J.Natn.Sci.Foundation Sri Lanka 2018 46 (1): 3-10
DOI: http://dx.doi.org/10.4038/jnsfsr.v46i1.8260

RESEARCH ARTICLE

Calculation of some of the nuclear properties of even-even $^{172-176}$Hf isotopes using IBM-1

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Revised: 21 May 2017; Accepted: 20 July 2017

Abstract: A description of the even-even Hf isotopes for A = 172 – 176 is carried out in the framework of the interacting boson model (IBM-1). The energy levels, B(E2) transition probabilities, electric quadrupole moment Q_L, and potential energy surface of those nuclei have been calculated. The calculated results are compared with the most recent experimental data. A good agreement for low lying energy states is obtained between experimental results and theoretical calculations. The contour plots of the potential energy surfaces show that the interested nuclei have rotational characters.

Keywords: B(E2) values, energy level, interacting boson model, quadrupole moment, potential energy.

INTRODUCTION

The proton-neutron interaction plays an important role in quadrupole correlations in nuclei. The stimulation energies of collective quadrupole nuclei are strongly dependent on the number of nucleons outside the closed shell (Abdul & Mushtaq, 2011). The interacting boson model constitutes a simple Hamiltonian capable of describing collective nuclear properties across a wide range of nuclei, based on general algebraic group theoretical techniques, which have found applications in problems in atomic, molecular, and high energy physics (Kellman & Herrick, 1980; Van Roosmalen *et al*., 1982). In the interacting boson model (IBM-1), proton and neutron-boson degrees of freedom are not distinguished. It has been successful in reproducing the nuclear collective levels in terms of s and d bosons, which are essentially the collective s and d pairs of valence nucleons (Otsuka *et al*., 1978; Eid & Stewart, 1985; Sharrad *et al*., 2013). As the s and d bosons span a six-dimensional Hilbert space, the IBM-1 Hamiltonian has a group structure U(6). The three limiting symmetries O(6), SU(3) and U(5) correspond to the geometrical shapes, γ-unstable rotor, symmetric rotor and spherical vibrator, respectively (Sethi *et al*., 1991). Hafnium isotopes with even number of neutrons N = 100, 102, 104 have been comprehensively investigated experimentally using different types of reactions. The excited states in the even $^{172-176}$Hf isotopes have been investigated from (γ, γ'), (d, p), (α, 2nγ), (α, 4nγ), (n, γ), (n, n'γ) and Coulomb excitation reactions (Ejiri & Hagemann, 1971; Zaitz & Sheline, 1972; Bushnell *et al*., 1974; Khoo *et al*., 1976; Kondurov *et al*., 1981; Hague *et al*., 1986; Raman *et al*., 1987; Raymond *et al*., 1993; Morikawa *et al*., 1995).

In this study, energy levels of ^{172,174,176}Hf isotopes have been calculated using IBM-1. The reduced transition probabilities [$B(E2)$], level of positive parity state, electric quadrupole moment Q_L and potential energy surface are calculated and compared with available experimental data.

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METHODOLOGY

In the IBM-1, the low energy collective properties of even-even nuclei are described by the interaction of s bosons (L = 0) and d bosons (L = 2). Moreover, the model assumes that the structure of the low-lying level is dominated by excitations among the valence particles outside the closed major shells (Eid & Stewart, 1985). The number of proton bosons N_π and neutron bosons N_ν is counted from the nearest closed shells, and the resulting total boson number is a strictly conserved quantity.

The IBM-1 Hamiltonian can be expressed as follows (Arima & Iachello, 1979; Atalay & Kaan, 2005) :

$$\mathrm{H} = \varepsilon_s(s^\dagger.\tilde{s}) + \varepsilon_d(d^\dagger.\tilde{d}) + \sum_{L=0,2,4}\frac{1}{2}(2L+1)^{\frac{1}{2}}C_L\left[[d^\dagger\times d^\dagger]^{(L)}\times\left[\tilde{d}\times\tilde{d}\right]^{(L)}\right]^{(0)} + \frac{1}{\sqrt{2}}v_2\left[[d^\dagger\times d^\dagger]^{(2)}\times\left[\tilde{d}\times\tilde{s}\right]^{(2)} + [d^\dagger\times s^\dagger]^{(2)}\times\left[\tilde{d}\times\tilde{d}\right]^{(2)}\right]^{(0)} + \frac{1}{2}v_0\left[[d^\dagger\times d^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)} + [s^\dagger\times s^\dagger]^{(0)}\times\left[\tilde{d}\times\tilde{d}\right]^{(0)}\right]^{(0)} + \frac{1}{2}u_0\left[[s^\dagger\times s^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)}\right]^{(0)} + u_2\left[[d^\dagger\times s^\dagger]^{(2)}\times\left[\tilde{d}\times\tilde{s}\right]^{(2)}\right]^{(0)} \quad ...(1)$$

