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LETTERS TO THE EDITOR

Comments on "Chaos in daisyworld" by X. Zeng et al.

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1. Introduction

Zeng et al. (1990, henceforth denoted ZPE) found chaotic behavior in daisyworld (Watson and Lovelock, 1983, henceforth denoted WL), a simple nonlinear model of an idealized world containing only species of daisies of various albedos. The planetary albedo depends on the coverage of each type of daisy, and the daisies have temperature-dependent growth rates, enabling feedback between the environment (temperature) and the biota (daisies).

The chaos obtained by ZPE results from using a discrete model and does not occur in the differential model conceived by WL. However, ZPE are unclear about the connection between the differential and discrete systems, misattribute their chaos to the coupling between biota and temperature, and fail to mention the unphysical behavior their model produces for some initial conditions. Further, we have discovered that long-term means of the chaotic states obtained by ZPE are close to the equilibrium solutions of the WL differential system and have small variance, stabilizing daisyworld against external forcing. The purpose of this letter is to elucidate these issues. Notation is as in ZPE unless otherwise defined.

2. Relationship between differential and discrete daisyworlds

Differential and discrete daisyworld systems are mathematically and physically distinct systems and should not be confused with each other. This is obscured in ZPE by using derivative notation in their difference equation (9), by calling both their system and the WL system by the same name and even stating, "The governing equations used in [ZPE] are the same as in [WL]", and by not including the time step in their analysis.

The physical relationship between the discrete and differential systems is in the lag of the forcing function, which we shall call the feedback. This feedback consists of a parabolic dependence on population density and a temperature dependence. In WL, both feedback factors operate instantaneously. In ZPE, *both the population and temperature dependences are assumed to be delayed by one generation.* The role of the temperature feedback will be examined in more detail in Section 3. Whether it is physically reasonable to delay the population feedback, which is used to simulate such things as competition for resources, is uncertain. When the delay causes the equilibrium condition to be unstable, the population swings from one generation to another can be extreme and in some cases result in negative values for the population, clearly an unphysical result. *Negative populations result for 48% of the physically possible initial conditions* ($a_1, a_2 > 0$, $a_1 + a_2 < p$) *for the parameter values generating the chaos depicted in Fig. 7 of ZPE.* (The other 52%, except for a set of measure zero, are in the basin of the chaotic attractor.)

The mathematical link between the discrete and differential systems is through the time step, or generation time as it is interpreted in ZPE. The differential system is continuous, thus a trajectory in phase (a_1, a_2, etc.) space cannot cross another, forever bounded by its neighboring trajectories. Since zero amounts of any species are invariant manifolds and the flow is inwards along the outer boundary $\sum_i a_i = p$, trajectories are trapped inside the physically possible region of phase space ($a_i \geqslant 0$ for all i, $\sum_i a_i \leqslant p$). In contrast, trajectories in the discrete system may jump around. For small time steps, they will be similar to the continuous trajectories of the differential system, slowing their velocity through phase space as they near the equilibrium point. As the time step is increased, they will shoot towards the equilibrium too fast and hop over it, only to overshoot in the opposite direction next time. When chaos is achieved, the system is always adjusting *toward* equilibrium, but it overshoots and never hits it. This jumping around also may cause trajectories to jump into regions of unphysical negative populations.

To explicitly see the role of the time step, consider the case of constant β (no temperature feedback) for one species, with $p = 1$. The governing equation for the differential system is

$$\mathrm{d}a/\mathrm{d}t = a[(1-a)\,\beta - \gamma], \tag{1}$$

which has analytic solution

$$a(t) = b/[1 + (b/a_0 - 1)\,e^{-t/\tau}], \tag{2}$$

where a_0 is the initial condition, and $b = 1 - (\beta/\gamma)$ is the equilibrium solution approached exponentially with e-folding time $\tau = 1/(\beta - \gamma)$, provided the growth rate exceeds the death rate ($\tau > 0$). The discrete system using (9) of ZPE yields the well-known logistic map,

$$x^{n+1} = \mu x^n (1 - x^n), \tag{3}$$

where the superscript denotes the generation and

$$x^n = (a^n/b)/(1 + \tau/\Delta t), \qquad \mu = 1 + \Delta t/\tau. \tag{4}$$

Note the appearance of the time step in the map parameter, μ. The discrete model has the *same* equilibrium as the continuous model, but as the time step is increased, the equilibrium becomes unstable, leading to bifurcations and chaos. The two-daisy problem with temperature feedback corresponding to Fig. 7 in ZPE follows exactly the same pattern of bifurcation as the time step is increased, bifurcating at similar values of the normalized time step ($\Delta t/\tau$) when τ is taken using the value of β at the equilibrium point.

