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The incompatibility of ice-core CO_2 data with reconstructions of biotic CO_2 sources (II). The influence of CO_2-fertilised growth

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ABSTRACT

The compatibility of ice-core CO_2 data with direct estimates of biotic CO_2 release is tested using a response function analysis that, apart from stationarity, leaves the oceanic uptake of CO_2 largely unconstrained. The role of CO_2-enhanced growth (which had not been considered in an earlier study of this type) is examined explicitly. It is found that allowing for CO_2-enhanced growth increases the discrepancy between the source estimates and the ice-core data. The proposed explanation is that some of the early increase in CO_2 concentrations reflects a recovery from a perturbation associated with the little ice-age.

1. Introduction

This paper describes an extension of an earlier study (Enting and Mansbridge, 1987a; hereafter denoted (I)) which revealed, in a model-independent manner, an incompatibility between CO_2 data from ice cores and direct estimates of release of biotic CO_2 from land-use changes. The role of the terrestrial biota in the atmospheric carbon budget has been a topic of considerable uncertainty for many years. Francey and Enting (1990) summarised the development of analyses based on carbon cycle modelling into three phases.

(i) A period around 1980 in which carbon cycle models calibrated by ^{14}C predicted a larger airborne CO_2 fraction than was being observed (see for example, Pearman, 1980; Siegenthaler, 1983).

(ii) A growing realisation that the present day atmospheric CO_2 budget will be strongly influenced by the history of past changes in releases. The analysis of this influence was formalised by Oeschger and Heimann (1983). Within the assumptions of a given carbon cycle model, the influence of past changes on the interpretation of present budgets is fully accounted for by deconvolution of CO_2 concentrations from bubbles trapped in polar ice. Such deconvolutions give the history of changes in CO_2 sources. Siegenthaler and Oeschger (1987) performed several such calculations. They obtained source estimates with a peak release (in the range 0.5 to 1.5 Gt C yr^{-1}) around 1900 and a current release in the range 0 to 1.0 Gt C yr^{-1}, depending on the choice of ocean model and the uncertainties assigned to the ice-core data.

(iii) More recently, studies using spatial distributions of CO_2 to give a "snapshot" of the current atmospheric budget have cast doubt on these earlier views of the atmospheric budget. Studies by Enting and Mansbridge (1989), Tans et al. (1989) and Keeling et al. (1989) have suggested that the largest CO_2 sink is in the mid-latitudes of the northern hemisphere even though, on the basis of area, oceanic sinks would be expected to be greatest in the southern hemisphere. An analysis by Tans et al. (1990) confirmed these modelling results and also used ocean surface p_{CO_2} data to estimate a current net oceanic uptake of CO_2 of around 0.5 to 1.0 Gt C yr^{-1}. Therefore they repeated the suggestion by Enting and Mansbridge (1989) that the northern hemisphere sink included a strong contribution from net biotic growth. A sink of at least 1 Gt C yr^{-1} was required

to balance their proposed atmospheric carbon budget.

In an attempt to estimate biomass changes directly, Houghton et al. (1983) analysed land-use changes. Using simple models of changes in the carbon content of various ecosystems, they estimated the history of net biotic carbon release for the period since 1860. The release estimates showed an overall growth, in contrast to the estimates from ice-core data which showed a peak release around 1900 (Siegenthaler and Oeschger, 1987). Houghton et al. suggested a net biotic release of around 2 Gt C yr^{-1} around 1980, in contrast to estimates of 0 to 1 Gt C yr^{-1} release calculated from ice-core data and differing even more from the Tans et al. (1990) result of a net biotic uptake of at least 1 Gt C yr^{-1}.

The "snapshot" approach based on analysing the spatial distribution of CO_2 applies only for the short recent period for which the data are available. However the analyses based on land-use changes and the deconvolution of ice-core data apply over the whole of the industrial period and so these two approaches can be subjected to a more detailed comparison. In (I), Enting and Mansbridge analysed the discrepancy between the carbon cycle modelling and the direct estimates of biotic release in terms of its implications for oceanic uptake. They expressed the relation between CO_2 sources and concentrations in terms of an oceanic response function, following the formalism described by Oeschger and Heimann (1983), thus considering only models with time-invariant (i.e., steady-state) mixing and CO_2 exchange varying linearly with partial pressure changes. They also applied rather weak (and physically reasonable) constraints on this oceanic response. Under these conditions, they were able to show that no such linear steady-state model of oceanic uptake could reconcile the ice-core data of Neftel et al. (1985) with the biotic release estimates of Houghton et al. (1983).

