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Modelling ocean carbon cycle with a nonlinear convolution model

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ABSTRACT

A nonlinear convolution integral is developed to model the response of the ocean carbon sink to changes in the atmospheric concentration of CO_2. This model can accurately represent the atmospheric response of complex ocean carbon cycle models in which the nonlinear behavior stems from the nonlinear dependence of CO_2 solubility in seawater on CO_2 partial pressure, which is often represented by the buffer factor. The kernel of the nonlinear convolution model can be constructed from a response of such a complex model to an arbitrary change in CO_2 emissions, along with the functional dependence of the buffer factor. Once the convolution kernel has been constructed, either analytically or from a model experiment, the convolution representation can be used to estimate responses of the ocean carbon sink to other changes in the atmospheric concentration of CO_2. Thus the method can be used, e.g., to explore alternative emissions scenarios for assessments of climate change. A derivation for the nonlinear convolution integral model is given, and the model is used to reproduce the response of two carbon cycle models: a one-dimensional diffusive ocean model, and a three-dimensional ocean-general-circulation tracer model.

1. Introduction

Concern over the rise in atmospheric CO_2 concentration has led to the use of models for global carbon cycle to predict the change in atmospheric CO_2 content as a function of the emission rate of CO_2 to the atmosphere. Carbon dioxide released to the atmosphere accumulates in the atmosphere, incorporates in the terrestrial biosphere and soils, and dissolves in the oceans. Global carbon cycle models have been used to estimate the size of the ocean and terrestrial biospheric carbon sinks as a function of the history of atmospheric CO_2 content (Siegenthaler and Oeschger, 1978; Harvey, 1989b; Wigley, 1993; Schimel et al., 1994).

Models for ocean carbon cycle range in complexity from ones that represent the oceans as several reservoirs to ones that represent the full three-dimensional mixing of the oceans. Ocean carbon cycle models have also included internal processes such as the production, settling and dissolution (respiration) of organic and inorganic (calcite) particulates which are byproducts of biological activity and are expected to have nonlinear effects on ocean carbon cycle (Broecker and Peng, 1982; Kheshgi, 1989; Sundquist, 1990; Sarmiento et al., 1993; Archer and Maier-Reimer, 1994; Jain et al., 1995). Different ocean processes affect the net oceanic uptake of carbon over different time scales. For example, El Niño is thought to be responsible for much of the interannual fluctuation in the ocean carbon sink (Volk, 1989; Winguth et al., 1994). Over time scales of a thousand or more years, the dissolution (or accumulation) of calcite sediments at the ocean floor is thought to affect the ocean carbon sink by altering ocean alkalinity (Sundquist, 1986). Scenarios for the future emissions of CO_2 from the burning of fossil fuels (Leggett et al., 1992) are expected to lead to changes in the atmospheric concentration of CO_2 over the next century. Time scales from decades to several centuries have been

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the focus of models used to assess the ocean carbon cycle's role in climate change (Schimel et al., 1994). Over these time scales important processes used in models include ocean circulation and mixing, ocean chemistry, and air/sea exchange (Enting et al., 1994). Over these time scales, however, changes in other processes—such as ocean circulation which interacts with changes in climate (Manabe and Stouffer, 1993)—may well lead to a more complex response (as illustrated by Gaffin et al., 1986). Therefore, caution in application of carbon cycle models to project the future magnitude of the ocean carbon sink is in order.

The goal of this study is to capture solutions of complex ocean carbon cycle models using a convolution integral (i.e., Green's function) approach. While there are many processes, such as algal blooms (Peng et al., 1987), which lead to large local air/sea fluxes of CO_2 in the oceans, the effect of ocean carbon cycle on climate is thought to be through the net ocean sink of carbon; therefore, this is the aspect of the solution that we wish to capture. The solution of a linear, time-invariant system can be represented exactly by a convolution integral (see, e.g., Subsection 8.1 of Peebles, 1993). However, models for ocean carbon cycle contain an important nonlinearity: the nonlinear dependence of the solubility of CO_2 on the carbon content of the seawater. Earlier applications of convolution models to ocean carbon cycle (Maier-Reimer and Hasselmann, 1987; Harvey, 1989b; Wigley, 1991) use the atmospheric response to finite impulses of various amplitudes to approximate the nonlinear response of the ocean sink to arbitrary emission rates. Siegenthaler (1993) described a new convolution integral approach that directly incorporates the nonlinearity due to CO_2 solubility. We develop methods to use this new approach which allows the capture of the response of an ocean tracer model from just one model response to an arbitrary CO_2 emission to the atmosphere. We then test this approach by comparison to other results of the same three-dimensional model for ocean carbon cycle.

