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RESEARCH ARTICLE

Astrophysics

Simulation of coloured particles radiation spectra from black holes

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Abstract: This work investigates the spectra of coloured particles radiated from Schwarzschild black holes and Kerr black holes as their mass decrease from ~10^{13} g to Planck mass. The emission rate formulas in terms of black hole mass, temperature, entropy, and spin are derived. The variation of spectra with time, and black hole parameters such as mass, temperature, entropy, and spin are obtained and discussed. During the last stages of black hole evaporation, black hole loses its mass at a drastic rate. When the orders of magnitude of particles were disregarded, the spectrum of quarks and gluons emitted from the 10^{13} g black hole over 3171 millennia was emitted from the 10^{8} g black hole within 2.5×10^{-4} s. It only took 3.5×10^{-25} s for a Schwarzschild black hole to reduce its mass from 10.00 g to 2.55×10^{-5} g. As the black hole reaches the end of its lifetime, the emission rates of the high energetic particles gradually recede and reach the limit of zero as Planck mass is reached. By the time the black hole begins to emit coloured particles its temperature is ~10^{12} K, but when it reaches the Planck mass its temperature has reached closer to the Planck temperature. As Planck mass is reached, the entropy of the black hole gradually decreases and gets very close to zero.

Keywords: Black hole entropy, black hole mass, black hole spin, black hole temperature, coloured particles, Hawking radiation.

INTRODUCTION

Hawking's discovery of particle emission by black holes (BHs) led to many further investigations (Carr, 1976; Page, 1976a; 1976b; 1977; MacGibbon & Webber, 1990). One of the earliest works in astrophysics on the theory of BH evaporation via particle emission was by Page and Hawking (1976), who examined the possibilities of detecting gamma rays produced by the evaporation of primordial black holes (PBHs). They predicted the gamma-ray spectra produced by PBHs and suggested that the best prospect for detecting a PBH would be to look for hard gamma rays that are expected in the final stages of the evaporation of the BH. Carr (1976) investigated how small PBHs could have contributed to the background density of gravitons, electrons, neutrinos, nucleons, and photons in the universe. He also predicted the spectrum of gravitons, electrons, and neutrinos which should result from PBH evaporations.

Page (1976a) calculated the emission rates for the known massless particles from a nonrotating, uncharged BH. He also estimated that a PBH would decay away within the present age of the universe if and only if its initial mass were less than $(5\pm1) \times 10^{14}$ g. In a subsequent paper (1976b), Page extended the calculations of the first paper to a rotating BH and the resultant evolution of the BH. He showed that angular momentum is given off a few times quicker than energy and thus, a swiftly spinning BH spins down to a nearly nonrotating state prior to giving up most of its mass.

A calculation done for the colour charge by Perry (1977) showed that a BH conserves colour and any fluctuation in the radiation process that generates a coloured BH will immediately be countered by the BH losing its colour in much the same way as a charged BH would lose its charge in the case of electromagnetism.

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Thus, the BH will remain neutral concerning colour guaranteeing a colourless final state for the emission products.

The first calculation of BH emission considering the particle decays and emission of quarks and gluons from BHs was done by MacGibbon and Webber. They assumed that quarks and gluons would be emitted by the BH above the Quark Gluon Deconfinement (QGD) temperature (~ 150–200 MeV) and the emitted quarks and gluons fragment into hadrons, which further decay into neutrinos, electrons, photons, etc., and found (1990) that owing to the tremendous rise in the degrees of freedom for quarks and gluons, and the huge number of final particles produced by the decay of each coloured particle, the quark and gluon emission dominates the evaporation of BHs beyond the QGD temperature (~ 150–200 MeV). They also showed that the emitted quarks and gluons are similar to those produced in e^+e^- collider events and therefore must decay in the same way into final colourless hadronic states.

A group theoretical approach was used by Kofoed (2015) to study the emission of coloured particles from sufficiently hot BHs. He performed the calculations using the bird track notation, a pictorial representation of SU(N) tensor calculations and compared quantum chromodynamics (QCD) to quantum electrodynamics. During more recent times Ukwatta *et al.* (2016) have investigated the emission of coloured particles from BHs approaching the end of their lifetimes and calculated the energy dependent PBH burst light curves in the GeV/TeV energy range.

The objective of this work is to derive emission rate formulas in terms of BH parameters such as mass, temperature, entropy, and spin and investigate the variation of primary spectra of coloured particles from Schwarzschild black holes (SBHs) and Kerr black holes (KBHs) of masses between ~10^{13} g and Planck mass (~10^{5} g) with time, mass, temperature, entropy, and spin. SBHs are non-rotating, uncharged BHs, and KBHs are rotating, uncharged BHs. More detailed definitions of SBHs and KBHs and their basic properties are presented in Methodology under Theoretical Formulation. We restrict the analysis to BHs of masses below ~10^{13} g because these are the only BHs that emit coloured particles. Furthermore, the classical description used in our analysis breaks down when the BH mass approaches Planck mass. All formulas used are in Planck units where = c = k_B = G =1 (= reduced Planck constant, c = speed of light in vacuum, k_B = Boltzmann constant, G = gravitational constant), unless stated otherwise. The available C code BlackHawk (Arbey & Auffinger, 2019; 2021) at *https://blackhawk.hepforge.org* was used to calculate the emission rates. The study of emission of coloured particles is important because coloured particles are being emitted more dominantly than other types of particles from the BHs of the considered mass range. And BHs of the considered mass range are the ones which can produce any observable bursts of Hawking radiation.

MATERIALS AND METHODS

Theoretical formulation

Schwarzschild black holes

Schwarzschild black holes (SBHs) are spherically symmetric, nonrotating, uncharged BHs which are described by the Schwarzschild metric in Schwarzschild coordinates (t, r, θ, ϕ),

$$ds^2 = \left(1 - \frac{r_H}{r}\right) dt^2 - \left(1 - \frac{r_H}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \quad ...(1)$$

r_H is the Schwarzschild radius of the BH defined by,

$$r_H = 2M \quad ...(2)$$

where *M* is the BH mass. Given that the spacetime is asymptotically flat, the boundary in spacetime between events that can communicate with distant observers and events that cannot, is called the event horizon. The area of the event horizon of a SBH is,

$$A = 4\pi r_H^2 \quad ...(3)$$

Hawking's area theorem (Hawking, 1971) says that the area of a BH cannot decrease under any process. This is the second law of BH mechanics. This is evocative of the second law of thermodynamics which states that the entropy of a closed system cannot decrease under any process. Bekenstein proposed that a BH has an entropy which is a finite multiple of its area. Later the Bekenstein-Hawking formula for the entropy of a BH was given as,

$$S_{BH} = \frac{1}{4} A = \pi r_H^2 \quad ...(4)$$

Hawking showed that the event horizon of any stationary BH must be a Killing horizon for some vector field. For every Killing horizon, there is an associated quantity called the surface gravity (Carroll, 2019). The surface gravity of a SBH is,

$$\kappa = \frac{1}{4M} \quad ...(5)$$

The zeroth law of BH mechanics states that the surface gravity of a stationary BH is constant over its event horizon and the third law of BH mechanics states that it cannot be reduced to zero by any finite sequence of operations (Bardeen *et al*., 1973).

Hawking showed (1975) that the gravitational fields of SBHs create particles from their event horizons and emit them to infinity as if they were hot bodies with temperature *T*, given by,

$$T = \frac{\kappa}{2\pi} \quad ...(6)$$

Using the definition of gravity given in equation (5) the temperature of the BH can be defined in terms of BH mass as,

$$T = \frac{1}{8\pi M} \quad ...(7)$$

A BH with angular velocity Ω and electric potential Φ emits particles with total energy between Q and $Q+dQ$ at a rate (MacGibbon & Webber, 1990),

$$d\dot{N} = \frac{\Gamma_s dQ}{2\pi\hbar}\left[exp\left(\frac{Q - n\hbar\Omega - q\Phi}{\hbar\kappa/2\pi\, c}\right) - (-1)^{2s}\right]^{-1} \quad ...(8)$$

Where Γ_s are the graybody factors for the particle species which are dependent on energy of the emitted particles,Q, and the mass of the BH, *M*. They have to be calculated numerically. Rewriting the equation using Planck units where $\hbar = c = 1$,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{Q - n\Omega - q\Phi}{\kappa/2\pi}\right) - (-1)^{2s}\right]^{-1} \quad ...(9)$$

For a SBH, $\Omega = \Phi = 0$. Hence, equation (9) reduces to,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{Q}{\kappa/2\pi}\right) - (-1)^{2s}\right]^{-1} \quad ...(10)$$

Taking into consideration all the degrees of freedom of a particle species, the emission rate formula can be written as,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{Q}{\kappa/2\pi}\right) - (-1)^{2s}\right]^{-1}.\, n_{dof} \quad ...(11)$$