The present paper extends the earlier analysis of (I) There are a number of reasons for doing this.

(i) The analysis by Houghton et al. is based on land-use changes. Any changes in carbon storage that do not result from land-use changes will not be included in the estimates. Enting and Mansbridge (1989) and Tans et al. (1990) have suggested that a mid-latitude biotic CO_2 uptake, possibly due to CO_2-enhanced growth, is necessary in order to account for the spatial distribution of CO_2 concentrations. Similarly, Keeling et al. (1989) have also suggested a strong CO_2-fertilisation effect and specifically used the results of Enting and Mansbridge (1987a) as a reason for regarding the estimates by Houghton et al. as incomplete.

(ii) Enting (1990) has noted that uncertainties in the biotic uptake of ^{14}C can imply very large uncertainties in the calibration of the oceanic uptake of CO_2 in carbon cycle models. The calibration appears to be extremely ill-conditioned. Given such uncertainties, an analysis that avoids making precise assumptions about oceanic processes in particularly valuable.

(iii) The Bern group have produced a revised record of CO_2 concentrations in ice (Friedli et al., 1986). Additionally, Houghton (personal communication) has calculated revised estimates of the carbon release from land-use changes giving both the "high release" and "low release" estimates. Each of these new estimates is lower than the original 1983 estimates. It seems worthwhile repeating the earlier analysis using these new data sets.

2. Analysis

The basic equation relating CO_2 sources to concentrations is

$$C(t) = C(t_0) + \int_{t_0}^{t} R(t-t')\, S(t')\, \mathrm{d}t', \qquad (1)$$

where $C(t)$ is the atmospheric CO_2 concentration, $S(t)$ is the net source/sink strength from non-ocean processes and $R(t)$ is a response function describing the oceanic uptake of CO_2. The use of such a response function assumes that the processes of oceanic uptake of CO_2 are stationary in time and are linear in the CO_2 perturbation. It also assumes a pre-industrial equilibrium characterised by $S(t) = 0$ for $t < t_0$. Since the more recent ice-core gives additional values for the 18th century, the time origin t_0 in the present study has been set to 1700. We regard the terrestrial source, $S(t)$, as being due to the sum of $F(t)$, the fossil carbon release, and $B(t)$, a net biotic release (or uptake if $B(t) < 0$).

The analysis of (I) used $F(t)$ as estimated by Rotty (1987), $B(t) = B_H(t)$ from Houghton et al. (1983) and m_i, the CO_2 concentrations measured in ice-cores by Neftel et al. (1985). The result was that with these data sets, there is no response function (subject to conditions (3), (4) and (5) below) for which expression (1) for $C(t_i)$ gives a value within range Δ of m_i, unless the tolerance Δ exceeded 3.91 ppmv. Specifically, for $\Delta < 3.91$, the constraints (3), (4), and (5) could not be satisfied simultaneously with the constraints:

$$-\Delta < \delta_i = C(t_i) - m_i < \Delta \qquad \text{for all } i. \tag{2}$$

The constraints on the response function $R(t)$ were

$$R(t) \geqslant 0, \tag{3}$$

$$\frac{d}{dt} R(t) \leqslant 0, \tag{4}$$

$$\frac{d^2}{dt^2} R(t) \geqslant 0. \tag{5}$$

The present analysis considers the more general source function:

$$S(t) = F(t) + D(t) - G(t), \tag{6}$$

where $F(t)$ is taken from Rotty (1987), $G(t)$ describes a CO_2-enhanced biotic growth contribution to the atmospheric carbon budget and $D(t)$ is a re-scaled version of the release $B_H(t)$ from Houghton et al. (1983) (or from Houghton's new estimates). These release estimates include the effects of removal of vegatation followed by either burning or decay, regrowth and changes in soil carbon as a result of such changes.

The growth term, $G(t)$, in equation (6) is modelled using the approach of Kohlmaier et al. (1987) which is expressed as

$$G_K(t) = P_0 \left(1 + \beta \frac{C(t) - C(t_0)}{C(t_0)} \right) - N_B/\tau, \tag{7}$$

where P_0 is the unperturbed value of the net primary productivity (NPP), N_B is the amount of terrestrial biotic carbon and τ is a characteristic turnover time for biotic carbon. The growth enhancement is described by the "beta factor", β. Kohlmaier et al. (1987) quote experimental values ranging up to $\beta \approx 0.5$.

