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Portfolio optimization using single index model and historical value at risk (VaR) and tail value at risk (TVaR): An empirical study on the Standard and Poor's Sri Lanka 20 (S&P SL20)

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Abstract

The core of modern investment management practice involves portfolio optimization and risk measurement, particularly when markets move erratically, such as with emerging economies. The Markowitz model provides a well-established theoretical framework, but it requires extensive calculation, which can be a barrier. The Single Index Model (SIM) provides another viable alternative with fewer problems. The practical application, however, still must be extensively researched, particularly regarding to Sri Lanka and its Standard and Poor's Sri Lanka 20 (S&P SL20) Index. In this paper, the optimization of the portfolios is carried out through the S&P SL20 Index using the Single Index Model, and the impact that the selection of returns has on the measurement of simple returns or log returns is examined. The paper also examines the evaluation of the risk associated with the portfolio through the historical estimation of Value at Risk (VaR) and its corresponding Tail Value at Risk (TVaR) The selection of time intervals is based on the performance of the stock market from February 1, 2022, to November 21, 2025, which includes the post-pandemic era, the 2022 Sri Lankan economic crisis, and the subsequent recovery. The results indicate that different return measures do not substantially affect the selection of assets, but they do affect the results in terms of how much weight each asset receives, along with differences in terms of expected return and risk calculations. There is a slightly better trade-off between risk and return in portfolios using simple returns, and a more statistically stable selection in portfolios using log returns. TVaR values are also significantly higher in all portfolios than VaR values, which shows a higher risk in these difficult financial environments. Overall, it supports using SIM in Sri Lanka and highlights the importance of specifying asset return and risk measures in dealing with emerging markets.

Keywords: Portfolio optimization; Return measurement; Single Index Model (SIM); Tail Value at Risk (TVaR); Value at Risk (VaR)

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Introduction

Portfolio theory has evolved substantially since the seminal work of Markowitz, whose mean–variance framework formalized the principle of diversification by optimizing the trade-off between expected return and risk (Markowitz,1952). While theoretically robust, the Markowitz model poses significant practical challenges due to its heavy computational requirements, particularly the need to estimate a large covariance matrix. These challenges become more pronounced in emerging and less liquid markets, where data limitations and market instability can undermine estimation accuracy (Elton & Gruber,1973).

Emerging markets are generally known for their relatively high volatility, time varying risk, and greater sensitivity to the state of the economy than in more developed markets. According to Harvey (1995) the risk-return profile of emerging-market stocks differs from that of developed-market stocks, whereas, according to Bekaert and Harvey (1995) emerging markets are subject to varying degrees of integration with global markets through time. Similar points are made by De Santis and İMrohoroğlu (1997) regarding return volatility and risk dynamics. In order to overcome some of the drawbacks associated with the mean variance approach that considers full covariance, Sharpe (1963) proposed the Single Index Model. In this model, it is assumed that the return on each security depends on the movement of a single index for the entire market. In other words, according to this model, the return on each security can be split into two parts, the systematic part that depends on the beta and the firm-specific part that depends on the residual term. This approach greatly reduces the number of risk estimates needed and facilitate practical portfolio construction and optimization. Elton et al. (1976) developed simple criteria for optimal portfolio selection using the excess-return-to-beta ratio and the cutoff-rate rule.

SIM proves to be more efficient in cases when the estimation of a full covariance matrix is less precise. Rather than estimating the covariance of every pair of stocks, the model needs the beta of each stock, its residual variance, expected return, risk-free rate, and variance of the market index. The efficiency of the model lies in its use for constructing portfolios in an empiric way, which is especially relevant in the context of relatively scarce information available and unstable market conditions. According to Rouwenhorst (1998), emerging markets have return factors, which influence the stocks' return rates along with their turnover factors. However, empirical evidence suggests that the SIM is still relevant in optimizing portfolios in the emerging markets in Asia. For example, Yusup (2022) compares the Mean-Variance Model and the SIM in the stock market in Indonesia and found out that the latter could generate useful portfolio construction insights at relatively low computational effort. Another study by Yusan and Riyadi (2024) compares the use of the Markowitz model and the SIM to build efficient portfolios for LQ45 stocks in Indonesia. They found that both models could be used to optimize portfolios and different model choices would lead to varied results. Furthermore, Wijaya et al. (2024) also use SIM to optimize portfolios for health sector stocks in Indonesia and found out that the approach was effective in identifying optimal portfolio components, expected return and risk. Most recently, Gunawan and Owen (2025) use a combination of fuzzy clustering and SIMs to generate portfolios for LQ45 stocks.