2. Development of a nonlinear convolution model

Emissions of CO_2 to the atmosphere (subscript a) at a rate $F_{e+b\Rightarrow a}(t)$ from the burning of fossil fuels (subscript e) plus changes in the total mass of carbon contained in the terrestrial biosphere (i.e., the net biospheric source, subscript b) contribute to the atmospheric carbon content. Note that the biosphere can be modeled as a system of equations as has been done, for example, by Harvey (1989a) and Wigley (1993); these equations then determine, in part, the emissions rate $F_{e+b\Rightarrow a}(t)$ as a function of time t. The concentration of carbon in the atmosphere and oceans (subscript o) are given respectively by $\tilde{N}_a(t, z, \boldsymbol{x})$ and $\tilde{N}_o(t, z, \boldsymbol{x})$ where z is the vertical coordinate, and $\boldsymbol{x}$ are the horizontal coordinates. For the time scales of interest (decades) the atmosphere is often modeled as well mixed so that $\tilde{N}_a(t, z, \boldsymbol{x}) \approx N_a(t)$. While the spatial dimension in the ocean must be retained to accurately model our understanding of the mixing of carbon within the oceans, we will demonstrate that a nonlinear convolution model (Green's function) representation of the net ocean carbon sink is a good approximation to the net ocean sink calculated by a spatially resolved ocean model.

Ocean carbon cycle models used to estimate the size of the ocean carbon sink over time scales of one to several hundred years are usually run as tracer models where the model is a system of linear differential equations which represents mixing within the ocean with the only nonlinearity being in the representation of CO_2 solubility which affects the mass transfer of CO_2 at the ocean/atmosphere boundary. When CO_2 is added to seawater, it is partitioned among the species CO_2, HCO_3^-, and CO_3^{2-}. However, CO_2 is not partitioned in the same proportion as the preexisting proportion, but rather goes disproportionately into dissolved CO_2. This causes the ocean partial pressure of CO_2 to increase faster relatively than the increase in total dissolved carbon (the carbon contained in all the dissolved inorganic species). This nonlinear dependence of equilibrium CO_2 solubility on the total dissolved carbon—or, alternatively, on the partial pressure of CO_2—is usually represented by the buffer factor $\xi(\tilde{N}_o(t, 0, \boldsymbol{x}), \tilde{T}(t, 0, \boldsymbol{x}), \tilde{T}A(t, 0, \boldsymbol{x}), \tilde{S}(t, 0, \boldsymbol{x}))$ which is dependent on the surface ocean concentration of total dissolved inorganic carbon $\tilde{N}_o(t, 0, \boldsymbol{x})$, surface temperature $\tilde{T}(t, 0, \boldsymbol{x})$, the titration alkalinity $\tilde{T}A(t, 0, \boldsymbol{x})$, and salinity $\tilde{S}(t, 0, \boldsymbol{x})$.

We neglect spatial dependence of the buffer factor because we do not expect this dependence to have a major effect on the global oceanic uptake of

carbon. Therefore, we use globally aggregated (no tildes) values to compute a global buffer factor, $\xi(\tilde{N}_o(t,0,\boldsymbol{x}),\ \tilde{T}(t,0,\boldsymbol{x}),\ \tilde{TA}(t,0,\boldsymbol{x}),\ \tilde{S}(t,0,\boldsymbol{x})) \approx \xi(N_o(t,0),\ T(t,0),\ TA(t,0),\ S(t,0))$ for use in constructing a convolution model for the oceans. The buffer factor is defined by:

$$\xi \equiv \left. \frac{\dfrac{[P_o(N_o(t,0)) - P_o(N_o(t_0,0))]}{P_o(N_o(t_0,0))}}{\dfrac{[N_o(t,0) - N_o(t_0,0)]}{N_o(t_0,0)}} \right|_{\substack{\text{constant alkalinity} \\ \text{salinity \& alkalinity}}}, \tag{1}$$

where P_o is the equilibrium partial pressure of CO_2 in seawater which is a function of carbon concentration N_o as well as T, TA and S: see Peng et al. (1987). Prior to the initial time t_0, often taken to be a pre-industrial time, the system is assumed to be in steady state. It is an idiosyncrasy of the way in which the buffer factor is defined, that the value of ξ depends on initial concentration $N_o(t_0, 0)$ for which different values have been used (Maier-Reimer and Hasselmann, 1987; Jain et al., 1995).