This formalism is modified to take into account the effect of forest clearing, including the secondary effects of regrowth and (to a low-order approximation) changes in soil carbon. We define α to be amount of terrestrial carbon, expressed as a proportion of the initial amount. We define a density-like quantity, ρ, such that

$$N_B(t) = \rho\alpha. \tag{8}$$

Since Houghton et al. (1983) did not take into account changes in the carbon density of uncleared forests, we set

$$B_H(t) = -\rho(t_0)\,\dot{\alpha}. \tag{9}$$

Strictly speaking we should use $B_H(t) = -\rho(t_1)\,\dot{\alpha}$ for some more recent time t_1, to be fully consistent with the approach of Houghton et al. However, in practice the difference is small and most of the effects of using $\rho(t_0)$ rather than $\rho(t_1)$ can be absorbed into the definitions of β, τ and $R(t)$. We have therefore used $\rho(t_0)$ to avoid having to solve a boundary value problem when integrating (9) and (11).

The Kohlmaier et al. formalism is taken as applying to a proportion α of the original biota so that

$$G(t) = \dot{\rho}\alpha = P_0\alpha \left(1 + \beta \frac{C(t) - C(t_0)}{C(t_0)} \right) - \rho\alpha/\tau, \tag{10}$$

whence

$$\dot{\rho} = P_0\beta \frac{C(t) - C(t_0)}{C(t_0)} - \frac{\rho(t) - \rho(t_0)}{\tau}. \tag{11}$$

A more refined analysis would disagregate the effects on $G(t)$ of changes in α, separating changes due to changes in standing biomass from changes due to reduction in soil carbon. The analysis by Houghton et al. suggests that soil carbon changes may represent about 30% of the net flux B_H. The approximation above assumes that reductions in soil carbon will imply a reduced ability to sequester any excess new production in the soil and that effect of this on $G(t)$ will be similar (as a proportion of the carbon changes) to the amount of reduction in $G(t)$ resulting from decreases in the standing biomass.

The equations for $\dot{\rho}$ and $\dot{\alpha}$ can be integrated, using the initial conditions $\alpha(t_0) = 1$ and $\rho(t_0) =$

1400 Gt C. The initial equilibrium defines $P_0 = \rho(t_0)/\tau$. The combined biotic source contribution is thus

$$-\dot{N}_B = D(t) - G(t) = \frac{\rho(t)}{\rho(t_0)} B_H(t) - G(t). \quad (12)$$

The inclusion of $G(t)$ forces us to assume a degree of smoothness in $C(t)$ that we did not require in the earlier analysis. The previous analysis required $C(t)$ only at a discrete set of times. However since $G(t)$ is defined by an integral over $C(t)$, $G(t)$ will be relatively insensitive to errors arising from our use of a smoothing spline fit to the ice core data.

As noted above, the time origin has been set to 1700, rather than to 1800 as in (I). The ice-core data of Friedli et al. show increasing CO_2 concentrations through the 18th century and so part of the present study will assess whether this increase is attributable to known sources, given the assumption of time-invariant ocean-uptake processes. It is necessary to construct source estimates for earlier times than are usually considered in carbon cycle studies. For the fossil carbon release, we use the exponential approximation given by Keeling (1973) for the period prior to 1860. For $B_H(t)$ we use a linear increase from 0 at 1700 through to 0.4 Gt C yr^{-1} in 1827.

As in (I), the consistency of the various conditions is tested by discretising the relations so that the analysis takes the form of a linear programming problem. The technical details are presented below in an appendix. We take Δ from (2) as a measure of the discrepancy between model and observations and seek to minimise Δ subject to the constraints (3), (4), (5) and the constraints:

$$-\Delta \leqslant C(t_0) + \int_{t_0}^{t_i} R(t-t')\, S(t')\, dt' - m_i \leqslant \Delta. \quad (13)$$

In (I), we considered cases where we constrained the large t limit of $R(t)$ to reflect the fact that the oceans would not, except possibly on time-scales of millenia, take up all of a CO_2 input into the atmosphere. All of the solutions found in the present study satisfied the corresponding constraint even though it was not imposed explicitly.

We begin by describing calculations based on the estimates from Houghton et al. (1983). The analysis has been applied for a range of values of β and τ. The linear programming formalism can not be used directly to minimise Δ over a range of values of β and/or τ because β and τ enter the problem non-linearly, multiplying the $R(t)$ values.

