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Description of a one-dimensional carbon cycle model calibrated using techniques of constrained inversion

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ABSTRACT

A quasi one-dimensional carbon cycle model is discussed. The ocean is divided into two zones; thus the mixing processes can be parameterized in terms of both eddy diffusion and advective circulation. All the model processes are parameterized so as to allow, so far as is possible, for independent geophysical (or biogeochemical) estimates of the model parameters. The model is then calibrated by using a constrained inversion approach in which the independent estimates of the parameters act as the constraints. This calibration scheme has proved stable and robust. The model is used for a number of studies in which various indirect observations of the carbon cycle constrain the range of possibilities for the history of biospheric change over the last 200 years. The estimates of current net releases of biotic carbon are small and possibly negative and all the estimates give negative values for the current rate of change of biotic carbon release. This is in conflict with results recently deduced from analysis of Mauna Loa CO_2 data using the unjustifiable assumption of a constant airborne fraction.

1. Introduction

This paper describes a one-dimensional carbon cycle model that has been developed within the Division of Atmospheric Research of CSIRO (Australia). The model was developed for three main reasons:

(i) to have a basis for interpreting the results of the Division's observational programs, particularly those involving ^{13}C;

(ii) to ensure that the air-sea exchange formalism that is used in the Division's two-dimensional atmosphere/mixed-layer model (Pearman et al., 1983; Pearman and Hyson, 1986) is consistent with the carbon cycle as a whole;

(iii) to establish boundary conditions for the two-dimensional modelling studies.

In the course of the program, an additional aim has emerged, *viz*,

(iv) to use the model to test the applicability of the constrained inversion calibration scheme proposed by Enting (1985a).

Some aspects of the program have been described previously. The present paper gives an overview of the model, its calibration and its current best estimates of biotic carbon releases. An account of an early version of the model was given by Pearman (1980). Enting and Pearman (1986) gave an analysis of the relative utility of the various data items used in the calibration. A detailed technical description of the model is given in the reports of Enting and Pearman (1982, 1983).

The layout of the remainder of this paper is as follows. Section 2 gives a summary of the structure of the model and the way in which the various processes are represented. Section 3 describes how the constrained inversion scheme of Enting (1985a) is used to calibrate the model. Section 4 lists the data that are used in the calibration. Section 5 describes a number of results of the modelling studies, particularly those concerning the interpretation of various indirect observations of the carbon cycle. It also discusses the reasons for the discrepancy between the present estimates of biotic carbon releases and the analysis of Elliott et al. (1985).

2. The model

2.1. Structure

The structure of the model is shown schematically in Fig. 1. It is a box model with a single atmospheric reservoir, two biospheric reservoirs and an ocean that is divided into warm and cold regions each of which has a surface layer and M sub-surface layers. All the calculations presented here use M = 6 in each region. The state of the system at any time is defined in terms of the amounts of ^{12}C, ^{13}C and ^{14}C in each of the reservoirs. The sources of ^{14}C are cosmic rays and nuclear testing. Radioactive decay is the sink for ^{14}C. Fossil fuel use is the source for ^{12}C and ^{13}C. The net release (or uptake) of biospheric carbon due to agricultural changes is represented by having a non-zero net flux between the atmosphere and the biospheric reservoirs.

Mathematically the model is represented as a set of ordinary differential equations giving the rates of change of the amounts of each carbon isotope in each reservoir. These equations have the general form

$$\frac{dN_i^\nu}{dt} = g_i^\nu(\{N_j^\mu\}, \{x_k\}, t) \tag{2.1}$$

where N_i^ν denotes the amount of isotope ν (ν = 12, 13 or 14) in reservoir i and x_k denotes the kth of K model parameters.

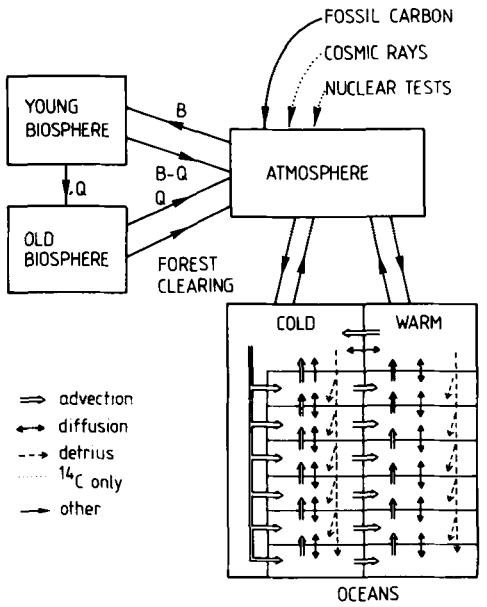

Fig. 1. Structure of the global carbon cycle model.

Table 1. *Symbols used to denote model processes*

Symbol	Process	Equations
ADV	Ocean advection	(2.22–2.26)
ASE	Air-sea exchange	(2.10–2.16)
BIO	Terrestrial biotic processes	(2.27–2.30)
BOM	Nuclear weapons ^{14}C source	(2.7)
COS	Cosmic ray ^{14}C source	(2.8)
DEC	^{14}C decay	(2.9)
DET	Ocean detrital flux	(2.17)
FOS	Fossil carbon release	(2.5a, b)
HDF	Horizontal eddy diffusion in ocean	(2.18)
VDF	Vertical eddy diffusion in ocean	(2.19–2.21)

In order to specify the model equations we expand (2.1) as

$$\frac{dN_i^\nu}{dt} = \sum_{j,\alpha} F_{ji;\alpha}^\nu + \sum_\beta S_{i;\beta}^\nu, \tag{2.2}$$

where $F_{ji;\alpha}^\nu$ represents the net flux of isotope ν from reservoir j to reservoir i due to process α, while $S_{i;\beta}^\nu$ represents the net source/sink strength for isotope ν in reservoir i due to process β. The symbols $F_{ij;\alpha}$, $S_{i;\beta}$ are used to denote the sums over all the isotopes. In these generic equations α, β represent the abbreviated labels assigned to the various processes as specified in Table 1. In specific equations, the general reservoir indices i, j are replaced by initials 'a' for atmosphere, 'y' for young biota, 'o' for old biota, 'r' for general ocean surface region, 'c' or 'w' for cold or warm surface layers and 'rn', 'cn, 'wn' for the respective nth subsurface layers.

