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Review on Mathematical Modelling of Boomerang for Novel Outfield Applications

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Abstract: A boomerang is an aerodynamically built object that, when thrown precisely, makes a quasi-circular flight and returns closely to its starting position. Numerous academics have analysed the boomerangs' motion and aerodynamics. None of the researchers in the field investigated the possibility of using the boomerangs for novel field applications rather than hunting and playing. Using boomerangs for field applications is beneficial in several ways because, unlike general drones, these boomerangs do not consume power continuously and low manufacturing cost. Using a boomerang in outfield applications is challenging because the external field is always unsteady and turbulent; therefore, a boomerang should be mathematically modelled for unsteady environment for precisely predict the trajectory. Few steady mathematical models were developed and validated in the reported literature and an unsteady mathematical model of boomerang was developed recently though it was not experimentally validated. The article reviews the mathematical modelling of boomerangs for certain novel outfield applications like agricultural inspection and weather monitoring. The depth of study in the mathematical modelling of boomerang is insufficient for developing novel applications. The article will aid the industrialists and researchers to advance in the field, and they can see the possibility of replacing the drones in novel outfield applications.

Keywords: Mathematical modelling, Boomerang, Novel application, Aerodynamics, Flight dynamics

1. Introduction

A boomerang is a basic aerodynamically built object that, when thrown precisely, makes a quasi-circular flight and returns closely to its starting position {1}. For millennia, boomerangs have captivated the interest and creativity of scientists, engineers, and individuals because of the mechanical structure that realises the complex shape and attractive movements of the boomerang [2]. Two different varieties of boomerangs, including a returning and a non-returning boomerang, were used by the Australian Aborigines as far back as 10,000 years ago [3]. Different varieties of boomerangs are being used, such as symmetric or asymmetric two-bladed ones and multi-bladed ones as well as alphabets shaped models like X, V, T, H, and Y. Different shapes of boomerang have distinct behaviours [4].

None of the researchers in the field investigated the possibility of using the boomerangs for some potential novel applications rather than hunting and playing. Unlike other drones and quadcopters, boomerangs don't need continuous power supply to move in the air, and they don’t require sophisticated manufacturing and maintenance. Replacing drones and quadcopters in novel outfield applications will save energy and money. The practical use of a boomerang for potential applications requires a perfect mathematical model to accurately predict and control the trajectory based on the dynamic environmental conditions.

The dynamic stability of the boomerang plays a vital role in applications with cameras, batteries, optical sensors, etc. The stability of the mathematical model will be analysed thoroughly to use the boomerang in outfield applications. The geometrical design of the

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boomerang plays a huge role in the boomerang trajectory because the radius of the trajectory, aerodynamic forces and moments are heavily dependent on the geometrical and inertial properties of the boomerang [1] [5]. Modifying the boomerang design is vital for adapting the gadgets to perform certain tasks. The movement of air-foiled blades produces lift and gyroscopic precession, which control the boomerang flight [5] [6] [7] [8].

The boomerangs are thrown manually by an experienced thrower to get them returned; it cannot be controlled once it is launched. Researchers have shown that, when the launching velocity and inclination angle alter, the flight trajectory varies significantly [4], [7]. The boomerang flight radius can be determined by the mathematical equations derived by several researchers [1] [5]. The radius plays a major role in replacing drones in certain applications because the drones can travel long distances as desired, but boomerang cannot.

The main scope of the article is reviewing the mathematical modelling of the boomerang to analyse the possibility of using the boomerang for novel outfield applications like monitoring and surveillance. As shown in Figure 1, the article mainly investigates the three major topics, such as the equations of dynamics of boomerang, the geometry of the boomerang design and the possibility of using boomerang for novel outfield applications.

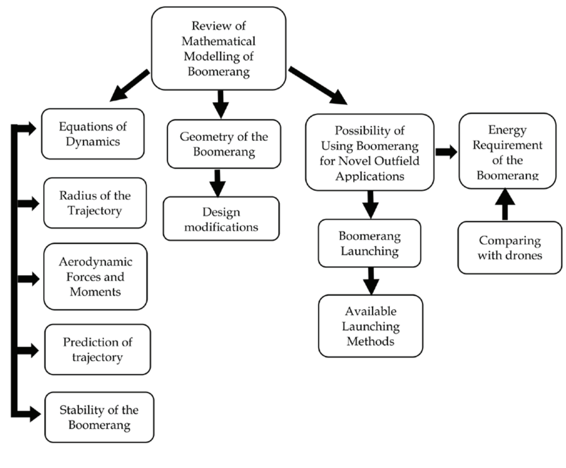

Figure 1 - Structure of the Article Body

2. Review of Mathematical Modelling of Boomerang for Novel Outfield Applications

The practicability of boomerang for outfield applications is investigated through the review of mathematical modelling and essential aspects of boomerangs.