The values considered were $\beta = 0.0$, 0.1, 0.2, 0.3, 0.4 and 0.5 with $\tau = 40$, 60, 80, 100, 120 and 140 years (except for the case $\beta = 0$ where the source is independent of τ). The results were $\Delta_{min} = 3.08$ ppmv for $\beta = 0$; the calculated $C(t)$ for this case is shown as the solid line in Fig. 1. Shifting the origin back to 1700 has reduced the discrepancy found in (I). For larger β, the smallest Δ_{min} values were found for $\tau = 140$ years. These were $\Delta_{min} = 3.17$ for $\beta = 0.1$ rising to $\Delta_{min} = 3.61$ for $\beta = 0.5$. It should be noted that this last case has a $G = 0.67$ Gt C yr^{-1} in 1980; this is small compared to the 2 to 3 Gt C yr^{-1} suggested by the analysis of Tans et al. (1990) (assuming a deforestation rate of 1 to 2 Gt C yr^{-1}). For smaller τ, implying larger P_0, the CO_2-enhanced growth rates were larger. The largest of those we considered was $G = 1.40$ Gt C yr^{-1} for the case $\beta = 0.5$, $\tau = 40$ yr. This case had $\Delta_{min} = 4.43$ ppmv and the calculated $C(t)$ is shown with long dashes in Fig. 1.

As in (I), the discrepancies show a systematic pattern with the observed data being lower at the ends of the record and higher in the middle. As in (I), it is this systematic pattern as much as the size of the discrepancy that indicates that the discrepancy is real and not just an artifact arising from the measurement uncertainty in the observed concentrations. The greater number of data points in the present study re-inforces this conclusion. Out of 22 points there is (in each case considered) a run of 15 points whose deviations, δ_i, are of a consistent sign—a result that is highly unlikely to have arisen by chance if the measurement errors are assumed to be independent.

The fitting procedure has allowed us to set the mean value of the curve by having $C(t_0)$ as a free parameter. Similarly, the ability to adjust $R(t)$ enables us to fit almost any (positive) mean trend. Therefore any discrepancy between the best fit is only to be expected in the quadratic or higher order components and so we use a quadratic fit to the residuals (δ_i from eq. (2)) as a measure of the statistical significance of the discrepancy. The appropriate quadratic function to fit is

