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ORIGINAL ARTICLE

Reduced Equations for K UTM Scale Factor and GNSS Usage in Chilean Cadastral Environments

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ABSTRACT

Topographic distances from global navigation systems (GNSS) equipment are being progressively involved in legal land parcel registration applications (Bramanto et al., 2019; Dabove, 2019; Pullar and Donaldson, 2022; Roberts, 2005; Tamrakar, 2013), a practice not entirely reliable from the surveying point of view of legal land registration. GNSS observations expressed as Universal Transverse Mercator (UTM) coordinates, are cartographic projection data and are not exactly what horizontal ground topographic distances indicate, which are valid for land tenure, deeds, rights purposes. A method for converting GNSS positions to horizontal topographic distances in cadastral environments preserving metric accuracy standards fixed for parcel registration according to Chilean authorities, is presented here. Complex equations to determine scale factor K from the UTM projection have been reduced to a set of constant coefficients, allowing replicating electronic distance measurement (EDM) performance in horizontal distances used in other experiences as Australia, by just providing coordinates and observed accuracies. All assumptions are aimed to simplify the property registration procedures in singular geographic environments like Chile, where urban settlements can be found at heights 2,000 m or more, being of utmost relevance to bear in mind differences between projected and ground coordinates. Applied experiences performed near Santiago de Chile at heights of 600 m over the MSL, yielded 3 mm difference in 500 m distance compared to measurements carried out by means of GNSS equipment in static mode.

1 Introduction

Before the existence of GNSS equipment, topographic surveying methods such as traversing or triangulations were the main source to provide UTM coordinates to ground objects. Angles and distances were taken on the field and later transformed to cartographic projection distances according to strict established sequences. These conversions were many times more time consuming than new technologies [i.e. "Geodetic Survey Stations near mount Spurr Volcano, Alaska" (March et al., 1986)]. Today, GNSS equipment reduced such processes by collecting cartographic coordinates in real time, saving the user carrying out explicit geometric conversions and transformations. This change remarkably speeded up actions as providing ground control point coordinates for aerial photogrammetry and others (Alkan et al., 2017). However, basic data as horizontal ground lines information were lost because they are implicitly within UTM distances. In Cadastral applications, horizontal distances ["most linear features in a 2D Cadastre are defined as a straight line between two points" (Thompson and van Oosterom, 2021)], parcel areas, corner coordinates and angles are common set of fundamental information for legal parcel registration. Therefore, recovering horizontal distances from GNSS data can even simplify more field tasks and complement traditional topographic procedures.

The flow diagram expresses how UTM distances were obtained from topographic works when no GNSS technology was available.

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* The previous version of the same article has been replaced with this version where **Section 2.4** has been corrected.

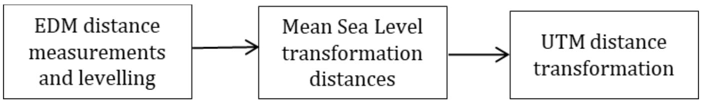

Fig. 1: Flow diagram for UTM distance conversion from topographic surveys

Using EDM involves laser equipment, assembled or being part of total topographic stations. These laser devices were designed to measure kilometres of distance according to light reflection into prisms. The inclined distance was subsequently converted to horizontal one, prior to its MSL conversion (Section 2.6) and UTM distance transformation (Section 2.7)

In order to recover topographic distances for cadastral applications and related accuracies, it is required to invert this data flow, hopefully simplifying equations to make the GNSS usage a fast and reliable tool for this aim.

2 Data Transformation

2.1 Earth Distances to Projection Transformed Lines

GNSS equipment provide accurate metric information in the form of northings and eastings coordinates at UTM graticule system, plus projected heights to a reference ellipsoid (transformed later into orthometric heights). Absolute coordinates of a position may vary by a few millimetres, showing a low dispersion and yielding reliable data when representing spatial positions and projected distances. However, these distances are not the same as the topographic ones measured on the ground surface that are required to declare and register parcel boundaries. Derived distances from UTM positions approximately represent shapes and dimensions, but they must be fully in accordance to topographic means to be considered as faithful source for land registration purposes. Metric differences can even reach meters per linear kilometre if high altitudes are dealt with, or when the projection factor "K" moves away from the unity, or if both sources components coincide. For scale factor variations, the problem can be visualized as shown in Fig. 2:

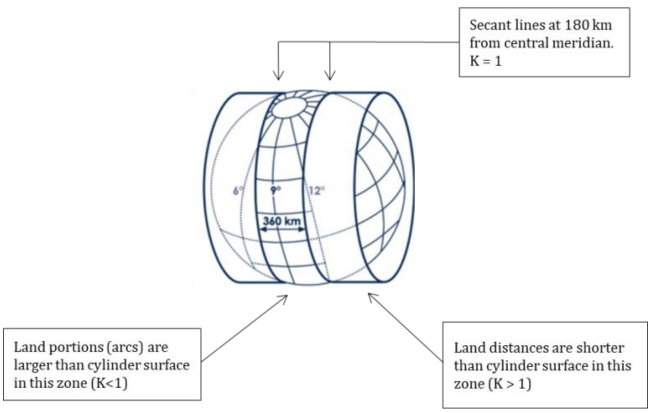

Fig. 2: UTM scale factor according to its position in a regular 6° longitude zone (Source: Jaud et al., 2020).

The "K" scale factor is a quantity derived from the UTM projection required to convert ellipsoidal or spherical earth distances into cylindrical cartographic expression. Scale factor can show its lowest value of 0.9996 ["with a reduction on the central meridian of one part in 2,500" (O'Keefe, 1952)] at the central meridian of UTM zone, whereas K= 1.00018 can be reached at the extreme zones, about 300 km distance from the central meridian (Snyder, 1987). Reduction must be performed between secant lines because the arc distances (on the earth surface) must be converted in a cylinder distance, which acts as a linear segment, shorter than the curved dimension. The opposite occurs out of secant zones, where earth distances must be expanded to be projected on the UTM surface, having scale factors greater than the unity. Only the perimeter of earth and cylinder holds the same value along the secant lines, being K = 1 at about 180 km from the central meridian (assigned as 500 km easting coordinate). The determination of the K factor value is of great importance to convert the reduced mean sea level distance into a UTM distance, which is achieved using conventional equations.

K factor calculation is then:

$$K = K_0\left[1 + \frac{(1+C)A^2}{2} + \frac{(5-4T+42C+13C^2-28e'^2)A^4}{24} + \frac{(61-148T+16T^2)A^6}{720}\right] \quad (1)$$

where:

K_0 = Scale factor at the Central Meridian (0.9996)

$A = (\lambda - \lambda_0)\cos\varphi$

C = e'^2 (second ellipsoid eccentricity)

$T = tan^2\varphi$ (φ latitude at the point)

2.2 Alternative Scale Factor Determination

An alternative perspective is proposed from conventional cartographic and geodetic equations to derive K values within a UTM zone, by means of a cubic interpolation with fixed values from the central meridian, secant lines, and a single selected intermediate point position within both sides of UTM zone. This solution is intended to provide accurate and simple computing procedures when programmed, excluding most of geodetic variables required by conventional equations and meeting cadastral legal planimetric tolerances standards (compared output scale factor from cartographic and simplified equations are presented in Table 1 and equations 4 and 5). Four constant transformation coefficients are then computed to solve the K factor for any further observed position by just providing the Easting information and keeping a unique northing argument. Numerical outputs of interpolated values are equal up to 6th decimal place of results from cartographic equations, which mean certainty when the K factor is applied.

Fig.3 shows a UTM zone where reference distances and K values were taken as components to carry out a cubic interpolation. Four cartographic conventionally computed values where selected to develop a continuous identical surface at each side of the central meridian.

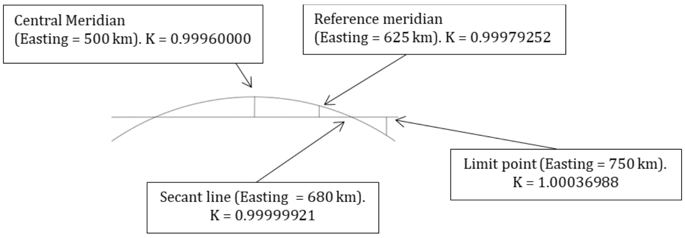

Fig. 3: Reference values for interpolation in a mid-side of a UTM zone.