2.1 Geometry of the Boomerang

The geometry of the boomerang plays a vital role in boomerang's aerodynamic performance and its returning ability. The aerodynamic performance of the boomerang is greatly influenced by its recognisable shape [11]. All the existing models contain the aerodynamic forces and moments and it is proven [5] that the aerodynamic forces and moments are dependent on geometrical parameters. Several types of boomerangs have been used to study the dynamics of the boomerang, such as 'L','V', I, 'Ω',' X', etc. [5] [11] [12] [13]. Most boomerangs in use have two wings attached to a central core. The wings have a small curvature, like an aerofoil. The boomerang is considered as a rotor working at a very high advance ratio which is the ratio of the free stream fluid speed to the rotor speed; there are no geometric restrictions placed on the boomerang to compare the behaviour of boomerangs of various shapes and sizes [14]. The distinctive designs of the boomerang will result in differentgyroscopic inertial torques, aerodynamic forces, and returning trajectories [15]. Mathematical models are used to express gyroscopic effects, while classical mechanics is used to define their physics [16] [17] [18].

Figure 2 shows different geometries of boomerangs that have been used to study the dynamics of the boomerangs. The geometry optimisation was studied by Rosario in 2013; he stated that the most significant parameters that influence the boomerang behaviour are linked to the wing profile, the dihedral of the arms and the angle between the two arms [11]. CFD and numerical simulation can be coupled to optimise the shape of the boomerang to attain an effective design for certain applications [19]. The geometrical configurations (blade pitch angle, joint angle) affect boomerang flight performance [20]. Few tuning methods were presented to improve the performance of boomerang, such as holes, cambering, undercutting, bending, twisting, aerofoil modification and add-ons [6] [21]. Since the geometry influences the aerodynamic forces and moments, the geometry modification is indispensable in applying boomerang in outfield activities because the design should contain cells, sensors, cameras, and special gadgets to perform particular tasks. The application of boomerangs needs precise