$$f(t) = 10^{-4} \times [(t-\bar{t})^2 - \overline{(t-\bar{t})^2}], \quad (14)$$

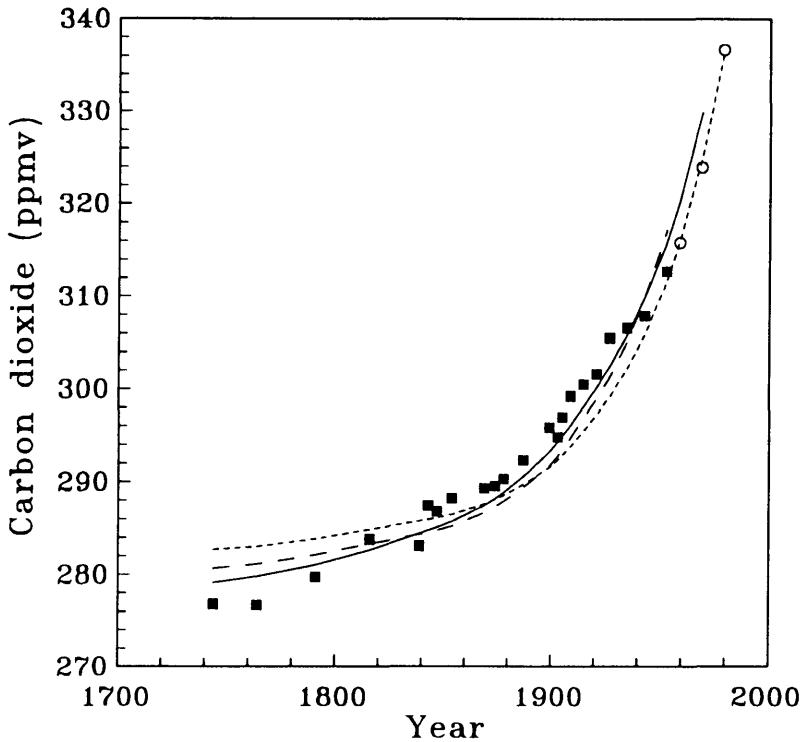

Fig. 1. Comparison of ice-core CO_2 data (solid squares) against best possible calculated fits, assuming specified sources and oceanic uptake processes that are stationary but otherwise almost arbitrary. Sources for the case shown as the solid curve fossil fuel use and forest clearing. For the dashed curves, these net sources are reduced by a component representing CO_2-enhanced growth with a "β-factor" of 0.5 applying to a productivity of 35 Gt C yr^{-1}. The curve with long dashes is fitted only to the ice-core data. The curve with short dashes is subject to the additional constraint of fitting direct observations from Mauna Loa (open circles) to within 1 ppmv.

where the bar denotes averaging over the Mt_i values. Under the assumption of normally distributed errors, the best fit is $\gamma f(t)$ with γ estimated by

$$\hat{\gamma} = \sum w_i \delta_i \qquad (15)$$

with $w_i = \sum f(t_i) / \sum f(t_i)^2$. For the δ_i of the $\beta = 0$ case we obtain $\hat{\gamma} = 3.55$. Under the null hypothesis that the δ_i are normally distributed about zero with standard deviation σ, $\hat{\gamma}$ would be normally distributed about zero with variance $\sigma^2 \sum (w_i)^2$, i.e., a standard deviation of 0.517σ. Taking the value $\sigma = 3$ ppmv from Friedli et al. (1986) means that the estimated value of γ is 2.3 times the standard deviation expected from under the null hypothesis. A discrepancy that large (of either sign) could be expected purely by chance with a probability of 0.022. Therefore we regard the discrepancy as significant.

One aspect of Fig. 1 that seems counter-intuitive is that increasing the amount of CO_2-enhanced growth increases the rate of change of concentration at the end of the record even though the sink from such CO_2-enhanced growth would be most active in the most recent times. In order to clarify this, we note that the fossil carbon release has been increasing with an e-folding time of about 25 years over much of the industrial period. The combination $F(t) + D(t)$ has increased with an e-folding time of about 50 years over the same period. For comparison, the ice-core record is characterised by an e-folding time of about 75 years (taking $C(0)$ as 277 ppmv). On this basis, the importance of

uptake by CO_2-enhanced growth has been declining relative to the releases. Because the calculated best-fits to $C(t)$, as shown in Fig. 1, are determined with $R(t)$ allowed to vary, they will, as noted above, all have approximately the observed mean gradient. As β is increased, the net source decreases and $R(t)$ is increased to maintain the observed mean concentration increase. The requisite change in $R(t)$ will be in proportion to the importance of $F(t)+D(t)$ relative to the concentration changes driving the CO_2-enhanced growth, averaged over the whole industrial period. Because the relative importance of the concentration changes has been declining, the current importance of concentration changes will be less than their average importance. Therfore an increase in $R(t)$ designed to keep the average rate of change constant (given increased growth) will be too large to preserve the recent growth rate.

Because of this counter-intuitive behaviour of the current rate of increase of CO_2, we have performed additional calculations in which the data set was expanded to include data from Mauna Loa. Such data were used in (I) but were found not to affect the results—in technical terms the constraints involving Mauna Loa data were inactive. For the present study, we use annual means, taken from Keeling et al. (1989), for 1959, 1969 and 1979. We assume that these represent long-term trends to within a precision of 1 ppmv, but that they have an unknown fixed offset ϕ with respect to the ice-core data. This offset will incorporate both calibration differences between the records and the effects of spatial separation. Thus we include the additional constraints

$$-1 \leqslant C(t_0)+\phi+\int_{t_0}^{t_j} R(t-t')\,S(t')\,\mathrm{d}t'-m_j \leqslant 1. \tag{16}$$

The inclusion of the unspecified offset, ϕ, means that the Mauna Loa data will act mainly as a constraint on the recent gradient of $C(t)$.