This Hamiltonian contains 2 terms of one-body interactions (ε_s and ε_d) and 7 terms of two-body interactions [C_L (L = 0, 2, 4), v_L (L = 0, 2), u_L (L = 0, 2)], where ε_s and ε_d are the single-boson energies, and C_L, v_L and u_L describe the two-boson interactions. However, it turns out that for a fixed boson number N, only one of the one-body terms and five of the two-body terms are independent, as it can be seen by noting $N = n_s + n_d$. Then the IBM-1 Hamiltonian in equation (1) can be written in general form as follows (Arima & Iachello, 1978; Iachello & Arima, 1978; Casten & Warner, 1988):

$$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \quad ...(2)$$

where

$\hat{n}_d = (s^\dagger, d^\dagger)$ is the total number of d_{boson} operator,

$\hat{P} = 1/2\left[(\tilde{d}.\tilde{d}) - (\tilde{s}.\tilde{s})\right]$ is the pairing operator,

$\hat{L} = \sqrt{10}[d^\dagger\times\tilde{d}]^1$ is the angular momentum operator,

$\hat{Q} = \left[d^\dagger\times\tilde{s} + s^\dagger\times\tilde{d}\right]^{(2)} + \chi\left[d^\dagger\times\tilde{d}\right]^{(2)}$ is the quadrupole operator (χ is the quadrupole structure parameter and take the values 0 and $\pm\frac{\sqrt{7}}{2}$ (Iachello & Arima, 1978; Casten & Warner, 1988),

$\hat{T}_r = [d^\dagger\times\tilde{d}]^{(r)}$ is the octoupole ($r = 3$) and hexadecapole ($r = 4$) operator, and

$\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy.

The parameters a_0, a_1, a_2, a_3 and a_4 designate the strength of the pairing, angular momentum, quadrupole, octoupole and hexadecapole interaction between the bosons, respectively.

RESULTS AND DISCUSSION

The results for energy levels and transition energy, the transition probabilities B(E2) values and quadrupole moment Q_L, and potential energy surface are discussed separately.

Energy levels and transition energy

The rotational limit of the IBM-1 has been applied for the $^{172\text{-}176}$Hf isotopes due to the values of the $E_{4_1^+}/E_{2_1^+}$ ratio ($E_{4_1^+}/E_{2_1^+} = 3.33$) (Iachello & Arima, 1978; Casten & Warner, 1988). Figure 1 shows that the $^{174\text{-}176}$Hf isotopes have a rotational (deformed nuclei) dynamical symmetry SU(3) respecting to IBM-1. The adopted Hamiltonian is expressed as follows (Iachello & Arima, 1978; Casten & Warner, 1988).

$$\hat{H} = a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} \quad ...(3)$$

In the framework of the IBM-1, the Hafnium (Z = 72) has 5 proton-bosons holes, and 9, 10, 11 neutron-bosons holes for $^{172\text{-}176}$Hf isotopes. The best fitting for the Hamiltonian parameters are presented in Table 1, which gives the best agreement with the experimental data (Singh, 1995; Browne & Junde, 1999; Basunia, 2006).

The calculated yrast band and experimental data of low-lying states for $^{172\text{-}176}$Hf are plotted in Figure 2. This figure shows that the IBM-1 calculations for ground band (energies, spin and parity) are in good agreement

with those of the experimental data (Singh, 1995; Browne & Junde, 1999; Basunia, 2006), but it is deviated in the high spin (energies).

The β and γ bands for even $^{172\text{-}176}$Hf nuclei are shown in Tables 2 and 3, respectively. These tables show that the IBM-1 calculations are in good agreement with the experimental data for these bands.

