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Novel Spanning-Tree Matrix Approach to Model and Optimize Large-Scale Tree-Shaped Water Distribution Networks

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Abstract: This study introduces the Spanning-Tree Matrix Approach, a novel computational method for modeling large-scale, tree-shaped water distribution networks. The method focuses on minimizing network cost while satisfying hydraulic design constraints. A case study demonstrated the integration of this approach with the Honey-Bee Mating Optimization algorithm to determine the optimal combination of pipe diameters. Results confirm the model's effectiveness and scalability for tree-shaped network configurations. The Spanning-Tree Matrix Approach is also versatile—it can accommodate various governing equations and design criteria, and can be easily embedded in modern stochastic optimization algorithms such as Genetic Algorithms, Simulated Annealing, and Ant-Colony Optimization. Due to its adaptability and simplicity, further research is encouraged to apply this approach to other spanning-tree hydraulic systems, including waster water networks, river or groundwater networks.

Keywords: Hydraulic Modeling, Optimization, Spanning-Trees, Water Distribution Networks

1. Introduction

The recent past has witnessed many developments in computational methods in water resources management, with particular interest in water distribution networks (WDNs) modeling and optimization. The influence of modern mathematics upon these developments of WDNs is inevitable. Rapid urbanization, scarcity of natural resources, and upscale-costs involved are identified to be some of the reasons for this continuous research interest on WDNs. Among many criteria for the design of WDNs, one of the widely-used is finding the combination of pipe diameters that satisfies the minimum-nodal-hydraulic-head and maximum-pipe-head-loss requirements while minimizing the cost involved. Hence, this is regarded as a combinatorial optimization problem. Due to the environmental conditions, the non-linear nature of the information associated with the hydraulic constrains, and the type of architecture being adopted, the modeling and optimization of WDN have, most of the time, governing models specific to their own. When existing hydraulic modeling packages do not facilitate the ability to model adaptable governing equations, the development of problem specific new computational methods by engineers and analysts becomes an indispensable interest.

2. Literature Review

A growing body of research shows that researchers have been using numerous mathematical methods to model and optimize water systems (Alporevits & Shamir [1]; Awe et al. [2]; Corte & Sörensen [3]; Lin et al. [7]; Cunha & Sousa [4]; Dijk et al. [5]; Hsu & Cheng [6]; Mohan & Babu [8]; Mohan & Babu [9];]; Senavirathna et al. [10]; Suribabu [11]; Cong & Zhao [12]).

As far as modeling of WDN is concerned, it is very noticeable that the use of a commercially available hydraulic modeling software package is still very common practice among the researchers.

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An example of such practice is the use of EPANET by Awe et al. [2], Mohan & Babu [9] and Suribabu [11]. These commercially available hydraulic modeling software packages of this sort provide users considerable convenience when it comes to modeling a WDN. However, oversimplified and/or linearized hydraulic governing equations in those packages may give off undesired, or rather less-accurate results upon simulation.

Thus, it has been a keen interest for researchers to develop novel computational methods that are highly adaptable to specific governing equations of the WDN problem being concerned, as the complexity grows.

Incorporation of graph theory to tackle WDN has been noticeable in the recent past. The work done by Cong & Zhao [12] has focused on applying a minimum spanning-tree algorithm, the Kruskal's algorithm, to find the minimum spanning-tree for a WDN that has fixed nodal positioning. Kruskal's algorithm, however, finds the minimum-weight spanning-tree by choosing edges with minimum weights while avoiding potential loops. Their study has assigned constant weights to the edges that represent the pipes, not taking into account the possibility of using commercially available pipes of different sizes. On the contrary to their study, it also needs to be noted that designing and optimizing WDNs should be done taking hydraulic constraints into consideration, as final goal is, as one can speculate, a successful WDN prediction model.

Corte & Sörensen [3], in their critical review, have shown that not only modeling, but also optimization of WDNs is much needed to reach the goals of delivering reliable water supply at optimum cost. In literature, there can be seen two main categories for WDN optimization algorithms, namely, deterministic-heuristic and stochastic-heuristic algorithms.

Deterministic-heuristic algorithms provide methods incorporating gradient and implicit information associated with the WDN to reach an optimal solution. The main advantage of algorithms of this kind is that it takes a very smaller number of iterations to reach the optimal solution compared to stochastic-heuristic algorithms. Moreover, deterministic-heuristic algorithms cannot guarantee if the solution so obtained is the global optimum solution. The studies done by Awe et al. [2], Hsu & Cheng [6], Lin et al. [7], Mohan & Babu [8], and Suribabu [11], provide their own yet similar criteria for employing deterministic-heuristic algorithms to reach near optimal solutions for WDNs.