Even though a number of minor details of the formulation have been changed, in the $\beta=0$ case the Mauna Loa data gave inactive constraints as in (I). Including the direct observations does not increase the discrepancy for $\beta=0$, $\beta=0.1$ or $\beta=0.2$ with $\tau \geqslant 80$ yr. The inclusion of the direct observations increases the discrepancy, Δ, as β increases beyond 0.2. For the high biotic uptake case ($\beta=0.5$, $\tau=40$ yr) $\Delta_{\min}$ has increased from 4.43 to 5.30 ppmv with the inclusion of the Mauna Loa data. Furthermore this case requires $\phi \approx -5$ ppmv, a correction of opposite sign to that required by the spatial gradient. We have performed a final series of calculations with ϕ constrained to be non-negative. This corresponds to assuming that any errors due to calibration differences or dating errors in the ice core have a combined effect that is less than the spatial gradient. The results of this calculation are that $\Delta_{\min}$ increased to 6.35 ppmv for the high biotic uptake case ($\beta=0.5$, $\tau=40$ yr). The "best-fit" CO_2 concentration for this case is shown using short dashes in Fig. 1.

The analyses based on the new estimates of B_H from Houghton (personal communication) have been restricted to cases in which the Mauna Loa data were included and ϕ was constrained to be non-negative.

For the "high release" case, in which 1.693 Gt C was estimated as the 1980 release, the $\beta=0$ case gave $\Delta_{\min}=3.96$ ppmv. For $\beta=0.5$ and $\tau=40$ yr, it was found that $\Delta_{\min}=5.78$ ppmv. For the "low release" case, in which 1.058 Gt C was estimated as the 1980 release, the $\beta=0$ case gave $\Delta_{\min}=$ 3.64 ppmv. For $\beta=0.5$ and $\tau=40$ yr, it was found tat $\Delta_{\min}=6.07$ ppmv. Thus the effect of using the revised estimates of carbon release from landuse change is to increase the discrepancy for the case without CO_2-induced growth but to decrease it slightly for those cases considered in which there was a large CO_2-induced growth.

3. Concluding remarks

The present study has significantly generalised the earlier result from Enting and Mansbridge (1987a) (paper (I)) which showed the CO_2 concentration record from ice-cores was incompatible with direct estimates of biotic CO_2 release if one assumed a linear steady-state ocean. The analysis above shows that increasing β, above the $\beta=0$ case considered in (I), to allow for CO_2-enhanced growth increases the discrepancy between the ice-core data and the source estimates. The discrepancy persists if the biotic release estimates from Houghton et al. (1983) are replaced by more recent estimates from Houghton (personal com-

munication). In (I), the possible reasons for the discrepancy were listed as: (i) systematic bias in the ice-core data; (ii) errors in the biotic (or less probably fossil) source estimates; (iii) the assumption of a pre-industrial equilibrium is incorrect; (iv) there is significant non-linearity in the oceanic uptake processes; (v) the assumption of steady-state oceanic processes is incorrect.

In (I), the favoured explanation was that there were errors in the biotic source estimates. Keeling et al. (1989) justified their use of a sink from CO_2-enhanced growth on the basis that the biotic source estimates must be incomplete as shown in (I). In fact the present analysis shows that assuming any CO_2-enhanced growth increases the discrepancy. If the discrepancy is due to errors or incompleteness in the biotic releases, then some cause other than CO_2-enhanced growth must be found. Two possibilities are biotic responses to climatic fluctuations and the possible fertilising effect of increases in biologically available nitrogen. (Of course to the extent that such changes in nitrogen follow the course of CO_2 increases, they are excluded as a possible explanation by the present analysis.)

In view of these results, we need to re-consider the other possibilities listed in (I). What has been shown is that, regardless of the details of the ocean process, it is not possible to explain the ice-core CO_2 record in terms of a system with time-invariant processes perturbed by a combination of fossil carbon release, CO_2-enhanced biotic growth and deforestation effects as given by Houghton et al. (1983). Either one of the time series ($F(t)$, $B_H(t)$ or $C(t)$) is incorrect or else some other process has occurred.

A possibility that needs to be considered seriously is that the CO_2 increases in the early part of the record reflect a recovery from a perturbation associated with the little ice-age. This is a combination of reasons (iii) and (v) above. The distinction between the two is mathematical rather than of physical significance. Such an effect would imply that the climatic influence on CO_2 that is known to operate on glacial-interglacial time-scales (Lorius et al., 1990) is paralleled by a similar effect (not necessarily from the same mechanism) on time-scales of centuries. Given the importance of such an effect which would close a feedback loop between CO_2 and climate, the acquisition of high quality ice-core CO_2 data covering the little ice age would seem to be very high priority. Isotopic data could provide a basis for helping distinguish biotic changes from oceanic changes.