Regarding Sri Lanka, the Colombo Stock Exchange becomes an ideal platform for investigating the concepts of portfolio optimization and downside risk. The Sri Lankan stock market has witnessed significant turbulence during the time of the COVID-19 pandemic and the economic crisis of 2022. IDW and A (2023) analyse volatility in the Sri Lankan stock market employing ARMA-GARCH models and provide empirical proof of volatility persistence in the All-Share Price Index. Additionally, Riyath (2025) examines the concept of volatility memory dynamics in the Colombo Stock Exchange using of ASPI and S&P SL20 indices and points out the necessity of conducting research on the phenomenon in normal times, during the COVID-19 period, and in times of crisis.

The differences between simple return and log return go beyond technical considerations. According to Hudson and Gregoriou (2014) the means calculated for returns based on logarithms might be different from the means calculated for returns based on simple rates of return, depending on the level of variance. Consequently, the choice of how returns are calculated might impact the expected return and the level of risk. In an extremely volatile country like Sri Lanka, this consideration becomes especially important due to possible discrepancies between the two methods resulting from market volatility, non-normally distributed returns, and volatility during crises. Simple-return-based and log-return-based SIM portfolios thus provide an opportunity to test the sensitivity of portfolio performance to return definition.

Value at Risk (VaR) is the most commonly used metric for calculating downside risks. VaR estimates the maximum possible loss within a particular period of time with a certain level of confidence. Bali and Cakici (2004) demonstrate that VaR has explanatory power regarding to expected stock returns. VaR has the drawback that it can identify a threshold loss but fails to assess the magnitude of losses above the threshold. The failure to determine the magnitude of loss is crucial in emerging markets because crises may cause abnormal market conditions. TVaR, otherwise known as Expected Shortfall, compensates for this problem by computing the expected loss when loss is beyond the threshold. Artzner et al. (1999) propose coherent risk measures and stress the importance of features like subadditivity in evaluating risk within a portfolio. Acerbi and Tasche (2002) elaborate on how Expected Shortfall is a coherent risk measure compared to VaR. It is worth noting that there is ongoing research involving VaR and Expected Shortfall today. For instance, Meng and Taylor (2019) devise new ways of estimating VaR and Expected Shortfall based on intraday low and range observations. Meanwhile, Trucíos and Taylor (2022) assess the accuracy of methods for forecasting VaR and Expected Shortfall of crypto currencies, The process of measuring risks needs backing from back testing as well. It is only meaningful if the estimated VaR gives valuable information on loss exceedances. According to Kupiec (1995), the unconditional coverage test is used for testing the reliability of risk models by comparing the number of expected VaR violations with observed cases. The method is further developed by Christoffersen (1998) in relation to assessing the accuracy of interval forecasting as well as conditional coverage. It is essential that the methods can help verify the practicality of VaR models.

These conditions provide a robust empirical setting to test the effectiveness of SIM, the sensitivity of portfolio outcomes to return measurement, and the behaviour of downside risk in an emerging market context. By addressing these dimensions jointly, this research contributes to the limited empirical literature on portfolio optimization in Sri Lanka, extends the application of the SIM to the S&P SL20 Index, and offers practical insights for investors, analysts, and policymakers regarding return measurement choices and risk management in emerging equity markets.

Methodology

Data preparation

The study utilizes daily closing price data for the companies included in the S&P SL20 Index, which represents the 2 January 2022 to 21 November 2025 largest and most liquid listed firms on the Colombo Stock Exchange. The data spans the period Daily closing prices were obtained from the CSE Data Library, which provides SL20 stock prices, the ASPI (All Share Price Index) as the market benchmark, and trading dates.

The dataset was sorted chronologically, and missing observations were carefully examined to distinguish between market-wide non-trading days and stock - specific missing values. Where prices were missing due non-trading or unavailable observations, the data were handled consistently to preserve the continuity of the time series. Market and stock data were sourced from CSE, while the risk-free rate was derived from the 3-month Treasury Bill rate reported by the Central Bank of Sri Lanka. The annualized rate was converted to a daily rate using the formula:

$$R_f^{daily} = (1 + R_f^{annual})^{1/365} - 1 \qquad (01)$$

yielding a daily rate of 0.000291456. This rate was held constant throughout the optimization period, consistent with the assumption of a stable short-term risk-free environment.