Here n_{dof} is the number of degrees of freedom of the particle species such as electric charge, colour, etc. and *s* is particle spin. Equation (5) gives the relationship between the surface gravity, κ and the mass of the BH, *M*. Using equation (5) particle emission rates can be written in terms of the energy of the emitted particles, *Q* and the mass of the BH, *M* as,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}[exp(8\pi MQ) - (-1)^{2s}]^{-1}.\, n_{dof} \quad ...(12)$$

Equation (7) gives the relationship between the surface gravity, κ and the temperature of the BH, *T*. It can be used to modify the equation (12) to write the emission rate formulas in terms of the energy of the emitted particles, *Q* and the temperature of the BH, *T* as,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{Q}{T}\right) - (-1)^{2s}\right]^{-1}.\, n_{dof} \quad ...(13)$$

Equation (11) can also be modified to be written in terms of the energy of the emitted particles, *Q* and the entropy of the BH, S_{BH} by using equations (2), (4) and (5). It can be given as,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(4Q\sqrt{\pi S_{BH}}\right) - (-1)^{2s}\right]^{-1}.\, n_{dof} \quad ...(14)$$

The mass of the Hawking evaporating SBH as a function of time can be written in the form of a differential equation (Arbey & Auffinger, 2019),

$$\frac{dM}{dt} = -\frac{f(M)}{M^2} \quad ...(15)$$

$f(M)$ is the Page factor which accounts for all directly radiated particle species by a BH of mass *M* and their quantum degrees of freedom. When the BH evaporates the value of *M* is decreased by a quantity equal to the total mass-energy of the radiated particles. By conservation of energy,

$$\frac{dM}{dt} = -\sum_i \int_0^{+\infty} \frac{d^2N_i}{dQdt} Qdt \quad ...(16)$$

where the summation over *i* is over all the directly emitted species. Thus, the Page factor can be obtained as,

$$f(M) = -M^2\frac{dM}{dt} = M^2\int_0^{+\infty}\sum_i \frac{\Gamma_s(Q,M)}{2\pi}\left[exp\left(\frac{Q}{\kappa/2\pi}\right) - (-1)^{2s}\right]^{-1}.\, n_{dof}QdQ \quad ...(17)$$

For SBHs the mass decay gets faster with time, as the temperature gets higher as the mass decreases.

Kerr black holes

Kerr black holes (KBHs) are an expansion of SBHs. They are uncharged but rotating and thus, have a nonzero spin. They are described by the Kerr metric which can be given in Boyer-Lindquist coordinates as,

$$ds^2 = (dt - a\sin^2\theta\, d\phi)^2 \frac{\Delta}{\Sigma} - \left(\frac{dr^2}{\Delta} + d\theta^2\right)\Sigma - - [(r^2 + a^2)d\phi - adt]^2 \frac{\sin^2\theta}{\Sigma} \quad ...(18)$$

where the coordinates r, θ, ϕ are standard oblate spheroidal coordinates. Here Σ and Δ are defined as,

$$\Sigma(r) = r^2 + a^2\cos^2\theta$$

$$\Delta(r) = r^2 - 2Mr + a^2 \quad ...(19)$$

and *a* is the BH spin defined by,

$$a = \frac{J}{M} \quad ...(20)$$

where *J* is the angular momentum of the BH. The reduced spin parameter can be defined as,

$$a^* = \frac{a}{M} \in [0, 1] \quad ...(21)$$

When $a^* = 1$, the BH is maximally rotating.
Kerr geometry has an inner and an outer horizon located at,

$$r_\pm = M \pm \sqrt{M^2 - a^2} \quad ...(22)$$

where $a \to 0$, KBH simplifies to a SBH.
The area of the event horizon of KBH located at r_+ can be given as (Page, 2005),

$$A = 4\pi(r_+^2 + a^2) \quad ...(23)$$

The Bekenstein-Hawking entropy of the rotating BH is thus defined as (Ibungochouba Singh *et al*., 2014),

$$S_{BH} = \frac{1}{4}A = \pi(r_+^2 + a^2) \quad ...(24)$$

The surface gravity of Kerr solution is defined as (Page, 2005),

$$\kappa = \frac{4\pi(r_+ - M)}{A} \quad ...(25)$$

The temperature of a rotating KBH is given by (Arbey & Auffinger, 2019),

$$T = \frac{1}{2\pi}\left(\frac{r_+ - M}{r_+^2 + a^2}\right) \quad ...(26)$$

The particle emission rates from a KBH can be derived from equation (9). For a KBH electric potential, $\Phi = 0$, and it has an angular velocity at the horizon defined by,

$$\Omega = \frac{a^*}{2r_+} \quad ...(27)$$

Thus, for a KBH equation (9) reduces to,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{Q - m\Omega}{\kappa/2\pi}\right) - (-1)^{2s}\right]^{-1} \quad ...(28)$$

In this case the graybody factors are dependent on the energy of the emitted particles, Q, the mass of the BH, *M*, and the reduced spin parameter, a^*. Taking into account all the degrees of freedom of a particle species, the emission rate formula is,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{Q - m\Omega}{\kappa/2\pi}\right) - (-1)^{2s}\right]^{-1} . n_{dof} \quad ...(29)$$

Similar to the derivation done for a SBH, the emission rate formula can be written in terms of the temperature of the KBH using equations (23), (25), and (26). It can be given as,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{Q - m\Omega}{T}\right) - (-1)^{2s}\right]^{-1} . n_{dof} \quad ...(30)$$

Similarly, emission rate formulas can be written in terms of the mass of the BH *M*, entropy S_{BH}, and reduced spin parameter a^* as,

$$\frac{d^2N}{dQdt} = \frac{\Gamma_s}{2\pi}\left[exp\left(\frac{2S_{BH}(Q - m\Omega)}{M\sqrt{1 - a^*}}\right) - (-1)^{2s}\right]^{-1} . n_{dof} \quad ...(31)$$

In KBHs both mass *M* and angular momentum *J* evolve with time. A rotating BH is more likely to emit particles with high angular momentum, and with a projection *m* of that angular momentum aligned with the BH spin. Hence, the angular momentum of the BH will be gradually extracted. The Page factor for a KBH is,

$$f(M, a^*) = -M^2\frac{dM}{dt} = M^2\int_0^{+\infty}\sum_i \frac{\Gamma_s(Q, M, a^*)}{2\pi} \left[exp\left(\frac{Q - m\Omega}{\kappa/2\pi}\right) - (-1)^{2s}\right]^{-1} . n_{dof}QdQ \quad ...(32)$$

The differential equation describing the evolution of BH angular momentum is,

$$g(M,a^*) = -\frac{M}{a^*}\frac{dJ}{dt}$$

$$= -\frac{M}{a^*}\int_0^{+\infty}\sum_i\sum_{dof}\frac{m\Gamma_s(Q,M,a^*)}{2\pi}$$

$$\left[exp\left(\frac{Q-m\Omega}{\kappa/2\pi}\right)-(-1)^{2s}\right]^{-1}.n_{dof}dQ \quad ...(33)$$

After obtaining $f(M,a^*)$ and $g(M,a^*)$, the evolution of a^* can be obtained as,

$$\frac{da^*}{dt} = \frac{d(J/M^2)}{dt} = \frac{1}{M^2}\frac{dJ}{dt} - 2\frac{J}{M^3}\frac{dM}{dt}$$

$$= a^*\frac{2f(M,a^*)-g(M,a^*)}{M^3} \quad ...(34)$$

Simulation

First the emission rate formulas in terms of black hole mass, temperature, entropy, and spin were derived for both Schwarzschild and Kerr BHs. Then the public C code BlackHawk (Arbey & Auffinger, 2019; 2021) was used to calculate the particle emission rates. Then separate Python scripts were written to simulate the variation of the emission rates of different particle species with time and different BH parameters. BHs radiate coloured particles once their temperatures exceed QGD temperature (~200 MeV). In this paper, all BHs radiating coloured particles are categorized into three categories based on their mass and temperature. (i) BHs beginning to emit coloured particles, (ii) Intermediate BHs emitting coloured particles, and (iii) BHs reaching the end of their lifetime.

For BHs beginning to emit coloured particles, BHs of mass ~10^{13} g were simulated. For intermediate BHs emitting coloured particles, BHs of mass ~10^8–10^{11} g were simulated. For a BH reaching the end of its lifetime, a BH of mass 10 g decaying away down to Planck's mass ~2.18×10^{-5} g was simulated. When simulating KBHs, the spin of the BH was given as 0.9999 to simulate the maximally rotating BHs.

To obtain the primary spectra, one million elementary particle energies were simulated. In the simulation, the minimum and maximum energies a directly emitted elementary particle was allowed to have were 10^{-7} GeV and 10^{21} GeV, respectively. However, this range was modified according to the mass of the BH. Variation of total primary spectra and spectra of coloured particles with time, variation of spectra of coloured particles with BH mass, the temperature, with BH entropy and with BH spin plots were generated for the investigation.