On the other hand, stochastic-heuristic, or at times called modern-stochastic-heuristic, methods neither take gradient nor implicit information associated with the WDNs. These kinds of algorithms evaluate the objective function at randomly taken different regions of the solution space to seek the feasible solution that minimizes the cost function. The main advantage of this method, as described earlier, is that it investigates the whole solution space to work out the global optimum solution. On the contrary to deterministic-heuristic methods, stochastic-heuristic methods perform extremely large number of objective function evaluations to reach this global optimum. The very first publication on these sort of evolutionary optimization algorithms is by Holland [13]. Some of the notable studies that utilize these optimization algorithms for WDNs include Simulated Annealing (SA) Approach done by Cunha & Sousa [4], Genetic Algorithm (GA) by Dijk et al. [5], and Honey-Bee Mating Optimization (HBMO) by Mohan & Babu [9]. Recently, Senavirathna et al. [10] have adapted the HBMO to facilitate a design criterion slightly different to what had been used by Mohan & Babu (2010), in order to address their case study. This paper, however, brings about a novel and simple, yet very effective computational-method, called Spanning-Tree Matrix Approach (STMA), to model tree-shaped WDNs. Proposed STMA also can function, similar to dynamic programming problems, as an adaptable sub-system in an optimization algorithm which utilizes non-linear energy governing equations in the WDNs that have tree-shaped architecture. A case study has also been presented to demonstrate the applicability of STMA in a modern stochastic optimization algorithm, the version of the HBMO algorithm adopted from Senavirathna et al. [10].

3. Methodology and Analysis

3.1 Formulation of the Optimization Model

Optimal WDN design problem focused in this study can be written as,

$$Min\ Z = \sum_{i=1}^{N} C_i(D, L) \qquad \ldots(1)$$

subjected to the hydraulic design constraints,

$$H_{R\ j} \geq H_{R\ j}^{min} \quad j = 1,2,3, \ldots, nd \qquad \ldots(2)$$

$$g_{FF\ i} \leq g_{FF\ i}^{max} \quad i = 1,2,3, \ldots, np \qquad \ldots(3)$$

where

Z = total cost involved with the WDN;

N = number of pipes in the water distribution network;

$C_i(D, L)$ = cost of the i^{th} pipe having diameter D and length L;

$H_{R\ j}$ = residual water head available at the j^{th} node;

$H_{R\ j}^{min}$ = minimum residual water head required at the j^{th} node;

$g_{FF\ i}$ = friction and fitting loss gradient in the i^{th} pipe;

$g_{FF\ i}^{max}$ = maximum allowable friction and fitting loss gradient in the i^{th} pipe;

nd = number of demand nodes; and

np = number of pipes.

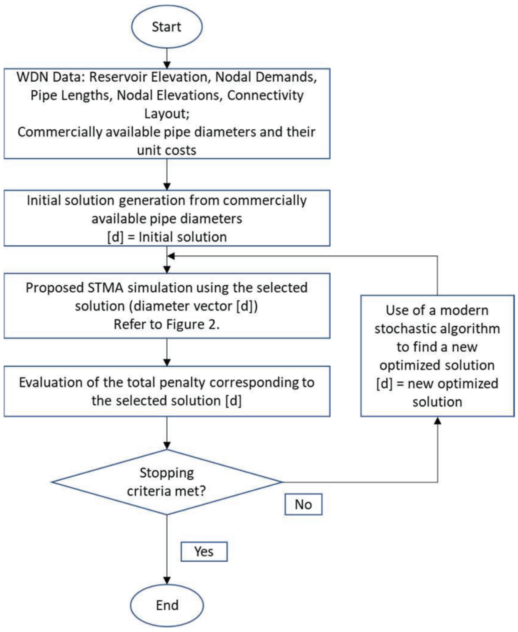

Figure 1 - Flowchart of the optimization algorithm considered in this study

The flowchart shown in Figure 1, depicts the algorithm that accommodates the above optimization model. It typically requires the WDN data, such as reservoir elevation, nodal demands, pipe lengths, nodal elevations, and the pipe connectivity layout. In addition to WDN data, commercially available pipe diameters and their unit costs are needed as the fulfillment of the data requirement.