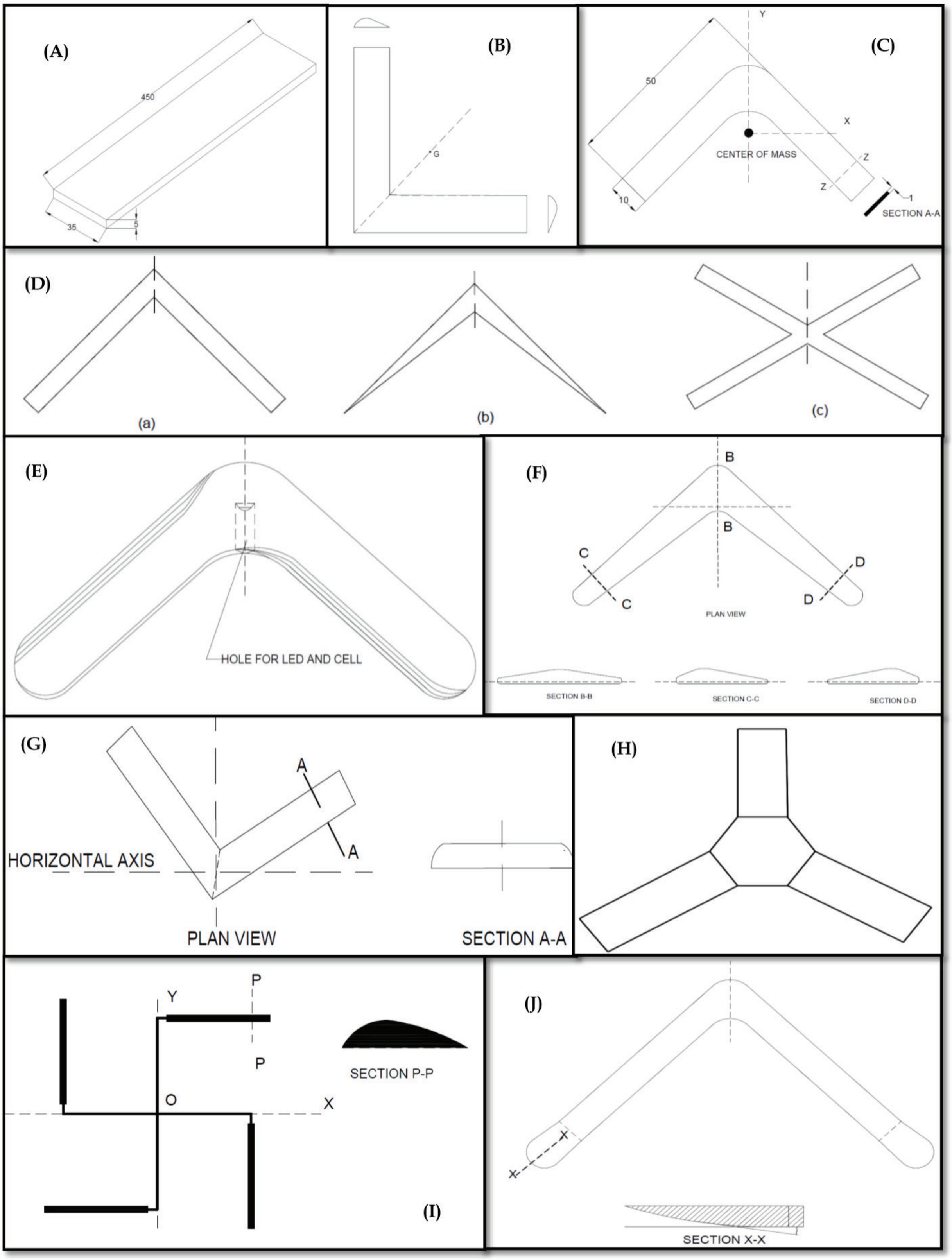

Figure 2 - Different Types of Boomerang Geometries Found in Literature [1,2,4,5,6,12,13,25,26,30]

controlling and sensing features to perform certain tasks. It needs to be modified to accommodate the boomerang with gadgets without affecting the aerodynamic performance of the boomerang.

2.2 Mathematical Modelling of the Boomerang

The mathematical modelling of boomerang is thoroughly reviewed to use it for novel applications. If boomerang is used to perform a monitoring or surveillance application, the path of the boomerang should be accurately predefined because it cannot be controlled once launched. The boomerang should be modelled to travel through the waypoints or predefined paths, which will be defined based on the required application where the elliptical path of the boomerang sufficiently addresses the need.

Before analysing the mathematical model of the boomerang, a proper coordinate system should be defined to indicate the boomerang's position and orientation. As shown in Fig. 3, there are mainly two kinds of coordinate systems used to define the dynamics of a boomerang such as fixed coordinate system (x,y,z – fixed to the earth) and body coordinate system (b_1,b_2,b_3 - fixed with the boomerang centre of gravity) [9]. In addition to the two stated coordinate systems, a few other intermediate coordinate systems are also used to define the dynamics of boomerangs [2] [20].

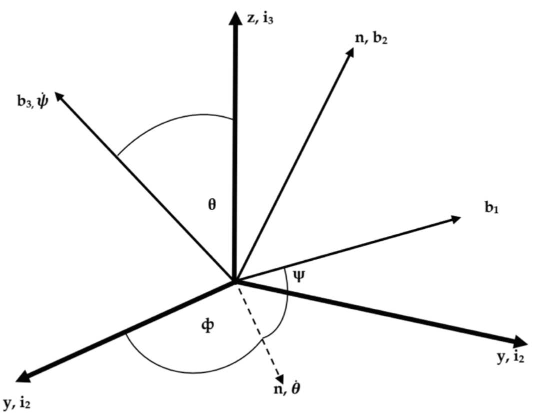

Figure 3 - Body and Inertial Coordinate Systems [5]

Three sets of Euler rotations are used to transform from one coordinate system to another [22]. Euler angles are usually denoted as θ, ψ, and ϕ. The transformation matrix is formed after three consecutive rotations to transform one variable to another while developing mathematical equations for dynamics [9]. In 1960, Felix Hess mathematically modelled a boomerang and validated it through a set of numerical simulations and experiments for different kinds of geometrical shapes of boomerangs and launch conditions [2]. Hess decided that, for rotation, the smooth lift and rolling-moment of a boomerang on air are proportional to ω^2 and ωV where V is translational velocity of the boomerang and ω is angular frequency of the boomerang. The main drawback of the Felix Hess model is that it cannot self-trim which contradicts with actual boomerang flight because it is obvious the actual boomerangs are dynamically stable for a period of time for different types. He further analysed the effect of wind in a boomerang flight, but it was not expanded to expected unsteady conditions. However, in his detailed investigation, the equations for boomerang flight dynamics are smoothened such that fast changes in physical quantities within one spin cycle are neglected.

In 1992, Restrepo developed a mathematical model for a straight boomerang capable of predicting the 3D dynamics based on several assumptions such as steady flow, neglecting the 3D aerodynamic forces and moments, etc. [12]. The outdoor environment is always unsteady and the latest investigations found that there will be considerable aerodynamic forces and moments in all three dimensions. Therefore, we can conclude that the assumptions made during the above model are challengeable for a traditional 'V" type boomerang.

Battipede (1999) analysed boomerang behaviour as a function of its throw parameters, geometric properties, and wind conditions, through a simulation program [24]. He concluded that unsteady conditions are dominant in-flight trajectory. Battipedde assumed the boomerang to be a rotating disc challengeable for an asymmetrical boomerang. Also the unsteady effect of wind is proven only for a limited range of input conditions.