Furthermore, we have used the root mean square deviation (RMSD) to calculate a deviation between IBM-1 energy levels and the experimental; $\mathrm{RMSD} = \left[\frac{1}{m}\sum(E_{Cal} - E_{Exp})^2\right]^{1/2}$, where m is the number of levels (Xu *et al*., 1989). Table 4 shows the RMSD between experimental and calculated energy levels.

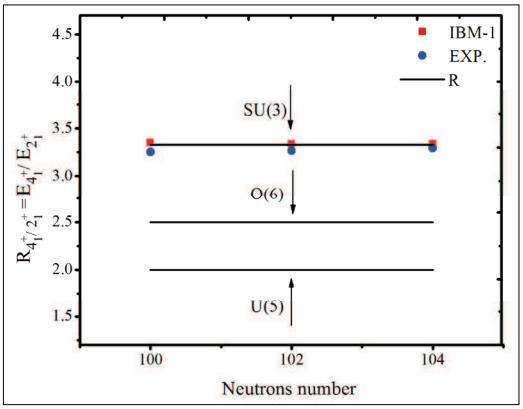

Figure 1: Comparison of the IBM-1 calculations with the available experimental data (Singh, 1995; Browne & Junde, 1999; Basunia, 2006) of the $E_{4_1^+}/E_{2_1^+}$ ratio for $^{172\text{-}176}$Hf nuclei

Table 1: Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV, except N and CHQ

A	N	ε	a_0	a_1	a_2	a_3	a_4	CHQ (χ)
^{172}Hf	14	0.000	0.000	0.0118	- 0.0105	0.000	0.000	- 1.33
^{174}Hf	15	0.000	0.000	0.0117	- 0.0095	0.000	0.000	- 1.33
^{176}Hf	16	0.000	0.000	0.0101	- 0.0121	0.000	0.000	- 1.33

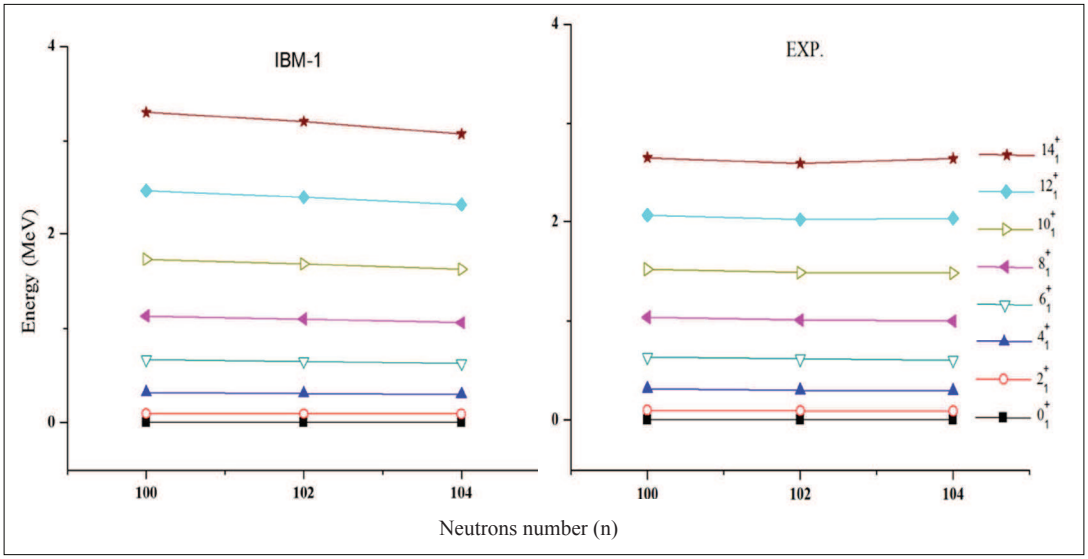

Figure 2: Comparison of the IBM-1 calculations with the available experimental data (Singh, 1995; Browne & Junde, 1999; Basunia, 2006) of the ground band for $^{172\text{-}176}$Hf nuclei

Table 2: β band (in MeV) for Hf nuclei (Singh, 1995; Browne & Junde, 1999; Basunia, 2006)