At the beginning, an initial solution [d], a combination of pipe diameters, is generated from the commercially available pipe diameters. The WDN is then simulated for [d] according to the proposed STMA. If the selected [d] is violating the design constraints above, the solution [d] is penalized by adding a penalty value to discourage its fitness for application in the design. If the stopping criterion is not met, it uses a modern stochastic algorithm to implement the optimization algorithm shown in the Figure 1, embedding the STMA for hydraulic modeling. The algorithm, as set up, iteratively finds a new optimized solution from the commercially available pipe diameters until the stopping criterion is met. The solution thus obtained can be regarded as the global optimum solution, or in other words, the optimal combination of pipe diameters that minimizes the cost of the WDN to the maximum extent.

Next, it can be discussed how STMA can be used to simulate a WDN that takes a tree-shaped architecture.

3.2 Spanning-Tree Matrix Approach

The goal of using STMA is to obtain the nodal-residual-head vector $[H_R]$ and the friction and fitting loss gradient vector $[g_{FF}]$ for the selected diameter vector [d]. Detailed derivations of each vector are elaborated in the following paragraphs. To compute the nodal residual head vector $[H_R]$ and $[g_{FF}]$ for a selected pipe diameter vector $[d]$, the STMA follows a structured sequence of matrix-based steps. Initially, the nodal water demand vector $[D]$ is used in conjunction with the transformation matrix $[T_D^Q]$ to determine the flow vector $[Q]$ in all pipes. Subsequently, the Hazen-Williams equation is employed to calculate the friction-loss gradient vector $[h_f]$. This vector is then scaled by the fitting-loss coefficient $[C_f]$ to obtain the combined $[g_{FF}]$. Pipe lengths are represented as vector $[L]$, and their element-wise product with $[g_{FF}]$ yields the head-loss vector $[F]$. A second transformation matrix $[T']$ is applied to $[F]$ to compute the total head-loss at each node, which is then subtracted from the reservoir elevation to derive the nodal head vector $[H]$. Finally, $[H_R]$ is determined by subtracting the nodal elevation vector $[E]$ from the computed $[H]$. These steps complete the STMA simulation cycle for any given diameter configuration, enabling further penalty evaluation and optimization iteration.

In order for the employed mathematical concepts in the computations to be clearly understood, it is convenient to consider a fairly small, tree-shaped, dummy WDN that has six consumer water demand nodes. As the flow occurs in one direction only, the spanning-tree of WDN here is a directed graph rooted at the reservoir node (see Figure 2).

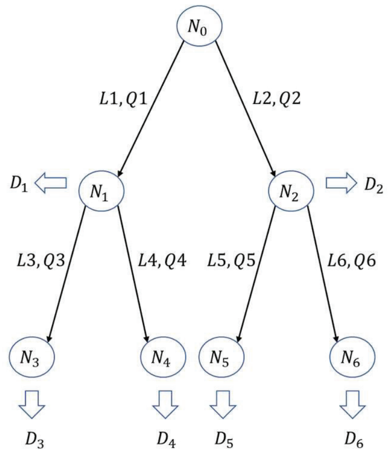

Figure 2 - The dummy WDN for the demonstration of the functionality of STMA

The reservoir, which is the root of the spanning-tree, is denoted by N_0. However, N_is $(i = 1,2,3,4,5,6)$ are the consumer demand nodes of the WDN; L_is $(i = 1,2,3,4,5,6)$ are the lengths of pipes; Q_is $(i = 1,2,3,4,5,6)$ are the flows in pipes in the indicated direction; and D_js $(j = 1,2,3,4,5,6)$ are the nodal water demands.
In matrix notation, the nodal water demand vector is $[D] = [D_j]$, $(j = 1,2,3,4,5,6)$, and pipeline flow vector is $[Q] = [Q_i]$, $(i = 1,2,3,4,5,6)$, respectively.
Then the flow vector $[Q]$, computed to facilitate the continuity equation, is given by,

$$[Q] = [Q_i] = [T_D^Q].[D] \quad \ldots(4)$$

where, $[T_D^Q]$ is not only a form of adjacency matrix that represents the connectivity of pipes through nodes, but also a linear-transformation matrix that transforms nodal demands in to pipeline flows. The matrix $[T_D^Q]$ for the WDN corresponding to the Figure 3 can be considered as equation 5. Intuitively, for example, if the pipe number 1 has a flow of Q_1, then Q_1 is equal to the summation of the demands of all the downstream nodes, resulting 1's at the corresponding columns and 0's for all the other entries of the first row of $[T_D^Q]$. Similarly, all the entries of all the other rows of $[T_D^Q]$ can be determined.

$$[T_D^Q] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(5)$$

The matrix form of the selected diameter solution vector can be seen as $[d] = [d_i]$, $(i = 1,2,3,4,5,6)$.

