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Joint Probability of Cyclonic Wind Speed and Maximum Rainfall for Cyclones Affected Sri Lanka

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Abstract: Tropical cyclones pose significant risks to coastal regions, with impacts including high wind speeds, heavy rainfall, and storm surges. This study aims to analyse the joint probability of cyclonic wind speed and maximum rainfall for cyclones affecting Sri Lanka using the Copula theory. Historical data from 33 cyclones that made landfall in Sri Lanka between 1900 and 2020 were analysed. Through exploratory data analysis and marginal distribution analysis, the Generalized Pareto distribution and Pearson type III distribution were identified as the best fits for cyclonic wind speed and rainfall, respectively. The study employed the Clayton copula to model the joint distribution of these variables, given its suitability for capturing moderate correlation and tail dependence. Optimisation of the copula parameters led to a synthetic dataset that validated the copula model's accuracy. The results revealed a moderate correlation between wind speed and rainfall. The joint return periods were calculated using the copula model, providing crucial insights into the likelihood of simultaneous occurrences of specific wind speeds and rainfall intensities. This statistical relationship offers valuable insights for coastal flood modelling and risk mitigation strategies in Sri Lanka, aiding policymakers and stakeholders in preparing for and mitigating the impacts of tropical cyclones.

Keywords: Tropical Cyclone, Cyclonic Wind Speed, Maximum Rainfall, Joint Probability, Copula Theory

1. Introduction

Tropical cyclones can be considered one of the most dangerous and costliest hazards in coastal regions [1,2]. Both the number of tropical cyclones and the intensity of cyclones have been observed to increase due to global climate changes [3]. Tropical cyclone intensity is projected to increase with climate warming by about 1-10% for a 2-degree Celsius global warming scenario [3]. Tropical cyclones are caused due to atmospheric changes and they contain high wind speeds, heavy precipitation and storm surges [4,5]. Projections of future climate change suggest widespread increases in storm surge hazard [6,7], extreme rainfall hazard [8,9], and the occurrence of compound flood events [10]. As the coastal regions may face floods due to various flood drivers, it is important to find the dependency between different flood drivers.

There are numerous methods to find the dependency between rainfall and storm surge. One widely used method is using physically based models, as demonstrated by Bacopoulos et al., 2017, Herdman et al., 2018, Kumbier et al., 2018, and Ray et al., 2011 [11-14]. Another common approach is using statistical analysis between these two flood drivers [15-19].

Most past studies have focused on building a correlation between flood drivers using observed values and then sampling the joint probability distribution to develop scenarios which will be modelled using a hydro dynamical model for analysing compound flood scenarios [20-24]. There are many statistical approaches to derive correspondence between flood drivers. Tao et al., 2013 [24] selected the Poisson bivariate compound maximum entropy distribution to establish the joint distribution of extreme water level and wave height in a typhoon period. Wahl et al. 2016 [25] utilised the likelihood of compound events of storm surge and heavy precipitation for the coastal areas of the United States (US).

A new bivariate compound extreme value distribution was proposed by Dong, 2017 [26] to describe the probability distribution of annual extreme wind speed and maximum rainfall intensity in the coastal areas that are affected by

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typhoons. Kwon et al., 2017 [27] analysed the correlation between annual maximum wind speed and rainfall by using the copula function in the typhoon danger zone.

Copulas are functions that join or couple multivariate distributions to their one-dimensional marginal distribution functions. Sklar, 1959 [28] first used the word copula in a statistical and mathematical context and introduced a theorem that is named after him. Copulas are statistical functions that link multivariate distribution margins to full joint distribution, allowing for the modelling of complex dependencies between variables. Equivalently, copulas are multivariate distributions whose marginals are uniform on the interval (0, 1) [29]. Copula gained popularity in hydrology and climatology after the works of Favre et al., 2004 [30], and Salvadori and De Michele, 2004 [31]. Among many copula families, Archimedean copulas, which include Clayton, Frank, and Gumbel copula, are widely used in hydrology analysis. Tang et al. 2022 [32] utilised the Archimedean copula family to find hazard probability to develop a high-resolution framework to assess the regional hazards of wind and rain and it was applied to Shenzhen, China. Ai et al. 2019 [33] utilised Archimedean for a joint probability analysis of compound floods from rainstorms and typhoon surges in the coastal areas of Jiangsu Province, China. Xu et al. 2014 [34] evaluated the combined risk of extreme precipitation and storm tides in the coastal region of Fuzhou, China, using statistical methods such as copulas. Lu et al. 2022 [35] constructed a bivariate probability distribution using copulas between river discharge and sea level to analyse the impact of mixed flood-generating mechanisms. They selected 15 locations on the western coast of China mainland and Hainan Island (1060E – 1200E and 180N – 320N).

Although Sri Lanka is in a region prone to cyclones, the frequency of cyclone landfalls is relatively low compared to neighbouring countries. Nevertheless, studying cyclones and compound flooding remains crucial due to the potential severity of these hazards. There is multiple research done for analysing the impact of cyclones and coastal flooding due to storm surges such as Dube et al. 2003 [36], Maduwantha and Wijerathne, 2019 [37]. However, the joint distribution of cyclones and precipitation is not analysed. This study aims to build a statistical relationship between the maximum sustained wind speed of a cyclone and the cyclonic rainfall using the Copula theory. Joint retain periods (T_{or} and T_{and}) were found based on Salvador and Michelle (2004) [31]. This statistical relationship can be utilised in coastal flood modelling and other risk mitigation measures for Tropical cyclones.

2. Study Area and Past Cyclonic Events

Sri Lanka is a tropical island located between 5^0 – 10^0 Latitude and 79^0 -82^0 Longitude. Sri Lanka, located in the Indian Ocean, is positioned near both the Bay of Bengal and the Arabian Sea, which influences its cyclonic activity. Sri Lanka can be considered a relatively small island with a total land area of about 65,000 km^2.

The formation of cyclones in these regions is strongly related to the seasonal migration of the Inter Tropical Convergence Zone (ITCZ) [38]. Even though Sri Lanka is affected only occasionally by storm surges compared to Bangladesh and India, some extreme losses occurred during historical events [36]. Thirty three (33) cyclones have made landfall in Sri Lanka from 1900 to 2020. Cyclones of 1964 and 1978 can be considered the most disastrous cyclonic hazards in Sri Lanka in recent history. The entire land area of the island was considered for this study.