Return calculation

Returns were computed via two methods, simple and log return which ensure robustness and comparability.

$$R_t = \frac{P_t}{P_{t-1}} - 1 \qquad (02)$$

Simple return captures the arithmetic percentage changes in price over a single period. It is widely used for short-term performance evaluation.

The log return is defined as:

$$R_t = ln\, \frac{P_t}{P_{t-1}} \qquad (03)$$

which is suitable for continuous compounding and possesses time additivity, making them particularly suitable for econometric modelling and long-term analysis. The differences between the simple and logarithmic returns are significant since these two types of returns have different properties. Simple returns are more intuitively interpretable as they correspond to the actual increase or decrease in wealth within one investment period. Logarithmic returns are characterized by statistical properties that are advantageous for empirical analysis and computational modelling and they allow for easy calculations of expected returns, beta coefficient, residual variance, and weights of the portfolio. The importance of the difference between these two types of returns becomes apparent during periods of economic and financial instability, price volatility, changes in policy and economic recovery.

Single Index Model (SIM)

The Single Index Model (SIM), developed by Sharpe (1963), relates individual security returns to the market index return. The fundamental premise is that most of a stock's movement can be explained by the overall market's movement, thereby simplifying the portfolio analysis process. For each stock i, the relationship is expressed as:

$$R_i = \alpha_i + \beta_i R_M + \epsilon_i \tag{04}$$

where:

R_i : Return on security i

α_i : Alpha or intercept term (stock's excess return independent of the market)

ϵ_i : Random error term (firm-specific risk) with $E(\epsilon_i) = 0$.

β_i : Beta coefficient (stock's sensitivity to market movements)

R_M : Return on the market index (ASPI for the Sri Lankan market)

The SIM separates total risk into two components.

Systematic risk (captured by Beta):

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)} \tag{05}$$

Interpretation of β:

$\beta > 1$ Stock is more volatile than the market (aggressive)

$\beta = 1$ Stock moves proportionally with the market

$\beta < 1$ Stock is less volatile than the market (defensive)

Unsystematic (idiosyncratic) risk:

Unsystematic risk is captured by the residual variance $\sigma^2_{\varepsilon_i}$, which reflects firm-specific risk not explained by market movements and this risk can be reduced by diversification.

SIM-based portfolio construction

Optimization of the Portfolio was done based on excess return to beta of the SIM. The objective of this approach is to find stocks that have high excess return with respect to their systematic risk.

The SIM portfolio optimization procedure includes the following steps, and it must also satisfy these two assumptions.

a. The residual error of the stock i is uncorrelated with the residual error of the stock j for all values of i and j;

$$Cov(\varepsilon_{i,}\varepsilon_{j,}) = 0 \quad i \neq j$$

This implies that firm-specific risk for one stock is unrelated to firm-specific risk for another stock. After the impact of the market return on the stock returns has been stripped away, any leftover variations in the returns on one stock would be unrelated to the leftover variations in the returns on another stock.

b. The residual error of the stock i is uncorrelated with market return.

$$Cov(\varepsilon_{i,}R_M) = 0$$

The assumption must be made for clear differentiation between systematic and unsystematic risks. The reason why the correlation between the residual and the return on market must be equal to zero is the existence of possible biases in estimation of beta coefficient which means that it cannot completely capture the effects of market movements.

Excess Return to Beta (ERB):

For each stock:

$$ERB_i = \frac{E(R_i) - R_f}{\beta_i} \tag{06}$$

where $E(R_i)$ is the expected return of stock i and R_f is the risk-free rate and R_f is the risk-free rate.

Rank stocks by ERB:

Stocks are ranked in descending order of ERB_i so that assets offering the highest excess return per unit of systematic risk are considered first. A_i and B_i**:**

For each stock:

$$A_i = \frac{\left(E(R_i) - R_f\right)\beta_i}{\sigma_{\varepsilon_i}^2} \tag{07}$$

$$B_i = \frac{\beta_i^2}{\sigma_{\varepsilon_i}^2} \tag{08}$$

$\sigma_{\varepsilon_i}^2$ is the residual variance of stock i.