Friction-loss-gradient of the i^{th} pipe, $h_{f\ i}$, is then calculated according to the Hazen-Williams Equation shown in equation 6.

$$h_{f\ i} = \frac{10.666 * Q_i^{1.85}}{C_{HW}^{1.85} * d_i^{4.87}} \quad \ldots(6)$$
$$i = 1,2,3,4,5,6$$

The matrix form of the set of friction-loss gradients for all the pipes can be identified as equation 7.

$$[h_f] = [h_{f\ i}] = \frac{10.666}{C_{HW}^{1.85}}.[[Q]^{o\,(1.85)}./\,[d]^{o\,(4.87)}] \quad \ldots(7)$$

where, $[Q]^{o\,(1.85)}$ is the notation for the vector obtained by the element-wise exponentiation of the elements in $[Q]$ to the power 1.85; and "./" is the operator "Hadamard Division" for denoting the element wise division between two vectors $[Q]^{o\,(1.85)}$ and $[d]^{o\,(4.87)}$.
The equations 4 and 7 can be combined to obtain the equation 8 as shown below.

$$[h_f] = [h_{f\ i}] = \frac{10.666}{C_{HW}^{1.85}}.[([T_D^Q].[D])^{o\,(1.85)}./\,[d]^{o\,(4.87)}] \quad \ldots(8)$$

To account for the fitting losses that occur in each pipe, friction-loss gradient vector is multiplied by the fitting-loss coefficient C_{ft} to obtain the $[g_{FF}]$, the gradient vector for both friction and fitting losses.

$$[g_{FF}] = C_{ft}.[h_f] \quad \ldots(9)$$

The pipe-length vector is denoted by $[L] = [L_i]$, $(i = 1,2,3,4,5,6)$.
The pipeline-head-loss vector $[F]$, is the product of $[L]$ and $[g_{FF}]$. The matrix notation of this element-wise multiplication of two vectors, $[L]$ and $[g_{FF}]$, is given by the "Hadamard Product $\odot$" as shown in equation10.

$$[F] = [F_i] = [L] \odot [g_{FF}] \quad \ldots(10)$$

The equations 9 and 10 can be merged to obtain the matrix equation 11.

$$[F] = [L] \odot C_{ft}.[h_f] \quad \ldots(11)$$

As the hydraulic-head-loss along the flow path is a continuous function, total algebraic summation of the head-loss around a closed path, is equal to zero. The mathematical notation of this law of conservation of energy is given by,

$$\sum_{i=1}^{np_L} F_i = 0 \quad \ldots(12)$$

where, npL is the number of pipes in a closed path.

As the focus here in this study is directed only towards the tree-shaped WDNs, the adaptation of the above equation 12 is shown in equation 13 as a matrix form.

$$[H] = E_0[1] - [T_H].[F] \quad \ldots(13)$$

where, $[H]$ is the nodal-head vector; E_0 is the elevation of the reservoir node N_0 measured from the mean sea level (MSL); [1] is the vector in which each element is equal to 1; $[T_H]$ is the transformation matrix that transforms pipeline-head-loss values $[F]$ into total head-loss values at all the nodes, and total head-loss values are given by $[T_H] * [F]$. The matrix expression of $[T_H]$, however, is given by equation 14.

$$[T_H] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(14)$$

It can be noticed, if N_4 is considered for example, head-loss occurs only through pipe numbers 1 and 4, hence first and fourth column entries of row number four are equal to 1, resulting all the other entries in that row being equal to zero. Likewise, entries of all the other rows of $[T_H]$ can be determined by mere layout of the WDN concerned.

By considering equations 11 and 13 together, the following expression (15) can be derived for $[H]$.

$$[H] = E_0[1] - [T_H].([L] \odot C_{ft}.[h_f]) \quad \ldots(15)$$

The nodal-elevation vector $[E]$ has entries E_js where E_j is the nodal-elevation of the j^{th} node measured from MSL. $[E]$ is given by $[E] = [E_j]$, $(j = 1,2,3,4,5,6)$.