In this study, it is assumed that when cyclones enter Sri Lanka, the climate of the entire island is impacted. This assumption is supported by the geographical size of Sri Lanka and the nature of cyclonic systems. Cyclones are large-scale weather systems with wide-ranging impacts affecting a significant portion of the island's territory. While the intensity of impacts may vary across regions, it is reasonable to assume that cyclonic events would influence the island's climate as a whole due to the size and magnitude.

For the past cyclonic events in Sri Lanka, data acquisition was done by multiple governmental and private entities on cyclones that made landfall in Sri Lanka. The following datasets were utilized for this analysis: Cyclonic trajectories, occurrence dates and wind velocity data. These were collected from the International Best Track Archive for Climate Stewardship (IBTrACS). Furthermore, comprehensive insights into cyclonic occurrences, their trajectories, and associated wind velocities were collected from the Department of Meteorological Sri Lanka

(DMSL) [39]. Based on these two data sources, all 33 cyclone events which made landfall in Sri Lanka were sorted out. Then maximum daily rainfall record of cyclone affected period was collected from DMSL. Figure 1 shows the distribution of maximum rainfall during a cyclone event concerning cyclonic wind speed.

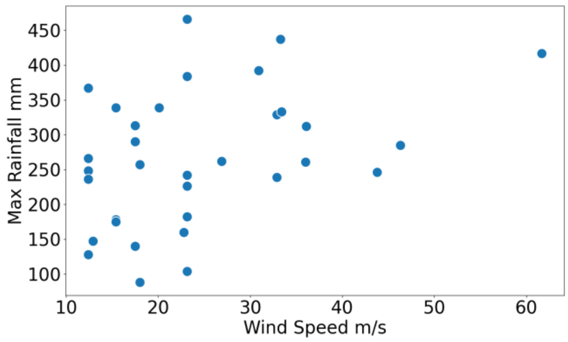

Figure 1 - Wind Speed vs. Maximum Rainfall for the Cyclones Made Landfall in Sri Lanka from 1900 to 2020

3. Methodology

The Sklar theorem introduced by Sklar [1959][28] was utilised to find the joint distribution between maximum sustained wind speed and maximum rainfall during cyclonic events in Sri Lanka. Sklar theorem denotes that for any multivariate distribution F(x_1, x_2,…… x_n) can be represented with the help of a unique copula (C):

$$F_{x_1,\ldots,x_n} = C\left(F_{x_1(x_1)}, \ldots., F_{x_n(x_n)}\right) \quad \ldots(1)$$

where $F_{x_i(x_i)}$denotes the i^th^ one-dimensional marginal distribution of the multivariate distribution. For continues distribution ($F_{x_i(x_i)}$), there is a unique copula function (C). Copulas of multivariate distributions can be extracted by taking:

$$C_{u_1,\ldots,u_n} = F\left(F^{-1}_{x_1(u_1)}, \ldots., F^{-1}_{x_n(u_n)}\right) \quad \ldots(2)$$

where $F^{-1}_{x_i(u_i)}$denotes the i^th^ one-dimensional inverse marginal distribution.

Before the joint probability analysis, an Exploratory Data Analysis (EDA) and Kernal analysis were conducted to gain an understanding of the overall structure and relationship between wind speed and rainfall data. Kernal plots were also developed for each variable as well as the joint distribution to visualize the distribution and dependencies. After that, Marginal Distribution Analysis was done for maximum sustained wind speed and maximum rainfall to identify the distribution that best captures the characteristics of the behaviour of each variable. Six marginal distributions were considered here: Normal, log normal, Pearson 3, Weibull, gamma and Generalized Pareto distribution. Optimization of the parameters for each distribution to find the best fit marginal distribution parameters was done by maximizing Likelihood Estimation. Log-likelihood function $ln\ (L(\theta|\ x)$ formulates as

$$\ln(L(\theta|x) = \sum\left[ln\left(f\left(x_i|\theta\right)\right)\right] \quad \ldots(3)$$

where $f(x_i|\theta)$is the probability density function (PDF). From six optimised marginal distributions, the best fit was selected using Root Mean Square Error (RMSE) and visual interpretation for both maximum sustained wind speed and maximum rainfall.

$$RMSE = \sqrt{\frac{1}{n}\sum_1^n (Pe_i - P_i)^2} \quad \ldots(4)$$

where Pe is the predicted data and P is the observation data. The best fit one-dimensional marginal distribution of these variables was utilised in the copula analysis. Wind speed and rainfall data were converted into the relevant marginal distribution value based on the selected distribution.

After the one-dimensional marginal distribution analysis, joint probability analysis was conducted for the data set based on the Copula theorem. Here, Archimedean copulas which are Clayton, Frank and Gumbel (Table 1) were considered in this study as these copulas are frequently used in hydroclimatic applications as they have a simple mathematical form and are specifically easy to sample from when considering only two variables [40].

An Archimedean copula C in n dimensions is defined using a generator function φ that is a continuous, strictly decreasing function from [0,1] to [0, ∞] with $\varphi(1) = 0$. The copula C is given by;

$$C_{u_1,u_2,\ldots,u_n} = \varphi^{-1}\left(\varphi(u_1) + \varphi(u_2) + \cdots + \varphi(u_n)\right) \quad \ldots(5)$$

where $\varphi^{-1}$is the pseudo-inverse of the generator function φ.
Considering the behaviour of these copula families, the Clayton copula is the best-suited copula family when the parameters have moderate correlation and some tail dependence.