ZPE achieve the effect of increasing the normalized time step by increasing the value of C in their eq. (3). However, C is an extraneous parameter because the normalized time step can be linearly increased by increasing Δt, and the effect of C on the value of the equilibrium can be reproduced by changing the values of γ_i. Moreover, it is illusory to claim, as ZPE do, that C is the strength of the coupling between the daisies and the environmental temperature, because C is merely a proxy for comparing the time between generations to the response time of the dynamical system.

3. The effect of temperature feedback

We will compare the behavior of daisyworld with and without temperature feedback for both the differential and discrete systems and consider the effect of delaying the temperature feedback while allowing instantaneous population feedback.

Differential daisyworld without temperature feedback (constant β) has either an infinite number of equilibria containing multiple species (when the ratios γ_i/β_i are equal) or no equilibria containing more than one species. The presence of the temperature feedback permits the possibility of an isolated equilibrium containing more than one species.

These results carry over to discrete daisyworld, since they have the same equilibria. The question for the discrete case is how the temperature feedback relates to the stability of the equilibria. With constant β, the problem becomes that examined by May (1974, 1976), for which chaos is possible. Thus, unstable equilibria leading to bifurcations and chaos are possible even in the absence of temperature feedback.

Now, consider the case where the temperature interaction is delayed but the population feedback is instantaneous. Two types of temperature delay were tested: (1) a continuous delay of duration t_{lag} in responding to the environmental conditions $[\beta = \beta(t - t_{\mathrm{lag}})]$, and (2) a discrete delay such that β remains constant for intervals of t_{lag} with updates to its current value after each interval. For the parameters used to generate the chaos shown in Fig. 7 of ZPE, equilibrium solutions are obtained for $t_{\mathrm{lag}} = 1$, and t_{lag} has to be increased to 3.5 for delay type (1) and 5 for delay type (2) to produce chaotic solutions. Furthermore, the chaotic attractor for each of these systems does not resemble the chaotic attractor for the ZPE system. Thus, the chaos obtained in ZPE is due to the lag of the population feedback and not the lag of the temperature feedback. The environmental temperature feedback is of only secondary importance in destabilizing an isolated multi-species equilibrium in discrete daisyworld.

4. Chaos in discrete daisyworld still upholds Gaia hypothesis

The *climate* of chaotic solutions to discrete daisyworld has almost the same mean as the stable equilibrium solutions of differential daisyworld, and these means have small variance. This should not be surprising because the chaotic attractor bounds the eventual states of the system, and the effect of the chaos is to cause the system to overshoot equilibrium, then overshoot back to the vicinity of where it came from before.

The climate of chaotic discrete daisyworld was examined with histograms and time-averaged means and variances of the planet effective temperature. Time averages were running means of 2–200 years applied on 2500 generations (after discarding the initial 300–500 generations). External forcing was applied by allowing the solar luminosity to vary, with a new time series com-