2.3 Cubic Interpolation

The algebraic linear expression for an exact-fit third degree polynomial (Lam, 1983; Tempfli, 1982) interpolation is:

$$C = A^{-1} \times F \tag{2}$$

where:

F = Scale factor argument for reference fixed values

A = Powered distance coefficients referred to a base point (easting = 625,000 m and easting = 475,000 m)

C = Four computed coefficients for subsequent interpolation within a UTM zone

$$A = \begin{bmatrix} 1 & x(i-1) & x(i-1)^2 & x(i-1)^3 \\ 1 & 0 & 0 & 0 \\ 1 & x(i+1) & x(i+1)^2 & x(i+1)^3 \\ 1 & x(i+2) & x(i+2)^2 & x(i+2)^3 \end{bmatrix}$$

The four reference values computed from geodetic equations for each UTM zone were:

f(1) k=0.99960000, (easting = 500,000 m)

f(2) k= 0.99979252, (easting = 625,000 m)

F(3) k=0.99999921, (easting = 680,000 m)

F(4) k=1.00036988, (easting = 750,000 m)

According to equation 2, the observation equations system is:

$$\overset{F}{\begin{bmatrix} 0.99960000 \\ 0.99979252 \\ 0.99999921 \\ 1.00036988 \end{bmatrix}} = \overset{A}{\begin{bmatrix} 1 & -125000 & 1.56 \times 10^{10} & -2 \times 10^{15} \\ 1 & 0 & 0 & 0 \\ 1 & 55000 & 3.03 \times 10^{9} & 1.66 \times 10^{14} \\ 1 & 125000 & 1.56 \times 10^{10} & 1.95 \times 10^{15} \end{bmatrix}} \times \overset{C}{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}}$$

Solution for each parameter yields a set of 4 constant values:

c1 = 0.9997922518

c2 = 3.08094E-09

c3 = 1.23151E-14

c4 = -9.0242E-23

For any easting value, scale factor K can be obtained by replacing, in A matrix, the powered to the base reference meridian distances and multiplying it times the C matrix. An example is provided here:

Table 1: Interpolated and cartographic output results

Eastings (m)	**Interpolated values**	**UTM cartographic equations**	**Difference**	**Residual (mm) per kilometre**
500000	0.999600	0.999600	9.66E-15	0.000
525000	0.999608	0.999608	-4.81E-08	-0.048
550000	0.999631	0.999631	-6.18E-08	-0.062
575000	0.999669	0.999669	-5.09E-08	-0.051
600000	0.999723	0.999723	-2.54E-08	-0.025
625000	0.999793	0.999793	5.00E-09	0.005
650000	0.999877	0.999877	3.03E-08	0.030
675000	0.999977	0.999977	5.06E-08	0.051
700000	1.000093	1.000093	5.61E-08	0.056
725000	1.000224	1.000224	4.69E-08	0.047
750000	1.000370	1.000370	3.04E-09	0.003

Differences in mm per 1 km distance can be observed in the last column, where a difference of less than 1 mm was detected at about 136 km from the central zone meridian. These residuals are fully inside the fixed tolerance ["a standard attempts to define the quality of the work in a way that is ideally independent of the equipment or technology in use" (Craig and Wahl, 2003)] for a point in urban areas established by the "Ministerio de Bienes Nacionales de Chile" (MBN), where the maximum allowed circular error is 0,050 m for land parcel vertex positions. Therefore, almost identical values with geodetic equations can be achieved if the polynomial coefficients are used, according to the following equations:

Eastings (>=500000 m): K= c1 + c2*(Easting-625000 m) + c3*(Easting - 625000 m)^2 + c4*(Easting -625000 m)^3 (4)

Eastings (<500000 m): K= c1 + c2*(375000 m - Easting) + c3*(375000 m -Easting)^2 + c4*(375000 m -Easting)^3 (5)

This assumption using the polynomial interpolation only takes into account computed values at the unique Northing UTM coordinate of 6000000 m (approximated northing average coordinate of continental Chilean territory) excluding any other latitude or northing argument.

2.4 The Latitude Argument

Chile is a country with an extended latitude, from about 18° to 56° south (from UTM northings= 8065 km to northings =3840 km). When considering the north south UTM expression, this requires only two UTM zones to cover the whole territory (18 and 19 zones). For the computation of cartographic conventional coordinates, it is demanded to register northings or latitude arguments when scale factor K is determined. That type of information could be neglected if the final outputs are within national cadastral standards used, according to the interpolated K factors approach. An empirical comparison between the UTM equation using the correct latitude (equation 1) versus interpolated coefficients was done in order to estimate numerical outputs from the two procedures. Latitude argument should not stand for a relevant change in K factor for medium extension latitude bands, meaning that a non-unique set of transformation parameters should be used for the whole country, simplifying the final horizontal topographic distance computation.

Table 2 shows empirical differences between UTM interpolated coefficients for a singular 6,000 km northing, and those extracted from the UTM geodetic equations, using both northing and easting arguments. Highlighted cells carry the maximum difference between cartographic and interpolated values.