The deconvolution of (1) can be expressed using an inverse operator $\boldsymbol{R}^{-1}$ acting on the function $Q(t) = C(t) - C(t_0)$). (The structure of $\boldsymbol{R}^{-1}$ as an integro-differential operator is described by Enting and Mansbridge, 1987b). The result of the present analysis is that the biotic source estimate from the ice-core data, $\hat{B}(t) = \boldsymbol{R}^{-1}Q(t') - F(t)$ is inconsistent with the estimate $D(t) - G(t)$ even though the analysis allows both variations in $R(t)$ and the parameters that define $G(t)$. Furthermore, adjusting $G(t)$ so as to reduce the discrepancy with the Tans et al. (1990) estimate of the current rate of biomass change, increases the overall discrepancy between $\hat{B}(t)$ and $D(t) - G(t)$.

In order to account for the discrepancy between the deconvolution $\hat{B}(t)$ and the more direct estimates, $D(t) - G(t)$, we have proposed that the oceanic uptake processes may have changed through time. This possibility leads to considerable ambiguities in the interpretation of atmospheric CO_2 budgets. Enting and Mansbridge (1991) noted that recent changes in oceanic uptake could be a reason for the low southern hemisphere oceanic uptake found in the various recent modelling studies (Enting and Mansbridge, 1989; Tans et al., 1989; Tans et al., 1990). An earlier modelling study by Pearman and Hyson (1986) had found a southern hemisphere CO_2 uptake that was larger and thus in better agreement with global carbon cycle modelling. Enting and Mansbridge (1991) noted that the data used in the 1986 study showed a different inter-hemispheric profile to the data used in the later studies. The difference most probably reflects problems of intercalibration between the different sampling networks, especially since data from Keeling et al. (1989) for the difference between Mauna Loa and the South Pole do not show any anomalous change around 1980. However the possibility remains that there has been a recent change in oceanic CO_2 uptake and that the low oceanic uptake found by Tans et al. (1990) is a very recent phenomenon.

The possibility of non-stationary oceanic CO_2 uptake processes must be taken seriously: firstly as an interpretation of 18th century CO_2 increases, secondly as a possible reason for the anomalous result found by Tans et al. (1990) and finally as a

complicating factor in predicting future CO_2 levels if the oceanic uptake is affected by CO_2-induced climatic change.

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5. Appendix. Technical details

In order to perform numerical calculations it is necessary to discretise the response function, $R(t)$. The discretisation used here differs from that used in (I) in that the discretisation interval increases with t so as to give greatest resolution for small t where the most rapid changes in $R(t)$ are expected. Furthermore linear interpolation is used to represent the integrands of (1). Using integer time values to indicate the midpoints of years in $C(t)$ and $S(t)$ and the ends of years in $R(t)$, integral (1) transforms to y:

$$C(j)-C(0)=\int_0^j R(t)\,S(j-t)\,\mathrm{d}t$$

$$=\sum_{m=0}^{j-1}\int_m^{m+1} R(t)\,S(j-t)\,\mathrm{d}t, \qquad \text{(A.1)}$$

whence

$$C(j)-C(0)=\sum_{m=0}^{j-1}\left\{\frac{1}{2}\left[R(m)\,S(j-m-1)\right.\right.$$

$$\left.+R(m+1)\,S(j-m)\right]+\frac{1}{3}\left[R(m)-R(m+1)\right]$$

$$\left.\times\left[S(j-m)-S(j-m-1)\right]\right\}. \qquad \text{(A.2)}$$

We define working variables x_1 to x_{53} where x_1 to x_{11} denote $R(0)$ to $R(10)$ at one year intervals, x_{12} to x_{21} denote $R(12)$ to $R(30)$ at 2-year intervals, x_{22} to x_{31} denote $R(34)$ to $R(70)$ at 4-year intervals, and x_{32} to x_{41} to denote $R(78)$ to $R(150)$ at 8-year intervals and x_{42} to x_{51} to denote $R(166)$ to $R(310)$ at 16-year intervals. The variable x_{52} denotes $C(0)$ and the final variable, x_{53}, denotes Δ.