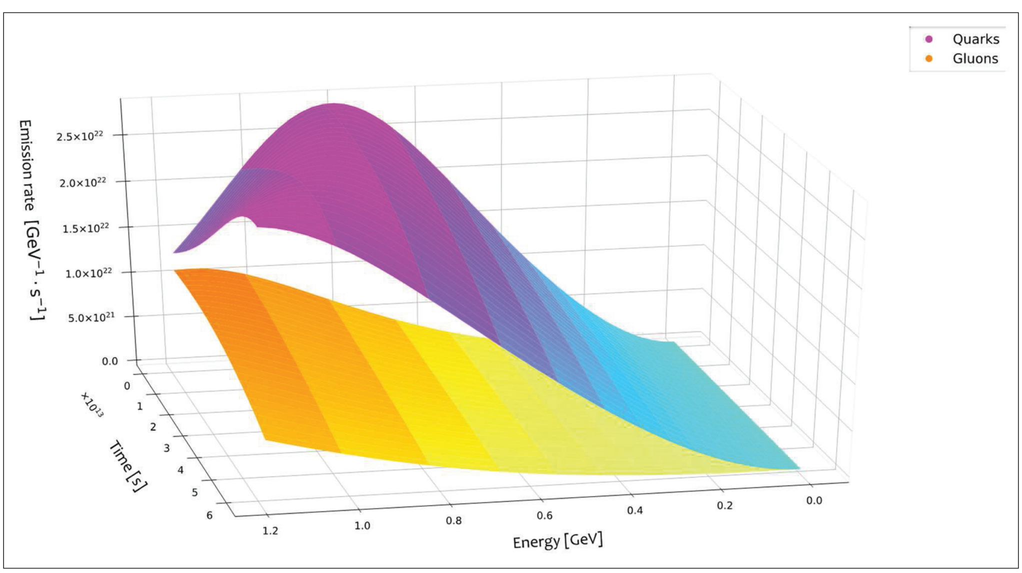

Figure 1: Variation of quark and gluon spectra from a SBH beginning to emit coloured particles with time. Mass of the BH is dropped from 5.29×10^{13} to 3.53×10^{13} g

RESULTS AND DISCUSSION

Variation of spectra with time

Schwarzschild black holes

Figure 1 presents the variation of primary spectra with time from a BH as the mass of the BH is dropped from 5.29×10^{13} to 3.53×10^{13} g within 6×10^{13} s. In the process, the temperature of the BH is increased from 200 to 300 MeV. Quarks, neutrinos, $e^{\pm}$, $\mu^{\pm}$, gluons, and photons are the only primary particle species that are being emitted. Results of instantaneous primary spectra also show that these are the only types of particles that are being emitted from a SBH of this temperature (Wijewardhana & Gamalath, 2021). Figure 1 shows that the emission of quarks dominates the spectrum when they begin to radiate. This provides proof for the fact that the emission of coloured particles dominates the BH evaporation spectrum above the QGD temperature. In Figure 1, it can be seen how the peak of the quark spectrum is reached at energies lower than the peak of the gluon spectrum. When the mass of the black hole drops further, its temperature increases and surpasses the rest mass energies of the more massive particle species such as $W^{\pm}$, Z^0 and Higgs bosons. The emission of those particles becomes significant when the mass of the black hole has reached down to ~10^{11} g and the temperature has increased to ~50 GeV.

Figure 2 denotes the coloured spectrum from a BH of mass 1.06×10^8 g for about 2.5×10^{-4} s. This BH mass is about half of the mass of a BH which is just hot enough to radiate coloured particles. All primary particle species, quarks, neutrinos, $e^{\pm}$, $\mu^{\pm}$, gluons, photons, Higgs bosons, $W^{\pm}$, and Z^0 are being emitted from the BH at this temperature. The emission of quarks far exceeds the emission of all other primary particles. The plot is drawn considering only a single quark flavour. When all twelve quark flavours are considered, the emission of quarks must dominate the primary spectrum from a BH radiating at this temperature. One striking feature is that, if the orders of magnitude of the energy axis are disregarded, Figure 2 which shows the coloured spectrum for 2.5×10^{-4} s is remarkably similar to the coloured spectrum shown in Figure 1, which is plotted for 6×10^{14} s. Hence, we can conclude that the spectrum of quarks and gluons that may be emitted from the 10^{13} g BH over 3171 millennia, will be emitted from the 10^8 g BH within 2.5×10^{-4} s. However, the energies of emitted particles will be hugely different. Energies of emitted quarks and gluons from the 10^8 g BH will be several orders of magnitude greater than the energies of quarks and gluons emitted from the 10^{13} g BH.

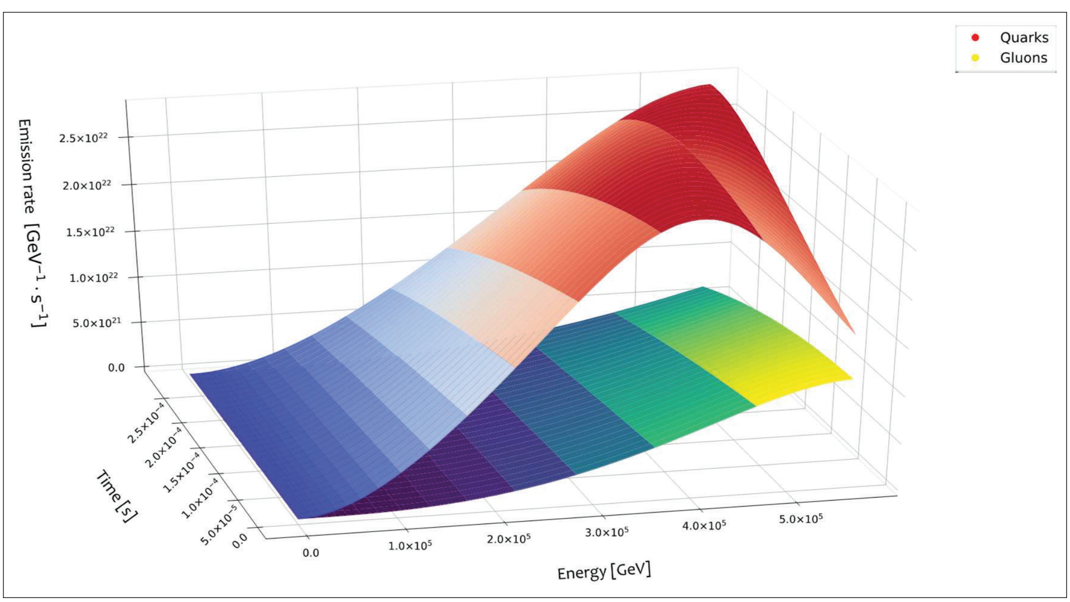

Figure 2: Variation of quark and gluon spectra from an intermediate SBH emitting coloured particles with time. BH mass is 1.06×10^8 g. Note that the energy axis is flipped compared to Figure 1

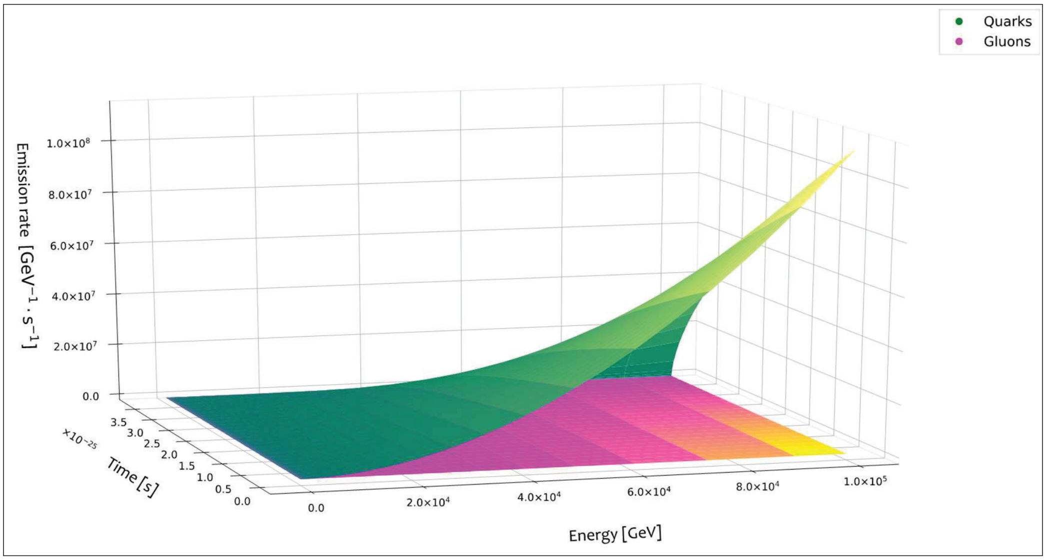

Figure 3: Variation of quark and gluon spectra with time from SBH reaching its end. BH mass is dropped to Planck mass from 10 g

Figure 3 presents the variation of quark and gluon spectra when the BH mass reaches down to Planck mass from 10 g. In Figure 3, time begins when the mass of the BH has dropped to 10 g and time counts-up till the BH mass reaches down to Planck mass. As can be seen from Figure 3, this only takes 3.5×10^{-25} s for the BH to reduce its mass from 10.00 g to 2.55×10^{-5} g. Hence, during the last stages of BH evaporation, it loses its mass at a drastic rate. When comparing with the rates of mass loss from the SBH as seen from the previous two cases, BH beginning to emit coloured particles and the intermediate BH emitting coloured particles, it can be concluded that these results agree with the fact that BHs lose mass at an ever-increasing rate. In Figure 3 also, quark emission dominates over gluon emission. It can be seen that, when comparing quark and gluon spectra, the emission of quarks at high energies far exceeds the emission of gluons at high energies.