Table 1 - Archimedean Copula Families and their Closed-Form Mathematical Description

Name	Mathematical Description	Parameter Range	Reference
Clayton	$max(u^{-\theta}+v^{-\theta}-1,0)^{1/\theta}$ …(6)	$\theta \in [-1,\infty]\backslash 0$	Clayton [1978] [41]
Frank	$-\frac{1}{\theta}ln[1+\frac{(exp(-\theta u)-1)(exp(-\theta v)-1)}{exp(-\theta)-1}]$…(7)	$\theta \in R\backslash 0$	Li et al. [2013] [42]
Gumbel	$exp\{-\left[(-ln(u))^{\theta}+(-ln(v))^{\theta}\right]^{1/\theta}\}$…(8)	$\theta \in [1,\infty]$	Li et al. [2013][42]

Gaussian copula is a copula family best suited for linear correlation while the Frank copula is not as flexible as the Clayton copula and may not be able to capture the tail dependence on the data as well. Among these Copula families the best fit for the data set was selected based on the correlation between maximum sustained wind speed and maximum cyclonic rainfall. 10% of the higher data set was considered for the tail dependency analysis. The following functions were considered to find the correlation between two variables.

Pearson correlation (r) which denotes as

$$r=\frac{\sum((x-\bar{x})\times(y-\bar{y}))}{\sqrt{\sum((x-\bar{x})^2)\times\sum(y-\bar{y})^2}} \quad \text{…(9)}$$

where x, y are two variables and $\bar{x}$, $\bar{y}$ are the mean of those variables.

Spearman Correlation (ρ) which denotes as

$$\rho = 1-\frac{6\times\sum d^2}{(n^2-1)} \quad \text{…(10)}$$

where d is the difference in the ranks of corresponding values and n is the number of data points. Kendall's Tau, which measures the strength and direction of association between two variables, based on the ranks of the data rather than their actual values, and is particularly useful for non-linear relationship was also calculated apart from above correlation functions. Kendall's tau (τ) is given by

$$\tau=\frac{n_c-n_d}{\sqrt{(n_0-n_1)(n_0-n_2)}} \quad \text{…(11)}$$

where n_c is the number of concordant pairs, n_d is the number of discordant pairs, n_0 is the total number of pairs, n_1 is the number of tied values in the first dataset and n_2 is the number of tied values in the second dataset. These parameters were found for the entire data set as well as the higher 10% for tail dependency. These parameters give a statistical measure of the correlation of two variables and the tail dependency. Based on these statistical results the best-fit copula family to describe the joint variation of maximum cyclonic wind speed and cyclonic rainfall was selected between Archimedean copulas.

After selecting the best-fit copula family for two variables the copula function was fit to the data set. The copula parameter was optimised using the likelihood function in case of finding the best fit copula parameter.

After finalising the Copula function for the marginal values of the observed data set, a synthetic copula parameter data set was generated and the joint Cumulative Distribution Function (CDF) values for each data set were found according to the optimised copula family. Then a back transformation for copula parameters was done using selected marginal distribution for wind speed and rainfall to convert copula parameters back to these measures. The accuracy of the copula model was assessed using the Kolmogorov-Smirnov (KS) test. Observation parameters and copula-generated parameters were compared.

Finally, primary return periods were calculated for the copula-based joint probability assessment. In bivariate statistical analysis, there are two formulas to calculate the joint return period, depending on whether only one event happens at a time or both events happen simultaneously. Both T_{OR} and T_{AND} cases defined by Salvador and Michelle (2004) [31] were considered here.

$$T_{u,v}^{OR}=\frac{\mu}{1-C(u,v)} \quad \text{…(12)}$$

$$T_{u,v}^{AND}=\frac{\mu}{1-u-v+C(u,v)} \quad \text{…(13)}$$

where T_{OR} is the return period or happening at least in one event and T_{AND} is the return period of both events. Here u, v are the copula parameters of each variable and $C_{(u,v)}$ is the copula function. Return period curves were generated based on these calculations to illustrate the joint probability of cyclonic events affecting wind speed and rainfall.

4. Results

The first step of the study is to have a basic understanding of the behaviour of two variables: maximum sustained wind speed and maximum rainfall during cyclonic events. Exploratory Data Analysis results which are shown in Table 2 show the basic statistical parameters.

Table 2 - Summary of Statistical Analysis for Wind Speed and Rainfall

Parameter	Wind Speed (m/s)	Maximum Rainfall (mm)
Mean	24.41	266
St. dev.	11.48	97
Min. value	12.4	88
25th percentile	15.4	182
50th percentile	23.15	261
75th percentile	32.9	333
Max. value	61.67	466

The mean and median wind speeds are 24.4 m/s and 23.2 m/s, respectively, indicating that the central tendency of wind speeds during cyclonic events is relatively high. The distribution of wind speeds, as indicated by the standard deviation (i.e.,11.48 m/s), shows significant variability, which suggests that cyclones affecting Sri Lanka can have a wide range of intensities. The rainfall data, with a mean of 266.3 mm and a median of 261 mm, shows a slightly lower variability as indicated by the standard deviation of 97.21 mm. The rainfall range (from a minimum of 88 mm to a maximum of 466 mm) indicates that cyclones can bring moderate and extremely heavy rainfall to the island.

The kernel density plots of the wind speed and maximum rainfall data are shown in Figure 2. The distribution of maximum wind speed (Figure 2(a)) can be observed to be right skewed, which denotes that cyclones with lower wind speed values can be observed more frequently. The distribution of maximum rainfall (Figure 2(b)) can be observed to be centred indicating a more symmetrical distribution.

Figure 3 shows the kernel distribution for the joint distribution of the two parameters. It can be observed that most of the cyclone events were concentrated at lower wind speeds (15 m/s - 30 m/s) and rainfall of 200 mm and 300 mm.

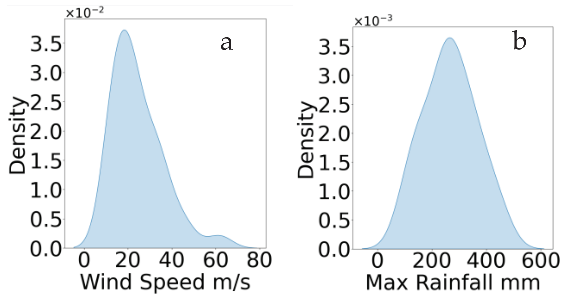

Figure 2 - Kernal Plots of Two Variables (a) Wind Speed and (b) Rainfall

Further analysis of the joint distribution reveals a peak in the probability density where the wind speed is around 20 m/s and rainfall are approximately 250 mm. This peak indicates the most likely scenario during a cyclone event.