As the residual water head H_R at each node is equal to the difference between the nodal-head and nodal-elevation, the matrix form is given by (16).

$$[H_R] = [H] - [E] \quad \ldots(16)$$

By merging the equations 15 and 16 together, equation 22 can be obtained.

$$[H_R] = E_0[1] - [T_H].([L] \odot C_{ft}.[h_f]) - [E] \quad \ldots(17)$$

As a summary, from the proposed STMA, $[g_{FF}]$ and $[H_R]$ are now computed for selected $[d]$. Hence, it is now possible to proceed with the penalty evaluations in order to continue with the algorithm discussed under Figure 1.

3.3 Warapitiya Service Zone, Sri Lanka

The general optimization algorithm that includes proposed STMA simulation described under Figure 1, was implemented for the WDN scheme planned for Warapitiya Service Zone, Sri Lanka, aiming for obtaining the combination of pipe diameters that minimizes the cost function while satisfying the design constraints, as per the optimization model formulation described. In this study the modern stochastic optimization algorithm Senavirathna et al. [10] have used, i. e., Honey-Bee Mating Optimization Algorithm (HBMO), was updated with the STMA component to facilitate the hydraulic simulations for the tree-shaped WDN. The minimum-allowable-nodal-hydraulic-head value H_R^{min} and the maximum-allowable-gradient for friction-and-fitting losses g_{FF}^{max} were considered to be 10 m and 0.005 m/m (meter per meter), respectively. The Hazen-Williams coefficient, C_{HW}, and the fitting loss coefficient, C_{ft}, were taken to be 130 and 1.15, respectively, for all the pipes.

Here, in fact, are two main attributes of Warapitiya WDN to be an ideal choice for STMA simulation. On the one hand, WDN of Warapitiya Service Zone has the layout architecture of a spanning-tree, and on the other hand, it has substantial number of demand nodes and pipes that it can be considered as a relatively "large-scale" WDN.

The data requirement, as identified for the optimization algorithm, included unit-costs of commercially available pipe sizes, reservoir water head, nodal-water-demands, pipe lengths, nodal-elevations, and the pipe-connectivity layout. The pipe layout of Warapitiya WDN can be seen from the Figure 3 as a directed-graph to identify the flow directions. It is to be noted that the symbology used under the Warapitiya Service Zone mean the same as that was utilized in STMA methodology explanation. Pipe length data are given in the Table 1.

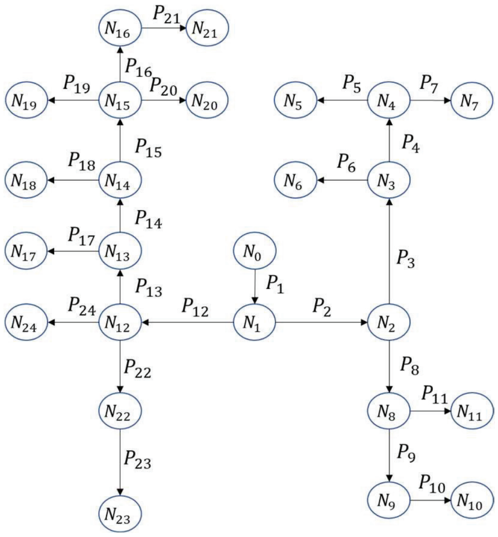

Figure 3 - Pipe layout directed-graph for WDN of Warapitiya Service Zone, Sri Lanka

Nodal-water demand data and nodal-elevation data, including the datum of reservoir elevation, can be referred to as listed in the Table 2. All these data, excluding unit-costs of commercially available pipes, were collected from the National Water Supply and Drainage Board (NWS&DB), Sri Lanka. Unit-cost data for different pipe sizes were obtained from Mohan & Babu [9], and shown in the Table 3. Corresponding unit-costs for commercially available pipe sizes used in Warapitiya WDN, however, were estimated by interpolating these data using Lagrange-Interpolation-Method.