Consider the carbon balance for the atmosphere (Oeschger et al., 1975)

$$\begin{aligned} \frac{\mathrm{d}N_a(t)}{\mathrm{d}t} &= \frac{\mathrm{d}n_a(t)}{\mathrm{d}t} \\ &= k_a \left\{ \frac{N_a(t_0)}{N_o(t_0,0)}\, \xi[N_o(t_0,0) \right. \\ &\quad + n_m(t),\ T(t,0),\ TA(t,0),\ S(t,0)]\, n_m(t) \\ &\quad \left. - n_a(t) \right\} + F_{e+b \Rightarrow a}(t), \end{aligned} \tag{2}$$

where

$$N_a(t) = N_a(t_0) + n_a(t), \qquad N_o(t,0) = N_o(t_0,0) + n_m(t),$$

and k_a is the globally aggregated air/sea exchange rate. The perturbations of atmospheric and surface ocean carbon content (from their values at initial time t_0) are $n_a(t)$ and $n_m(t)$. The rate at which additional carbon enters the ocean from the atmosphere is then equal to

$$F_{a \Rightarrow o} = -k_a \left(\frac{N_a(t_0)}{N_o(t_0,0)}\, \xi n_m - n_a \right). \tag{3}$$

The dependence of rate $F_{a \Rightarrow o}(t)$ on temporal variations in temperature (caused by the temperature dependence of the buffer factor) is retained in this approach, although temperature is not varied in the examples described in the following sections. However, the use of globally aggregated quantities for the air/sea exchange rate, the buffer factor, and ocean mixing (which are spatially variable in nature) is an approximation that will neglect the nonlinearities arising from the interplay between these quantities. Currently, for instance, the buffer factor varies from a value of about 14 in polar regions, where bottom water is formed, to a value of about 10 in equatorial waters, which make up the bulk of the ocean surface area (Holmén, 1992). Since the mixing of surface to deep water occurs at different rates in these different regions, while the nonlinearity of the buffer factor differs with surface water conditions (namely temperature), the net nonlinear effect will be somewhat different in nature than it is in a model using globally aggregated properties. The adequacy of this approximation will be checked by comparison to the ocean-general-circulation tracer model (OGCTM) of Maier-Reimer and Hasselmann (1987) which includes the effects of spatially variable surface temperature on buffer factor in combination with a three-dimensional representation of ocean circulation and its influence on the mixing of the tracer, dissolved inorganic carbon.

In this simple model (3) for the ocean carbon sink, the path for all carbon entering the ocean is assumed to be via gas exchange at the air/sea interface. In nature, there are other paths. For instance, the runoff of dissolved carbon into the oceans is thought (Sarmiento and Sundquist, 1992) to be an important path. While the runoff path, for example, is not modeled explicitly in (3), runoff does result in a net transfer of carbon from the atmosphere to the *surface* ocean water. Since we expect the ocean sink to be limited (over time scales of decades to centuries) by the mixing of surface and deep water, we expect that approximating this path as air/sea exchange will not lead to significant error in the net ocean sink. We are not suggesting that these other paths are not sizable, only that they can be approximated in a model by the air/sea exchange path for the purpose of estimating the response of the net ocean carbon sink to changes in atmospheric carbon dioxide.

The transport of a tracer in the ocean may be

described by a system of linear equations with constant coefficients[†]; and so the response of the ocean surface (mixed layer) concentration $n_m(t)$ to carbon rate $F_{a\Rightarrow o}[n_m(t), n_a(t)]$ can be represented by the convolution integral

$$n_m(t) = \int_{t_0}^{t} \{F_{a\Rightarrow o}[n_m(\zeta), n_a(\zeta)]\ C_o(t-\zeta)\}\ \mathrm{d}\zeta. \quad (4)$$

The nonlinear effect of the buffer factor is contained in $F_{a\Rightarrow o}(t)$ which represents the transfer of carbon across the ocean/atmosphere boundary, and is defined by (3). Note that since $F_{a\Rightarrow o}(t)$ is a nonlinear function of $n_m(t)$, (4) is a nonlinear convolution problem that can be solved for the perturbation of ocean surface concentration, $n_m(t)$, from the steady state value of $N_o(t_0, 0)$. A physical interpretation of the kernel C_o is the response of the mixed layer concentration to an impulse of carbon into the mixed layer from the atmosphere with no prior or subsequent net carbon exchange with the atmosphere. This kernel can also be calculated from other model experiments.