2.2. Initialization

For any given set of parameters the model is integrated numerically from an assumed equilibrium initial state. This equilibrium state is determined by the solution of the set of simultaneous equations

$$g_i^\nu(\{N_j^\mu\}, \{x_k\}, t_0) = 0 \tag{2.3}$$

for the N_j^μ. These equations are easily solved since the functions F_i^ν are only weakly non-linear in the N_j^μ. The equations (2.3) do not determine a unique equilibrium state unless some of the initial conditions are specified; it is necessary to specify some aspect for each of the distributions of ^{12}C

and ^{13}C. Since these tracers are conserved in the model, it is possible to have a pre-industrial equilibrium with any amounts of these isotopes. We have chosen to specify the initial composition of the atmosphere and express it as a CO_2 mixing ratio and an atmospheric $\delta^{13}C$. These initial values are regarded as being two of the model parameters. With this usage, the set of parameters $\{x_k\}$ (which we also write as a vector, $\boldsymbol{x}$), is sufficient to determine a unique equilibrium initial state. It was, however, found by Enting and Pearman (1982) that the convergence of the iterative equilibration procedure could be improved if the set of equations was made overcomplete by adding the specification of the amount of ^{14}C present in the assumed natural equilibrium, i.e.,

$$\sum_i N_i^{14} = \omega/\lambda. \tag{2.4}$$

Here ω is the rate at which cosmic rays produce ^{14}C and λ is the radioactive decay rate of ^{14}C. However, since eq. (2.4) can be derived from the dynamical equations for ^{14}C, the only new parameters specifically required for initialization are C_0, the initial atmospheric CO_2 concentration, and δ_0, the initial atmospheric $\delta^{13}C$.

2.3. *Sources and sinks*

The sources and sinks of carbon into the system are from fossil carbon which gives a source of ^{12}C and ^{13}C only and cosmic rays and nuclear testing which form ^{14}C from ^{14}N in the upper atmosphere. There is also a sink of ^{14}C in all reservoirs due to natural radioactive decay.

The release rate of carbon from fossil sources has been estimated by Keeling (1973a) and Rotty (1981). This function is denoted $f(t)$ and is shown in Fig. 2. The ratio of ^{13}C to ^{12}C in the release $f(t)$ is based on a piece-wise linear approximation to the data given by Tans (1981). Thus

$$S^{12}_{a;FOS} = (1 - r(t))\, f(t), \tag{2.5a}$$

$$S^{13}_{a;FOS} = r(t)\, f(t), \tag{2.5b}$$

where $r(t)$ is related to the $\delta^{13}C$ data of Tans (1981) by

$$\delta(t) = (r(t)/((1 - r(t))\, R_s) - 1) \times 1000, \tag{2.6}$$

and the standard ratio $R_s = 0.0112372$. The approximation $\delta(t)$ is shown in Fig. 3. Thus we

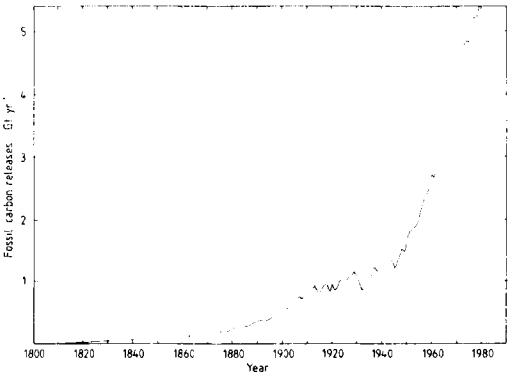

Fig. 2. Fossil carbon release function used in the model. Based on Rotty (1981) and Keeling (1973a).

have not introduced any free parameters into our description of the fossil carbon release.

The production of ^{14}C from nuclear testing is related to energy yields estimated by Enting (1982). The input function is expressed as a function $n(t)$ representing the energy yield over each 3-month period and is converted to ^{14}C production using a scale factor Λ. Thus

$$S^{14}_{a;BOM} = \Lambda n(t). \tag{2.7}$$

The source of ^{14}C by cosmic rays is described by a parameter ω, giving the production rate which is taken as constant in time in the reference version of the model. Thus

$$S^{14}_{a;COS} = \omega. \tag{2.8}$$

The natural radioactive decay of ^{14}C occurs in all reservoirs at a known rate so that

$$S^{14}_{i;DEC} = -\lambda N_i^{14}, \quad \text{for all } i \tag{2.9}$$

where $\lambda^{-1} = 8267$ years.

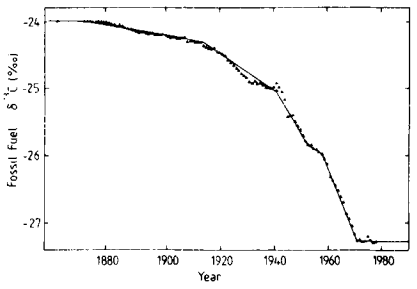

Fig. 3. Linearized approximation to the $\delta^{13}C$ of fossil fuel based on the data by Tans (1981) which is shown as triangles. Note that the extrapolation beyond Tans' data does not influence the calibration.

The sources and sinks in the model have thus been described in terms of the two free parameters ω and Λ with all other aspects being regarded as relatively well-known quantities. The effects of changing agricultural practices are not regarded as a net source or sink for the atmosphere-ocean-biosphere system but are treated as a biosphere-atmosphere transfer in Subsection 2.5 below.

2.4. *Air-sea exchange*

The air-sea exchange formalism is based on the work of Deacon (1977). If the reservoir index r is used to denote the ocean surface in the warm (w) or cold (c) region and a denotes the atmosphere, then the gross air-sea flux (in mol m^{-2}) is

$$\phi_{ar} = k_r \tilde{C}_a \tag{2.10a}$$

with a return flux

$$\phi_{ra} = k_r P_r, \tag{2.10b}$$

where $\tilde{C}_a$ and P_r are the CO_2 partial pressures in the atmosphere and region r of the ocean surface respectively, both expressed in atmospheres. The quantity k_r is usually expressed as a transfer velocity or piston velocity. However, following Deacon, we put

$$k_r = 0.011(1 + 0.007T_r)u_r^* \quad \text{mol m}^{-2}\,\text{s}^{-1}, \tag{2.11}$$

where T_r is the ocean surface temperature in °C and u_r^* is the effective friction velocity for region r in m s^{-1}. This form of k_r is a piston velocity multiplied by a density factor. The effective averaging of u_r^* is discussed in Section 4 below.

