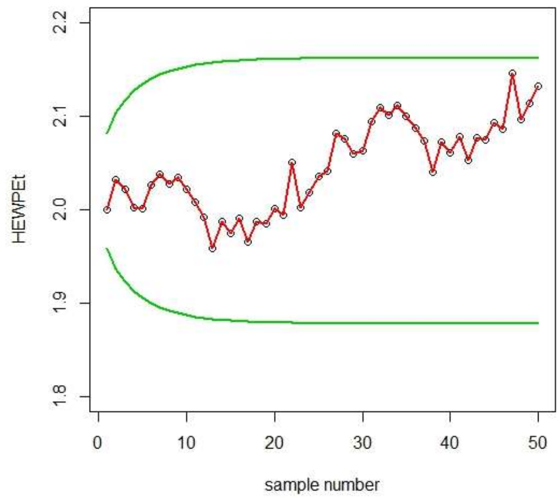

Figure 7: Simulated data of the proposed HEWMA control chart under PE.

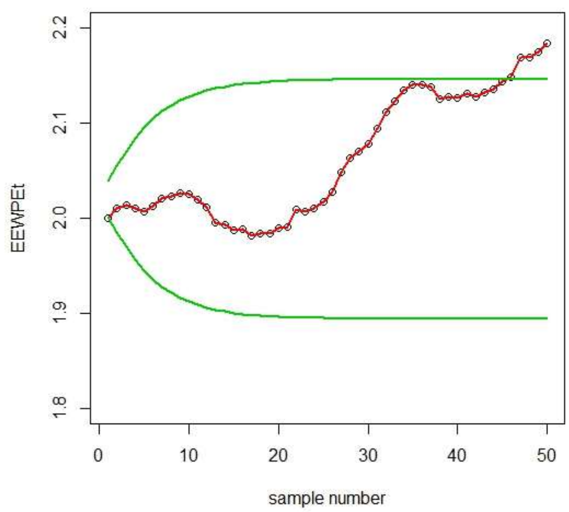

Figure 8: Simulated data of the proposed EEWMA control chart under PE.

Real life applications

Data set 1

The first set of data is taken from Montgomery (2012) about the automobile ring piston diameter size. Each sample or subgroup consists of five piston rings. We object to controlling the flow width of the resist in the process by using proposed control charts.

We compare HEWMA and EEWMA and from Figure 9 and Figure 10, we observe that EEWMA performs better than HEWMA. This indicates that EEWMA under PE provides more applicable to detect any shift in the process early when the underlying process is based on BL2PFD.

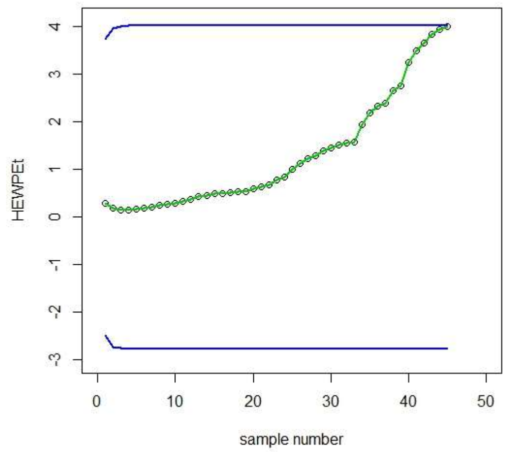

Figure 9: Automobile engine piston rings data of the HEWMA control chart under PE

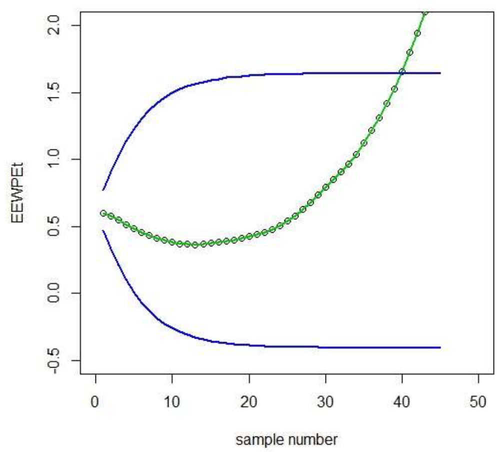

Figure 10: Automobile engine piston rings data of the EEWMA control chart under PE