By solving (4) for the mixed layer response to an arbitrary change in atmospheric concentration, the size of the ocean carbon sink $F_{a\Rightarrow o}(t)$ can be calculated with account for the nonlinear effect of the buffer factor. This idea can also be applied to calculate the ocean uptake of other tracers such as ^{14}C, as was suggested by Siegenthaler (1993).

The atmospheric carbon mass balance (2) can be rewritten as

$$\frac{\mathrm{d}N_a(t)}{\mathrm{d}t} = \frac{\mathrm{d}n_a(t)}{\mathrm{d}t} = F_{e+b\Rightarrow a}(t) - F_{a\Rightarrow o}(t). \quad (5)$$

This differential equation can be integrated in time. To do this, the net emissions from the burning of fossil fuels and the biosphere $F_{e+b\Rightarrow a}(t)$ must be evaluated, which may require solution of additional equations. In addition, the ocean uptake $F_{a\Rightarrow o}(t)$ is evaluated by (3) which requires the mixed layer concentration $n_m(t)$ as well as an expression for the buffer factor ξ. The mixed layer concentration is evaluated by the nonlinear convolution integral (4). Together these equations can be used to evaluate the atmospheric response to emissions of carbon dioxide to the atmosphere. To evaluate the nonlinear convolution integral requires an expression of the ocean response kernel $C_o(\Delta t)$.

3. Calculation of the ocean response C_o

In the examples that follow, we use two different methods for calculating the ocean response kernel $C_o(\Delta t)$. In the first method an analytical expression for the kernel is derived; this approach is only applicable to ocean carbon cycle models which yield to analytical solution. The second approach, which we describe in this Section, is to calculate the kernel from a response of an ocean carbon cycle model to an arbitrary change in atmospheric concentration.

Consider the response of a global carbon cycle model to an emission rate of carbon $F_{e+b\Rightarrow a}(t)$. Note that in the examples which follow we compare results from emission impulses even though an arbitrary emission rate $F_{e+b\Rightarrow a}(t)$ can be specified. To find the response $N_a(t)$ to carbon emission $F_{e+b\Rightarrow a}(t)$, a run of a full global carbon cycle model (e.g., an OGCTM) could be carried out; in example #2 (below) we use an OGCTM response $N_a(t)$ reported by Maier-Reimer and Hasselmann (1987).

Using (5) and the result $N_a(t)$ from a full carbon cycle model and the emissions function $F_{e+b\Rightarrow a}(t)$, the response $F_{a\Rightarrow o}(t)$ of the ocean to the atmospheric release can be calculated from:

$$F_{a\Rightarrow o}(t) = F_{e+b\Rightarrow a}(t) - \frac{\mathrm{d}N_a(t)}{\mathrm{d}t}. \quad (6)$$

Next, using the relation (3) between the net ocean sink $F_{a\Rightarrow o}(t)$ and the atmosphere and mixed layer conditions, we solve the nonlinear equation

$$F_{a\Rightarrow o}(t) + k_a \left\{ \frac{N_a(t_0)}{N_o(t_0, 0)}\, \xi[n_m(t) + N_o(t_0, 0), T, TA, S]\ n_m(t) - n_a(t) \right\} = 0, \quad (7)$$

for the unknown response $n_m(t)$ of the mixed layer given the net ocean sink $F_{a\Rightarrow o}(t)$ calculated from (6) and the atmospheric response $n_a(t)$ calculated

[†] If the tracer equations have time varying coefficients, then $C_o = C_o(t, \zeta)$ in eq. (4) and (4) is then an integral equation but not a convolution. This could be the case if ocean circulation were modeled as a time varying field.

by the full model. At this point we can solve the linear problem for $C_o(t)$:

$$n_m(t) - \int_{t_0}^{t} F_{a\Rightarrow o}(\zeta)\, C_o(t-\zeta)\, d\zeta = 0. \tag{8}$$

This is a deconvolution problem to calculate the ocean kernel $C_o(\Delta t)$ from the information generated by the full model result.