2.5 UTM Distance Conversion

The expression used to convert an earth surface distance to a projection distance is:

$$UTM\ distance\ (UTMd)\ = Reduced\ mean\ sea\ level\ distance\ (RD) \times scale\ factor\ (K) \quad (6)$$

And hence

$$RD\ = \frac{UTMd}{K} \quad (7)$$

Once the K factor is obtained, euclidean distances between UTM coordinates can be expressed in the form of reduced mean sea level magnitudes, requiring a subsequent transformation to take them to horizontal topographic distances.

2.6 Horizontal Topographic Distance Reduction

Conversion from earth surface to a cartographic surface or vice versa is performed assuming a regular and continuous shape for the earth body. This is the case for an ellipsoid or sphere where all topographic distances are reduced prior to their cartographic transformation. Conversion from earth surface to a horizontal distance involves knowing average heights above the mean sea level of two points (Fig. 4). If small portions of lands are used (case of urban cadastral measurements of less than 2 km) a sphere for earth's surface is suitable to be the considered as the basis for calculations. Moreover, if short distances are measured, land portions can be treated as a flat expression.

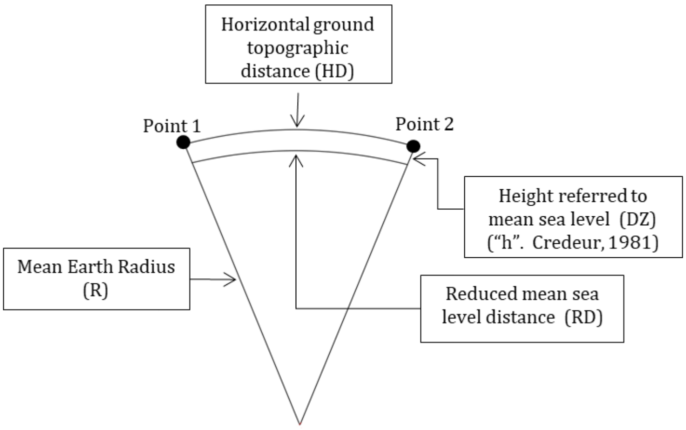

Fig. 4: Topographic horizontal distance between two points on the earth surface and its reduced expression at MSL

As expressed, if distance is short, two similar triangles can be established:

$$\frac{HD}{RD} = \frac{(R+DZ)}{R} \quad (8)$$

Then:

$$RD = \frac{(R \times HD)}{(R+DZ)} \quad (9)$$

The UTM distance provided by GNSS equipment is required to be returned to its original horizontal topographic distance, so the subsequent equation should be:

$$HD = \frac{RD \times (R+DZ)}{R} \quad (10)$$

For HD computation, orthometrics heights must be provided besides the UTM coordinates. It can be seen from the equation (9) that the ratio for RD changes as much as the DZ value increases. In high-lands, observed differences between HD and RD can reach to almost one meter per kilometre, making it an unavoidable action to apply corrections to this systematic distortion. Table 3 shows the combined effect of scale factor K and height in mean sea level distances reduction.

Table 2: K scale factor differences from Northings = 6000 km and other northing arguments

	Distance to central meridian (km)	25 (475/525)	50 (450/550)	75 (425/575)	100 (400/600)	125 (375/625)	150 (350/650)	175 (325/675)	200 (300/700)	225 (275/725)	250 (250/750)	275 (225/775)
Northings	8000000	0.99960773	0.99963085	0.99966954	0.99972362	0.99979314	0.99987811	0.99997852	1.00009436	1.00022563	1.00037231	1.00053441
	6750000	0.99960771	0.99963084	0.99966940	0.99972338	0.99979278	0.99987843	0.99997852	1.00009344	1.00022447	1.00037090	1.00053276
	5500000	0.99960770	0.99963077	0.99966923	0.99972308	0.99979231	0.99987693	0.99997852	1.00009229	1.00022305	1.00036917	1.00053067
	4000000	0.99960770	0.99963068	0.99966902	0.99972271	0.99979174	0.99987612	0.99997586	1.00009100	1.00022141	1.00036720	1.00052844
	Interpolated K values	0.99960765	0.99963074	0.99966925	0.99972317	0.99979251	0.99987723	0.99997734	1.00009283	1.00022368	1.00036988	1.00053143
	Residuals * 1 km (mm)	0.078	0.112	0.291	0.443	0.639	0.880	1.179	1.534	1.953	2.427	2.995

Table 3: Effect of reduction/expansion combined effect of height over MSL and K scale factor on 1000 m of horizontal distances