The gross fluxes are required for the purpose of calculating the effect of isotopic fractionation. We put

$$F^{13}_{ar;ASE} = -F^{13}_{ra;ASE} = A_r(\phi_{ar} f_{as} N_a^{13}/N_a - \phi_{ra} f_{sa} N_r^{13}/N_r), \tag{2.12a}$$

$$F^{14}_{ar;ASE} = -F^{14}_{ra;ASE} = A_r(\phi_{ar} f^*_{as} N_a^{14}/N_a - \phi_{ra} f^*_{sa} N_r^{14}/N_r), \tag{2.12b}$$

$$F^{12}_{ar;ASE} = -F^{12}_{ra;ASE} = A_r(\phi_{ar} - \phi_{ra}) - F^{13}_{ar;ASE} - F^{14}_{ar;ASE}. \tag{2.12c}$$

The quantity A_r is the surface area of region r and the f_{as}, f_{sa}, f^*_{as}, f^*_{sa} are isotopic fractionation factors. The fractionation factors for ^{13}C and ^{14}C are connected by

$$f^*_{as} = 2f_{as} - 1, \tag{2.13a}$$

$$f^*_{sa} = 2f_{sa} - 1. \tag{2.13b}$$

The ocean partial pressures P_r, expressed in atmospheres, are calculated from

$$P_r = \frac{(2[\sum CO_2]_r - [A])^2}{\alpha(T_r)\kappa_{eq}(T_r)([A] - [\sum CO_2]_r)} \tag{2.14}$$

(see Broecker, 1974). In this expression α is the CO_2 solubility, $[A]$ is the alkalinity and κ_{eq} is the equilibrium coefficient of the reaction

$$H_2O + CO_2 + CO_3^{=} \rightleftharpoons 2HCO_3^-.$$

For the temperature dependence we use

$$\kappa_{eq}(T) = 1780 - 20T \tag{2.15}$$

(from Broecker and Peng, 1974) and

$$\alpha = 65.0/(1 + 0.0387T + 0.000456T^2) \quad \text{mol m}^{-3}\,\text{atm}^{-1} \tag{2.16}$$

(Broecker, 1974; Hodgman, 1958).

It is through these expressions (and to a lesser extent through k_r) that the ocean surface temperature affects the behaviour of the model.

The free parameters in the formulation are thus T_c and T_w the temperatures, u_c^* and u_w^* the effective wind-stresses, $[A]$ the alkalinity, f_{as} and f_{sa} the fractionation factors and d_m the mixed layer depth which is used in relating the carbonate contents N_r, to the concentrations $[\sum CO_2]_r$ required for relation (2.14).

2.5. *Ocean mixing*

The ocean mixing of carbon occurs by a detrital flux, a vertical eddy diffusion, a diffusive flux between the two surface layers and an advective circulation parameterized in terms of two velocities.

If the surface layer of region r is indexed by r and the nth subsurface layer by rn, the detrital flux is

$$F_{r,rn;DET} = -F_{rn,r;DET} = D^* A_r/M(A_c + A_w), \quad n = 1, M. \tag{2.17}$$

The quantity D^* represents the total detrital flux from the surface layers. As indicated by eq. (2.17), the detrital flux is assumed to be divided

between regions in proportion to their area and to contribute equally to each subsurface layer. The isotopic composition of this flux is determined from the isotopic composition of the surface reservoir, depleted by a factor δ_{DET} for ^{13}C and $2\delta_{DET}$ for ^{14}C.

The diffusive mixing between the surface layers is regarded as diffusion between two reservoirs with centres a distance $\Delta_{0;45}$ apart, mixing across a rectangular boundary of dimensions d_m by L_{30} and is written as

$$F^v_{cw;HOR} = -F^v_{wc;HOR} = K_h(C^v_c - C^v_w)\,d_m L_{30}/\Delta_{0;45}$$
$$= K_h(N^v_c/A_c - N^v_w/A_w)\,L_{30}/\Delta_{0;45}, \qquad (2.18)$$

where C^v_r is the carbonate concentration for isotope v in region r, L_{30} is the length of the 30° parallels separating the ocean regions and $\Delta_{0;45}$ is the distance between the "midpoints" (0° and 45°) of the regions.

The vertical eddy diffusion formulation allows for a depth dependence in the diffusion coefficient. We put

$$K_v(z) = K_v + zK'_v \qquad (2.19)$$

where the depth z is measured from the ocean surface, and the diffusion coefficient is required at points z_i representing the midpoint between the centre of layer i and the layer above it (i.e., layer $i-1$, the mixed layer being taken as layer zero).

For region r, the vertical eddy diffusion flux is

$$F^v_{ri,r(i+1);VDF} = -F^v_{r(i+1),ri;VDF}$$
$$= K_v(z_{i+1})(C^v_{ri} - C^v_{r(i+1)})\,A_r/\tfrac{1}{2}(d_i + d_{i+1})$$
$$= 2K_v(z_{i+1})(N^v_{ri}/d_i - N^v_{r(i+1)}/d_{i+1})/(d_i + d_{i+1}), \qquad (2.20)$$

where d_i is the thickness of layer i, i.e.,

$$d_0 = d_m, \qquad (2.21a)$$

$$d_i = (3730 - d_m)/M, \quad i \neq 0. \qquad (2.21b)$$

The advective flux is regarded as a sum of two components which are described in terms of their two upwelling parameters. Their combined effect is to have the upwelling velocity decrease linearly with depth but, at any depth, be the same in each region. For each component of the circulation, the cold surface box is regarded as connected to all the cold subsurface layers as shown in Fig. 1. One component of the circulation injects water into the lowest cold layer, from which there is a further transport into the lowest layer of the warm region. This contribution is characterized by the upwelling velocity, v, being the same in each region and at each depth. The fluxes in regions c and w are in the ratio $0.9A_c : A_w$. The upwelling in the cold region is taken as only 90% of the area as the other 10% is regarded as a region of downflow.

The second component of the circulation injects equal amounts of cold surface water into each cold subsurface layer. Some of each injection flows on to the corresponding warm subsurface layer so that the upwelling velocity from this component is the same in each region and directly proportional to the distance from the ocean floor. Its value at the bottom of the mixed layer is denoted v^*.