It is observable that, with respect to both quarks and gluons, the emission rates of particles with exceedingly small energies are close to zero throughout the entire course of time considered. The highest overall emission rate is observed at the beginning of the considered timespan. This is because from the timespan considered, this is the time at which the BH possesses the most mass and energy. At this time BH emits highly energetic particles having energies of the order of 10^5 GeV the most. As time increases, even the emission rates of the highly energetic particles gradually recede and reach the limit of zero as Planck mass is reached. The emission rates of low energy particles are comparatively low throughout the entire timespan considered. However, after a long time has passed and the BH has reached its end, the emission rates of all particles of all energies reach the limit of zero. This is because when the BH has reached its end it has an exceedingly small total energy remaining. The spectra of quarks and gluons when plotted separately look similar to each other. However, when examining the emission rates there are striking differences in the orders of magnitude. Even though when compared together, the spectrum of gluons look flattened out in comparison as seen in Figure 3, the spectra of both quarks and gluons look similar to each other in their corresponding scales. This shows that when the last moments of the BH are reached, quark emission would dominate over gluon emission. Ukwatta *et al*. (2016) have also obtained this domination of quark emission over gluon emission at higher energies.

Kerr black holes

Figure 4 is plotted for a KBH with the same initial mass considered for the SBH beginning to radiate coloured particles. In Figure 4, the BH mass is reduced from 5.29×10^{13} to 4.85×10^{13} g. The BH had a spin of 0.99 when its mass was 5.29×10^{13} g and throughout the entire timespan depicted in Figure 4, its spin has receded down only to 0.90. If the BH were maximally rotating when it reached QGD temperature, its primary spectrum would be this.

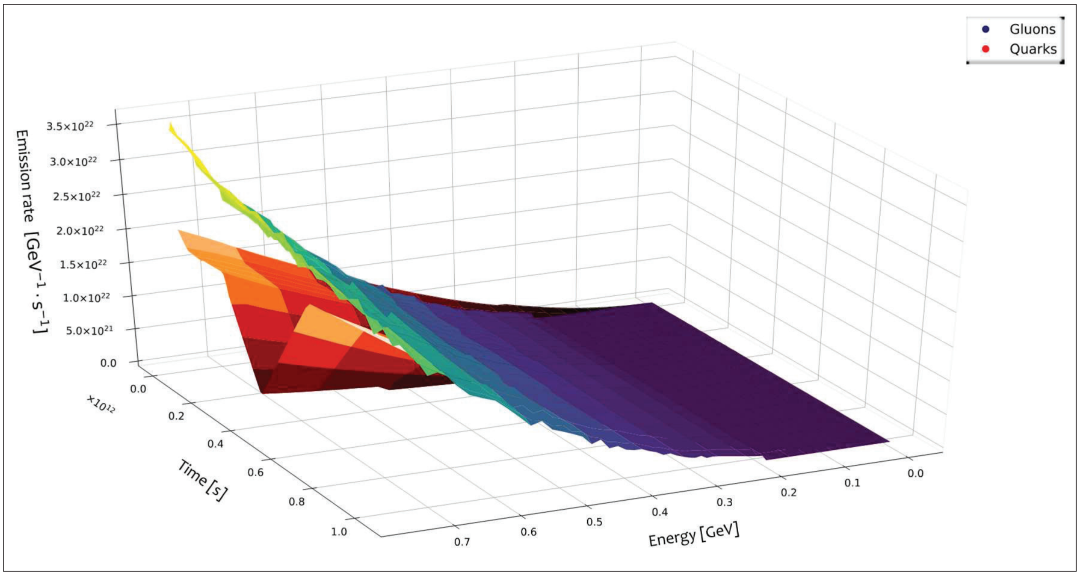

Figure 4: Variation of quark and gluon spectra from a maximally rotating KBH beginning to emit coloured particles with time

All quarks, gluons neutrinos, $e^{\pm}$, $\mu^{\pm}$, and photons are emitted. The emission of $\mu^{\pm}$ resulted in the same curve as $e^{\pm}$. $\tau^{\pm}$ was not emitted in the plotted energy range as their rest mass is 1.78 GeV/c^2. Because of larger variations than in the case of the SBH, only a time span of 1×10^{12} s is considered in Figure 4. From Figure 4, it is clear that in this considered energy range, the emission of quarks dominates over the emission of other denoted particle species. Figure 4 shows the emission spectra of quarks and gluons from a BH which starts its mass with 5.29×10^{13} g and emits particles for 1×10^{12} s. In the considered range of energies, between 0.0 GeV and 0.7 GeV, the gluon emission rate dominates over the quark emission rate. It was apparent that the emission of gluons dominates the primary spectrum in the considered energy range.

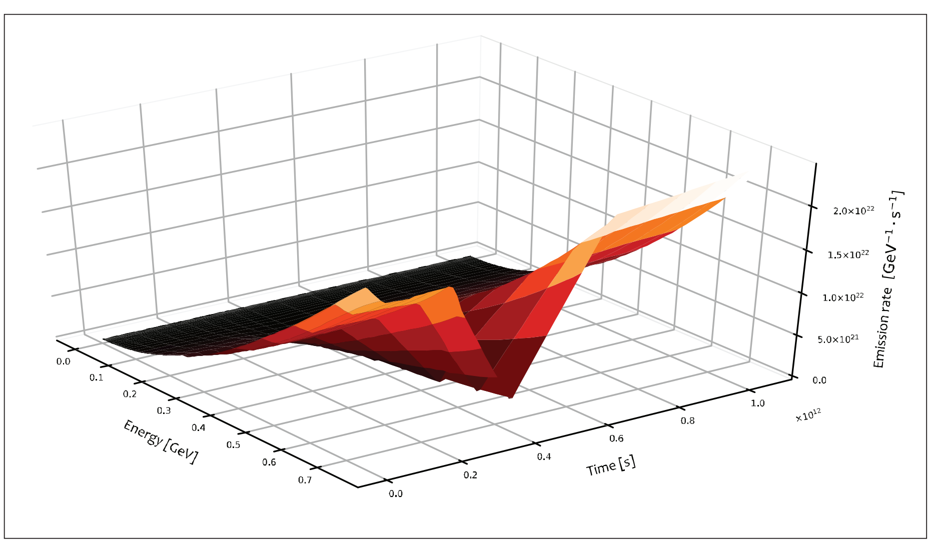

Figure 4(a): Variation of quark spectrum from a maximally rotating KBH beginning to emit coloured particles with time

Figure 4(a) shows the quark spectrum from KBH which starts emission with 5.29×10^{13} g and continues to emit particles for 1×10^{12} s. When comparing this quark spectrum from the KBH with the quark spectrum from the SBH, this dip seen in Figure 4(a) is a notable feature of the KBH spectrum. Figure 4(b) shows the gluon emission spectrum from KBH, which starts its mass with 5.29×10^{13} g and emits particles for 1×10^{12} s. When comparing with the smooth curve observed for the SBH, it can be seen that KBH has many irregularities in the gluon spectrum. These irregularities can be observed in instantaneous spectra as well (Wijewardhana & Gamalath, 2021). Another notable feature that can be observed in Figure 4(b) is that during the entire time span considered, the emission rate of gluons with energies below 0.2 GeV is zero. This is because the temperature above which the gluons can exist as free particles is the QGD temperature, 0.2 GeV. This was observed in the instantaneous spectra as well (Wijewardhana & Gamalath, 2021). In Figure 4(a) and 4(b), variations of quark and gluon spectra are plotted for a shorter time period of 1×10^{12} s. The variation of the spectra for a longer time period of 3.62×10^{13} s are given in Figure 4(c) and 4(d).