J^{π}	IBM-1	Exp.	IBM-1	Exp.	IBM-1	Exp.
	^{172}Hf		^{174}Hf		^{176}Hf	
0^+	0.8505	0.8713	0.8481	0.8281	1.2255	1.1499
2^+	0.9449	0.9524	0.9189	0.9002	1.2173	1.2266
4^+	1.1652	1.1295	1.1330	1.0622	1.4243	1.3902
6^+	1.5115		1.4695	1.3074	1.7498	1.6286
8^+	1.9836		1.9284		2.1941	1.9327
10^+	2.5816		2.5101		2.7579	2.2948
12^+	3.3055		3.2146		3.4421	

Table 3: γ band (in MeV) for Hf nuclei (Singh, 1995; Browne & Junde, 1999; Basunia, 2006)

J^{π}	IBM-1	Exp.	IBM-1	Exp.	IBM-1	Exp.
	^{172}Hf		^{174}Hf		^{176}Hf	
2^+	0.9449	1.0753	0.9403	1.2268	1.3156	1.3413
3^+	1.0394	1.1809	1.0107	1.3365	1.3061	1.4458
4^+	1.1653	1.3047	1.1554	1.4489	1.5260	1.5403
5^+	1.3226	1.4629	1.2859	1.6584	1.5722	1.7278
6^+	1.5115	1.6215	1.4937		1.8569	1.862
7^+	1.7318		1.6836		1.9570	2.1065
8^+	1.9836		1.9553		2.3092	2.2848
9^+	2.2669		2.2039		2.4610	
10^+	2.5816		2.5407		2.8837	
11^+	2.9278		2.8469		3.0848	

Table 4: Root means square deviation (RMSD) between experimental and calculated energy levels

A	RMSD (MeV)		
	g band	β band	γ band
^{172}Hf	0.252	0.024	0.132
^{174}Hf	0.156	0.090	0.321
^{176}Hf	0.175	0.224	0.095

Reduced transition probabilities B(E2) and quadrupole moment Q_L

The general form of the electromagnetic transitions operator in IBM-1 is (Iachello & Arima, 1978; Casten & Warner, 1988; Yazar & Erdem, 2008):

$$\hat{T}^{(L)} = \gamma_0[\hat{s}^+ \times \hat{s}^{\sim}]^{(0)} + \alpha_2[\hat{d}^+ \times \hat{s}^{\sim} + \hat{s}^+ \times \hat{d}^{\sim}]^{(2)} + \beta_L[\hat{d}^+ \times \hat{d}^{\sim}]^{(L)} \quad ...(4)$$

where γ_0, α_2 and β_L (L = 0, 1, 2, 3, 4) are parameters specifying the various terms in the corresponding operators. Equation (5) yields transition operators for E2 transitions with appropriate values of the corresponding parameters. The E2 transition operator must be a Hermitian tensor of rank two and therefore, the number of bosons must be conserved. With these constraints, there are two possible operators in the lowest order. The general E2 operator can be written as (Kassim & Sharrad, 2014):

$$T^{E2} = \alpha_2 [d^{\dagger} s + s^{\dagger} d]^{(2)} + \beta_2[d^{\dagger} d]^{(2)} \quad ...(5)$$

where ($s^{\dagger}$, $d^{\dagger}$) and (s, d) are creation and annihilation operators for s and d bosons, respectively (Sharrad *et al*., 2012), while α_2 and β_2 are two adjustable parameters that measure the strength of each term. The electric transition probabilities B(E2) values are defined in terms of reduced matrix elements (Iachello & Arima, 1978; 1987; Yazar & Erdem, 2008):

$$B(E2, J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \langle J_f \| T^{E2} \| J_i \rangle \right|^2 \quad ...(6)$$

For calculation of the absolute B(E2) values, the parameters α_2 and β_2 of equation (5) were adjusted accordingly to reproduce the experimental $B(E2; 2_1^+ \rightarrow 0_1^+)$. Table 5 shows the values of α_2 and β_2 parameters, which were obtained in the present calculations. The comparison of calculated values of B(E2) transitions with experimental data (Singh, 1995; Browne & Junde, 1999; Basunia, 2006) are given in Table 6, for all isotopes under study.

Table 6 shows that, in general most of the calculated results in IBM-1 are reasonably consistent with the available experimental data, except for a few cases that deviate from the experimental data.