Thus the two components of the circulation are parameterized by the contribution that they make to the upwelling velocity at the bottom of the mixed layer. In each case there is a return flux from the warm surface layer to the cold surface layer. This is given by

$$F^v_{wc;ADV} = -F^v_{cw;ADV} = (v + v^*)\,N^v_w/d_m. \qquad (2.22)$$

The injections into the cold layers are

$$F^v_{c,cn;ADV} = -F^v_{cn,c;ADV}$$
$$= v^*\,N^v_c(A_w + 0.9A_c)/A_c d_m M,$$
$$n = 1 \text{ to } M-1, \qquad (2.23a)$$
$$= (v + v^*/M)\,N^v_c(A_w + 0.9A_c)/A_c d_m,$$
$$\text{for } n = M. \qquad (2.23b)$$

The transfer from cold to warm regions in subsurface layers is

$$F^v_{cn,wn;ADV} = -F^v_{wn,cn;ADV}$$
$$= v^*\,N^v_{cn}A_w/MA_c d_n, \quad n = 1 \text{ to } M-1, \qquad (2.24a)$$
$$= (v + v^*/M)\,N^v_{cn}A_w/A_c d_n, \quad n = M. \qquad (2.24b)$$

In the warm region, the flux associated with upwelling is

$$F^v_{w(n+1),wn;ADV} = -F^v_{wn,w(n+1);ADV}$$
$$= (v + v^*(M-n)/M)(N^v_{wn}/d_n + N^v_{w(n+1)}/d_{n+1})/2$$
$$\text{for } n = 0 \text{ to } M-1, \qquad (2.25)$$

using a centred differencing scheme.

In the cold region, the flux associated with upwelling is

$$F^{v}_{c(n+1),cn;\mathrm{ADV}} = -F^{v}_{cn,c(n+1);\mathrm{ADV}} = 0.45(v + v^*(M-n)/M)(N^{v}_{cn}/d_n + N^{v}_{c(n+1)}/d_{n+1}). \quad (2.26)$$

The free parameters for ocean mixing are D^*, δ_{DET}, K_h, K_v, K'_v, v and v^*. The use of centred-differencing is accurate to second-order in the inter-layer spacing and so our results will be only weakly dependent on M, the number of layers. Although centred-differencing is more accurate than upstream-differencing, it is more prone to instability in both the integration of the equations and in the equilibration of the model. Our centred-difference treatment of vertical advection is only stable because there is a vertical diffusion in the model. In order to ensure stability in the horizontal advection, upstream differencing was required. This is effectively equivalent to including an extra horizontal diffusive mixing. Since we never change the horizontal resolution in our model, there is always a fixed relation between the advective velocity and the "numerical diffusion" and so no additional parameters are needed to describe the diffusive behaviour.

2.6. *The biosphere*

The terrestrial biosphere is modelled by two reservoirs denoted "young" and "old". The atmospheric CO_2 taken up by the biosphere all passes into the "young" reservoir initially and some passes on to the old reservoir. The natural return rates to the atmosphere from each reservoir are chosen to balance the uptake. In addition there is an anthropogenic flux $b(t)$ from the "old" biosphere to the atmosphere. This represents the net results of processes such as forest clearing. The flux $b(t)$ is an unknown function of time and is parameterized as a quadratic spline by putting

$$b(t) = \sum_{i=1}^{7} a_i B_i(t), \quad (2.27)$$

where the $B_i(t)$ are quadratic B splines (de Boor, 1978). The splines have nodes at 25-year intervals from 1815 with the ith B-spline being zero outside the range $1790 + 25i \leqslant t \leqslant 1865 + 25i$. The splines are normalized, each having a time integral of 1, so that the a_i represent the amounts of carbon that each B-spline pulse contributes to the release.

The carbon uptake, i.e., the net primary production (NPP) is taken as a constant B. However the transfer, Q, through the old biosphere is taken as proportional to the size of this reservoir by giving it a fixed natural turnover time τ. Thus

$$F_{\mathrm{yo;BIO}} = -F_{\mathrm{oy;BIO}} = Q = N_o/\tau, \quad (2.28)$$

$$F_{\mathrm{oa;BIO}} = -F_{\mathrm{ao;BIO}} = Q + b(t). \quad (2.29)$$

The isotopic composition of these two total fluxes is the isotopic composition of the source reservoir. The exchange between the "young" biosphere and the atmosphere must be separated into gross fluxes in order to allow for fractionation effects. Thus:

$$F^{13}_{\mathrm{ay;BIO}} = -F^{13}_{\mathrm{ya;BIO}} = f_{\mathrm{ab}} B N^{13}_{\mathrm{a}} \Big/ \left(\sum_v N^v_{\mathrm{a}}\right) + (Q-B) N^{13}_{\mathrm{y}} \Big/ \left(\sum_v N^v_{\mathrm{y}}\right), \quad (2.30a)$$

$$F^{14}_{\mathrm{ay;BIO}} = -F^{14}_{\mathrm{ya;BIO}} = (2f_{\mathrm{ab}} - 1) B N^{14}_{\mathrm{a}} \Big/ \left(\sum_v N^v_{\mathrm{a}}\right) + (Q-B) N^{14}_{\mathrm{y}} \Big/ \left(\sum_v N^v_{\mathrm{y}}\right), \quad (2.30b)$$

$$F^{12}_{\mathrm{ay;BIO}} = -F^{12}_{\mathrm{ya;BIO}} = Q - F^{13}_{\mathrm{ay;BIO}} - F^{14}_{\mathrm{ay;BIO}}. \quad (2.30c)$$

The size of the young biosphere N_y thus remains constant. The old biosphere is specified by its size after the forest clearing (i.e., roughly as at present) as N_* and so it is initialized to $N_* + \sum a_i$.

The free parameters in the description of the biosphere are thus f_{ab}, B, τ, N_y, N_* and a_i for $i = 1$ to 7.

3. Constrained inversion calibration

The general principles of using constrained inversion techniques to calibrate carbon cycle models have been given by Enting (1985a). The

discussion in that paper was in terms of a modified verison of the model of Oeschger et al. (1975) but some preliminary results from the present model were given, based on the technical report of Enting and Pearman (1983).