These quantities captured the trade -off between expected return, systematic risk and unsystematic risk and much needed for determining the optimal cutoff rate (C^*).

Cutoff rate C^*:

The cumulative values of A_i and B_i are used to compute a sequence of cutoff points. The optimal cutoff C^* is obtained using a market-variance-based formulation and defined as the maximum cutoff value obtained from the ranked securities. Stocks with ERB_i greater than C^* are included in the optimal portfolio. This criterion ensures that only stocks contributing positively to the portfolio's risk-adjusted return are selected.

$$C_i = \frac{\sigma_M^2 \sum_{j=1}^{i} A_j}{1 + \sigma_M^2 \sum_{j=1}^{i} B_j} \quad (09)$$

Initial weights W_i:

After identifying the selected securities, the initial weight component for each selected stock was calculated as,

$$W_i = \frac{(ERB_i - C^*)\beta_i}{\sigma_{\varepsilon_i}^2} \quad (10)$$

where W_i represents the unnormalized weight component of stock i. Stocks with higher excess return to beta ratios and lower residual variances receive larger initial weight components.

Normalize weights:

The initial weights are normalized to ensure that portfolio weights sum to 1 (100% of the portfolio).

$$w_i = \frac{W_i}{\sum_{j=1}^{N} W_j} \quad (11)$$

Where N is the number of stocks in the optimal portfolio. This normalization ensures proper portfolio composition: all weights are non-negative (no short selling), and their sum equals 1.

Return of portfolio:

The return of the optimized portfolio at time t was calculated as the weighted average of the returns of the selected stocks, which reflecting the overall performance of the optimized SIM -based portfolio.

$$R_{p,t} = \sum_{i=1}^{n} (w_i R_{i,t}) \quad (12)$$

where $R_{p,t}$ is the portfolio return at time t, w_i is the portfolio weight of stock i, and $R_{i,t}$ is the return of stock i at time t.

Risk-adjusted performance evaluation

Other risk-adjusted performance metrics were also considered apart from the already known expected returns, variance, standard deviations, and betas since just the expected returns and variance may fail to accurately reflect the efficiency of the portfolio, especially if it is in emerging and unpredictable markets.

The Sharpe Ratio was used to measure excess return per unit of total risk

$$SR_p = \frac{E(R_p) - R_f}{\sigma_p} \tag{13}$$

where SR_p is the Sharpe Ratio, $E(R_p)$ is the expected portfolio return, R_f is the risk-free rate, and σ_p is the portfolio standard deviation. A higher Sharpe Ratio indicates better compensation for total risk.

The Treynor Ratio was used to measure excess return per unit of systematic risk

$$TR_p = \frac{E(R_p) - R_f}{\beta_p} \tag{14}$$

where TR_p is the Treynor Ratio and β_p is the portfolio beta. A higher Treynor Ratio indicates better compensation for market-related risk.

Estimating risk measures

This stage focuses on estimating downside risk measures using the historical simulation approach.

Determination of percentiles

Percentile estimation is used to identify the expected portfolio loss that an investor may incur at a given significance level α. The location of the percentile is calculated as:

$$P_\alpha = \alpha \times n \tag{15}$$

where P_α represents the position of the α-percentile and n is the number of observations.

Estimating Value at Risk (VaR)

The VaR technique gauges the amount of possible losses for a certain investment period at a particular confidence level. Historical VaR is calculated based on confidence levels of 95% and 99% in this research. The 95% confidence level captures moderate downside risks, whereas the 99% confidence level captures tail events or extremes.

$$\mathrm{VaR}_{1-\alpha} = -V_0 P_\alpha \sqrt{t} \tag{16}$$

where V_0 denotes the initial investment value, P_α is the α-percentile return, and t represents the holding period.

Estimating Tail Value at Risk (TVaR)

TVaR, also known as Expected Shortfall which measures the expected loss conditional on the loss exceeding the VaR threshold. It provides additional information about the severity of losses in the tail of the distribution.

$$\mathrm{TVaR}_{1-\alpha}(X) = E\,[X \mid X > \mathrm{VaR}_{1-\alpha}(X)] \tag{17}$$

where X denotes the portfolio loss random variable.