4. Example 1: one-dimensional diffusion model

A simple model for the ocean carbon cycle is a mixed layer capping a diffusive medium (Oeschger et al., 1975). This model makes a convenient example because the ocean kernel $C_o(t)$ can be derived analytically. Consider the model for the mixed layer carbon concentration n_m (mol-C/m^3):

$$h\frac{dn_m}{dt} = \frac{1}{A\Omega} F_{a\Rightarrow o}(t) + \kappa \left[\frac{\partial n_o(t,z)}{\partial z}\right]\Bigg|_{z=h}. \tag{9}$$

In this model values of the mixed layer depth h, the vertical eddy diffusivity κ, the ocean area A, the ocean surface carbon concentration $N_o(t, 0)$, and the molecular weight of carbon Ω are given in Table 1. The deep ocean is modeled by a one dimensional diffusion equation

$$\frac{\partial n_o}{\partial t} = \kappa \frac{\partial^2 n_o}{\partial z^2}, \tag{10}$$

with the boundary conditions

$$n_o = n_m, \qquad \text{at} \quad z = h; \tag{11}$$

$$n_o = 0, \qquad \text{at} \quad z = \infty \qquad \text{and at} \quad t < 0. \tag{12}$$

For a diffusion model, the time required for the fluctuation of atmospheric carbon to diffuse to the finite depth $d \approx 4000$ m of the ocean is approximately $\tau \equiv d^2/\kappa = 4000$ year; it is appropriate to neglect the finite depth in boundary condition (12) as long as the time scale of interest is much less than τ. The kernel C_o is the response of system (9)–(12) to the impulse $F_{a\Rightarrow o}(t) = \delta(t)$ at time

Table 1. *Nomenclature and parameter values*

Symbol	Definition	Value, units
n_m	perturbed ocean surface carbon concentration	mol-C/m^3
$F(t)$	carbon mass rate	Gt C/year
$C_o(t)$	dimensional ocean kernel	mol-C/m^3/Gt-C
$K(t)$	dimensionless ocean kernel	$C_o(t)\, V\Omega$
Ω	molecular weight of carbon	$1.2 \cdot 10^{-14}$ Gt-C/mol-C
A	surface area of the oceans	$3.62 \cdot 10^{14}$ m^2
V	ocean volume	$1.35 \cdot 10^{18}$ m^3
ρ	seawater density	1025 kg/m^3
S	salinity	$35^{\dagger}$ ‰
TA	titration alkalinity	$2420^{\dagger}$ μmol-C/kg
T	base temperature	20°C
h	mixed-layer depth (diffusion model)	$75^{\ddagger}$ m
κ	vertical eddy diffusivity (diffusion model)	$3987^{\ddagger}$ m^2/year
Ξ	carbon mass of one ppm of atmospheric CO_2	2.123 Gt C/ppm
$P_a(0)$	base atmospheric partial pressure of CO_2	$265^{\dagger}$ ppm
$N_a(0)$	base atmospheric carbon mass	$563^{\dagger}$ Gt C
$N_o(t_0, 0)$	base ocean surface carbon concentration	2.094* mol-C/m^3
λ	bulk mass transfer coefficient	$0.05^{\dagger}$ mol-C/m^2/year/ppm
k_a	air/sea exchange rate	$0.10^{\dagger}$ year$^{-1} = \lambda\Omega A/\Xi$

† Values consistent with Maier-Reimer and Hasselmann (1987).
‡ Values consistent with Oeschger et al. (1975).
* Carbon concentration at equilibrium with $P_a(0)$.

$t=0$, where $\delta(t)$ is the delta impulse function, which has the analytical solution

$$C_o(t) = \frac{1}{A\Omega h} e^{(\kappa t/h^2)} \operatorname{erfc}(\sqrt{\kappa t}/h)$$

$$\xrightarrow[\lim h^2/\kappa t \to 0]{} \frac{1}{A\Omega\sqrt{\pi\kappa t}}, \qquad (13)$$

where erfc is the complementary error function (the limit is useful for numerical evaluation of the kernel for times greater than 140 years). The kernel is the response of the mixed layer concentration given here in units of mol-C/m^3 to the impulse in Gt-C.