Equations of motion of the wind turbine model (4-wings) boomerang are derived and examined numerically by Alexander [25]. He modelled only for undisturbed air and for a symmetrical shape of boomerangs but in practice it is not possible all the time. In addition, he further assumed the aerodynamic forces acting only on the wings. However, analysing the behaviour of a single, typical boomerang might help to understand broad elements of the return process. Azumacreated nonlinear equations of motion for boomerang flight dynamics and numerically solved them [26]. Hence, he proved that the motion consists of short and long-period oscillations, and the flight depends on the throwing and geometrical parameters.

Russo tried to optimise the geometries of boomerang, but he mainly followed the Felix Hess model. The boomerang geometry is optimised regarding the return ability, required energy and range [11]. The primary flaws in this model are that it mostly adopted *Felix Hess's* concept, which has more conflicts with real-world situations. Later, in 2012, Vassbergproduced a model for a 'V' shape boomerang capable of predicting the 2D trajectory of the boomerang while assuming the level circular flight path [9]. Dynamic stability is the main characteristic of the Vassberg’s mathematical model. He mainly experimented his mathematical model only in steady wind conditions and without wind conditions which is impossible for a practical application on the unsteady outfield environment. Another major issue with the Vassberg’s model is the level circular assumption as we all know the actual flight of the boomerang is not on a level all the time.

In 2013, Bigot invented an algorithm to find the altitude and movement of an L-shape boomerang and concluded that initial conditions greatly influence boomerang behaviour [27]. He considered only the steady flow of wind and validated for only “L” shape of boomerangs.

Hubbard [28] investigated the initial conditions that make the boomerang return. The returning behaviour of an individual boomerang depends on its particular aerodynamic and inertial properties.

In 2019, Prasad produced a mathematical model while considering the airflow reversal [10]. Furthermore, in his research, he proved the existence of the pitching moment. However, Prasad assumed the level circular flight path and the undisturbed wind conditions. In 2020, Prasad extended his studies and modified his mathematical model by adding drag force and moment [29]. The main drawback of Prasad's model is that it is not valid for unsteady flow, and the trajectory was assumed as 2D.

A mathematical model of the boomerang for unsteady flow was studied in 2023 by Ratheesan and team where they developed an unsteady model and numerically simulated for different wind conditions [30]. The model is capable to predict the trajectory in different wind conditions. The main challenges behind the modelling are the 2D assumption of the trajectory and the lack of experimental validation. The entire mathematical model considered only the symmetric shapes of a boomerang which is not possible in a boomerang application in an outfield environment. None of the researchers experimentally validated the model to the level of using it in outfield applications.

More work is to be focused on the dynamic response of boomerangs based on flight dynamics and analytically investigate the motion of boomerangs under various initial conditions [4]. When using a boomerang for a novel application, precise and regulated starting inputs are needed depending on the changing wind conditions to follow a predetermined elliptical path set by specific waypoints. The current mathematical models cannot predict the 3D trajectory of a boomerang in an unsteady environment. Therefore, use of boomerang for precise applications is not possible in industry at the moment.

2.2.1 Aerodynamic Forces and Moments Acting on the Boomerang

A boomerang's flight is controlled by aerodynamic forces and the moments it experiences during its flight. These thrusts and moments strongly influence boomerang stability and returning path. Therefore, a proper method to find the forces and moments is essential to predict the trajectory using numerical simulation. Initially, the aerodynamic forces and moments were found using a set of laboratory experiments and practical measurements [2]. BET is a method broadly used to predict the moments and forces on a blade element by assuming each blade section to act as a steady 2D aerofoil [30].

Equations for lift force, drag force, rolling moment, pitching moment and drag moment are derived using the BET [31]. The forces acting on the boomerang can have fluctuations with a period equal to the rotation period [32]. The time-averaged equations are used to find the aerodynamic equations [9]. Hunt analysis is applied to any boomerang but is mainly used for the cross-shaped 'X' boomerang [33].

In 2012, Vassberg proposed a BET to define the aerodynamic forces and moments with some geometrical freedoms such as non-tapered, uniform cross section and symmetrical. Later in 2019 and 2020 Prasad came up with a few reasonable modifications to the BET by introducing a reversible airflow effect and a pitching moment [6]. A reverse flow region is created when relative airflow moves from the trailing edge to the leading edge [34]. Later in 2020, Prasad modified the BET using the drag force and the drag moment. In 2021, Justin Tahmassebpur proved Prasad's model developed in 2020 by wind tunnel experiments for non-dimensionalized lift [35].