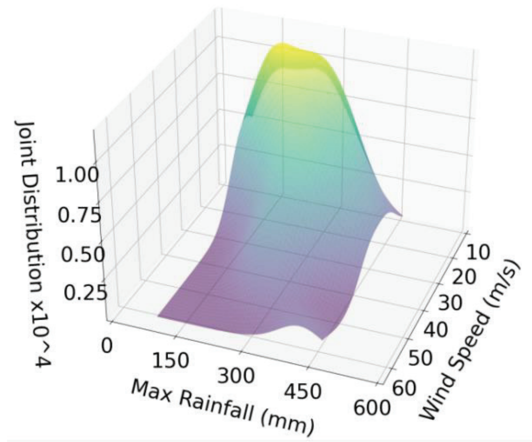

Figure 3 - Kernal Distribution of Joint Distribution of Cyclonic Wind Speed and Maximum Rainfall

Marginal Distribution Analysis (MDA) was done for both variables using Normal, Lognormal, Gamma, Weibull, Pearson type III, and generalised Pareto distribution. Marginal distribution parameters for each distribution were found using the log-likelihood function. Table 3 shows the optimised marginal distribution parameter for maximum cyclonic wind speed and maximum rainfall.

Visual interpretation and statistical measures were used to determine the best-fitting marginal distributions. The comparison of marginal cumulative distribution functions (CDFs) with empirical CDFs for both wind speed and rainfall are shown in Figure 4. Here, the CDF for all six marginal distributions was compared with the empirical CDF for maximum wind speed and rainfall.

Table 3 - Optimized Parameters for Each Distribution

Distribution	Parameters					
	Wind Speed			Rainfall		
	Shape	Scale	Location	Shape	Scale	Location
Generalised Pareto	0.30	12.40	10.12	-1.17	-1.23	547.54
Weibull	0.31	12.40	0.88	0.16	88.00	1.27
Normal	-	24.41	11.31	-	266.30	95.77
Log Normal	4.10	12.40	0.09	5.35	88.00	0.41
Pearson type III	2.59	20.44	10.41	0.21	266.30	95.91
Gamma	0.21	12.40	8.15	232.17	-1191.00	6.28

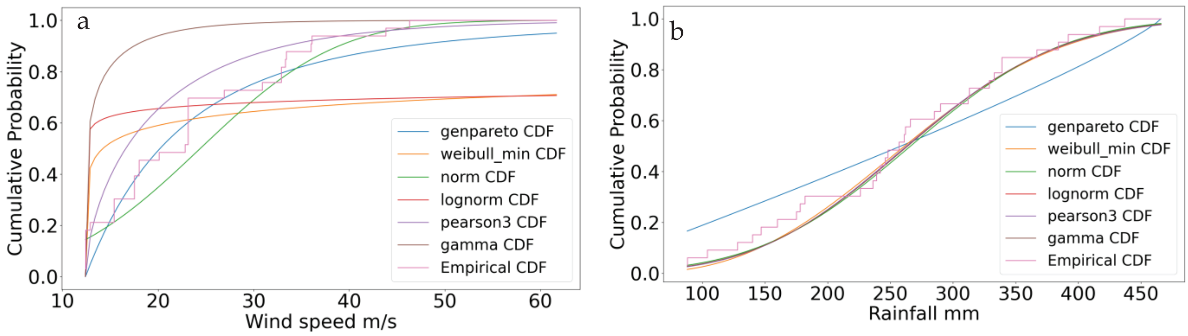

Figure 4 - Comparison of Marginal CDFs with Empirical CDF (a) Wind Speed and (b) Rainfall

Table 4 - Statistical Analysis of Marginal Distribution for Wind Speed and Rainfall

	Wind Speed			Rainfall		
	RMSE (m/s)	AIC	BIC	RMSE (mm)	AIC	BIC
Generalised Pareto	0.063	238.2	242.7	0.097	402.4	406.9
Weibull	0.306	57.8	62.3	0.427	485.2	489.7
Normal	0.084	257.7	260.7	0.035	398.7	401.7
Log-Normal	0.320	219.3	223.7	0.413	509.0	5134
Pearson type III	0.144	107.4	111.8	0.034	400.6	405.1
Gamma	0.366	63.0	67.5	0.034	400.6	405.1

Here we can see Generalised Pareto, Normal and Pearson type 3 distributions have better agreement with Empirical CDF for maximum wind speed (Figure 4 (a)). For maximum rainfall, all the distributions show good agreement.

Generalised Pareto Distribution was selected as the best fit marginal distribution for maximum sustained wind speed based on the statistical measures as well as the flexibility in fitting the upper tails of data. Pearson type 3 distribution was selected for maximum rainfall considering the ability to handle skewed data.

To assess the fit of the selected marginal distributions, QQ plots were generated (Figure 5) for theoretical quantiles and Ordered values of both maximum wind speed and maximum rainfall. It has been observed that there is a good agreement between the theoretical and empirical values for rainfall. This is also the case for lower wind speeds. However, when it comes to higher wind speeds, the agreement between theoretical and empirical values is not very good.

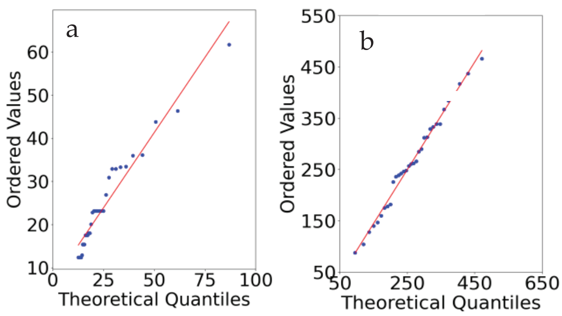

Figure 5 - QQ Plots for (a) Wind Speed and (b) Rainfall

The correlation between maximum wind speed and maximum rainfall shows a moderate correlation, with a Pearson coefficient of 0.40, Spearman coefficient of 0.35 and Kendall's Tau value is 0.243.

For tail dependency analysis, 10% of the higher data set was considered. The correlation coefficient for the tail is 0.97 and the tail dependency coefficient is 0.07. These stats show a moderate correlation between wind speed and rainfall. Tail dependency analysis focuses on the upper 10% of data points to examine how well the joint distribution captures the correlation between extreme wind speeds and rainfall. The QQ plots for the tail dependency of the two variables show that they are not perfectly linearly correlated, but they do have some tail dependence.