Table 1 - Lengths of pipes being deployed for Warapitiya WDN

Pipe (P_i)	Length (L_i) / (m)	Pipe (P_i)	Length (L_i) / (m)
P_1	250	P_{13}	260
P_2	160	P_{14}	630
P_3	740	P_{15}	300
P_4	080	P_{16}	530
P_5	290	P_{17}	850
P_6	350	P_{18}	520
P_7	130	P_{19}	290
P_8	330	P_{20}	620
P_9	160	P_{21}	140
P_{10}	180	P_{22}	300
P_{11}	410	P_{23}	1280
P_{12}	030	P_{24}	1160

Nodal-water demand data and nodal-elevation data, including the datum of reservoir elevation, can be referred to as listed in the Table 2. All these data, excluding unit-costs of commercially available pipes, were collected from the National Water Supply and Drainage Board (NWS&DB), Sri Lanka. Unit-cost data for different pipe sizes were obtained from Mohan & Babu [9], and shown in the Table 3. Corresponding unit-costs for commercially available pipe sizes used in Warapitiya WDN, however, were estimated by interpolating these data using Lagrange-Interpolation-Method.

Table 2 - Nodal-water-demands and nodal-elevations of Warapitiya WDN

Node (N_j)	Demand (D_j) / (m^3/ day)	Elevation (E_j) / (m)	Node (N_j)	Demand (D_j) / (m^3/day)	Elevation (E_j) / (m)
N_0	-	506.0	N_{13}	133.13	479.0
N_1	3.75	485.0	N_{14}	105.00	472.0
N_2	381.56	475.0	N_{15}	116.25	462.5
N_3	292.50	455.0	N_{16}	61.88	445.0
N_4	148.13	460.0	N_{17}	61.88	450.0
N_5	48.75	455.0	N_{18}	28.13	474.0
N_6	11.25	450.0	N_{19}	33.75	450.0
N_7	91.88	460.0	N_{20}	28.13	452.0
N_8	101.25	480.0	N_{21}	20.63	450.0
N_9	45.00	480.0	N_{22}	76.88	475.0
N_{10}	18.75	455.0	N_{23}	71.25	450.0
N_{11}	43.13	476.0	N_{24}	58.13	450.0
N_{12}	71.25	482.0	-	-	-

Table 3 - Unit-cost data for corresponding pipe sizes (Mohan & Babu [9])

Diameter / (mm)	Unit-cost
25.4	2
50.8	5
76.2	8
101.6	11
152.4	16
203.2	23
254.0	32
304.8	50
355.6	60
406.4	90
457.2	130
508.0	170
558.8	300
609.6	550

The unit-cost data in Table 3 were interpolated as per the Lagrange-Interpolation-Polynomial method, and the interpolated unit-costs for commercially available pipe diameters are listed in the Table 4.

Table 4 - Commercially available pipe sizes and their corresponding interpolated unit-costs

Diameter / (mm)	Interpolated unit-cost
055	5.0259
079	8.4781
097	10.6801
140	14.0679
198	22.5046
246	29.6739

4. Results and Discussion

From Table 5, it can be observed the contrast between the optimal design solution obtained by HBMO algorithm and the deterministically-obtained solution implemented by NWS&DB. It can be noted that the total cost involved with the optimal design solution obtained by HBMO algorithm was 100172 units whereas the total cost involved with the solution implemented by NWS&DB was 107588 units. The optimal design solution obtained by HBMO algorithm highlighted in Figure 1, therefore, was lower in cost compared to randomly guessed solutions.

Table 6 and Table 7 span down to tabulate the results, $[g_{FF}]$ and $[H_R]$, obtained at the STMA simulation stage, corresponding to the global optimal solution obtained by HBMO algorithm. The left-hand side of the Table 6 shows that the design constraint $g_{FF\ i} \leq g_{FF\ i}^{max}$ is satisfied for all the pipes $(i = 1,2, \dots ,24)$ in the spanning-tree network by delivering the g_{FF} value of each pipe less than $0.005\ m/m$, whereas the right-hand side shows that the design constraint $H_{R\ j} \geq H_{R\ j}^{min}$ is satisfied for all the nodes $(j = 1,2, \dots ,24)$ in the spanning-tree network by delivering the H_R value of each node greater than $10\ m$. Hence, the application of the global optimal solution for [d] is appropriate in terms of the satisfaction of the design constraints.

Although in Table 6 it is mentioned only these constraints (2) and (3), all the intermediate attributes of the STMA can be varied to achieve desired optimal-design of the spanning-tree pipe network.

To summarize, the results of the study demonstrate that, HBMO algorithm together with the proposed STMA simulation were able to deliver the combination of pipe diameters that minimizes the cost of the given spanning-tree pipe configuration network while satisfying the design constraints being concerned.