BET derivations developed by Vassberg and Prasad are based on non-tapered blades with zero offsets to the point of rotation. Later in 2019, Prasad concluded that a pitching moment exists and cannot be neglected [10]. The aerodynamic forces and moments significantly rely on the geometrical properties (radius, cross section of the blades) of the boomerang and the density of the air field. The dynamics of the boomerang is totally relying on the forces acting on the boomerang including the aerodynamic forces and moments. The assumption made during the aerodynamic forces and moments may cause errors in prediction of path using the same mathematical model. All of the above scholars assumed a uniform cross section during the mathematical modelling which may cause deviations with the simulated paths. If anyone tries to use the boomerang in field applications, the geometry of the boomerang strictly influences the travelling path. The optimization of boomerang shape was studied by the Russo in 2012 [11]; it shows the profile of the blade cross section, the angle between the blades and dihedral angle make heavy impact on the aerodynamic performances.

2.2.2 Radius of the Boomerang Path

One of the most important performance parameters of a boomerang is the flight path radius of curvature, R_p, and several researchers have predicted the R_p for a boomerang. The actual "boomerangs" were effective weapons with a maximum range of 427.2 m, but the radius of returning boomerangs was smaller (119m) than the non-returning [9]. The applications like environmental monitoring and surveillance need a high-range boomerang.
The trajectory of the boomerang was assumed as a circular path of radius R_p; also, the boomerang is assumed to reach the steady-state level flight [9].

$$V = \dot{\phi} R_p$$

Here,ϕ - right ascension angle.Further, it was assumed that the rotation plane maintains a constant inclination-angle θ with horizon.

Finally, he found that R_p is independent of the boomerang's velocity (V) or rotational speed (ω). Hence the radius of the boomerang's flight path can be determined by its design. For an equilibrium value of θ for a flat circular flight trajectory, R_p is given by equation (1)[9].

$$R_p = \left[\frac{3m \sin\theta}{\rho R C C_{l0}}\right] \ldots (1)$$

where m – mass of the boomerang, R – radius of the boomerang and C – chord length of wings, ρ- density of air and C_{l0}- coefficient of lift with zero angle of attack. Hess has found a different formula to find the radius of curvature of the trajectory [2]. Assuming the angle of incidence is stationary Ψ = 0. The following equation will give the radius of the flight path (R_p),

$$R_p = \frac{\lambda m}{\mu . R^2} . \frac{(1/X)}{T_{1x}} \ldots (2)$$

μ, λ - Constants depending on the geometry, T_{1x}- Torque about x axis.

In 2016, Fu and team predicted another relationship for R_p of a V-shaped boomerang [1]. At any instant, the trajectory of the boomerang is approximately an arc, where$V_h d\theta = R . dt$,

$$R_p = \frac{3I \cos\alpha}{\rho C_L A R^2} \ldots (3)$$

where V_h- Component of linear velocity in rotation plane,
A – Area of the boomerang,
C_L- Lift coefficient of boomerang,
I – Moment of inertia of boomerang,
α – Angle between the boomerang's angular velocity and the horizontal plane.

$$R_p = \frac{3I \cos\alpha}{\rho C_L A R^2} \quad \ldots (4)$$

Hunt[32] predicted the radius of the pathR_p,

$$R_p = \frac{4J}{\rho C_L \pi R^2} \ldots (5)$$

where J- Polar moment of inertia
Later in 2014, Pomeroy [3] combined equation (5) and the Vassberg's equation (1) and improved the radius for high angle of attack α,

$$R_p = \left[\frac{4m \sin\theta}{3\pi\rho R^2 (C_{l0} + C_{l\alpha}\alpha)}\right] \quad \ldots (6)$$

where, $C_{l\alpha}$- coefficient of lift with αangle of attack.

If we look at the above equations for the radius of curvature R_p, we can conclude that the path's radius depends on the boomerang's geometrical properties and the air density. The design of the boomerang is essential to have a high-radius flight path. Optimising the geometrical properties like the radius of the wing, chord length, and mass of the boomerang will result in the desired flight radius. However, several assumptions are used to predict R_p, such as level circular flight, negligible angle of attack and constant angle of incidence. Those assumptions are challengeable in practical use. However, field applications need higher radius than the predicted radius of the general boomerangs. Even though the boomerang radius can be controlled by inserting some power source while the boomerang is in the air, no other literature has been found regarding powered boomerangs.

2.2.3 Stability of the Boomerang

It describes the transient motion involved in recovering the equilibrium state after the disturbance [40]. If the motion of the boomerang is not stable, the boomerang may lose its spin rate and descend trembling like a wounded bird; such instability may occur due to bad launching or a bad shape of the boomerang. The boomerang may wobble if the motion is unstable [2]. Dynamic stability is indispensable for a boomerang for field applications like photography, monitoring and surveillance. The dynamic stability of the boomerang is essential because the boomerangs donot have a pilot or remote control; once a boomerang is launched, no one can control it. So, it is necessary to ensure dynamic stability before launch for specific applications.