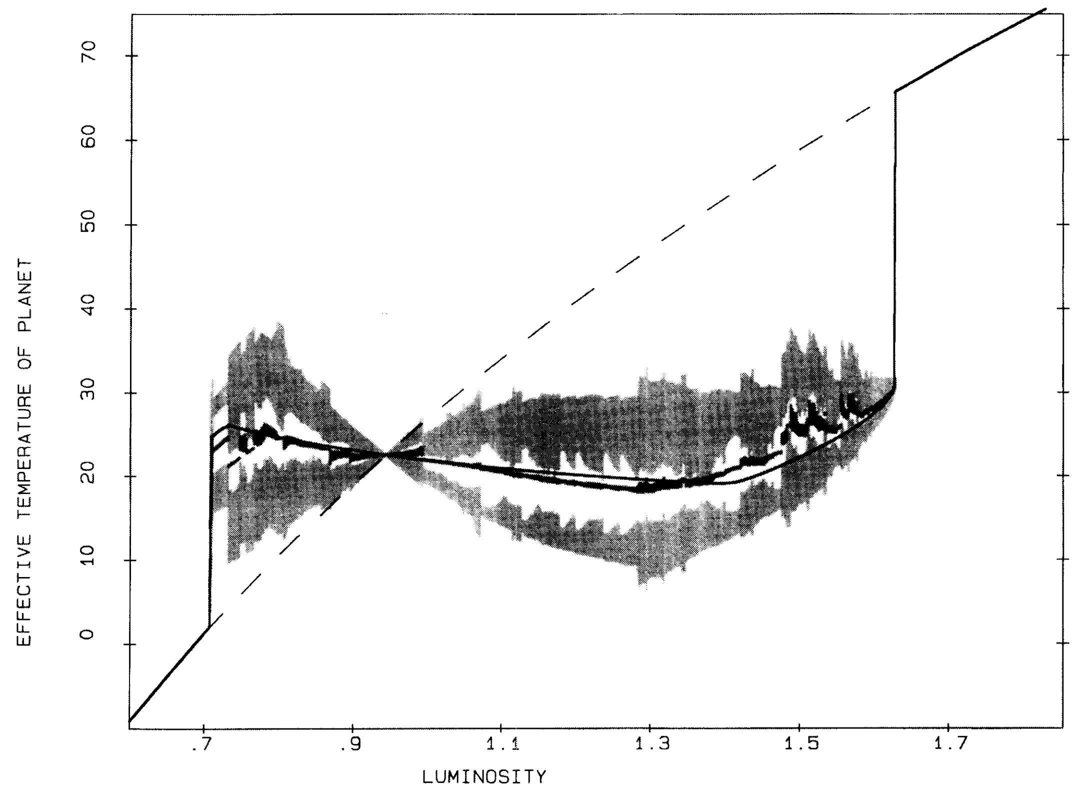

Fig. 1. Climate of daisyworld as external forcing is varied. Solid line is solution to differential daisyworld, showing homeostasis as luminosity is increased. Dark shading covers temperatures within one standard deviation of the 30-generation means of chaotic solutions to discrete daisyworld, showing that the 30-generation climate for chaotic-states also exhibits homeostasis. Light shading covers temperatures of individual generations within one standard deviation of the means for temperatures above and below the median, roughly indicating the distribution of temperatures for the chaotic states. Most values are much closer to the stable climatic conditions than to the temperature the planet would have without any daisies (dashed line). Parameter values were: $C = 4$, $p = 1$, $A_1 = 0.25$, $A_2 = 0.75$, $A_{gf} = 0.5$, $q' = 20$, $\gamma_1 = \gamma_2 = 1.0$, Δt adjusted to near the minimum needed for chaos for each value of luminosity.

puted for each luminosity. The initial population was the equilibrium solution found by using a small time step, except a minimum value was taken for any species with a smaller equilibrium population. The time step was then adjusted from a large first guess until a chaotic solution was found. Chaos could not be achieved without adjusting the time step as the luminosity was increased.

Results for the parameter values used to generate the chaos depicted in Fig. 7 of ZPE are shown in Fig. 1. The bold line is the equilibrium condition approached from all initial conditions possessing both species in differential daisyworld. It shows harsh conditions for which no daisies can exist for very low and very high luminosities. As the luminosity is altered such that daisies can survive, the equilibrium temperature jumps to a value close to the optimal temperature for daisy growth and remains close to optimal until the luminosity again reaches a point where daisies can no longer survive. The dark shading covers temperatures within 1.0 standard deviations of the mean for 30-generation time averages for chaotic solutions of discrete daisyworld. Observe that the climate means are very close to the equilibrium solutions and the standard deviations are very small. Thus, homeostasis of the climate is maintained, upholding the conclusions of WL even for chaotic behavior in discrete daisyworld.

Also shown in Fig. 1 is the temperature the planet would have without daisies (dashed) and the spread of temperatures (not time-averaged) for the chaotic states (light shading). The light shading covers values within 1.0 standard deviations of the means of the temperatures above and below the median. This was done because the distribution of temperatures for the chaotic cases was multi-modal with large concentrations of points a considerable distance from the mean. Notice that even the wildly fluctuating chaotic states are usually much closer to conditions favorable for daisies than the planet would be in the absence of daisies.

The variance of the time-averaged temperatures drops off slightly slower than $1/n^2$, where n is the number of generations in the running average. This is much faster than the $1/n$ from averaging members of a normally distributed random sample.

5. Conclusions

Discrete daisyworld is differential daisyworld but with a time lag of one generation in *both* the population feedback *and* the environmental temperature feedback. As the generation time is increased, the equilibrium becomes unstable, leading to bifurcations and chaos in the solutions to discrete daisyworld. This chaos is due primarily to the lag in the population feedback. Additionally, the discrete model with some parameter values produces unphysical negative populations for a large fraction of physically possible initial conditions.

The presence of daisies stabilizes the climate against external forcing even with chaotic states in the discrete model of daisyworld. The average climate over 30 generations is extremely close to the equilibrium climate of differential daisyworld. Thus, discrete daisyworld does not contradict the Gaia hypothesis, but rather supports the conclusions WL made based on differential daisyworld.

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