	Distance to central meridian (km) /(Eastings - UTM)	**0/500**	**25 (475/525)**	**50 (450/550)**	**75 (425/575)**	**100 (400/600)**	**125 (375/625)**	**150 (350/650)**	**175 (325/675)**	**200 (300/700)**	**225 (275/725)**	**250 (250/750)**
	K scale factor	**0.999600**	**0.999608**	**0.999631**	**0.999669**	**0.999723**	**0.999792**	**0.999897**	**0.999977**	**1.000093**	**1.000224**	**1.000370**
Height above MSL (m)	**Reduced distanced to MSL (m)**	**UTM distance**										
0	1000	999.600	999.608	999.631	999.669	999.723	999.792	999.897	999.977	1000.093	1000.224	1000.370
250	999.961	999.561	999.569	999.592	999.630	999.684	999.753	999.858	999.938	1000.054	1000.185	1000.331
500	999.922	999.522	999.530	999.553	999.591	999.645	999.714	999.819	999.899	1000.015	1000.146	1000.291
750	999.882	999.482	999.490	999.513	999.551	999.605	999.674	999.779	999.859	999.975	1000.106	1000.252
1000	999.843	999.443	999.451	999.474	999.512	999.566	999.635	999.740	999.820	999.936	1000.067	1000.213
1250	999.804	999.404	999.412	999.435	999.473	999.527	999.596	999.701	999.781	999.897	1000.028	1000.174
1500	999.765	999.365	999.373	999.396	999.434	999.488	999.557	999.662	999.742	999.858	999.989	1000.135
1750	999.725	999.326	999.334	999.356	999.394	999.448	999.517	999.622	999.702	999.818	999.949	1000.095
2000	999.686	999.286	999.294	999.317	999.355	999.409	999.478	999.583	999.663	999.779	999.910	1000.056
2250	999.647	999.247	999.255	999.278	999.316	999.370	999.439	999.544	999.624	999.740	999.871	1000.017
2500	999.608	999.208	999.216	999.239	999.277	999.331	999.400	999.505	999.585	999.701	999.832	999.978
2750	999.569	999.169	999.177	999.200	999.238	999.292	999.361	999.466	999.546	999.662	999.792	999.938
3000	999.529	999.130	999.138	999.161	999.198	999.252	999.321	999.426	999.506	999.622	999.753	999.899
3250	999.490	999.090	999.098	999.121	999.159	999.213	999.282	999.387	999.467	999.583	999.714	999.860
3500	999.451	999.051	999.059	999.082	999.120	999.174	999.243	999.348	999.428	999.544	999.675	999.821
3750	999.412	999.012	999.020	999.043	999.081	999.135	999.204	999.309	999.389	999.505	999.636	999.782
4000	999.373	998.973	998.981	999.004	999.042	999.096	999.165	999.270	999.350	999.465	999.596	999.742
4250	999.333	998.934	998.942	998.965	999.003	999.057	999.125	999.230	999.310	999.426	999.557	999.703
4500	999.294	998.894	998.902	998.925	998.963	999.017	999.086	999.191	999.271	999.387	999.518	999.664

Table 4: Variation of 1 cm per meter corresponding to the combined effect of scale factor and reduction to MSL

	Distance to central meridian (km) /(Eastings - UTM)	**0/500**	**25 (475/525)**	**50 (450/550)**	**75 (425/575)**	**100 (400/600)**	**125 (375/625)**	**150 (350/650)**	**175 (325/675)**	**200 (300/700)**	**225 (275/725)**	**250 (250/750)**
Height (m) above MSL	**Reduced distanced to MSL (m)**	**Horizontal topographic distances**										
0	1000	25.000	25.510	27.100	30.211	36.101	48.077	97.087	434.783	107.527	44.643	27.027
250	999.961	22.767	23.190	24.496	27.011	31.623	40.448	70.306	160.674	186.020	54.126	30.235
500	999.922	20.901	21.257	22.349	24.423	28.133	34.909	55.107	98.549	688.788	68.725	34.306
750	999.882	19.318	19.621	20.548	22.288	25.337	30.705	45.311	71.071	404.563	94.103	39.643
1000	999.843	17.957	18.219	19.016	20.497	23.047	27.404	38.473	55.576	156.369	149.191	46.947
1250	999.804	16.776	17.004	17.696	18.972	21.137	24.745	33.429	45.629	96.917	359.803	57.549
1500	999.765	15.741	15.941	16.548	17.658	19.519	22.556	29.554	38.703	70.221	874.191	74.335
1750	999.725	14.826	15.004	15.540	16.515	18.131	20.723	26.485	33.602	55.056	197.362	104.941
2000	999.686	14.012	14.170	14.648	15.511	16.928	19.166	23.993	29.690	45.279	111.242	178.380
2250	999.647	13.282	13.425	13.852	14.622	15.875	17.826	21.930	26.594	38.451	77.449	594.132
2500	999.608	12.625	12.754	13.139	13.829	14.945	16.662	20.194	24.082	33.413	59.405	446.542
2750	999.569	12.030	12.146	12.495	13.118	14.118	15.640	18.712	22.005	29.542	48.180	162.293
3000	999.529	11.488	11.595	11.912	12.477	13.377	14.737	17.433	20.257	26.476	40.524	99.171
3250	999.490	10.993	11.091	11.381	11.895	12.711	13.932	16.318	18.767	23.986	34.968	71.402
3500	999.451	10.539	10.629	10.895	11.365	12.108	13.211	15.337	17.481	21.924	30.752	55.783
3750	999.412	10.121	10.204	10.449	10.881	11.559	12.561	14.468	16.360	20.189	27.443	45.772
4000	999.373	9.735	9.812	10.038	10.436	11.059	11.971	13.691	15.374	18.709	24.778	38.807
4250	999.333	9.378	9.448	9.658	10.026	10.599	11.435	12.994	14.501	17.431	22.584	33.683
4500	999.294	9.045	9.111	9.306	9.647	10.177	10.945	12.365	13.721	16.316	20.748	29.754