Table 5: Parameters (in eb) used to reproduce B(E2) values for $^{172\text{-}176}$Hf nuclei

A	N	α_2	β_2
^{172}Hf	14	0.1004	- 0.2969
^{174}Hf	15	0.0940	- 0.2780
^{176}Hf	16	0.0980	- 0.2898

Table 6: Experimental and the IBM-1 values of B(E2) for $^{172\text{-}176}$Hf nuclei (in e^2b^2)

$J_i \rightarrow J_f$	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{172}Hf		^{174}Hf		^{176}Hf	
$2_1^+ \rightarrow 0_1^+$	0.8748	0.8754	0.8735	0.8775	1.0689	1.0726
$2_2^+ \rightarrow 0_1^+$	0.0000	-	0.0000	0.0013	0.0000	0.0005
$2_2^+ \rightarrow 0_2^+$	0.6845	-	0.0053	-	0.0059	-
$2_3^+ \rightarrow 0_2^+$	0.0270	-	0.7092	-	0.8647	-
$4_1^+ \rightarrow 2_1^+$	1.2353	-	1.2351	-	1.5129	-
$4_1^+ \rightarrow 2_2^+$	0.0000	-	0.0000	0.0416	0.0000	0.0140
$4_2^+ \rightarrow 2_2^+$	0.0294	-	0.4197	-	0.5198	-
$6_1^+ \rightarrow 4_1^+$	1.3320	-	1.3352	-	1.6384	-
$6_2^+ \rightarrow 4_2^+$	0.7624	-	0.8094	-	1.0047	-
$8_1^+ \rightarrow 6_1^+$	1.3512	-	1.3596	-	1.6728	-
$8_2^+ \rightarrow 6_2^+$	0.9183	-	0.9438	-	1.1756	-
$10_1^+ \rightarrow 8_1^+$	1.3299	-	1.3452	-	1.6611	-
$10_2^+ \rightarrow 8_2^+$	0.0278	-	0.9801	-	1.2267	-
$12_1^+ \rightarrow 10_1^+$	1.2806	-	1.3045	-	1.6185	-
$12_2^+ \rightarrow 10_3^+$	0.9011	-	0.0021	-	0.0022	-
$2_1^+ \rightarrow 2_1^+$	1.2529	-	1.2506	-	1.5299	-
$4_1^+ \rightarrow 4_1^+$	1.1390	-	1.1368	-	1.3904	-
$6_1^+ \rightarrow 6_1^+$	1.1162	-	1.1139	-	1.3618	-
$8_1^+ \rightarrow 8_1^+$	1.1078	-	1.1053	-	1.3504	-
$10_1^+ \rightarrow 10_1^+$	1.1038	-	1.1008	-	1.3438	-
$12_1^+ \rightarrow 12_1^+$	1.1015	-	1.0980	-	1.3386	-

The quadrupole moment (Q_L) is an important property for nuclei that can determine if the nucleus is prolate ($Q_L > 0$), deformed oblate ($Q_L < 0$) or spherical ($Q_L = 0$) shape. The electric quadrupole moments of the nuclei can be derived from the transition rate B(E2, $L_i \rightarrow L_f$) values according to equation (7) (Kassim & Sharrad, 2014):

$$Q_L = [16\pi/5]^{1/2}[L(2L-1)/(2L+1)(L+1)(2L+3)]^{1/2} \\ [B(E2, L_i \rightarrow L_f)]^{1/2} \quad ...(7)$$

where L is the angular momentum. Table 7 presents the calculation of the electric quadrupole moment Q_L within the framework of IBM-1 for the even-even Hf nuclei. The presented results for Q_L are compared with previous experimental results (Stone, 2005).

Potential energy surface (PES)

In recent years, the potential energy surface (PES) by Skyrme means-field method was mapped onto the PES of the IBM-1 Hamiltonian (Robledo *et al*., 2009; Bentley & Frauendorf, 2011; Nomura *et al*., 2011). The expectation value of the IBM-1 Hamiltonian with the coherent state ($|N,\beta,\gamma\rangle$) is used to create the IBM-1 energy surface (Iachello & Arima, 1978; Sharrad *et al*., 2012). The state $|N,\beta,\gamma\rangle$ is a product of boson creation operators ($b_c^\dagger$) over the boson vacuum $|0\rangle$, i.e.,

$$|N,\beta,\gamma\rangle = 1/\sqrt{N!}\left(b_c^\dagger\right)^N|0\rangle \quad ...(8)$$

with

$$b_c^\dagger = (1+\beta^2)^{-\frac{1}{2}}\left\{s^\dagger + \beta\left[\cos\gamma\left(d_0^\dagger\right) + \sqrt{1/2}\sin\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right]\right\} \quad ...(9)$$