The model involves 31 free parameters, (e.g., u_r^*, K_h etc.) as described in Section 2 and as listed in Table 2. The constrained inversion formalism is designed to obtain estimates of the parameters (which are denoted generically as x_k, $k = 1, K$) by fitting a set of observations m_j, $j = 1, J$. The observations will correspond to carbon concentrations and isotopic ratios in various reservoirs at various times. In other words the model predictions (denoted generically as y_i) are functions of the $N_j^v(t)$. Given a model that predicts each m_i to have a value $y_i(\boldsymbol{x})$ for a given set of parameters $\boldsymbol{x}$, the estimates $\hat{\boldsymbol{x}}$ are the parameter values that minimize:

$$\theta = \sum_{j=1}^{J} (y_j(\boldsymbol{x}) - m_j)^2/v_j^2 + \gamma \sum_{k=1}^{K} (x_k - q_k)^2/w_k^2. \quad (3.1)$$

In eq. (3.1), v_j is the standard deviation of observation v_j, q_k is an independent prior estimate of x_k and w_k is the standard deviation of the estimate q_k. In all calculations presented here the additional weighting factor γ will be 1. The use of other γ values is discussed by Enting (1983).

The use of parameter estimates based on minimizing (3.1) can be justified in a number of ways:

Table 2. *Parameters used in the model, together with prior estimates q_k, standard deviations u_k for these estimates and posterior (calibrated) parameter values $\hat{x}_k$*

Symbol	Quantity	Units	Prior q_k	s.d. w_k	Best-fit value x_k
C_0	Preindustrial CO_2	ppmv	270	50	264.32
Λ	^{14}C from weapons $\times 10^{26}$	atoms/megaton	2.00	0.50	1.401
δ_0	Preindustrial $\delta^{13}C$	‰	−6.00	3.0	−5.896
a_1	Biospheric release coefficient	Gt	0.0	10.0	0.07
a_2		Gt	15.0	20.0	15.14
a_3		Gt	32.5	40.0	31.38
a_4		Gt	37.5	60.0	15.40
a_5		Gt	57.5	80.0	27.08
a_6		Gt	102.5	100.0	−35.08
a_7		Gt	87.5	100.00	−20.37
T_c	Cold ocean temperature	°C	10.3	5.0	10.80
T_w	Warm ocean temperature	°C	25.2	2.0	26.47
A	Alkalinity	mol m^{-3}	2.369	0.021	2.351
v	Uniform upwelling	m yr^{-1}	1.8	1.35	0.210
v^*	Varying upwelling	m yr^{-1}	2.5	2.0	0.526
K_v	Vertical diffusion	m^2 yr^{-1}	3,500	2,000	5,283
K_v'	$\frac{d}{dt}$ (vertical diffusion)	m yr^{-1}	0	1.0	0.242
K_h	Horizontal	m^2 s^{-1}	25,000	20,000	23,807
ω	^{14}C from cosmic rays	mol s^{-1}	1.484×10^{-5}	3.7×10^{-6}	1.3307×10^{-5}
u_c^*	u_{cold}^*	m s^{-1}	1.7	1.2	1.569
u_w^*	u_{warm}^*	m s^{-1}	0.8	0.5	1.308
D^*	Detrital fallout	Gt yr^{-1}	2.0	3.0	3.537
N_y	Young biosphere	Gt	140	70	169
N_*	Old biosphere	Gt	1,400	700	1,040
B	NPP	Gt yr^{-1}	100	50	103.2
τ	Old biosphere turn-over time	yr	60	30	67.0
δ_{DET}	δ_{13} depletion of detritus	‰	−20	10	−20.07
d_m	Mixed layer depth	m	100	60	100.06
f_{as}	Fractionation factors	—	0.99795	0.00025	0.99795
f_{sa}		—	0.99005	0.0025	0.99005
f_{ab}		—	0.982	0.004	0.9828

(i) it is treating the independent estimates as simply additional data, to be regarded on the same footing as the carbon cycle data m_j (cf. Rodgers, 1977).

(ii) It is a form of Bayesian estimation (Box and Tiao, 1973) with the q_k and w_k defining a multivariate normal prior distribution for the x_k.

(iii) It is a form of constrained inversion (Twomey, 1977) based on ridge regression (Hoerl and Kennard, 1970) which is a particular form of biased estimation (Goldstein and Smith, 1974). This constrained inversion approach to calibration has a number of specific advantages over various other calibration techniques. In particular it stabilizes poorly-posed calibration problems, it gives a natural way of handling the determination of unknown functions, and it leads to a convenient unified approach to sensitivity analysis. Each of these advantages has been described in general terms by Enting (1985a) and each is illustrated by some aspect of the present calculations.

The stabilization of the calibration problem is illustrated by comparing the results presented by Enting and Pearman (1982) with the later results presented here and in greater detail by Enting and Pearman (1983). The earlier calibration did not use any constraints and used carbon cycle data to obtain estimates of 13 of the model parameters. The unconstrained calibration was found to be relatively unstable even though many fewer parameters are involved.

In constrained inversion it is common to look at what Jackson (1972) calls "the marginal utility of data" or the extent of the influence that the observations have on the statistical fit. Such considerations can be applied to unconstrained fits but they are most commonly confined to a consideration of the influence of "outlier" data points. In order to consider the influence of the observations in a quantitative manner it is necessary to specify the quantity on which potential influences act. In the notation of Enting (1983) we denote this quantity by Z, or $Z(\boldsymbol{x})$ to indicate its dependence on the model parameters. As a concrete example Enting and Pearman (1986) considered Z to be the predicted atmospheric CO_2 concentration in the year 2050 assuming a fossil carbon release that grows at 2.25% per year from 1979 to 2000 and is constant thereafter.

The quantities of interest in studying the influence of data are how much Z can vary without giving more than some specified degree of disagreement with the observations. If this disagreement is specified as a restriction of θ, the sum of squares of residuals (see 3.1), then one can introduce a Lagrange multiplier β and minimize

$$\phi(\boldsymbol{x}) = \theta(\boldsymbol{x}) - 2\beta Z(\boldsymbol{x}). \tag{3.2}$$

Enting (1983) has shown how to use this Lagrange multiplier form to calculate $\mathrm{Var}(\hat{Z})$. The statistical uncertainties are taken as defined by $\{w_k\}$ and $\{v_k\}$ with $\gamma = 1$. In this case, if $\hat{\boldsymbol{x}}(\beta)$ is the value of $\boldsymbol{x}$ that minimizes $\phi(\boldsymbol{x})$ for a given β, then

$$\mathrm{Var}(\hat{Z}) = (Z(\hat{\boldsymbol{x}}(\beta)) - Z(\hat{\boldsymbol{x}}(0)))/\beta \tag{3.3}$$

to within a linear approximation. This formalism essentially extracts a particular projection of the covariance matrix for the estimates $\hat{\boldsymbol{x}}$, i.e., it identifies those aspects of the uncertainty in $\hat{\boldsymbol{x}}$ that are important in a given context (Enting, 1983).