Data set 2

We use the data of fracture toughness which was used by Arifa *et al*. (2017). The data are:

0.025, 0.089, 0.089, 0.250, 0.311, 0.345, 0.476, 0.565, 0.567, 0.656, 0.674, 0.675, 0.675, 0.769, 0.837, 0.839, 0.842, 0.864, 0.885, 0.911, 0.912, 0.983, 1.048, 1.059, 1.077, 1.173, 1.257, 1.277, 1.299, 1.321, 1.350, 1.355, 1.459, 1.488, 1.572, 1.573, 1.708, 1.726, 1.746, 1.763, 1.774, 1.827, 1.837, 1.850, 1.880, 1.887, 1.881, 1.931, 1.955, 2.004, 2.040, 2.090, 2.109, 2.133, 2.21, 2.246, 2.287, 2.323, 2.347, 2.351, 2.495, 2.526, 2.991, 3.025, 3.267, 3.404, 3.484, 3.743, 3.745, 3.914, 4.8073, 5.400, 5.443, 5.529, 6.554, 9.09..

We construct HEWMA and EEWMA control charts for the above data. For example, in Figure 11 and Figure 12, we observe that EEWMA is better to be used than HEWMA when the distribution of process is based on BL2PFD.

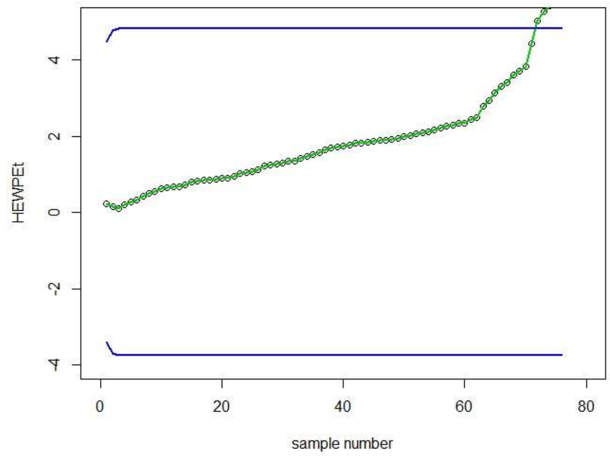

Figure 11: Fracture toughness data of the HEWMA control chart under PE

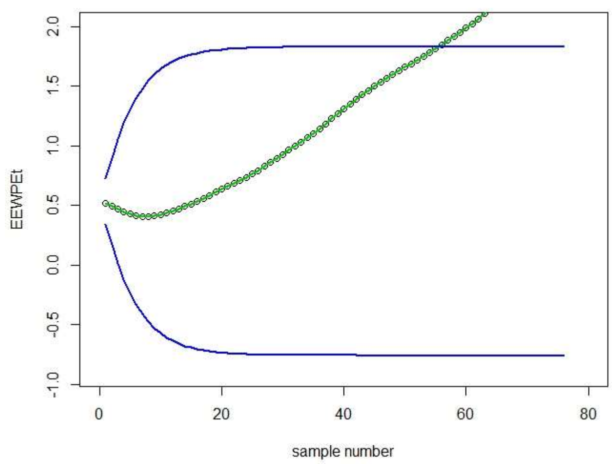

Figure 12: Fracture toughness data of the EEWMA control chart under PE

CONCLUSION

In this research paper, some modifications to existing control charts to monitor the shape parameter of BL2PFD are proposed, these modified control charts are named as modified Shewhart control chart, modified EWMA control chart, modified HEWMA control chart, modified extended EWMA control chart. The shape parameter of BL2 PFD is estimated through percentile estimator. The study of the proposed control charts for the processes using BL2 PFD as baseline distribution are done through simulation study and real-life applications.

We showed the application of BL2PFD in the field of reliability and engineering sciences using the proposed control charts. We conclude that among our proposed charts, EEWMA performs better than other control charts when the distribution of underlying process is BL2PFD.

Data Availability: The data used to support the findings of this study are included in the article.

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