Hess[2] stated that theoretical criteria for stability could not be generated except for sufficiently fast rotation, and the boomerang's greatest principal moment of inertia should be sufficiently greater than the middle one. Hess model is not dynamically stable. Vassberg [5[has analysed the condition of the boomerang stability and provided a necessary condition to satisfy. Think about a boomerang that is flying on a level, circle, a general relationship for the trim state can be found:

$$\overline{(C_M : C_L)} \equiv \frac{C_M}{C_L} = \left[\frac{I}{mR^2}\right] \sin\theta X \qquad \ldots (7)$$

Here, $C_L$and $C_M$are the time-averaged lift and rolling moment coefficients, respectively. However, $\overline{(C_M : C_L)}$ is also a function of angle-of-attack [40].

Condition for dynamic stability is given by:

$$\frac{d\left(\frac{C_M}{C_L}\right)}{d\alpha} < 0 \qquad \ldots (8)$$

It was proven that boomerangs are stable for $X \geq 1$and dynamic stability is an intrinsic property of the boomerang.

So we can design a dynamic stable boomerang with the help of the above Vassberg's research and the optimization of boomerang geometrical parameters.

From 1960 to 2021, we could see a noticeable improvement in the field of mathematical modelling of boomerangs, Initially Felix Hess analysed the dynamics with unsteady wind and he did not consider the all three dimensional aerodynamic forces and moments [2]. The next considerable advancement in the field happened only in 2012 by Vassberg who analysed the dynamics with consideration of time averaged aerodynamic forces and moments [5]. The best characteristic of Vassberg's model is dynamically stable for a range which is similar to a practical scenario. Even it is dynamically stable it is not compatible with unsteady wind conditions; he also neglected the drag and the pitching moment in his detailed study. Later in 2019-2021 Prasad invented the existence of pitching moment and drag moments which may lead to a considerable advancement in the field [6]. His model is not capable to predict the trajectory of a boomerang for actual unsteady environmental conditions.

Finally in 2023, a mathematical model predicted the trajectory in unsteady wind environment but it was not validated numerically. From the above review we can conclude that the available mathematical models are not suitable for unsteady environmental condition which is essential for outfield applications. Still there is a huge gap in this field to overcome to use the boomerang for outfield applications. A future model should be dynamically stable for a range of unsteady outfield conditions to use for an outfield application.

2.3 Boomerang as Energy-Efficient Alternatives to Drones

The boomerangs donot consume power continuously like a drone and it can be a proper replacement for adrone. The possibility of using boomerangs for novel outfield applications is analysed with the comparison of drones with boomerangs because all of the possible boomerang applications are being fulfilled by the drones nowadays.

Table 1 - Comparison of Drones and Boomerang

Criteria	Drones/Quadcopters	Boomerangs
Types of novel outfield applications	It is suitable for both load-carrying and no-load applications. Mainly possible to do all kinds of applications like environmental monitoring, package delivery, fire detection/fighting, surveying, filming, and traffic surveillance [36]. Photography and filming are done using drones because of the stability and the remote controlling feature.	It cannot work with load-carrying and long-distance applications because the heavy loads may disturb the boomerang's aerodynamic performance and return ability. Mainly it is feasible only for monitoring and surveillance-related applications. The filming or photography kind of application is challengeable for a boomerang because we cannot control the rotation and the movement remotely.
Cost of Manufacturing and Operation	Higher than the cost of a boomerang because it contains rotating parts, batteries, motors, controllers and the body.	Very low cost compared to drones and quadcopters because it doesnot contain rotating parts; therefore, operation and maintenance are cheaper than drones.
Energy requirement	Needs continuous power to launch and move.	Only needs energy to launch the boomerang
Range of path	It can operate at very high ranges as desired with a capable battery	Limited range only because it consumes power only at launch. It can be increased with powered boomerangs.
Environmental feasibility	Disturb wildlife or interfere with natural environments [36]. Drones are noisy and disturbing, scaring away insects and birds. The concentration of drones increases the noise impact [37]. The non-disposable wastages of drones severely affect the earth.	It won't create much noise as drones. It contains very few parts, and it is possible to make the boomerang with wood which is not harmful to the environment.

Table 2 - Possible Novel Outfield Applications of Boomerangs

Novel Outfield Applications	Descriptions
Disaster management	Boomerangs equipped with radars, sensors and cameras can help to identify the damage of disasters like floods and tsunamis without human involvement.
Filming/Photography	This can be done only with the dynamically stable boomerang with gadgets.
Remote sensing	Experts apply drone technology to collect images in isolated places, such as mountaintops and coastlines. Boomerang can replace these kinds of drone applications to save energy.
Smart farming	Boomerangs could be attached in smart farming to acquire specific information through spraying pesticides, detecting illness, planning for irrigation, identifying weeds, and monitoring the plants.
Fire detection	Drones can replace fire detection in risk areas though boomerangs can be used. Boomerangs can be made with fire-resistive materials.
Military surveillance	To monitor the enemy's movement or activities

2.3.1 Comparison of Boomerangs with Drones or Quadcopters

Nowadays the drones are involved in all kinds of monitoring applications like surveillance and delivery and some risky jobs like fire fighting and disaster monitoring. The boomerangs and drones are compared in Table 1 for outfield application under several criteria, such as possible application, cost of manufacturing and maintenance, energy requirement, possible range, and environmental feasibility. Boomerangs can be used in various commercial, disaster management, and agricultural, military and academic applications. Table 2 explores the novel applications of boomerang.