Enting and Pearman (1986) described two different criteria by which the utility of data can be measured. The first criterion is to look at the size of each contribution to θ. Because of the quadratic form of θ, the largest residuals will have the largest influence on the best fit parameter values and thus on the predictions Z. The definition of θ ensures that this comparison will be comparing different data (m_j and q_k) in terms of the dimensionless scale defined by their prior uncertainties (v_j and w_k). The second, and more useful, criterion looks directly at the influence of the data on a prediction Z and quantifies the change of the residuals with the variational parameter β. Enting and Pearman (1986) have given a ranking of the parameters and the observations according to each of these criteria, assessed according to their importance in predicting future CO_2 concentrations. They also pointed out that the study of the influence of data could be extended to a consideration of the potential utility of measurements that have not yet been made. In other words the calculations of utility are to act as a form of experimental design. They performed a number of such studies by including hypothetical data into the calibration process and determining the extent to which the value and precision of the hypothetical data influenced the value and precision of the model predictions.

4. The data used in the calibration

Calibration of the model requires a set of observations $\{m_j\}$ to which the corresponding calculated quantities $\{y_k\}$ are fitted. An optimal weighting requires that the $\{v_j^2\}$, i.e., the variances of the $\{m_j\}$, be used as weights as indicated in (3.1). The constrained inversion procedure supplements these data by taking the $\{q_k\}$, prior estimates of the x_k, as additional data to be fitted. As for the m_j, an optimal estimation requires that the $\{w_k^2\}$, i.e., the variances of these prior estimates, are to be used as weights.

One departure that we have made from this optimal weighting scheme concerns certain poorly known parameters. In these cases we have artificially increased the w_k so as to reduce the risk of biasing the estimation with incorrect information. Our aim is to approximate what a Bayesian analysis (Box and Tiao, 1973) would call a "non-informative prior distribution". We have also used unreasonably large w_k in cases where a reasonable prior distribution would be highly skewed about a preferred value. In such cases we have taken w_k to cover the plausible range in both directions. These changes make the calculations simpler by assuming that the prior estimates are effectively normally distributed. However this simplicity is achieved at the expense of a slightly reduced efficiency of parameter estimation. The choice of the sets $\{y_j\}$, $\{v_k\}$, $\{q_k\}$ and $\{w_k\}$ is described in detail by Enting and Pearman (1983). The various data items that are used are as follows.

(i) *Biospheric Suess effect data*

This is taken from Lerman et al. (1970) and has the biosphere $\Delta^{14}C$ as 0‰, −5‰, −12‰ and −22‰ in 1890, 1910, 1930 and 1950 with standard deviations of 5‰ in each case.

(ii) *Atmospheric CO_2 concentration*

This is based on Keeling et al. (1982) with a correction of 1.9 ppmv (Fraser et al., 1983) to approximate an atmospheric average. Data from the period 1964–1969 is omitted because of the problems described by Keeling et al. (1982). The corrected values used (in ppmv) are 313.76, 316.81, 323.61, 327.92, 331.73 and 334.46 at 1959.5, 1963.5, 1970.5, 1973.5, 1977.5 and 1980.5 years AD, with standard deviations of 0.3 ppmv for each point.

(iii) *Atmospheric radiocarbon levels*

This data is taken from Nydal et al. (1980). The period of major nuclear testing is deliberately excluded because at that time the atmospheric ^{14}C distribution can not be adequately represented by a single well-mixed reservoir. The values that were used are 650‰, 580‰, 500‰, 450‰, 420‰ and 350‰ at times 1967, 1969, 1971, 1973, 1975 and 1977. Standard deviations of 50‰ are taken for each value.

(iv) *Atmospheric $\delta^{13}C$*

The values used are -7.24 ± 0.05‰ at time 1978.5 and -6.69 ± 0.13‰ for 1956.2 (Keeling et al., 1979). These values, particularly that of 1956, involve significant corrections to remove the effects of seasonal variation and other sampling effects. (A subsequent revision of the 1956 value by Keeling et al., 1980, makes no significant change to our calibration).

(v) *Ocean carbonate levels*

The $\sum CO_2$ levels for the cold and warm surface regions are taken as 2.132 ± 0.102 mol m^{-3} and 2.009 ± 0.041 mol m^{-3} for time 1973.5. The $\sum CO_2$ levels for the cold and warm regions of the deep ocean are taken as 2.337 ± 0.041 mol m^{-3} and 2.358 ± 0.062 mol m^{-3} again for 1973.5. These numbers are based on GEOSECS data given by Takahashi et al. (1981).

(vi) *Ocean radiocarbon*

The ocean $\Delta^{14}C$ is based on the work of Broecker et al. (1960) which gives -58 ± 10‰ for surface waters in both cold and warm regions in 1955, and Nydal et al. (1980) which gives 150 ± 70‰ and 100 ± 50‰ for warm surface regions at 1968.5 and 1978.5. The GEOSECS program (Stuiver and Östlund, 1980; Östlund and Stuiver, 1980) gives values at 1973.5 of -150 ± 50‰ for the deep oceans in both regions and 120 ± 50‰ for the warm surface and 150 ± 50‰ for the cold surface.

(vii) *Initial conditions*

The values of the initial conditions (and all the subsequent data items) are the $q_k \pm w_k$ of eq. (3.1). The other items above were the $m_j \pm v_j$. The

two types of data are listed together because the calibration scheme treats both types of information on an equal footing. The initial conditions require the two quantities C_0 and δ_0. Both of these are given broad (i.e., relatively non-informative) prior distributions so as to avoid biasing the model results. We initially take $C_0 = 270 \pm 50$ ppmv and $\delta_0 = -6 \pm 3‰$.