A dimensionless kernel is given by $K(t) \equiv C_o(t)\, V\Omega$ where V is the ocean volume, and Ω is the molecular weight of carbon. Recall that the kernel represents the response of the mixed layer concentration a time t after an impulse of carbon to the mixed layer for a model ocean where there is no net exchange of carbon between the ocean and atmosphere. The factor $V\Omega$ used to define the dimensionless kernel $K(t)$ was chosen so that $K(t)$ will have a value of one if the carbon influx into the ocean surface is mixed uniformly throughout the entire ocean volume. The dimensionless ocean kernel $K(t)$ is shown in Fig. 1 for parameter values listed in Table 1. The value of $K(t)$ decreases monotonically from a maximum value of $V/Ah = 50$ (the ratio of the mean ocean depth to the mixed layer depth) at time $t=0$ to a value of 1.5 at $t=500$ years as shown in Fig. 1.

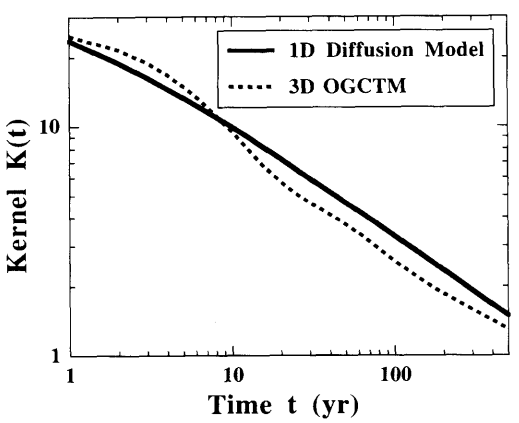

Fig. 1. Ocean kernels $K(t)$ for the one-dimensional mixed-layer/diffusive-ocean carbon cycle model found by analytical solution (solid line), and for Maier-Reimer and Hasselmann's (1987) OGCTM constructed by deconvolution of a response to a finite impulse (141 Gt C) to the atmosphere (dashed line).

The maximum value of the surface water response $K(0)$ of this model is at the time of the impulse. If there were a finite mixing rate in the mixed layer of the ocean, then the value of $K(0)$ would be indefinite. This would not, however, present a problem with this approach since the short time values of $K(t)$ would only affect the response of the ocean carbon sink over short time scales.

5. Example 2: 3-dimensional ocean general circulation tracer model (OGCTM)

Maier-Reimer and Hasselmann (1987) presented results of an ocean carbon cycle model created by modelling carbon as a passive tracer including the nonlinear effect of the buffer factor. Modelling carbon in this way results in a linear system of equations for the mixing of carbon in the ocean interior, making their model a candidate for the nonlinear convolution given by (4).

Maier-Reimer and Hasselmann (1987) did report functional fits of their model atmospheric response to finite impulses to the atmosphere. In this section we find the ocean kernel C_o for their model by solving the deconvolution problem (6)–(8) for the smallest impulse of 141 Gt C (equal to one quarter of the atmospheric carbon content) that Maier-Reimer and Hasselmann (1987) reported. This impulse was applied to a steady pre-industrial atmosphere containing 265 ppm CO_2 (Table 1). To do this we have adopted the function for the buffer factor taken from Peng et al. (1987), although Maier-Reimer and Hasselmann (1987) used an alternative function. To evaluate the buffer factor, we have chosen values of temperature and salinity listed in Table 1. The first step in the calculation of C_o is to use (6) to calculate $F_{a\Rightarrow o}(t)$ for an impulse ($F_{e+b\Rightarrow a}(t) = 0$ for $t>0$). $F_{a\Rightarrow o}(t)$ for $t>0$ is equal to the negative time derivative of the atmospheric response N_a which can be calculated analytically from Maier-Reimer and Hasselmann's (1987) response function which they approximate as a sum of exponentials. Next we solve the nonlinear equation (7) for n_m. Finally we solve the deconvolution problem (8) for the ocean kernel C_o. The deconvolution calculation was

performed using forward differencing with small (0.01 year) time steps so that numerical truncation errors were of negligible magnitude compared to the effect of the nonlinearity of the buffer factor, or the differences between models.