2.7 Combined Effect of Scale Factor K and Mean Sea Level Reduction

If $UTMd = RD \times K$ (11)

then this equation can be reversed and simplified as:

$$HD = \frac{UTMd}{K_a} \times \left(\left(\frac{MDz}{R}\right) + 1\right) \quad (12)$$

Where

K_a= Average scale factor from two points on the earth surface

MDz = Average mean sea level height from two points on the earth surface

According to Table 3, if a distance of one kilometre is measured at 3,250 m height over the mean sea level, this quantity will be decreased to 999.40 m only due to reduction to mean sea level procedure. If the easting coordinate is 575 or 425 km, the scale factor will be 0.999669 and hence the UTM distance will be reduced to 999.159 m. This change from a horizontal topographic distance of 1,000 m (provided by EDM equipment) with respect to UTM distance (yielded by GNSS equipment) makes it necessary to transform every GNSS longitude expression to the UTM distance (highlighted cells correspond to those falling beyond 180 km from the central meridian).

From the previous example, it can be assumed that at 3,250 m height and at 75 km distance (eastings = 425/575 km) on the central meridian of a UTM zone, the UTM distance is shorter by 1 cm with respect to the horizontal distance (11,895 m) measured on the surface.

3 Cadastral Implications Under GNSS Equipment Use

Legal cadastre in Chile is shared by several institutions, unlike other countries that gather spatial information and statistical information by one institution. The MBN is in charge of public properties, either rural or urban, and establishes technical standards for land surveys. It is mentioned that every urban parcel vertex must be in an error of no more than 0.05 m, not including angles between parcel vertexes. A similar situation applies to the "conservador de bienes raíces" (land tenure office) who deals with private land tenure affairs and require topographic plans to be registered as reliable documents jointly with legal documents. However, there are no technical standards associated, with most cases trusting in the accurate work performed by surveyors to convert geographical data into topographic plans.

A remarkable experience to bear in mind about cadastral standards has been achieved in Australia, where the New South Wales government clearly establishes in its document "Surveying's General Directions (n° 9)" (also referred in "Cadastral Integrity Unit Audit Survey Procedures" 2012.) tolerances and procedures to use GNSS equipment in cadastral applications, suggesting minimum distances and values for lines, coordinates and angles in land parcel measurements under COORS based observations. Both GNSS ground data collection modes analysis and declared restrictions for minimum distances to be measured to preserve angular consistence are encompassed as well. An interesting aspect mentioned is the use of GNSS receivers to be used under RTK or NRTK mode with similar behaviour than a 10 mm + 50 ppm accuracy total station as a valid method for cadastral tasks. A declared accuracy of a GNSS station no better than 2 cm (at 95%) is assumed as class "C", suitable to measure distances equal or greater than 400 m long between GNSS points, preserving parcel shape consistency. Distances shorter than 100 m are highly recommended to be checked by using complementary better performance total stations.

Table 5: New South Wales table comprising recommendable accuracies for GNSS equipment in RTK and NRTK mode (Fenwick, 2014). (Dickson (2012) addresses this topic establishing one sigma to express the point error ellipse).