(viii) *Sources and sinks*

The fossil carbon source is assumed to be known exactly in the present modelling study so that the only free parameters concern the sources of ^{14}C. In the present study the cosmic-ray production rate is taken as a constant ω although the computer routines provide for time-varying functions $\omega(t)$. Following the discussion by O'Brien (1979) we take $\omega = (1.482 \pm 0.370) \times 10^{-5}$ mol s^{-1}. The production of ^{14}C from nuclear testing is modelled using the energy-yield data compiled by Enting (1982) (Fig. 4). The sole free parameter is nominally the number of ^{14}C atoms produced per megaton energy yield but the deviations must also allow for the uncertainties in the yields. We put $\Lambda = (2 \pm 0.5) \times 10^{26}$ ^{14}C atoms (megaton yield)$^{-1}$.

(ix) *Air-sea exchange*

The ocean mixed layer depth is taken as 100 ± 60 m. The fractionation factors are:

$f_{as} = 0.99795 \pm 0.00025$,

$f_{sa} = 0.99005 \pm 0.00025$,

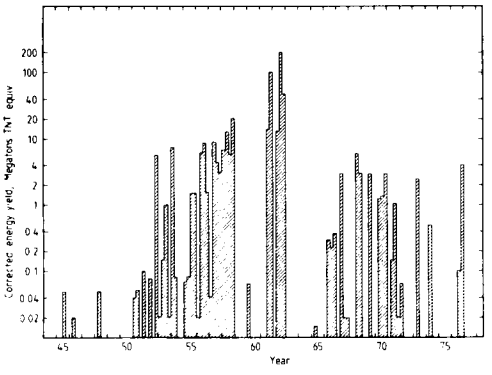

Fig. 4. Estimated energy yield of nuclear weapons tests over 3-month periods. Based on compilation by Enting (1982) and including corrections for surface effects.

based on the work of Siegenthaler and Münnich (1981). The ocean temperatures were constructed by averaging values in Table 65 of Defant (1961) and are taken as $T_c = 10.3 \pm 5$°C and $T_w = 25.2 \pm 2$°C. The uncertainties represent the degree of spatial variability but do not take seasonal variability into account. The alkalinity, $[A]$, is taken as 2.369 ± 0.0021 mol m^{-3} based on Takahashi et al. (1981).

The friction velocities u_c^* and u_w^* must be treated with some care. The model uses the linear u^* dependence discussed by Deacon (1977) but later work (see Liss, 1983) suggests a more rapid growth at larger u^* values. Enting and Pearman (1983) describe the use of averaging procedures that lead to effective values

$u_c^* = 1.7 \pm 1.2$ m s^{-1},

$u_w^* = 0.8 \pm 0.5$ m s^{-1}.

The uncertainties reflect both the uncertainties in the determination of mean wind stresses by Hellerman (1965, 1967) and the uncertainties in the actual dependence of the gas exchange rate on u^*.

(x) *Ocean mixing*

The detrital flux is put at 2 ± 3 Gt yr^{-1}, again using a relatively non-informative prior distribution. The calculations deriving this, using data from Broecker (1974), are given by Enting and Pearman (1983). The stable isotopic depletion is given as $20 \pm 10‰$ based on Sackett et al. (1965).

The horizontal diffusion constant is $K_h = (2.5 \pm 2) \times 10^4$ m^2 s^{-1} based on work by Okubo and Ozmidov (1970) who studied the scale dependence of horizontal diffusion coefficients. For the component of advection involving injection into the bottom layer, we use $v = 1.80 \pm 1.35$ m yr^{-1} based on an estimated rate of formation of Antarctic Bottom Water of (20 ± 15) sverdrup (1 sverdrup = 10^6 m^3 s^{-1}). Various estimates of this quantity ranging from 2 sv to 40 sv are listed by Enting and Pearman (1983). The modelling study by Kuo and Veronis (1973) suggests an upwelling velocity of 4.7 m yr^{-1} immediately below the thermocline. To bring our total up towards this figure we use $v^* = 2.5 \pm 2$ m yr^{-1}. The vertical eddy diffusion is taken as $K_v = 3500 \pm 2000$ m^2 yr^{-1} based on a variety of estimates listed by Enting and Pearman (1983). It

should be noted that the higher values suggested by Broecker et al. (1980) are for a purely diffusive mixing. Some of this mixing will be achieved in our model by the advective processes. There is virtually no information concerning an appropriate value for K_v', especially in a model that includes advection. We put $K_v' = 0 \pm 1$ m yr^{-1}.

(xi) *The biosphere*

Enting and Pearman (1982) show how to choose the biospheric reservoir sizes and transfer rates so that the form with transfer only into the young biosphere had the same age distribution (Bolin and Rodhe, 1973) as the model used by Keeling (1973b) which has short-lived and long-lived biospheric reservoirs connected to the atmosphere with no connections between the two biospheric reservoirs. On the basis of that analysis we use

Size of young biosphere,	$N_y = 140 \pm 70$ Gt
NPP,	$B = 100 \pm 50$ Gt yr^{-1}
Size of old biosphere,	$N_* = 1400 \pm 700$ Gt
Turnover time of old biosphere,	$\tau = 60 \pm 30$ yr.

The estimates by Olson (1982), Schlesinger (1977), Woodwell et al. (1978) and Whittaker and Likens (1973) were considered when estimating the uncertainties. The prior estimates of the anthropogenic flux, $b(t)$, are based on the work of Moore et al. (1981). In each case we choose the uncertainties to be comparable to the values themselves. The spline fit causes some distortion of the curve of Moore et al. We use $a_i = 0 \pm 10$, 15 ± 20, 32.5 ± 40, 37.5 ± 60, 57.5 ± 80, 107.5 ± 100 and 87.5 ± 100 (in Gt for $i = 1$ to 7 respectively).

5. Results of the calibration: biospheric release

The result of applying the constrained inversion formalism to the data listed in the previous section is the set of parameters estimates given in Table 2. Also listed are the prior estimates and their standard deviations. Table 3 gives the corresponding information for the observations and the model predictions. In each case the agreement is very good with far fewer values lying outside the ± 1 sd range than would be expected for independent errors. This agreement is a partial validation of the internal consistency of our modelling procedure. The various discrepancies in the fit are discussed below. As described by Enting (1985a) the estimation procedure proved to be numerically stable unlike the earlier unconstrained estimation of a subset of 13 parameters (Enting and Pearman, 1982). Enting and Pearman (1986) give a ranking of the parameters according to the influence that the prior estimates have and according to the extent to which these prior estimates influence predictions of future CO_2 levels. They also describe a similar ranking of influence of the carbon cycle data.