Thus the linear programming problem is to minimise x_{53} subject to the constraints (A.3), (A.4), (A.5), (A.7) and (A.8), i.e.:

$$x_j \geqslant 0 \qquad \text{for} \quad j=1 \text{ to } 53 \qquad \text{(A.3)}$$

For $j=1$ to 51, this follows from (3). For $j=52$ and 53, positivity is inherent in the definitions of $C(0)$ and Δ. The positivity constraints always apply to the individual variables in linear programming problems and so they are not explicitly entered into the program:

$$x_j - x_{j-1} \leqslant 0$$

$$\text{for} \quad j=2 \text{ to } 51 \text{ from (4)}, \qquad \text{(A.4)}$$

$$(a+1)\,x_j - a x_{j-1} - x_{j+1} \leqslant 0$$

$$\text{for} \quad j=2 \text{ to } 50 \text{ from (5)}, \qquad \text{(A.5)}$$

where $a=2$ for $j=11$, 21, 31 or 41 and $a=1$ otherwise. For a specified source function, the expression for $C(j)$ above is a linear combination of the $R(j)$ and therefore it is also a linear combination of the x_k:

$$C(m)=C(0)+\sum_{j=0}^{310} X_m(j)\,R(j)$$

$$=x_{52}+\sum_{k=1}^{51} Y_m(k)\,x_k. \qquad \text{(A.6)}$$

The $X_m(j)$ represent the source contributions from (A.2) and the $Y_m(k)$ are set to $\sum_j X_m(j)\,T_{jk}$ where the T_{jk} represent the expressions for $R(j)=\sum_k T_{jk}x_k$ determined from linear interpolation of the x_k. The constraints defined by the observational data are thus

$$\sum_{k=1}^{51} Y_m(k)\,x_k + x_{52} - x_{53} \leqslant C_{\mathrm{obs}}(m)$$

$$\text{for} \quad m=1 \text{ to } M, \qquad \text{(A.7)}$$

$$\sum_{k=1}^{51} Y_m(k)\,x_k + x_{52} + x_{53} \geqslant C_{\mathrm{obs}}(m)$$

$$\text{for} \quad m=1 \text{ to } M, \qquad \text{(A.8)}$$

where $M = 22$ using the data from Friedli et al. (1986). We also apply the equality constraint

$$x_1 = R(0) = 0.471 \text{ ppmv per Gt C.} \quad \text{(A.9)}$$

In practice, the first derivative constraint (A.4) is required only for $j = 51$. All other cases follow automatically from (A.5). Therefore we have $51 + M$ "$\leqslant$" inequalities, M "$\geqslant$" inequalities and one equality. For the calculations involving Mauna Loa data, we introduced an additional working variable, x_{54}, to represent the combination $C(0) + \phi$. (The combination will satisfy the positivity requirement regardless of the sign of ϕ). Thus the constraints for the three Mauna Loa data points are:

$$\sum_{k=1}^{51} Y_m(k)\, x_k + x_{54} \leqslant C_{\text{obs}}(m) + 1$$

$$\text{for} \quad m = M + 1 \text{ to } M + 3, \quad \text{(A.10)}$$

$$\sum_{k=1}^{51} Y_m(k)\, x_k + x_{54} \geqslant C_{\text{obs}}(m) - 1$$

$$\text{for} \quad m = M + 1 \text{ to } M + 3. \quad \text{(A.11)}$$

For the case where ϕ is constrained to be positive, we let x_{54} represent ϕ and replace constraints (A.10) and (A.11) by:

$$\sum_{k=1}^{51} Y_m(k)\, x_k + x_{52} + x_{54} \leqslant C_{\text{obs}}(m) + 1$$

$$\text{for} \quad m = M + 1 \text{ to } M + 3, \quad \text{(A.12)}$$

$$\sum_{k=1}^{51} Y_m(k)\, x_k + x_{52} + x_{54} \geqslant C_{\text{obs}}(m) - 1$$

$$\text{for} \quad m = M + 1 \text{ to } M + 3. \quad \text{(A.13)}$$

The linear programming problems were solved using the Pascal procedures given by Press et al. (1986) modified by (i) using "call-by-reference" (i.e., *var* declarations) for the constraint array (in order to save space), (ii), converting type *real* variables to the 80-bit reals supported in Turbo-Pascal, and (iii) resetting the tolerance variable *eps* to 10^{-10} and using eps in the Pascal procedure SIMP2 in the same way that it was used in the corresponding Fortran procedure given by Press et al. The last two modifications were introduced to overcome numerical difficulties that were encountered. These are presumed to arise from the fact that many of the concentration constraints (A.6) are numerically close to one another so that the simplex, over whose surface the linear programming algorithm progresses, has shallow "corners" with poorly defined "edges". In order to check the numerical calculations, the solutions were substituted back into (A.6).

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