Station Density (km) \ Point and (Relative) Error Ellipse	0.010 m (0.014 m)	0.020 m (0.028m)	0.030 m (0.042m)	0.040 m (0.056m)	0.050 m (0.070 m)	0.060 m (0.084m)	0.070 m (0.096 m)
0.1	C	D	E	E	–	–	–
0.2	C	D	E	E	E	–	–
0.4	B	C	D	D	E	E	E
0.6	B	C	C	D	D	E	E
0.8	A	B	C	C	D	D	D
1	A	B	B	C	C	D	D
2	A	A	B	B	C	C	C
5	2A	2A	A	A	A	B	B
10	3A	2A	2A	2A	A	A	A

Observations performed under these conditions do not confirm (or do not indicate) whether the combined scale factor K and height mean sea level reduction are taken into account. Since the geographic landscape in Chile often involves important height changes, it is of crucial importance to consider these distortions prior, so as to assess the observations quality from Table 5.

4 Observed Experience

As an example of reverse conversion (from UTM to horizontal topographic distance), a closed traverse performed near the Santiago city, using the total station with observed temperature and pressure coefficients is showed here,. Angles and traverse sides resulting from the topographic work where matched to a Local Tangent Plan (LTP) containing two Local Transverse Mercator (LTM) and UTM coordinate monuments (measured under static GNSS mode). Expected topographic outputs should meet deltas from previous conversion of LTM, which also should fit the horizontal distance achieved using equation 12.

Table 6: Topographic LTP traverse coordinates from GNSS observations

Vertex	Point	Angle	Compens	Azimuth	HD avg	ΔE	ΔN	ΔE correct	ΔN correct	ΔEc	ΔNc	EASTINGS	NORTHINGS
G03	T06	34.8430	34.8390	237.6591								**191318.62**	**3299433.08**
1.454	P3			202.8201	233.9505	**-10.3601**	-233.7210	-0.0004	0.0140	-10.3597	-233.7350		
P3		160.7805	160.7764									191308.2563	3299199.3460
1.57	P2			242.0437	150.9848	-92.6214	-119.2379	-0.0034	0.0071	-92.6180	-119.2450		
P2		194.1544	194.1503									191215.6383	3299080.1010
1.376	P1			247.8933	137.2398	-93.7793	-100.2008	-0.0034	0.0060	-93.7759	-100.2068		
P1		115.3970	115.3930									191121.8624	3298979.8943
1.542	T06			332.5004	89.6950	-78.2583	43.8273	-0.0029	0.0026	-78.2554	43.8247		
T06		94.8453	94.8413									**191043.6070**	**3299023.7190**
T06				37.6591								**191043.607**	**3299023.719**
		600.020257	600			-275.0191	-409.3323	-0.0101	0.0297	-275.0090	-409.3620	**-275.009**	**-409.362**
	error	0.02025712			Error	0.0101	-0.0297						
	eu	-0.0040514			Sum VA	275.0191	496.9870		HD	493.1604		**0.0000**	

Topographic distance obtained by traversing was 493.141 m, being adjusted to 493.160 m sourced from the LTM coordinates. If equations 4 or 5, and 12 are applied from UTM coordinates, a topographic horizontal distance of 493.157 m was obtained, with only a 0.003 m difference from the LTM system. The UTM data were:

Point	Eastings (m)	Northings (m)	Height (m)	UTMD
G3	328628.154	6299688.183	459.482	
T06	328360.106	6299274.299	477.740	493.102

According to equation 12:

$$HD = \frac{493.102}{0.999962160} \times \left(\left(\frac{468.611}{6371000}\right) + 1\right) = 493.157\ m$$

5 Conclusions

I. After observing numerical and empirical results, it was noticed that there is a true concordance between conventional cartographic equations and derived simplified interpolated values. They mostly have the same values up to the 6th decimal place for the UTM scale factor K, which assures a reliable distance transformation. The third degree polynomial accepted to obtain K can neglect singular northing information for extended portions of lands and for urban cadastral works, taking into account national accuracy standards established for this aim. Geodetical works such as control networks or land displacement assessment demand exclusively of traditional geodetic/cartographic equations, not constituting this experience a valid alternative for such type of tasks.

II. Angular issues involved in geodetic measurements like the (t - T) coefficient when topographic observations are done have been neglected due to scarce influence for short distances. Large scale Cadastral works accuracy standards allow to omit this factor when long geodetic distances primarily aimed.

III. Topographic ground distances can be effectively converted from UTM coordinates under proposed equations alongside Australian GNSS and total stations accuracies standards to meet Chilean cadastral requirements.