The dimensionless kernel $K(t)$ found by deconvolution is also shown in Fig. 1. Note that this kernel represents the ocean surface water response to an impulse of carbon with no net exchange with the atmosphere. This kernel represents primarily the effects of internal ocean mixing as a linear process. The deconvolution procedure collapses the interplay between 3D ocean mixing and 2D air/sea exchange into a globally aggregated kernel in combination with a buffer factor function and the convolution model (7–8). This implies that the same buffer factor used to create the kernel should be used to evaluate the convolution model in order to get accurate results. The comparison of the kernels from the two models shows that the OGCTM exhibits less mixing over short-time intervals and greater mixing over long-time intervals than does the 1D diffusion model. Using the kernels of the two models, the atmospheric response to emissions $F_{e+b\Rightarrow a}(t)$ can now be calculated by solving the ordinary differential equation (5) along with the nonlinear convolution integral (4).

6. Response of nonlinear convolution models

The nonlinear convolution models based on the two kernels shown in Fig. 1 are used to calculate the response of the ocean sink to finite impulses of CO_2 to the atmosphere. The results of the two examples can then be compared to each other and to Maier-Reimer and Hasselmann's (1987) responses of their OGCTM. This approach can also be used to calculate the response $F_{a\Rightarrow o}(t)$ to arbitrary emission functions $F_{e+b\Rightarrow a}(t)$ for use in, for example, assessments of climate change. The atmospheric response was calculated by solving (1)–(4) using the forward difference method to integrate the differential equation (2) and evaluate the convolution integral (4).

The response $F_{a\Rightarrow o}(t)$ of the convolution model, of course, should not be expected to represent nature any better than that of the model on which it is based. In nature, one might expect, for instance, that changes in climate will lead to changes in ocean circulation that will be a feedback to carbon cycle; and a feedback such as this one could be both nonlinear and, important over the time scales of interest. At this stage there remains significant uncertainty in estimating future carbon cycle. As understanding develops, ocean carbon cycle models will evolve, and so

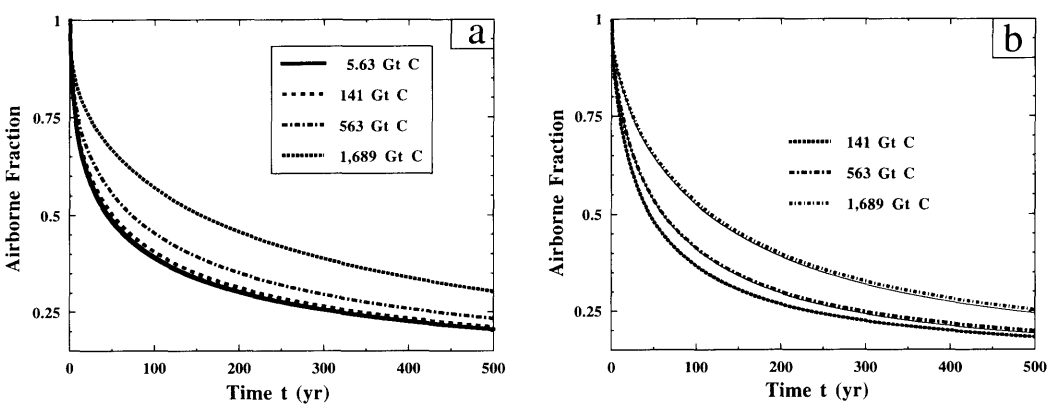

Fig. 2. Airborne fraction responses to impulses of CO_2 to the atmosphere at initial time $t = 0$. Before the initial time the carbon cycle is specified to be in steady state with an atmospheric concentration of 265 ppm CO_2. Responses calculated with (a) a nonlinear convolution model based on the kernel given in eq. (13) of the one-dimensional mixed-layer/diffusive-ocean carbon cycle model. Impulses of size 5.63, 141, 563, and 1689 Gt C shown. In panel (b) the responses calculated with a nonlinear convolution model based on a kernel deconvolved from Maier-Reimer and Hasselmann's (1987) OGCTM 141 Gt C response are compared to Maier-Reimer and Hasselmann's (1987) responses (shown by thin solid lines) for 141, 563, and 1689 Gt C impulses.