Results

After calculating the expected returns and variances of SL20 stocks and the ASPI market index, regressions were estimated to obtain alphas, betas, and residual variances for each stock against market returns. (Table 1)

Table 1: Alpha, beta, and residual variances of stock simple returns

Ticker	Stock Name	Alpha	Beta	Residual Variance
VONE	VALLIBEL ONE PLC	-0.000509	1.980290	0.000457
SAMP	SAMPATH BANK PLC	0.000632	1.001599	0.000296
RCL	ROYAL CERAMICS LANKA PLC	-0.001405	1.639854	0.000393
NTB	NATIONS TRUST BANK PLC	0.001520	0.754896	0.000335
NDB	NATIONAL DEVELOPMENT BANK PLC	0.000353	1.036256	0.000387
MELC	MELSTACORP PLC	0.000693	1.089960	0.000283
LOLC	L O L C HOLDINGS PLC	-0.001671	1.842497	0.000508
LLUB	CHEVRON LUBRICANTS LANKA PLC	0.000197	0.704123	0.000293
LIOC	LANKA IOC PLC	0.000178	1.482159	0.000939
LFIN	LB FINANCE PLC	0.000452	0.943326	0.000311
JKH	JOHN KEELLS HOLDINGS PLC	-0.000861	0.595093	0.001033
HNB X	HATTON NATIONAL BANK PLC	0.000699	0.810281	0.000376
HNB	HATTON NATIONAL BANK PLC	0.000757	0.856594	0.000246

HAYL	HAYLEYS PLC	-0.000295	1.469047	0.000313
DIAL	DIALOG AXIATA PLC	0.000952	0.408804	0.000283
DFCC	DFCC BANK PLC	0.000744	0.870292	0.000457
COMB X	COMMERCIAL BANK OF CEYLON PLC	0.000620	0.884482	0.000315
COMB	COMMERCIAL BANK OF CEYLON PLC	0.000523	0.957942	0.000215
CFIN	CENTRAL FINANCE COMPANY PLC	0.000787	0.997799	0.000381
CCS	CEYLON COLD STORES PLC	-0.000152	0.590050	0.001417

The average correlation between the absolute residuals was calculated to be 0.120 for the simple return model and 0.116 for the log return model (Table 2). Both are low figures, and the results imply that the co-movement of the residuals was minimal, once we control for the market returns. The variance of the residuals can thus be estimated as being only about 1.44% and 1.35% respectively, based on the square of each of the two correlations mentioned above. Considering that perfectly uncorrelated residuals are not expected from financial returns data, the findings suggest that the SIM residual independence assumption holds quite well empirically.

Table 2: Diagnostic assessment of SIM residual independence assumption

Diagnostic measure	Simple returns	Log returns
Average absolute residual correlation	0.120	0.116
Squared average correlation	0.0144	0.0135

After performing SIM portfolio optimization using both simple and log returns, two optimized portfolios were constructed (Table 3).

Table 3: List of optimized portfolio stocks

Stock Ticker (From Simple Return)	Stock Ticker (From Log Return)
NTB	NTB
DIAL	DIAL
HNB	HNB
DFCC	CFIN
HNB X	HNB X
CFIN	DFCC
COMB X	MELC
MELC	COMB X
SAMP	SAMP

The optimization selected the same nine stocks under both return calculation methods, though with slight variations in the ordering and inclusion of CFIN/DFCC and MELC/COMB X. This consistency suggests that the core composition of the optimal portfolio is robust to the choice of return metric.

To further evaluate portfolio performance, the Sharpe Ratio and Treynor Ratio were calculated using Equation (13) and Equation (14), respectively. For the simple-return portfolio, the expected portfolio return was 0.0016, the daily risk-free rate was 0.000291456, the portfolio standard deviation was 0.01382, and the portfolio beta was 0.7361 (Table 4).

Table 4: Portfolio weights and performance metrics

Metric	Simple Return Portfolio	Log Return Portfolio
Number of Selected Stocks	9	9
Cutoff C*	0.0010	0.0008
Portfolio Beta	0.7361	0.7152
Expected Return	0.0016	0.0014
Portfolio Variance	0.000191	0.000193
Standard Deviation	0.01382	0.013895

Using these values, the Sharpe Ratio was calculated as 0.0955, while the Treynor Ratio was calculated as 0.0018. For the log-return portfolio, the expected portfolio return was 0.0014, the daily risk-free rate was 0.000291456, the portfolio standard deviation was 0.013895, and the portfolio beta was 0.7152. Based on these values, the Sharpe Ratio was calculated as 0.0832, while the Treynor Ratio was calculated as 0.0016 (Table 5).