2.3.2 Boomerang Launching Methods

2.3.2.1 Investigation on boomerang kinematics

The main objective is to study how the kinematic motion of an over-hand launch fixes the initial conditions of the boomerang's kinetics. The throw (translational and rotational speeds and angle of inclination) and the wind influence the elliptical path and its return ability of the boomerang [4]. The boomerang should be launched perpendicularly for a close elliptical path [36] [9]. The rotation plane is nearly vertical, with only a small lay-over angle. Hence, the rotation axis is nearly horizontal, which agrees with θ^0 of the rotation plane in the range of 70◦ - 80◦ [9].

2.3.2.2 Existing Boomerang Launchers

The boomerang field applications need an accurate travelling path to perform the task as defined, therefore the boomerang should be thrown precisely to follow a particular path which is not easy for the throwers to maintain accuracy. Therefore, a proper launching method is necessary to maintain the boomerang launch's accuracy and repeatability. Only a few launchers were developed in the past, although those are not suitable for field applications [21][38] [39]. The developed boomerang cannot control the throwing parameters, so there is a need to control the launch of the boomerang by an automated or semi-automated launcher mechanism to launch it for field-related applications.

2.3.3 Material Selection of Boomerang for Field Applications

Balsa wood is the main boomerang material used by the Aborigines which has very low density and it can stay for long time in the air [5]. If we are trying to use it for different outfield application, we are supposed to modify the shape of the boomerang to accommodate the power source and the controllers. The modification of balsa wood with small thickness (<12 mm) is difficult to perform and also the balsa wood can absorb water when we use it on rainy days. The heavy materials like metals can affect the return ability of the boomerang because the aerodynamic forces become negligible when compared to the weight of the boomerang. The modification of Balsa wood is not that much easy, therefore we can propose a PLA material to 3D print the boomerang and the required modifications. The density of the material significantly influences the performance of the boomerang. The metals are strong and durable but cost of making is high. Plastic material is not suitable because it deforms easily and damageable.

If boomerangs are used in the above outfield applications, they are supposed to satisfy the expected characteristics like stability, high range, low cost, fewer energy consumptions precision of trajectory, accuracy in launching, flexibility of the design to have gadgets (sensors, cells and controllers, etc).

2.3.4 A Case Study: Using Boomerang for Environmental Monitoring Application (Temperature, Wind Speed & Direction and Humidity Measuring)

The application of boomerangs needs precise controlling and sensing features to perform certain tasks. The sensors and microcontrollers are added to the boomerang to perform tasks such as monitoring and surveillance. Shape of the boomerang needs to be modified to accommodate the boomerang with gadgets without affecting the aerodynamic performance of the boomerang.

Arduino combined boomerang can be used to measure the boomerang's temperature, velocities, acceleration, and position. The design modifications significantly change the boomerang's performance in several ways, such as changing the range, flying time, altitude, etc. The possible modifications are listed below:

1. Sensor hole
2. Circuit board – Carved cut
3. Undercut to increase the lift [16].

The energy requirement of the boomerang is compared to a drone with equal weight. The energy requirement of the drone is the energy (translational plus rotational) necessary to throw the boomerang.

For example, consider a boomerang and a drone traveling through the same flight path of a radius of 200 m. Assume the boomerang travels in the air for 50 m, the launch velocity is 12.5 ms^{-1}, the initial frequency is 11 Hz, the mass of the boomerang is 100 grams, and the radius of the boomerang is 0.2 m. The energy required for the boomerang,

$$\text{Total Energy (E)} = \text{Rotational Energy(RE)} + \text{Kinetic Energy(KE)}$$

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}mV^2$$

Boomerang was assumed as a rotating disc,

$$I = \frac{1}{2}mR^2 = \frac{1}{2} \times 0.1 \times 0.2^2 = 0.002 \text{ kgm}^2$$

$$E = \frac{1}{2}.0.002 \times 30^2 + \frac{1}{2} \times 0.1 \times 12.5^2$$

$$E = 0.9 + 7.8125$$

$$E = 8.7125 \text{ J}$$

The energy required to operate the drone for 240 seconds is as follows. Still, we assume it takes a little more time than actual time (60 seconds) for rising and landing. Assume the drone's mass is the same as the boomerang's (100 grams) and the battery capacity of the drone is 3.7 V, 8000mAh.