Table 2. *Comparison of model results: the airborne fractions of impulse responses*

	Impulse mass	Airborne fraction at time after impulse 20 years	100 years	500 years
1D diffusion model	5.63 Gt C	0.6107	0.3887	0.2052
	141 Gt C	0.6269	0.4036	0.2113
	563 Gt C	0.6737	0.4509	0.2323
	1689 Gt C	0.7641	0.5677	0.3021
OGCTM (Maier-Reimer and Hasselmann, 1987)	141 Gt C	0.6442	0.3669	0.1820
	563 Gt C	0.6906	0.4105	0.1916
	1689 Gt C	0.7749	0.5240	0.2441
Convolution model based on OGCTM	141 Gt C	0.6440	0.3667	0.1820
	563 Gt C	0.6913	0.4133	0.1982
	1689 Gt C	0.7788	0.5311	0.2529

should methods to capture the solutions of such models for use in assessment of climate change.

The response of the nonlinear convolution model based on the 1D diffusion model reproduces the direct numerical solution of the model differential equations to within the error of the numerical method. Fig. 2a shows the response of atmospheric CO_2 to impulses of 5.63, 141, 563, and 1689 Gt C (equal to 0.01, 0.25, one, and three times the atmospheric carbon content, respectively). The airborne fraction of the impulse (the fraction of the initial impulse mass that remains in the atmosphere) decreases from a value of one at time $t = 0$ over the 500 years shown. The nonlinear effect of the buffer factor, which increases with the partial pressure of CO_2, causes larger impulses to result in larger values of airborne fraction.

Responses of the nonlinear convolution model based on the OGCTM kernel are compared to the three (141, 563 and 1689 Gt C) impulse responses given by Maier-Reimer and Hasselmann (1987) in Fig. 2b. The 141 Gt C impulse response is reproduced to within the error of the numerical method, since this is the same impulse that was used to create the kernel shown in Fig. 1 by deconvolution. For the two larger impulses, the same kernel is used along with the nonlinear buffer factor function to calculate the airborne fraction responses. These responses are compared in Fig. 2b, and values listed in Table 2, to the responses reported by Maier-Reimer and Hasselmann (1987), and the comparison shows only small differences between the results. The differences could be caused by several effects:

1. Spatial variability of the buffer factor due to temperature effects are not included in the convolution model. The buffer factor in cold (bottom water forming) regions may have increasing importance, compared to the buffer factor in warm regions, as the length of time after the impulse increases. The buffer factor is less sensitive to temperature when at high pCO_2 than at low pCO_2. This leads to a nonlinearity that is not represented exactly by a buffer factor aggregated for the entire ocean surface.
2. Different buffer factor function was used in this study than were used by Maier-Reimer and Hasselmann.
3. Maier-Reimer and Hasselmann's responses were approximated as a sum of exponentials which incurs some error.

The small differences between the solutions of the convolution model and the OGCTM notwithstanding, we have demonstrated that we can capture the nonlinear behavior of a complex model with a nonlinear convolution model.

7. Conclusions

A nonlinear convolution model for the ocean carbon sink has been developed. In this approach the convolution model can be used to reproduce a more complex ocean carbon cycle model. New understanding of the ocean carbon sink can be incorporated by changing the kernel of the convolution integral without change in the structure

of the model. Earlier applications of convolution models to global carbon cycle (Maier-Reimer and Hasselmann, 1987; Harvey, 1989b; Wigley, 1991) used the atmospheric response to impulses of various amplitudes to estimate the response of the ocean sink to arbitrary emission rates. The approach used here directly incorporates the nonlinear effect of the buffer factor and can capture the response of Maier-Reimer and Hasselmann's (1987) ocean tracer model from just one model response. The extent to which a reduced-form model, such as the class of models based on convolution integrals, can capture the response of ocean carbon cycle models with other nonlinear effects remains to be seen. We demonstrate this method by reproducing atmospheric responses to finite size emission impulses given information from just one impulse. We propose that ocean carbon cycle models be represented and intercompared not by their atmospheric response, but rather by their mean ocean-surface-water response $C_o(t)$, which can subsequently be incorporated in a nonlinear convolution model for global carbon cycle. In this way other factors such as differences in buffer factor functions, scenarios for atmospheric CO_2 concentrations, and terrestrial biosphere effects can be removed to allow direct intercomparison of ocean carbon cycle models. In addition, this approach can be applied to the ocean uptake of other tracers like radiocarbon as was suggested by Siegenthaler (1993), to further evaluate tracer model performance.

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