Table 5: Summary of risk- adjusted performance ratios

Measure	Simple Return Portfolio	Log Return Portfolio
Sharpe Ratio	0.0955	0.0832
Treynor Ratio	0.0018	0.0016

The higher Sharpe Ratio of the simple-return portfolio indicates that it generated a higher excess return per unit of total risk compared with the log-return portfolio. Similarly, the higher Treynor Ratio of the simple-return portfolio indicates that it provided better compensation per unit of systematic market risk. Therefore, both risk-adjusted performance measures suggest that the simple-return portfolio performed marginally better than the log-return portfolio during the sample period as verified by simple return portfolio recording a higher expected return and slightly lower variance.

Given the superior risk–return profile of the simple-return portfolio, downside risk was evaluated using the Historical Simulation method. Daily portfolio returns were computed as the weighted sum of constituent stock returns (using the simple-return weights), sorted in ascending order, and the worst 5% of observations were used to estimate risk measures for an initial capital of LKR 1,000,000,

The downside-risk analysis showed that the potential losses of the simple-return portfolio increased as the confidence level became more stringent. For an initial investment of LKR 1,000,000, the portfolio recorded a VaR of LKR 18,161.69 and a TVaR of LKR 25,242.36 at the 95% confidence level, indicating that losses exceeding LKR 18,161.69 were expected on about 5% of trading days, with an average loss of LKR 25,242.36 beyond that threshold. At the 99% confidence level, the VaR and TVaR increased to LKR 29,625.20 and LKR 36,185.83, respectively (Table 6).

Table 6: Historical VaR and TVaR estimates for the simple-return portfolio

Portfolio	VaR 95%	TVaR 95%	VaR 99%	TVaR 99%
Simple-return portfolio	LKR 18,161.69	LKR 25,242.36	LKR 29,625.20	LKR 36,185.83

The results also show that TVaR is higher than VaR at both confidence levels. This is expected because VaR identifies only the threshold loss, whereas TVaR measures the average loss beyond that threshold. Therefore, the higher TVaR values suggest that losses exceeding the VaR threshold may be considerably more severe than the threshold loss itself. This finding highlights the importance of using TVaR together with VaR when evaluating portfolio risk in volatile emerging markets such as Sri Lanka.

VaR back testing was conducted to assess whether the estimated VaR thresholds were reliable when compared with realized portfolio losses. At the 95% confidence level, the expected violation rate is 5%, the expected number of violations was 46.20. The actual number of observed violations was 47, producing an actual violation rate of 5.09%. This value is very close to the expected violation rate of 5%, indicating that the 95% VaR estimate is empirically reliable (Table 7).

Table 7: VaR back testing results for the simple-return portfolio

Portfolio	Confidence Level	Expected Violation Rate	Expected Violations	Actual Violations	Actual Violation Rate
Simple-return portfolio	95%	5%	46.20	47	5.09%
Simple-return portfolio	99%	1%	9.24	10	1.08%

The expected violation frequency at 99% confidence level is 1%, which gives us an expected violation of 9.24. The actual violation frequency is 1.08%, since the actual number of violations is 10. This figure is again very close to the expected violation of 1%, and hence, we can assume that the VaR measure at 99% confidence level is also acceptable. Thus, based on the back-testing analysis, the Historical VaR measure is reasonably estimated.

Overall, the VaR back testing results strengthen the reliability of the downside-risk analysis. The observed violation rates at both confidence levels are close to their expected theoretical rates, meaning that the estimated VaR thresholds are not substantially underestimating or overestimating portfolio risk.

Discussion

This study presents a broad empirical evaluation of portfolio optimization and risk measurement using the SIM for the S&P SL20 Index, with a emphasis on return measurement and evaluation of downside risk. The results of the regressions indicate large variations in alpha, beta, and residual variances across stocks of SL20, reflecting differences in market responsiveness and firm-specific risk. Several stocks reveal betas lower than unity, indicating they exhibit defensive characteristics while others tend to respond more strongly to market movements. Theoretically, these findings correspond to the basics of SIM, which assume that systematic market risk is dominant in explaining returns of assets, particularly in the equity markets of a country like Sri Lanka, which is relatively concentrated.