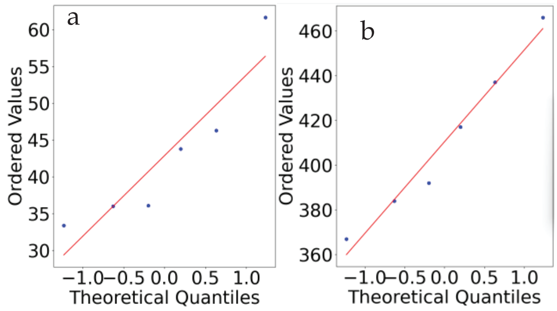

Figure 6 - QQ Plots for Tail Dependence (a) Wind Speed and (b) Rainfall

The QQ plot for the tail dependency of wind speed shows that while there is some alignment (Figure 6 (a)), it is not perfectly linear, indicating that extreme wind speeds might not be perfectly predicted by the current model. Tail dependency on rainfall shows a better agreement in Figure 6 (b).

Both correlation and tail dependency analyses indicate a moderate relationship between maximum sustained wind speed and maximum rainfall during cyclonic events. This relationship, though not perfectly linear, is sufficiently significant to consider using copula models for joint probability distribution. Among the Archimedean copulas, the Clayton copula was identified as the most suitable due to its ability to capture the dependency structure between the two variables effectively. To optimize the copula parameter (θ), the log-likelihood function was maximized, resulting in a parameter value of 0.53.

Figure 7 shows the log-likelihood function maximising when the copula parameter (θ) reaches an optimal value of 0.53. With the optimised copula parameter following equation is finalized as the Clayton copula function for the Joint Probability distribution between maximum sustained cyclonic wind speed and maximum cyclonic precipitation for Sri Lanka based on cyclones that made landfall in Sri Lanka from 1900 to 2020.

$$f_{(x)} = max(u^{-\theta} + v^{-\theta} - 1,0)^{1/\theta} \; ; \; \theta = 0.53 \ldots (14)$$

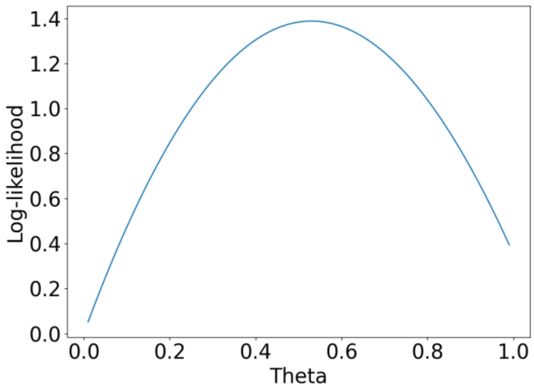

Figure 7 - Log-Likelihood Function for the Copula Parameter for Optimization

A synthetic data set was generated using the copula function. The correlation parameters for this synthetic data set, including Pearson and Spearman coefficients, were found to be 0.31 and 0.35, respectively, with Kendall's Tau value at 0.21. These correlation values align well with the observed data correlation parameters, indicating that the copula model effectively captures the dependency structure between wind speed and rainfall. Figure 8 shows the joint cumulative distribution for Clayton Copula.

The graph in Figure 8 depicts the copula CDF, where both axes represent the standardised scales of wind speed and rainfall, and the height represents their cumulative probability. The colour gradient indicates the probability density, with warmer colours signifying higher probabilities. This visualization confirms the model's capability to represent the most probable combinations of wind speed and rainfall during cyclonic events.

Then the copula parameters of the synthetic data set were back-transformed to wind speed and rainfall using the marginal distribution functions (Figure 9).

Figure 9 compares the observed data with the synthetic data generated using the copula model. The plot shows the relationship between rainfall and wind speed, with the observed data points marked in grey and the copula-generated data points in pink. This comparison demonstrates that the density of the synthetic data is higher in the range of the observed data density is higher. Which shows the copula model effectively simulating the joint behaviour of the two variables.

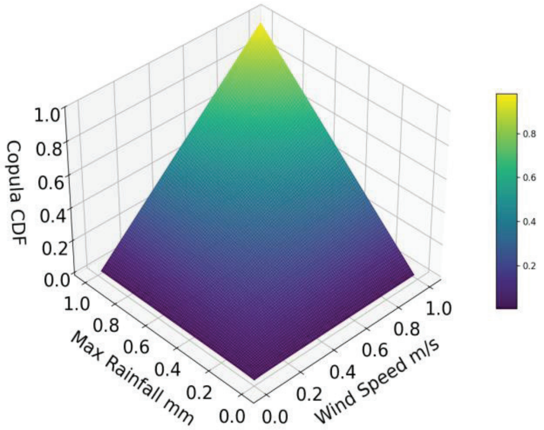

Figure 8 - Joint CDF for Clayton Copula

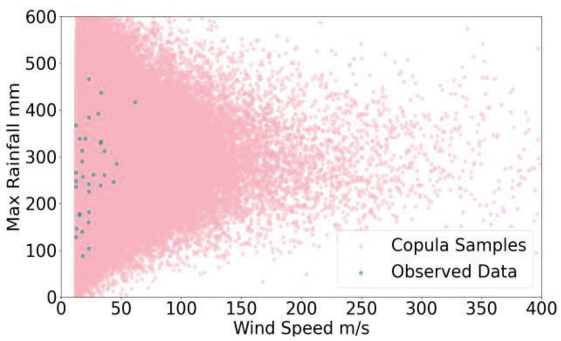

Figure 9 - Relationship between Rainfall and Wind Speed for Observed and Synthetic Values

The upper limits of the parameters are extended in the synthetic data set as 400 m/s for maximum sustained wind speed and 600 mm for maximum daily rainfall, covering extreme values. The Goodness to fit test was done using the KS test and the results are shown in Table 5. For both wind speed and rainfall, the p-values are relatively high, suggesting that there isn't strong evidence to conclude that these datasets significantly deviate from the expected distributions.