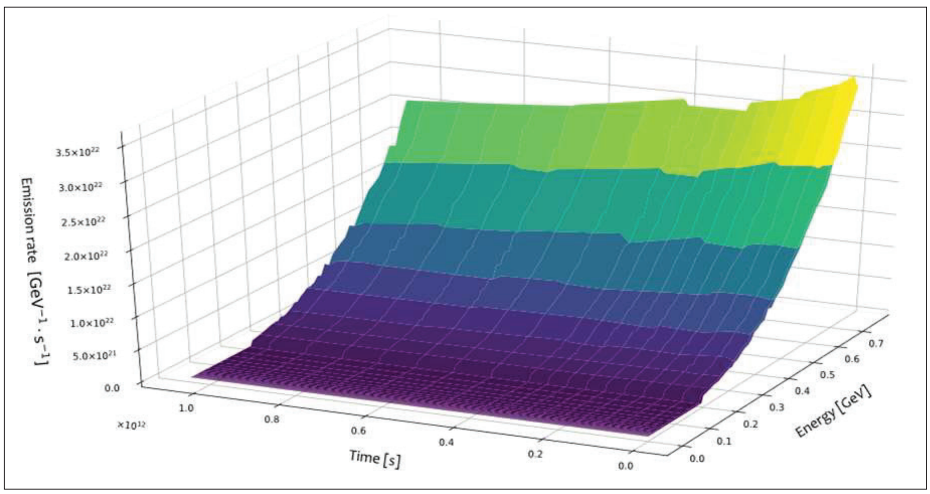

Figure 4(b): Variation of quark spectrum from a maximally rotating KBH beginning to emit coloured particles with time

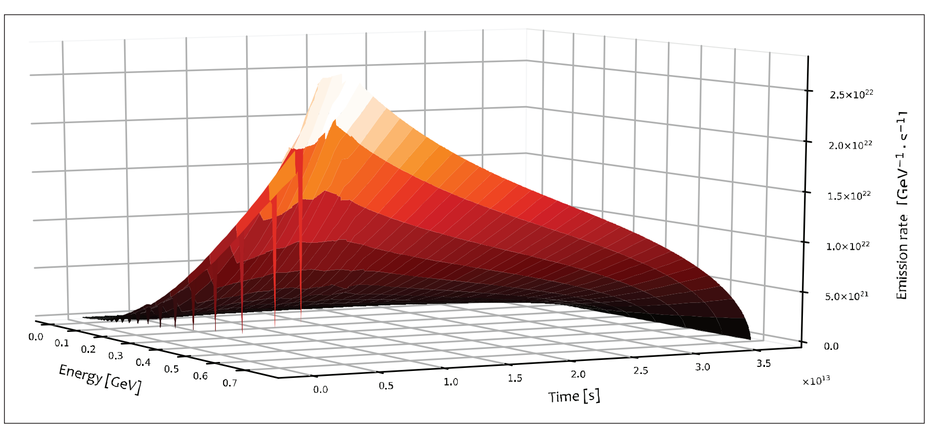

Figure 4(c): Variation of quark spectrum from a maximally rotating KBH beginning to emit coloured particles with time for an extended time span

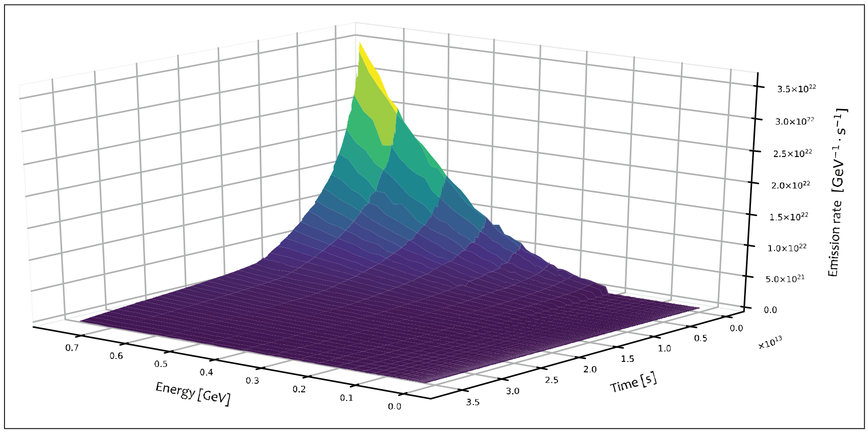

Figure 4(d): Variation of gluon spectrum from a maximally rotating KBH beginning to emit coloured particles with time for an extended time span

In Figure 7(c) plotted for an extended time period, it can be seen that there are many dips in the quark spectrum close to zero in the time axis. The dip seen in Figure 4(a) is corresponding to these minute dips observed in Figure 4(c). In fact, the spectra of KBHs have many fluctuations and variations like these.

The quark and gluon spectra presented respectively in Figure 4(c) and Figure 4(d) are plotted for the same range of energy and time. The resultant quark and gluon spectra are quite different from each other.

The variation of spectra of coloured particles is presented in Figure 5. Figure 5 shows the variation of quark and gluon spectra of a KBH of mass 1.06×10^8 g with time for 1.7×10^{-6} s. The range of time represented in the figure of the KBH is two orders of magnitude less than the range of time represented in the figure of the SBH. But the variations in spectra of the KBH are drastically large. In contrast to the SBH, the KBH emits gluons at higher rates than quarks in the energy range considered.

Figure 6 shows the variation of quark and gluon spectra of a BH with time when the BH mass reaches down to Planck mass, and a non-rotating state from a 10 g maximally rotating state. In Figure 6, the gluon spectrum looks flattened out in comparison to the quark

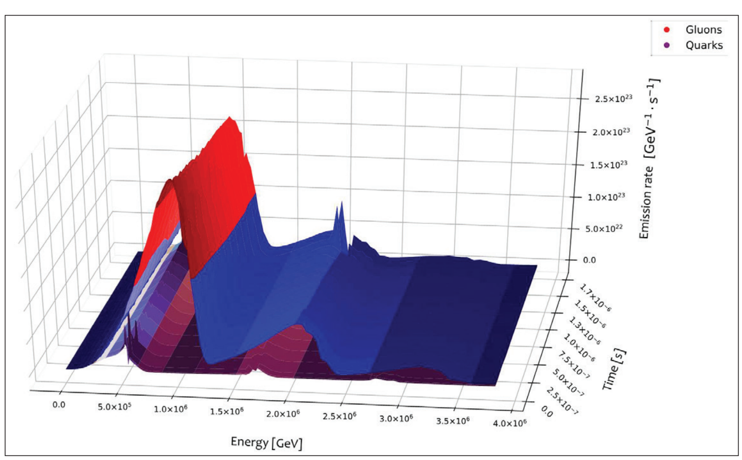

Figure 5: Variation of quark and gluon spectra from an intermediate maximally rotating KBH emitting coloured particles with time

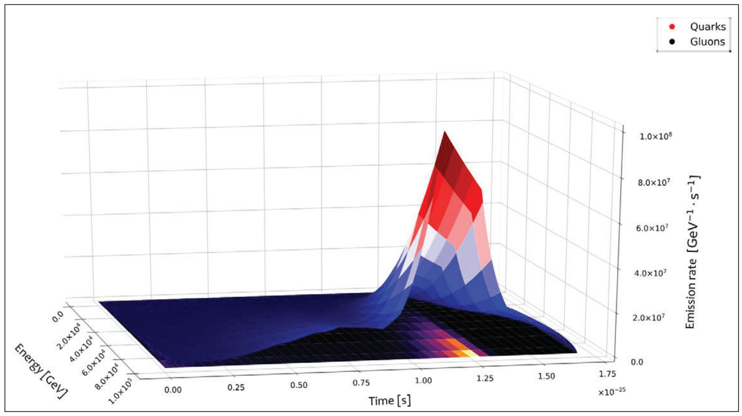

Figure 6: Variation of quark and gluon spectra with time from maximally rotating KBH reaching its end

spectrum. It can be noted that the emission rates of both quarks and gluons have been reduced from the usually observed order of magnitude 10^{22}.

Variation of spectra with black hole mass

Schwarzschild black holes

Figure 7 shows the variation of the spectra of quarks and gluons with the mass of the BH as the mass of the BH is reduced from 5.29×10^{13} to 3.53×10^{13} g. This is the same range of masses that the BH depicted in Figure 1 has. It can be observed that as the mass of the BH is decreased, the peak of the quark spectrum shifts towards higher energies. That the gluon emission rate is zero below 0.199 GeV can also be observed. This is because gluons cannot exist as free particles below the QGD temperature which is 0.2 GeV. The shapes of the surfaces are a bit similar to those observed in Figure 1. This is because the mass of the BH is a function of time. Similar to Figure 1, in Figure 7 also the quark emission dominates over gluon emission.