Table 5 - Comparison between the solution obtained by HBMO+STMA and the guessed solution implemented by NWS&DB

Pipe P_i	Optimal Solution [d] by HBMO / (mm)	Guessed solution implemented by NWS&DB / (mm)	Pipe P_i	Optimal Solution [d] by HBMO / (mm)	Guessed solution implemented by NWS&DB / (mm)
P_1	198	246	P_{13}	140	198
P_2	198	198	P_{14}	140	140
P_3	198	198	P_{15}	97	140
P_4	97	140	P_{16}	97	079
P_5	79	079	P_{17}	79	079
P_6	79	055	P_{18}	97	055
P_7	79	079	P_{19}	79	055
P_8	97	140	P_{20}	79	055
P_9	79	079	P_{21}	55	055
P_{10}	97	055	P_{22}	79	097
P_{11}	55	079	P_{23}	55	079
P_{12}	198	198	P_{24}	55	079

Table 6 - Tabulation of the optimal solution given by HBMO algorithm and constraint $g_{FF\ i}$ satisfaction status obtained by STMA

Pipe P_i	Optimal Solution [d] by HBMO / (mm)	$g_{FF\ i}$ / (m/m)	Design Constraint $g_{FF\ i} \leq g_{FF\ i}^{max}$ satisfied?
P_1	198	0.0040	Yes
P_2	198	0.0014	Yes
P_3	198	0.0004	Yes
P_4	097	0.0034	Yes
P_5	079	0.0003	Yes
P_6	079	0.0000	Yes
P_7	079	0.0011	Yes
P_8	097	0.0019	Yes
P_9	079	0.0006	Yes
P_{10}	097	0.0000	Yes
P_{11}	055	0.0016	Yes
P_{12}	198	0.0008	Yes
P_{13}	140	0.0021	Yes
P_{14}	140	0.0010	Yes
P_{15}	097	0.0028	Yes
P_{16}	097	0.0003	Yes
P_{17}	079	0.0005	Yes
P_{18}	097	0.0000	Yes
P_{19}	079	0.0002	Yes
P_{20}	079	0.0001	Yes
P_{21}	055	0.0004	Yes
P_{22}	079	0.0027	Yes
P_{23}	055	0.0004	Yes
P_{24}	055	0.0028	Yes

Table 7 - Tabulation of the constraint $H_{R\ j} \geq H_{R\ j}^{min}$ satisfaction status obtained by STMA

Node N_j	$H_{R\ j}$ / (m)	Design Constraint $H_{R\ j} \geq H_{R\ j}^{min}$ satisfied?
N_1	20.0088	Yes
N_2	29.7801	Yes
N_3	49.4855	Yes
N_4	44.2133	Yes
N_5	49.1135	Yes
N_6	54.4775	Yes
N_7	44.0688	Yes
N_8	24.1675	Yes
N_9	24.0770	Yes
N_{10}	49.0731	Yes
N_{11}	27.5114	Yes
N_{12}	22.9846	Yes
N_{13}	25.4312	Yes
N_{14}	31.7940	Yes
N_{15}	40.4496	Yes
N_{16}	57.7719	Yes
N_{17}	53.9765	Yes
N_{18}	29.7702	Yes
N_{19}	52.8990	Yes
N_{20}	50.8724	Yes
N_{21}	52.7147	Yes
N_{22}	29.1779	Yes
N_{23}	48.9941	Yes
N_{24}	51.7607	Yes

5. Conclusions

Aiming for deriving a hydraulic model that has adaptable hydraulic governing equations, this study was carried out to utilize the proposed Spanning-Tree Matrix Approach (STMA) that can model "large-scale", "tree-shaped" water distribution networks. Taking the WDN of Warapitiya Service Zone in Sri Lanka as a case study, it was tested the use of the STMA model embedded in the Honey-Bee Mating Optimization (HBMO) algorithm to find the combination of pipe diameters that minimizes the cost of the network. As the STMA approach utilizes experimentally established hydraulic governing equations, the results show that the STMA is successful in modeling a tree-shaped WDN of given design criteria, and proved its applicability as a hydraulic model in optimization algorithms.

Furthermore, proposed STMA can be updated with desired governing equations or design criteria of specific interest. Authors recommend further research on employing STMA for other types of spanning-tree hydraulic networks, such as waste water networks, river networks and ground-water networks, given the adaptability of STMA for such spanning-tree hydraulic networks.

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