Table 5 - KS Test Results for Wind Speed and Rainfall

	Wind Speed	Rainfall
KS Statistic	0.152	0.084
KS p value	0.40	0.96

This means that, based on this test, wind speed and rainfall data may reasonably follow the expected distributions. Therefore, this synthetic data set is reliable in representing real-world scenarios.

Joint return periods were found according to Salvador and Mitchelle, 2004 [31]. Both T_{or} and T_{and} were found and contour lines were presented in the wind speed and rainfall distribution (Figure 10). Joint return periods describe the likelihood of simultaneous occurrences of specific wind speeds and rainfall intensities and are critical for understanding the risks associated with cyclonic events.

The colour contours in Figure 10 show the joint Kernal distribution of two parameters. Here we can observe that wind speed and rainfall for certain T_{and} values are always higher than the same T_{or} value. The colour contours in both plots indicate the density of the joint distribution, with warmer colours representing higher densities. Notably, the T_{and} values are consistently higher than the T_{or} values, implying that the probability of both extreme wind and significant rainfall occurring together is lower than the probability of either happening independently.

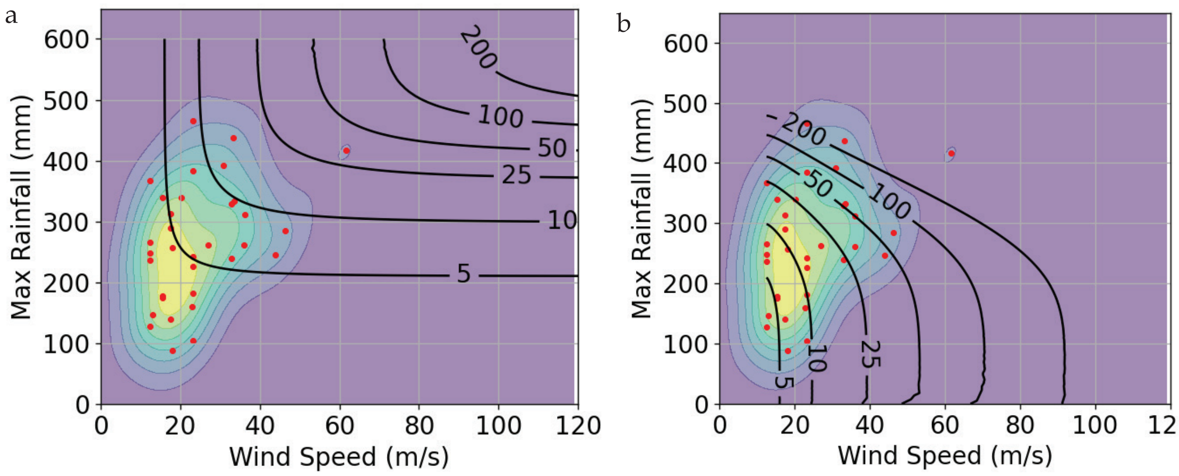

Figure 10 - Contour Plots of (a) T_{and} and (b) T_{or} for Distribution of Wind Speed and Rainfall

5. Conclusion

Tropical cyclones pose significant risks to coastal regions, with their impacts including high wind speeds, heavy precipitation, and storm surges. Understanding the joint probability distribution of cyclonic wind speed and rainfall is crucial for effective risk assessment, mitigation, and coastal flood modelling. The study aims to establish a statistical relationship between maximum sustained wind speed and cyclonic rainfall using the Copula theory, focusing on cyclones affecting Sri Lanka.

Through data acquisition and analysis, it was found that cyclones affecting Sri Lanka exhibit a moderate correlation between maximum wind speed and rainfall. Exploratory data analysis revealed key statistical parameters and distributions for both variables, with the Generalized Pareto distribution identified as the best-fit marginal distribution for wind speed and the Pearson type III distribution for rainfall.

Applying the Copula theory, the Clayton copula was selected as the most suitable copula family, considering its ability to capture moderate correlation and tail dependence between wind speed and rainfall. Through parameter optimization and validation, the effectiveness of the Clayton copula model in capturing the joint probability distribution of cyclonic wind speed and rainfall was demonstrated.

These findings provide valuable insights into the joint occurrence of cyclonic wind speed and rainfall in Sri Lanka, with implications for coastal flood modelling and risk management strategies. By understanding the probabilistic relationship between these two critical variables, policymakers and stakeholders can better prepare for and mitigate the impacts of tropical cyclones on coastal communities.

Xu et al. (2014) [34] established joint probability distribution through best fit copulas for different subsequence for coastal cities in eastern China. They selected Gumbel Copula and Frank Copula for two different sub sequences. Gori et al. (2020) [43] developed joint probability of peak storm tide and maximum rainfall intensity non-parametric kernel density estimate. This approach was chosen because the upper tail behaviour of the data is significantly different compared to the rest of the distribution, and fitting a parametric copula to the entire data set could bias the tail estimate.

Further research could explore the application of our methodology to other regions prone to cyclones, as well as the integration of additional variables such as storm surges for a more comprehensive risk assessment framework. Additionally, refining the Copula model with more extensive datasets and considering the effects of climate change on cyclonic behaviour could enhance the accuracy and reliability of future projections.

In conclusion, this study contributes to the growing body of knowledge on tropical cyclone risk assessment and management, providing valuable insights for enhancing resilience and preparedness in vulnerable coastal region.

References

1. Klotzbach, P. P. J., Bowen, S. G., PielKe, R. G. R., & Bell, M. “Continental U.S. Hurricane Landfall Frequency and Associated Damage: Observations and Future Risks”, *Bulletin of the American Meteorological Society*, Vol. 99, 2018,pp. 1359–1376. https://doi.org/10.1175/BAMS-D-17- 0184.1.

2. Peduzzi, P., Chatenoux, B., Dao, H., De Bono, A., Herold, C., Kossin, J., Moutin, F., and Nordbeck, O. “Global Trends in Tropical Cyclone Risk”, *Nature Climate Change*, Vol. 2,2012, pp. 289–294. https://doi.org/10.1038/nclimate1410.

3. Knutson, T., S. Camargo, S. J., Chan, J. C. L., Emanuel, K., Ho, C. H., Kossin, J., Mohapatra, M., Satoh, M., Sugi, M., Walsh, K., & Wu, L., “Tropical Cyclones and Climate Change Assessment: Part I, Detection and Attribution”,*Bulletin of the American Meteorological Society*, Vol/ 100, No.10, 2019, pp. 1987-2007.