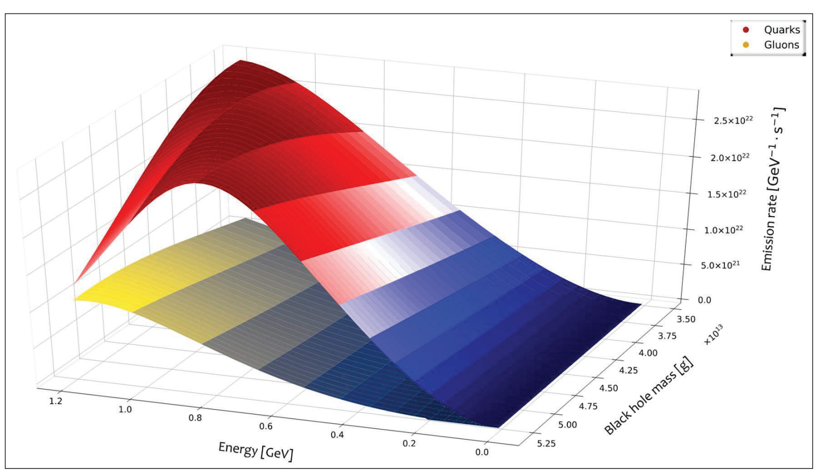

Figure 7: Variation of quark and gluon spectra from a SBH beginning to emit coloured particles with BH mass

Figure 8 shows the quark and gluon spectrum from a BH with a mass of 1.06×10^8 g, which starts emitting particles, and shows how the spectra vary as the BH gradually loses mass and reaches the Planck mass. As the mass of the SBH was reduced from 10 g to Planck mass, the resultant spectrum looked similar to Figure 3.

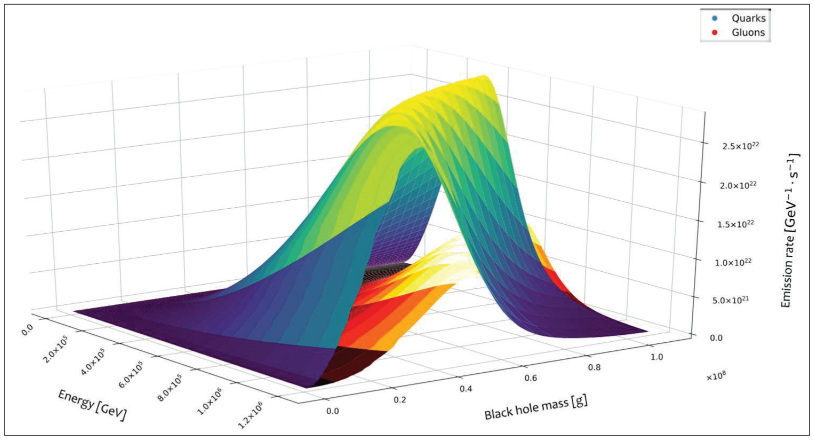

Figure 8: Variation of quark and gluon spectra from an intermediate SBH emitting coloured particles with BH mass

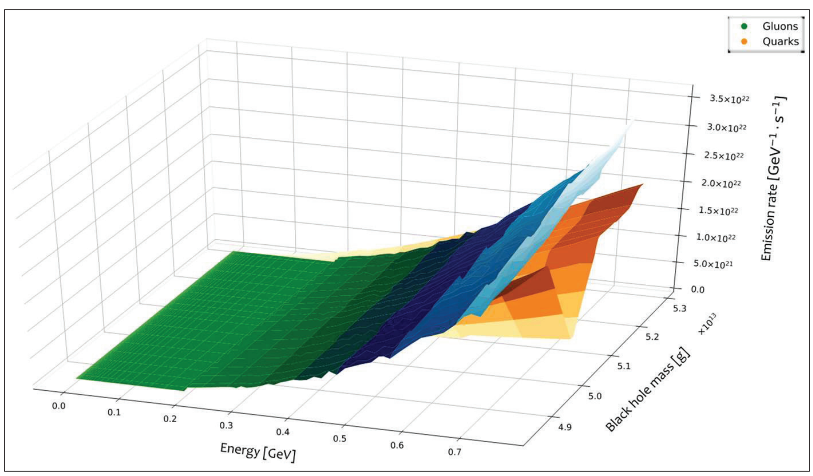

Figure 9: Variation of quark and gluon spectra from a maximally rotating KBH beginning to emit coloured particles with BH mass

Kerr black holes

Figure 9 shows the variation of the spectra of quarks and gluons with the mass of the BH as the mass of the BH is reduced from 5.29×10^{13} to 4.85×10^{13} g, which is less than the range of masses of the BH depicted in Figure 7. This is because the variation in quark and gluon spectra is higher when the mass of the BH is larger and therefore the high mass area was plotted for better comprehensibility. In the range of energies considered in Figure 9, the emission of gluons is more dominant than the emission of quarks.

Figure 10 shows the variation of quark and gluon spectra of a BH of initial mass 1.06×10^8 g with BH mass as the BH mass is reduced down to 0.75×10^8 g. Unlike the case the of SBH, KBH has many fluctuations in the spectra. Hence, the variation of spectra in a small range of BH mass is denoted. In this considered range of energies, emission of quarks is dominant when the BH mass is comparatively small in the range of BH mass considered, and emission of gluons is dominant when the BH mass is comparatively large. It can be seen that the spectrum of gluons does not have many fluctuations as in the spectrum of quarks in the mass range considered. Even in instantaneous spectra, it was seen that in some energies, one of quark or gluon spectra varied smoothly while the other had many fluctuations.

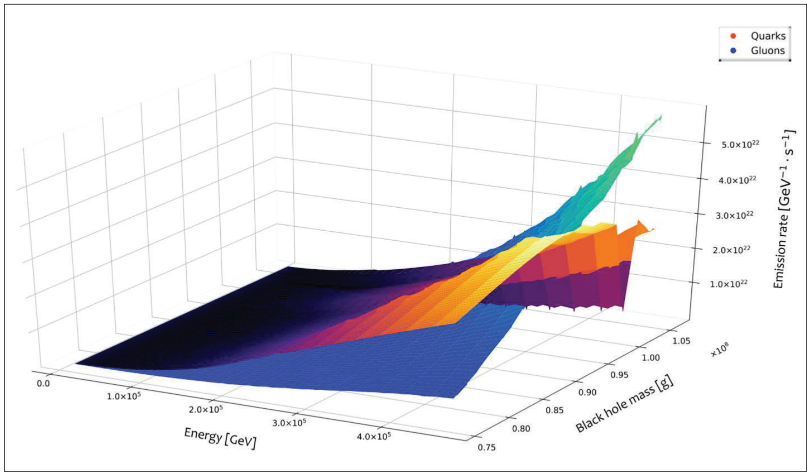

Figure 10: Variation of quark and gluon spectra from an intermediate maximally rotating KBH emitting coloured particles with BH mass

When the BH's mass is reduced from 10 g to Planck mass it was observed that the variations of spectra with BH mass were similar to the variation of spectra with time presented in Figure 6.

Variation of spectra with temperature

Schwarzschild black hole

Figure 11 shows the variation of quark and gluon spectra of a BH with temperature in Kelvins. When the temperature varies between 2.423×10^{12} and 3.5×10^{12} K as given in the figure, the variation in energy units is between 200 and 300 MeV. Thus, it is a BH that has just started to exceed the QGD temperature and beginning to emit quarks and gluons. Hence, the minimum temperature, in Kelvins, of a SBH emitting coloured particles would be $\sim 10^{12}$ K.

Figure 12 shows the variation of quark and gluon spectra with the temperature of the BH. In energy units, the temperature of this BH is 100 TeV. The graph is plotted for the energies with maximum emission rates. It can be seen that the quark emission exceeds the gluon emission at peak emission rates.