The validity of SIM assumptions is essential for reliable portfolio construction. The low average absolute correlations among residuals and the negligible correlation of residuals with market returns-both for simple and log returns-suggest that the two critical assumptions of residual independence and orthogonality with the market are fairly satisfied. This empirical validation reinforces confidence in the applicability of SIM to the Sri Lankan market.

The portfolio beta values provide important insight into the risk profile of the optimized portfolios. The simple-return portfolio recorded a beta of 0.7361, while the log-return portfolio recorded a beta of 0.7152. Since both beta values are below one, the optimized portfolios can be interpreted as defensive portfolios. Indicating that both portfolios are less sensitive to movements in the broader market index. This result is particularly meaningful in the Sri Lankan context, as the sample period includes post-pandemic recovery, the 2022 economic crisis, interest-rate adjustments, inflationary pressure, and subsequent market stabilization. For risk-averse investors, portfolios with beta values below one may be attractive because they provide lower exposure to broad market fluctuations.

The comparison of the portfolios shows that the simple-return portfolio performed slightly better over the study period. The expected return of the simple-return portfolio was 0.0016, which was higher than the 0.0014 recorded for the log-return portfolio. Based on these results, the simple-return portfolio appears to offer a slightly more favourable risk-return balance. One possible reason is that simple returns represent the actual percentage change in portfolio value over a single period, making them more intuitive and practical for evaluating daily investment performance.

Also, these results consistently indicate that the portfolio constructed using simple returns performed slightly better than the portfolio based on log returns. Higher Sharpe and Treynor Ratios suggest that the simple-return portfolio provided better risk-adjusted performance and more favorable compensation for both total and systematic risk. The diagnostic analysis showed cross-sectional correlations among residuals, supporting the applicability of the SIM, although some common influences beyond market movements may still exist.

The downside-risk analysis revealed that potential losses increase at higher confidence levels, with TVaR values exceeding VaR values in all cases. This indicates that losses beyond the VaR threshold can be considerably larger, highlighting the importance of considering tail risk in portfolio management. The VaR back testing results closely aligned with expected violation frequencies, confirming the reliability of the estimated risk measures.

Overall, the findings have important implications for investors, fund managers, and policymakers. The results indicate that the choice of return measure can influence portfolio weights and performance outcomes, emphasizing the importance of clearly specifying the return methodology used in portfolio construction. In addition, the VaR and TVaR analyses highlight the value of incorporating downside-risk measures into investment decision-making and market-risk monitoring. However, the findings should be interpreted in light of certain limitations, including the exclusion of transaction costs, taxes, liquidity constraints, and rebalancing costs. Future studies could address these factors and explore alternative portfolio optimization approaches to provide a more comprehensive understanding of risk and return in the Sri Lankan equity market.

Conclusion

This study investigates portfolio optimization and risk assessment of the S&P SL20 Index using the Single Index Model, focusing on return measurement and downside risk through Historical VaR and TVaR. By integrating these aspects within a unified framework, the study addresses gaps in Sri Lankan financial literature.

The results confirm the effectiveness of the SIM in the Sri Lankan stock market. Despite market inefficiencies, liquidity constraints, and data limitations, the model's assumptions hold reasonably well for SL20 stocks, supporting its use as a computationally efficient alternative to the Markowitz mean–variance framework.

A key finding is that the choice between simple and logarithmic returns does not affect the optimal stock selection, indicating that systematic risk (beta) dominates portfolio construction. However, return specification influences portfolio weights, expected returns, and risk levels.

The simple return-based portfolio shows slightly better performance in terms of higher expected return and lower variance over the sample period, suggesting greater practical relevance for short- to medium-term investors in emerging markets where price movements are discrete.

Incorporating Historical VaR and TVaR adds a downside risk perspective beyond volatility. The results show that TVaR exceeds VaR, highlighting potential extreme losses during stress periods and reinforcing its importance in capital risk assessment.

Overall, this research presents one of the first applications of the Single Index Model combined with VaR and TVaR for the S&P SL20 Index, demonstrating its usefulness for portfolio optimization and risk management in Sri Lanka. Future work may extend the analysis by incorporating alternative optimization models.

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Conflict of interest statement

The authors declare no conflict of interest.

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