4. Sondermann, M., Chou, S. C., Tavares, P., Lyra, A., Marengo, J.A. and Cunha, A.P., “Projections of Changes in Atmospheric Conditions Leading to Storm Surges Along the Coast of Santos, Brazil”, *Climate,* Vol. 11, No. 09, 2023, p. 176. https://doi.org/10.3390/cli11090176.

5. Lin, N., Emanuel, K. A., Smith, J. A., &Vanmarcke, E., “Risk Assessment of Hurricane Storm Surge for New York City”. *Journal of Geophysical Research: Atmospheres*, Vol. 115, 2010, D18121. http://dx.doi.org/10.1029/2009jd013630.

6. Marsooli, R., Lin, N., Emanuel, K., & Feng, K.,“Climate Change Exacerbates Hurricane Flood Hazards along US Atlantic and Gulf Coasts in Spatially Varying Patterns”, *Nature Communications*, Vol. 10, 2019, pp. 1–9. https://doi.org/10.1038/s41467-019-11755-z.

7. Emanuel, K., "Assessing the Present and Future Probability of Hurricane Harvey's Rainfall", *Proceedings of the National Academy of Sciences*, Vol. 114, No, 48, 2017,pp. 12,681–12,684. https://doi.org/10.1073/pnas.1716222114.

8. Knutson, T. R., McBride, J. L., Chan, J., Emanuel, K., Holland, G., Landsea, C., Issac Held, Kossin, James P., Sirivastava, A. K., and Masato Sugi., "Tropical Cyclones and Climate Change", *Nature Geoscience*, Vol. 3, 2017,pp. 157–163. https://doi.org/10.1038/ngeo779.

9. Bevacqua, E., Maraun, D., Vousdoukas, M. I., Voukouvalas, E., Vrac, M., Mentaschi, L., & Widmann, M., "Higher Probability of Compound Flooding from Precipitation and Storm Surge in Europe under Anthropogenic Climate Change", *Science Advances*, Vol. 5, 2019, pp. 1–8. https://doi.org/10.1126/sciadv.aaw5531.

10. Bacopoulos, P., Tang, Y., Wang, D., & Hagen, S. C., "Integrated Hydrologic-Hydrodynamic Modelling of Estuarine-Riverine Flooding: 2008 Tropical Storm fay". *Journal of Hydrologic Engineering*, Vol. 22, 2017. https://doi.org/10.1061/(ASCE)he.1943-5584.0001539.

11. Herdman, L., Erikson, L., & Barnard, P., "Storm Surge Propagation and Flooding in Small Tidal Rivers during Events of Mixed Coastal and Fluvial Influence", *Journal of Marine Science and Engineering*, Vol. 06, 2018, 158.https://doi.org/10.3390/jmse6040158.

12. Kumbier, K., Carvalho, R. C., Vafeidis, A. T., & Woodroffe, C. D., "Investigating Compound Flooding in an Estuary using Hydrodynamic Modelling: A Case Study from the Shoalhaven River, Australia", *Natural Hazards and Earth System Sciences*, Vol. 18, 2018, pp. 463–477. https://doi.org/10.5194/nhess-18-463-2018.

13. Ray, T., Stepinski, E., Sebastian, A., & Bedient, P. B., "Dynamic Modelling of Storm Surge and Inland Flooding in a Texas Coastal Floodplain", *Journal of Hydraulic Engineering*, Vol. 137,2011, pp.1103–1110.https://doi.org/10.1061/(ASCE)HY.1943-7900.0000398

14. Cao, Y., Zhang, W., Zhu, Y., Ji, X., Xu, Y., Wu, Y., & Hoitink, A. J., "Impact of Trends in River Discharge and Ocean Tides on Water Level Dynamics in the Pearl River Delta",*Coastal Engineering*, Vol. 157, 2020, 103634. https://doi.org/10.1016/j.coastaleng.2020.103634.

15. Couasnon, A., Eilander, D., Muis, S., Veldkamp, T. I. E., et al., "Measuring Compound Flood Potential from River Discharge and Storm Surge Extremes at the Global Scale and Its Implications for Flood Hazard", *Natural Hazards and Earth System Sciences Discussions*, 2019, pp. 1–24. https://doi.org/10.5194/nhess-2019-205.

16. Hendry, A., Haigh, I. D., Nicholls, R. J., Winter, H., Neal, R., Wahl, T., et al., "Assessing the Characteristics and Drivers of Compound Flooding Events Around the UK Coast", *Hydrology and Earth System Sciences*, Vol. 23, 2019,pp. 3117–3139. https://doi.org/10.5194/hess-23-3117-2019.

17. Moftakhari, H. R., Salvadori, G., AghaKouchak, A., Sanders, B. F., & Matthew, R. A., "Compounding Effects of Sea Level Rise and Fluvial Flooding", *Proceedings of the National Academy of Sciences*, Vol. 114, 2017, pp. 9785–9790. https://doi.org/10.1073/pnas.1620325114.

18. Ward, P. J., Couasnon, A., Eilander, D., Haigh, I. D., Hendry, A., Muis, S., Veldkamp, T. I. E., Winsemius, H. C., and Whal, T., "Dependence between High Sea Levels and High River Discharge Increases Flood Hazard in Global Deltas and Estuaries", *Environmental Research Letters*, Vol. 13, No. 08, 2018, 084012. https://doi.org/10.1088/1748-9326/aad400.

19. Wu, W., McInnes, K., O'Grady, J., Hoeke, R., Leonard, M., and Wesrta, S., "Mapping Dependence Between Extreme Rainfall and Storm Surge", Journal of Geophysical Research: Oceans, 123(4), 23 March 2018, pp. 2461-2474. https://doi.org/10.1002/2017JC013472.

20. Hendry, A., Haigh, I. D., Nicholls, R. J., Winter, H., Neal, R., Wahl, T., et al., "Assessing the Characteristics and Drivers of Compound Flooding Events Around the UK Coast", *Hydrology and Earth System Sciences*, Vol. 23, 2019, pp. 3117–3139.https://doi.org/10.5194/hess-23-3117-2019.