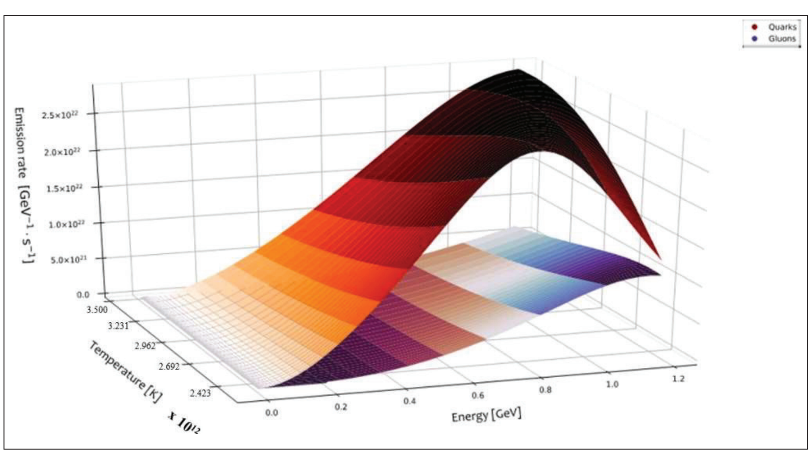

Figure 11: Variation of quark and gluon spectra from a SBH beginning to emit coloured particles with the temperature

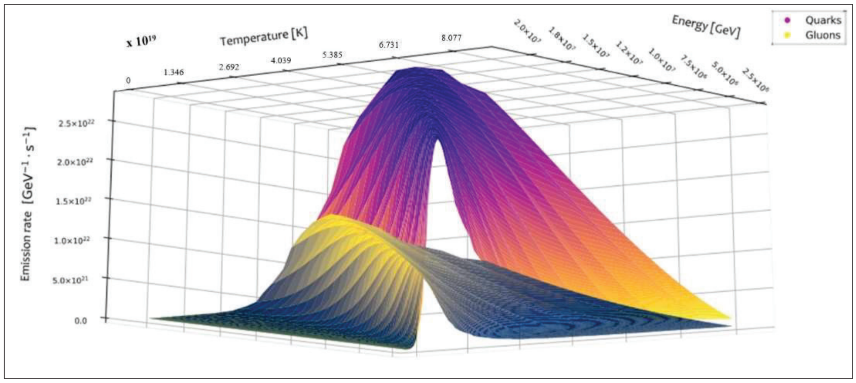

Figure 12: Variation of quark and gluon spectra from an intermediate SBH emitting coloured particles with the temperature

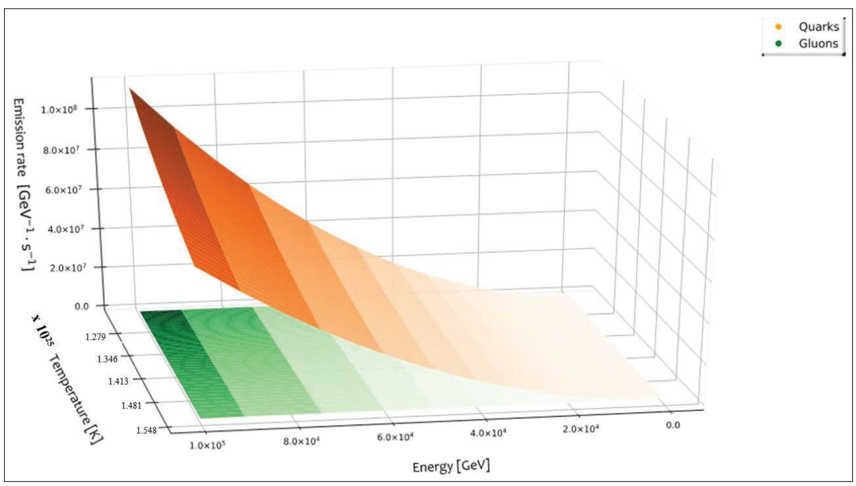

Figure 13: Variation of quark and gluon spectra from SBH reaching its end with the temperature

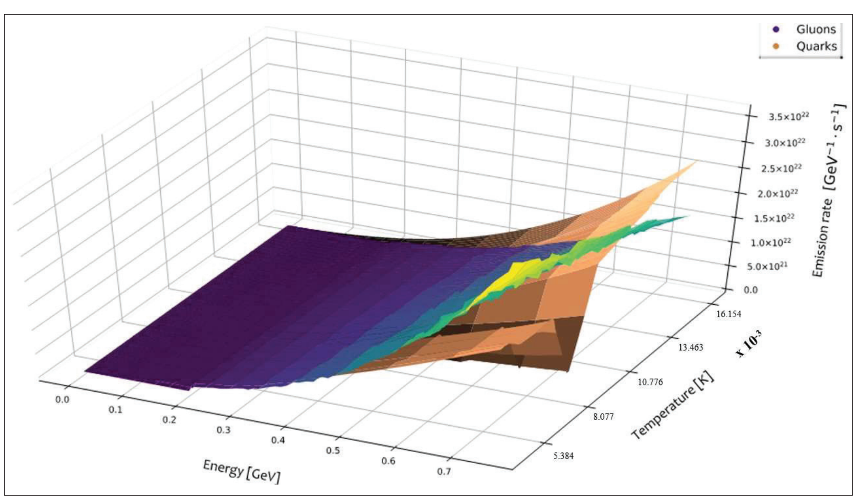

Figure 14: Variation of quark and gluon spectra from a maximally rotating KBH beginning to emit coloured particles with the temperature

Figure 13 shows the variation of quark and gluon spectra with the temperature of a BH as the BH reaches Planck mass. One notable feature is that now the temperature of the BH has reached ~10^{25} K, which is considerably closer to Planck mass when compared with a BH, which is just hot enough to radiate coloured particles.

Kerr Black Holes

Figure 14 gives the variation of quark and gluon spectra from a maximally rotating KBH with temperature. In the Kelvin scale, temperature varies between 5.384 × 10^{-3} and 16.154 × 10^{-3} K. When comparing Figure 13 and Figure 14, there is a striking difference in the orders of magnitudes of temperature scales. A rotating KBH appears to be colder than a SBH of the same mass. However, the range of temperatures considered in Figure 14 is also exceedingly small. This is because the large variations in spectra at lower temperatures of KBH make the spectra in the same range incomprehensible.

Variation of spectra with black hole entropy

Schwarzschild black hole

Figure 15 shows the variation of quark and gluon spectra with the entropy of BHs as the BH entropy varies between 1.50 × 10^{28} to 3.50 × 10^{28} cal. K^{-1}. As Figure 15 denotes a BH that started emitting particles with a mass that corresponds to a temperature that is just hot enough to radiate coloured particles, it can be observed from this graph that a SBH emitting coloured particles would have a minimum entropy of ~ 1.50 × 10^{28} cal. K^{-1}. Figure 15 shows the variation of quark and gluon spectra until the BH entropy reaches 3.50 × 10^{28} cal. K^{-1}.

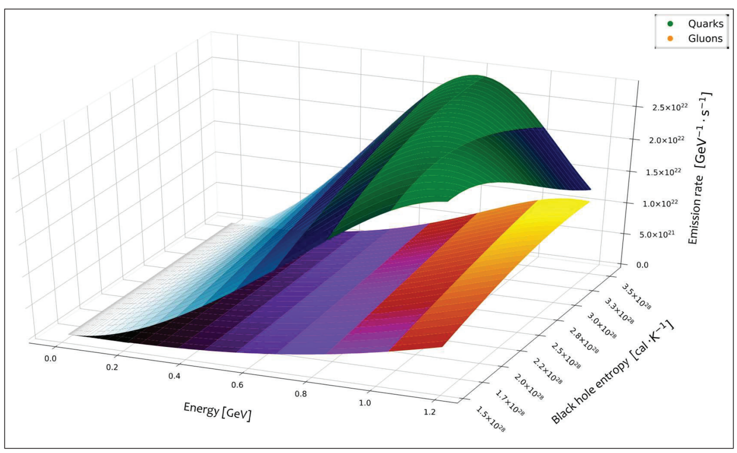

Figure 15: Variation of quark and gluon spectra from a SBH beginning to emit coloured particles with BH entropy

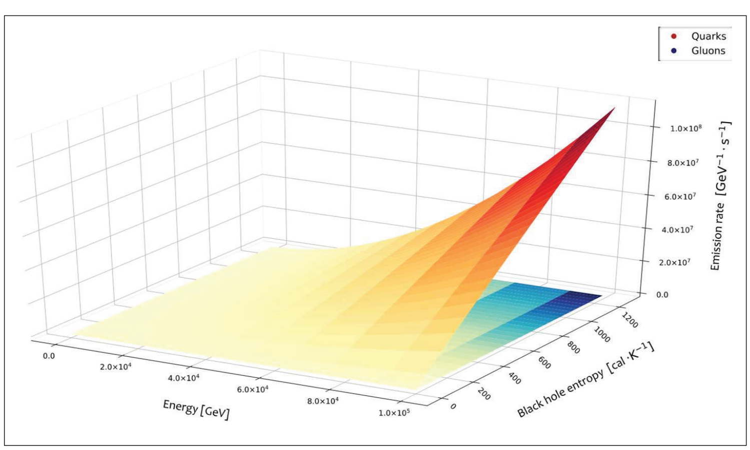

Figure 16: Variation of quark and gluon spectra from SBH reaching its end with BH entropy

When the quark and gluon emission from a SBH of mass of about 10^8 g was considered, it was observed that such a SBH would possess an entropy of the order of 10^{16} cal. K^{-1}.

Figure 16 shows the variation of quark and gluon spectra with the entropy of the BH when the BH reaches Planck mass from 10 g of mass. When the BH's mass is about 10 g, it possesses an entropy of about 1256.64 cal. K^{-1}. As Planck mass is reached, the entropy of the BH gradually decreases and gets very close to zero. Correspondingly, quark and gluon emission rates also gradually decrease and reach the limit of zero.