Assume it has two batteries with 50 % efficiency,

$$50\text{ \% of Total Energy (E)} = \text{Electrical Energy(EE)}$$

$$E = 2 \times VIt$$

$$E = 2 \times 2 \times 3.7 * 8000 * 10^{-3} \times \frac{1}{3600} \times 300$$

$$E = 9.87\text{J}$$

Difference of energy requirement = 9.87-8.7125=1.156J

Percentage of energy saving=$\frac{1.156 \times 100}{9.87}$% =11.7%

The above calculation proves that boomerang drones' energy requirement is 11.7 % lower than the general drones. Therefore the proper design of the boomerang will reduce the power requirement further. Not only the energy required but also the manufacturing cost and the operation and maintenance cost of the drones are higher than the boomerangs.

2.3.5 Barriers to the Boomerang Applications

The perfect 3D unsteady mathematical model is compulsory to predict the trajectory before launching because then only we can measure the weather conditions on predefined points. Boomerang has a low flight range, even though it can be solved using powered boomerangs. The boomerang should be dynamically stable to withstand the turbulence of the wind. The material should have the water resist property to withstand the rain and humidity while monitoring. The remote controlling of the boomerang is not yet developed therefore the rotating and moving while it is on air is not possible. The technologically improved simulators, précised sensors, controllers and the manufacturing methods can enhance the application of the boomerang in future.

3. Discussion

This paper presents a review of mathematical modelling of boomerang for novel outfield applications. By reviewing the mathematical model, we conclude that the mathematical modelling of boomerang 3D dynamics for unsteady flow was not clearly defined and validated, and also, the study on the purpose of using boomerang for field application is not yet started. Even though several boomerang shapes were designed and analysed, those studies weren't done in the context of field application. Also, there is a need for boomerang launchers in future boomerang related applications. Those powered boomerangs can travel long distances through the waypoints we predefine before the launch. If a boomerang is made with high range and energy efficiency, it can be replaced in almost all applications. Achieving high range is challengeable with the existing designs of returning boomerang. Proper geometry optimisation is needed for the best design with high range capacity. The propellers and the power source can be attached to the boomerang, providing additional thrust and torque whenever the boomerang loses its energy.

Felix Hess, Vassberg, and Prasad are the main authors who proposed the advanced mathematical model of a boomerang [2, 5, 6]. The Hess model is not self-trim, and the Vassberg model brought a broad knowledge in aero dynamical forces and moments. Prasad extended the knowledge with reversal airflow effect, drag force, and pitching moment. Prasad (2020) produced the latest proven model available in the literature to predict the trajectory of the boomerang. Even though it contains a few assumptions, those are impossible in real cases, such as the level circular flight assumption, wind, and uniform cross-section. The model developed in 2023 was capable to predict the dynamics in unsteady environments though it was not validated through experiments. If the boomerang is used in the outer environment, it should travel in an unsteady environment so we can clearly define the 3D path of the boomerang in an unsteady environment. The total cost of a boomerang application is comparatively lower than the total cost of a drone because the drone's initial cost, operation and maintenance cost is much higher than that of a boomerang. Therefore, using a boomerang is economically feasible for long time use. Most of the boomerangs are made out of wood which doesn't harm the environment; on the other hand, the drones are made out of plastics and electronic parts that are harmful to the environment.

The geometry of the boomerang could be modified to have sensors, cells and controllers. The boomerangs can have holes and undercuts to attach the cells, sensors, controllers, cameras and special features to perform such interesting tasks. Special type of batteries can be designed in the shape of a boomerang to maintain the aero-dynamical properties of the boomerang unchanged. The maintenance and initial costs are also lower than the boomerangs, therefore, we can conclude the boomerangs are more efficient than the drones.

4. Conclusions

No mathematical model was found to predict the trajectory of the boomerang perfectly. The mathematical model for the unsteady 3D motion of the boomerang is not well addressed. Strict consideration of the aerodynamic forces and moments governing a boomerang's flight motion is essential for accurate prediction of a flight. The boomerang is better than a drone in several factors such as cost, operation, maintenance and environmental friendliness, but the gap in the mathematical modelling, prediction of path and dynamic stability of boomerang are huge barriers for the application of a boomerang. Further studies are needed to overcome these drawbacks, and then only the boomerangs can be used for outfield applications such as environmental monitoring, agriculture inspection, military surveillance, etc. The practicability of the boomerang in field application is not 100% possible with the existing knowledge from the mathematical modelling and related fieldworks. The future researchers can study the 3-dimensional trajectory of a boomerang in unsteady environmental conditions and the invention of remote controllable powered boomerangs help to use the boomerangs in outfield applications.

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