IV. In high zones of the north of Chile (Calama / Chuquicamata) a combined effect of the minimal reduce scale factor K (0.9996) and altitudes greater than 2,000 m are registered. Differences of 1 cm can be detected for horizontal distances around 13 m , which gets increased for small towns close to 4,000 m altitude (Parinacota, Putre), where 1 cm is reduced for distances of only 9 m, simply due to the combined effect of scale factor and sea level distance reduction. These systematic values should be corrected prior to including the GNSS observed accuracies in order to emulate a total station (as the stated in paragraph Section 2.8) working at that height and in accordance with existing cadastral standards.

V. Geographic conditions of the north of Chile (are also shared by other countries in South America: Bolivia (UTM zone 19, 20, 21 S), Perú (UTM zone17, 18, 19 S). In Asia, countries such as Buthan (UTM zone 45 N), and Nepal (UTM zone 44, 45 N), Pakistan, South China present site similarities to the South American countries. However, cadastral regulations may vary from one country to another. The technical approach for measurements presented here, however, could be applied under similar operation conditions (cartographic projection, GNSS measurement mode) in order to provide alternative but reliable outputs in the field of cadastre actions.

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Author Contributions

Conceptualization, methodology, analysis, writing - original draft preparation, E.S-M., and writing - review and editing, M.C-S. All authors have read and agreed to the published version of the manuscript.

Conflict of Interest

The authors declare no conflict of interest.

References

Alkan, R. M., Murat Ozulu, I., İlçi, V., Engin Tombuş, F., Şahin, M., 2017. Usability of GNSS technique for cadastral surveying. In: T. Yomralioglu, J. McLaughlin, (Editors), Cadastre: Geo-Information Innovations in Land Administration, Springer, Cham, pp. 77-91.

Bramanto, B., Gumilar, I., Taufik, M., Hermawan, I. M. D., 2019. Long-range single baseline RTK GNSS positioning for land cadastral survey mapping. E3S Web of Conferences, Vol. 94. EDP Sciences, pp. 01022.

Craig, B. A., Wahl, J. L., 2003. Cadastral survey accuracy standards. Surveying and Land Information Science, 63(2): 87-106.

Dabove, P., 2019. The usability of GNSS mass-market receivers for cadastral surveys considering RTK and NRTK techniques. Geodesy and Geodynamics, 10(4): 282-289. DOI:https://doi.org/10.1016/j.geog.2019.04.006

Dickson, G., 2012. Control surveys: Why things are the way they are and not the way you think they should be. Proceedings of Association of Public Authority Surveyors Conference (APAS2012), Wollongong, New South Wales, Australia, pp. 12-28.

Fenwick, W., 2014. Looking at Cadastral GNSS from a New Angle. Proceedings of Association of Public Authority Surveyors Conference (APAS2014). Pokolbin, New South Wales, Australia, pp. 42-53.

Jaud, Š., Donaubauer, A., Heunecke, O., Borrmann, A., 2020. Georeferencing in the context of building information modelling. Automation in Construction, 118: 103211. DOI:https://doi.org/10.1016/j.autcon.2020.103211

Lam, N. S.-N., 1983. Spatial interpolation methods: A review. The American Cartographer, 10(2): 129-150. DOI:https://doi.org/10.1559/152304083783914958

March, R., Mayo, L., Trabant, D., 1986. Geodetic Survey Stations Near Mount Spurr Volcano, Alaska. US Department of the Interior, Geological Survey.

O'Keefe, J. A., 1952. The universal transverse mercator grid and projection. The Professional Geographer, 4(5): 19-24.

Pullar, D., Donaldson, S., 2022. Accuracy issues for spatial update of digital cadastral maps. ISPRS International Journal of Geo-Information, 11(4): 221. DOI:https://doi.org/10.3390/ijgi11040221

Roberts, C., 2005. GPS for cadastral surveying: Practical considerations. Proceedings of SSC 2005 Spatial Intelligence, Innovation and Praxis: The national Biennial Conference of the Spatial Sciences Institute. Citeseer, Melbourne: Spatial Sciences Institute.

Snyder, J. P., 1987. Map projections - A working manual. US Government Printing Office.

Tamrakar, R. M., 2013. Potential use of GPS technology for cadastral surveys in Nepal. Journal on Geoinformatics, Nepal, 12: 33-40.

Tempfli, K., 1982. Applied interpolation methods - Lecture notes. ITC publications.

Thompson, R., van Oosterom, P., 2021. Bi-temporal foundation for LADM v2: Fusing event and state based modelling of Land administration data 2D and 3D. Land Use Policy, 102: 105246. DOI:https://doi.org/10.1016/j.landusepol.2020.105246