Kerr black holes

Figure 17 shows the variation of quark and gluon spectra with the entropy of the BH as the BH entropy varies between 2.00×10^{28} to 2.12×10^{28} cal. K^{-1}. Both Figure 15 and Figure 17 are plotted for a BH of initial mass 5.29×10^{13} g. The starting entropy values of plots imply that a maximally rotating KBH would have greater entropy than an SBH with the same mass.

Moreover, it was observed that that the entropy of a maximally rotating KBH having a mass of about 10^8 g is of the order of 10^{16} cal. K^{-1}. The emission of gluons dominated over the emission of quarks.

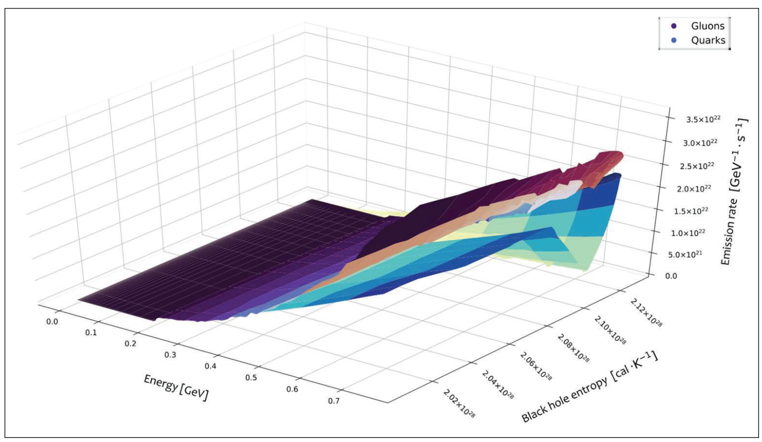

Figure 17: Variation of quark and gluon spectra from a maximally rotating KBH beginning to emit coloured particles with BH entropy

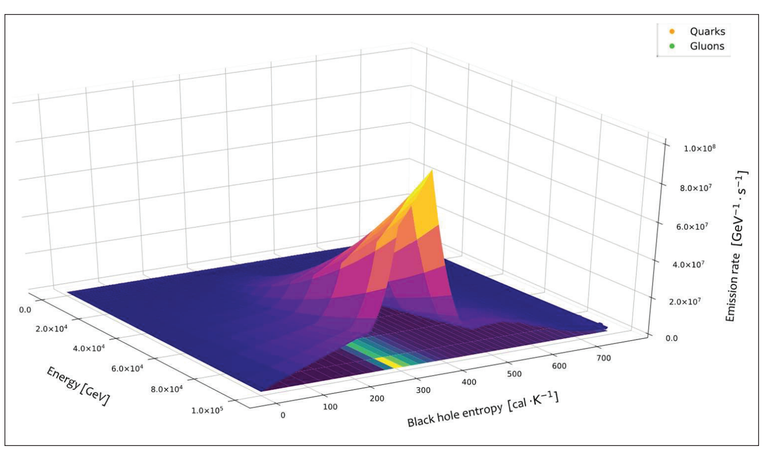

Figure 18: Variation of quark and gluon spectra from maximally rotating KBH reaching its end with BH entropy

Figure 18 shows the variation of quark and gluon spectra with the entropy of BH when the KBH reaches Planck mass from 10 g of mass. By inspecting the ranges of entropy values, it can be noted that when the BH reaches Planck mass, an SBH possesses more entropy than a KBH.

Variation of spectra with black hole spin

Kerr black hole

Figure 19 shows the variation of quark and gluon spectra with spin, of a BH with an initial mass of 5.29×10^{13} g, which loses its spin through the course of its lifetime before its mass is reduced down to Planck mass. Hence it is a plot that shows how the quark and gluon spectra vary as a rotating KBH that emits coloured particles loses its spin.

Figure 20 shows the variation of quark and gluon spectra with time for a BH that was maximally rotating when its mass was 10 g and gradually lose its spin as the BH evaporates and finally reaches Planck mass. It can be observed that the emission rates of coloured particles are comparatively higher for lower BH spins.

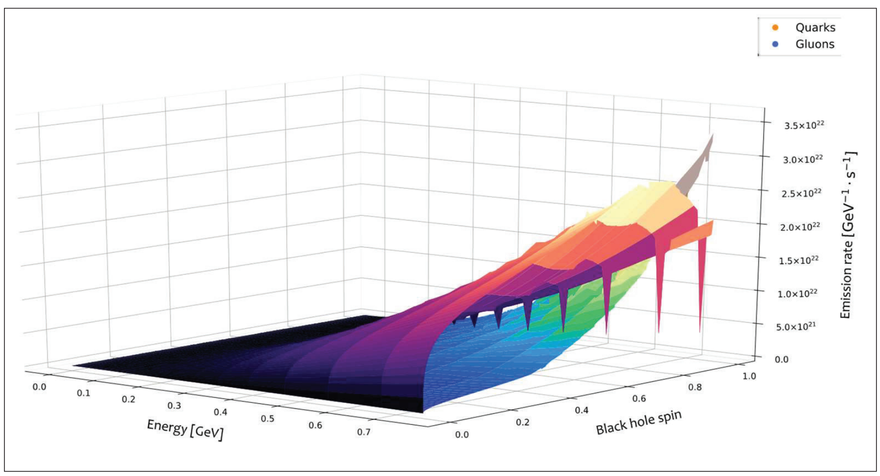

Figure 19: Variation of quark and gluon spectra from a maximally rotating KBH beginning to emit coloured particles with BH spin

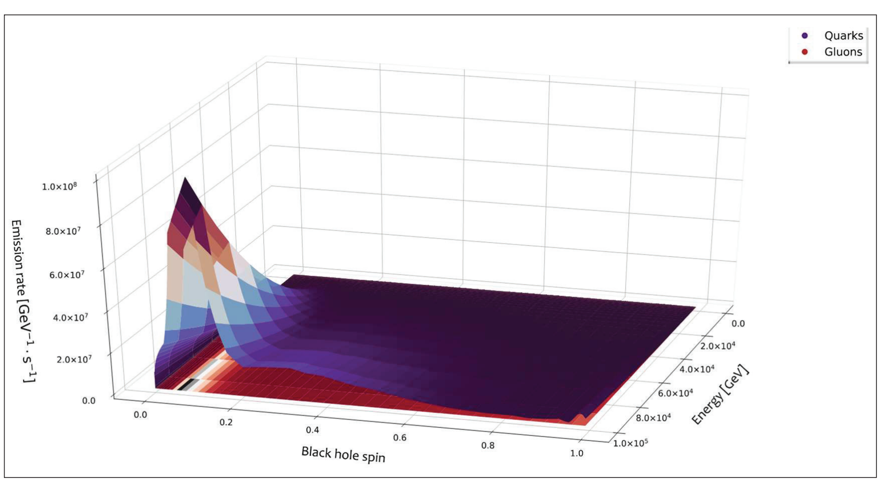

Figure 20: Variation of quark and gluon spectra from rotating KBH reaching its end with BH spin

CONCLUSIONS

The emission rate formulas in terms of black hole mass, temperature, entropy, and spin were derived and the Hawking spectra of coloured particles radiating from BHs of different masses, temperatures, entropies, and spin were investigated. Only BHs of mass ~10^{13} g or less are capable of radiating coloured particles. Emission of particles from BHs of mass ranging from ~10^{13} g to Planck mass, 2.18×10^{-5} g was investigated focusing mainly on the emission of coloured particles. Even though it is tempting to know what the spectrum would be like as the BH reaches the zero mass limit, the classical laws break down at Planck mass. Therefore, Planck mass is the lower limit of BH mass that can be investigated. Quarks, neutrinos, $e^{\pm}$, $\mu^{\pm}$, gluons, and photons are the only primary particle species that are being emitted from a BH of mass ~10^{13} g. Results show that, when the orders of magnitude of particles are disregarded, the spectrum of quarks and gluons that may be emitted from the 10^{13} g BH over 3171 millennia will be emitted from the 10^{8} g BH within 2.5×10^{-4} s. It was also observed it only takes 3.5×10^{-25} s for a SBH to reduce its mass from 10.00 g to 2.55×10^{-5} g. Hence, during the last stages of BH evaporation, it loses its mass at a drastic rate. As the BH reaches the end of its lifetime, the emission rates of the highly energetic particles gradually recede and reach the limit of zero as Planck mass is reached. It was noted that, by the time the BH begins to emit coloured particles its temperature is ~10^{12} K, but when it reaches the Planck's mass its temperature has reached ~10^{25} K. That is much closer to the Planck temperature which is ~10^{32} K, than the BH's original temperature. From the results, a rotating KBH appears to be colder than an SBH of the same mass. Moreover, results imply that a maximally rotating KBH has a greater entropy than an SBH with the same mass. As Planck mass is reached, the entropy of the BH gradually decreases and gets very close to zero.

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