The energy surface, as a function of β and γ, has been given by equation (10).

$$E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4) \quad ...(10)$$

where the α_i's are related to the coefficients C_L, ν_2, ν_0, u_2 and u_0 of equation (1). β measures the total deformation of a nucleus; when $\beta = 0$, the shape is spherical, and when $\beta \neq 0$ the shape is distorted. γ is the amount of deviation from the focus symmetry and correlates with the nucleus. If $\gamma = 0$ the shape is prolate, else if $\gamma = 60$ the shape becomes oblate.

Table 7: Electric quadrupole moment Q_L (in eb)

Q_L	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{172}Hf		^{174}Hf		^{176}Hf	
2_1^+	- 1.8971	-	- 1.8953	-	- 2.0963	- 2.10(2)
2_2^+	1.7001	-	1.7141	-	1.9059	-
4_1^+	- 2.4144	-	- 2.4121	-	- 2.6676	-
6_1^+	- 2.6559	-	- 2.6531	-	- 2.9335	-
8_1^+	- 2.7956	-	-	- 3.0865	- 2.7924	-
10_1^+	- 2.8868	-	- 2.8829	-	- 3.1851	-
12_1^+	- 2.9509	-	- 2.9461	-	- 3.2530	-

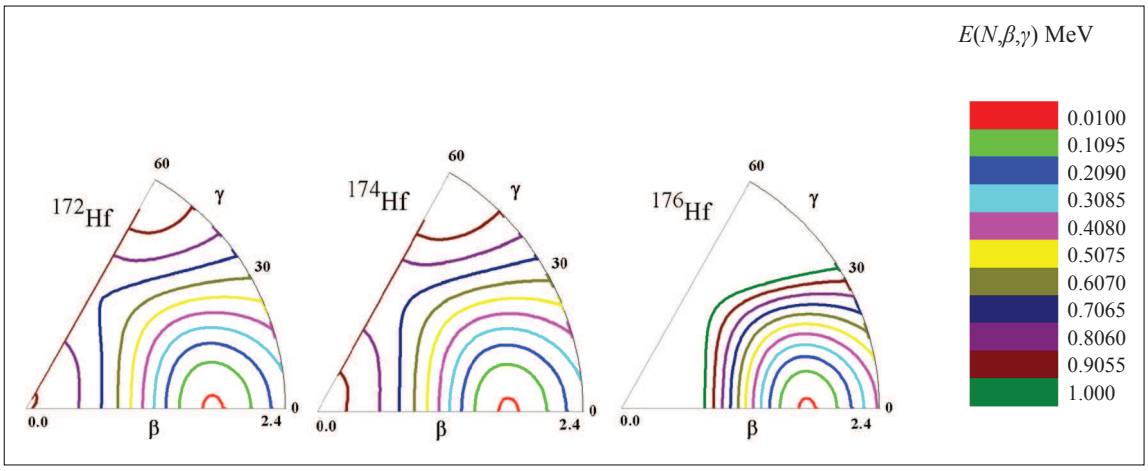

Figure 3: The potential energy surfaces for $^{172\text{-}176}$Hf nuclei

The calculated potential energy surfaces are presented in Figure 3 for the even-even $^{172-176}$Hf. In this figure, all nuclei are deformed and have rotational-like characters. The prolate deformation is deeper than oblate in all nuclei.

CONCLUSION

The even$^{172-176}$Hf isotopes in the IBM-1 have total bosons numbers of 14, 15, 16, respectively. They are considered fully rotational (fully deformed) nuclei, with dynamical symmetry SU(3). The low-lying positive parity states (energy levels), transition energy, electric transition probabilities B(E2), and electric quadrupole moment Q_L obtained for these isotopes using IBM-1 were compared with the available experimental data. The potential energy surfaces for Hf isotopes shows that all nuclei are deformed and have dynamical symmetry SU(3) characters.

Acknowledgement

The authors thank the Department of Physics, College of Science, University of Kerbala, Iraq for supporting this work.

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