21. Zheng, F., Westra, S., & Sisson, S. A., "Quantifying the Dependence Between Extreme Rainfall and Storm Surge in a Coastal City", *Water Resources Research*, Vol. 50, No. 3, 2014, pp. 2053-2073. https://doi.org/10.1002/2013WR014616.

22. Van Den Hurk, B., Van Meijgaard, E., De Valk, P., Van Heeringen, K. J., & Gooijer, J., "Analysis of a Compounding Surge and Precipitation Event in the Netherlands", *Environmental Research Letters*, Vol. 10, 2015, 035001. https://doi.org/10.1088/1748-9326/10/3/035001.

23. Zellou, A., & Rahali, H., "Assessment of the joint impact of extreme rainfall and storm surge on the risk of flooding in a coastal area", Journal of Hydrology", Vol. 19, No. 9, 2019, pp. 2011-2024. https://doi.org/10.5194/nhess-19-2011-2019 .

24. Tao, S., Dong, S., Wang, N., Soares, C.G., "Estimating Storm Surge Intensity with Poisson Bivariate Maximum Entropy Distributions Based on Copulas". *Natural Hazards*, Vol. 68, 2013pp. 791–807.

25. Wahl, T., Jain, S., Bender, J., Meyers, S. D., & Luther, M. E., "Increasing Risk of Compound Flooding from Storm Surge and Rainfall for Major US Cities", *Nature Climate Change*, 2015, pp. 1–6.https://doi.org/10.1038/NCLIMATE2736

26. Dong, S., Jiao, C. S., Tao, S.S., "Joint Return Probability Analysis of Wind Speed and Rainfall Intensity in Typhoon-Affected Sea Area", *Natural Hazards*, Vol. 86, 2017, pp. 1193–1205.

27. Kwon, T. and Yoon, S., "Analysis of Extreme Wind Speed and Precipitation using Copula", *In Proceedings of the EGU General Assembly Conference*, Vienna, Austria, April 2017, pp. 23–28.

28. Sklar, M, *Fonctions de re´partition a` n dimensions et leursmarges,* Universite´ Paris., 1959.

29. Balakrishna, N., & Lai, C. D., *Continuous Bivariate Distributions*., 2009,pp. 1–684. https://doi.org/10.1007/b101765.

30. Favre, A. C., El Adlouni, S., Perreault, L., Thiemonge N., and Bobee B., "Multivariate Hydrological Frequency Analysis using Copulas", *Water Resources Research*, Vol. 40, No. 1, 2004, pp. 1–12. https://doi.org/10.1029/2003WR002456.

31. Salvadori G, De Michele, C., "Frequency Analysis via Copulas: Theoretical Aspects and Applications to Hydrological Events", *Water Resource,*Vol. 40, No. 12, 2004, W12511. https://doi.org/10.1029/2004wr003133.

32. Tang, J., Hu, F., Liu, Y., Wang, W., and Yang, S., "High-Resolution Hazard Assessment for Tropical Cyclone-Induced Wind and Precipitation: An Analytical Framework and Application", Sustainability , Vol. 22, No. 03, 2022, pp. 987-1003. https://doi.org/10.3390/su142113969.

33. Ai, P., Yuan, D., & Xiong, C., "Copula-Based Joint Probability Analysis of Compound Floods from Rainstorm and Typhoon Surge: A Case Study of Jiangsu Coastal Areas, China", *Sustainability,* Vol. 10, No. 07, 2018, https://doi.org/10.3390/su10072232.

34. Xu, K., Ma, C., Lian, J., and Bin, L., . "Joint Probability Analysis of Extreme Precipitation and Storm Tide in a Coastal City under Changing Environment", PLoSONE, 9(10), 2014. https://doi.org/10.1371/journal.pone.0109341.

35. Liu, Q., Xu, H., and Wang, J.., "Assessing tropical cyclone compound flood risk using hydrodynamic modelling: a case study in Haikou City, China", Natural Hazards, Vol. 112, 2022, pp. 665–675. https://doi.org/10.5194/nhess-22-665-2022.

36. Dube, S.K., "Storm Surge Forecasting in the Bay of Bengal and Arabian Sea", *In Antarctic Geoscience, Ocean-Atmosphere Interaction and Paleoclimatology(Eds. S. Rajan and P.C. Pandey),* National Centre for Antarctic & Ocean Research, Goa, India, 2003.

37. Maduwantha, P. and Wijayaratna, N., "An Estimation of Storm Surge Heights of Colombo Coastline"*Proceedings of the 2019 Moratuwa Engineering Research Conference (MERCon)*, 2019.https://doi.org/10.1109/MERCon.2019.8818798.

38. Dube, S. K., Rao, A. D., Sinha, P. C., Murty, T. S. and Bahulayan, N., " Storm Surge in the Bay of Bengal and Arabian Sea: The Problem and its Prediction", *Natural Hazards*, Vol. 16, No. 01, 1997, pp. 283-304. https://doi.org/10.54302/mausam.v48i2.4012.

39. DMSL, "Department of Meteorology, Sri Lanka. Tropical Cyclones; Cyclone Events 1900-2000"*Department of Meteorology, Government of Sri Lanka*, 2010.

40. Joe, H., *Dependence Modeling with Copulas*, CRC Press, Boca Raton, Fla, 2014.

41. Clayton, D. G., "A Model for Association in Bivariate Life Tables and its Application in Epidemiological Studies of Familial Tendency in Chronic Disease Incidence", *Biometrika*, Vol. 65, No. 1, 1978, pp. 141–151.

42. Li, C., V. P. Singh, and A. K. Mishra, "A Bivariate Mixed Distribution with a Heavy-Tailed Component and its Application to Single-Site Daily Rainfall Simulation", *Water Resource*. Res., Vol. 49, 2013 pp. 767–789. https://doi.org/10.1002/wrcr.20063.

43. Gori, A., Lin, N., & Xi, D., "Tropical Cyclone Compound Flood Hazard Assessment: From Investigating Drivers to Quantifying Extreme Water Levels", *Earth's Future*, Vol. 08, 2020, e2020EF001660.https://doi.org/10.1029/2020EF001660.