The history of CO_2 concentrations corresponding to the parameter estimates is shown as the solid line in Fig. 5. For comparison we also show the recent measurements of CO_2 concentrations in polar ice obtained by Neftel et al. (1985) and Pearman et al. (1986). We also show (as the dashed curve) the CO_2 concentration history obtained by using $C_0 = 280 \pm 10$ ppmv, to correspond to the values measured in ice-cores. It should be noted that apart from the initial value of the dashed curve, the ice-core data has not been included in the data set used for calibration mainly because of the remaining uncertainties concerning the effective age of the gas. In particular the occurrence of anomalous imper-

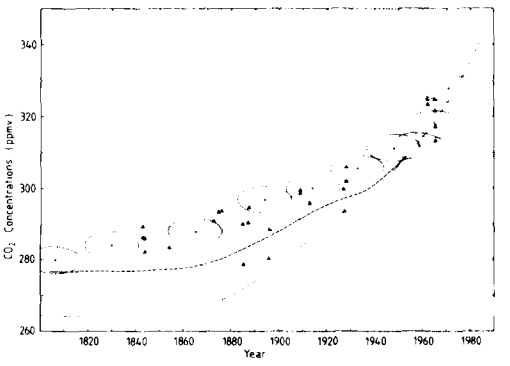

Fig. 5. Atmospheric CO_2 concentration for standard set of parameters (solid curve) and parameters estimated with the initial concentration set to 280 ± 10 ppmv (dashed curve). The ice-core data from Neftel et al. (1985) (ellipses) and Pearman et al. (1986) (triangles) are shown for comparison.

Table 3. *Data fitted in model calibration, m_i with assumed standard deviation u_i; for the estimated parameters $\hat{x}$ as listed in Table 2, the model prediction of m_i is $y_i(\hat{x})$*

Quantity	Reservoir	Units	Date	Data m_i	s.d. u_i	Prediction $y_i(\hat{x})$
C_a	Atmosphere	ppmv	1959.5	313.76	0.30	313.63
C_a	Atmosphere	ppmv	1963.5	316.81	0.30	317.06
C_a	Atmosphere	ppmv	1970.5	323.61	0.30	323.74
C_a	Atmosphere	ppmv	1973.5	327.92	0.30	327.25
C_a	Atmosphere	ppmv	1977.5	331.73	0.30	332.26
C_a	Atmosphere	ppmv	1980.5	336.46	0.30	336.35
$\Delta^{14}C$	Atmosphere	‰	1967.0	650	50	655
$\Delta^{14}C$	Atmosphere	‰	1969.0	580	50	594
$\Delta^{14}C$	Atmosphere	‰	1971.0	500	50	525
$\Delta^{14}C$	Atmosphere	‰	1973.0	450	50	451
$\Delta^{14}C$	Atmosphere	‰	1975.0	420	30	393
$\Delta^{14}C$	Atmosphere	‰	1977.0	350	30	347
$\Delta^{14}C$	Young biosphere	‰	1890.0	0	5	0.8
$\Delta^{14}C$	Young biosphere	‰	1910.0	−5	5	−4.9
$\Delta^{14}C$	Young biosphere	‰	1930.0	−12	5	−12.6
$\Delta^{14}C$	Young biosphere	‰	1950.0	−22	5	−19.7
$\Delta^{14}C$	Cold ocean surface	‰	1955.5	−58	10	−65.9
$\Delta^{14}C$	Cold ocean surface	‰	1973.5	100	50	132.9
$\Delta^{14}C$	Warm ocean surface	‰	1955.5	−58	10	−61.1
$\Delta^{14}C$	Warm ocean surface	‰	1968.5	150	70	144.7
$\Delta^{14}C$	Warm ocean surface	‰	1973.5	120	50	134.9
$\Delta^{14}C$	Warm ocean surface	‰	1978.5	100	50	105.6
$\Delta^{14}C$	Cold deep	‰	1973.5	−150	50	−141.5
$\Delta^{14}C$	Warm deep	‰	1973.5	−150	50	−183.9
$\delta^{13}C$	Atmosphere	‰	1956.2	−6.69	0.13	−6.75
$\delta^{13}C$	Atmosphere	‰	1978.5	−7.24	0.05	−7.23
$[\sum CO_2]$	Warm surface	mol m^{-3}	1973.5	2.009	0.041	2.093
$[\sum CO_2]$	Cold surface	mol m^{-3}	1973.5	2.132	0.102	2.173
$[\sum CO_2]$	Warm deep	mol m^{-3}	1973.5	2.337	0.041	2.328
$[\sum CO_2]$	Cold deep	mol m^{-3}	1973.5	2.358	0.062	2.357

meable layers in the firn may limit the precision with which dates can be assigned to gas samples (see Schwander and Stauffer, 1984; Neftel et al., 1985; Enting and Mansbridge, 1985). Each of the concentration curves obtained from the model lies somewhat lower than the concentrations measured in the ice cores, particularly for the period around 1950. This result reflects our use of the ecosystem model calculations (Moore et al., 1981) as prior estimates of $b(t)$. The ice-core data would be fitted better if the peak in the release was somewhat later than in our reconstructions.

Figs. 6 and 7 give further details of the agreement between the model predictions y_i and the observations m_i that are fitted. Fig. 6 is an enlargement of the last 35 years of the atmospheric CO_2 records shown in Fig. 5. The error bars denote the ranges $m_i \pm u_i$ for the times at which observed concentrations were fitted. Similarly, Fig. 7 shows that model values for atmospheric $\Delta^{14}C$ for the period 1945–1980. The model predictions for $\Delta^{14}C$ all pass within the 1 s.d. ranges shown. For CO_2 concentrations some of the model predictions lie outside the s.d. range. Fig. 6 indicates that this discrepancy is due to short time-scale fluctuations in the atmospheric CO_2 that are not represented in the model.

Fig. 8 shows the various estimates of the biotic release $b(t)$. The prior estimate (based on Moore et al., 1981) is shown as the chain curve. The reference calibration, corresponding to the parameters in Table 2, is shown as the solid curve. This corresponds to the CO_2 history shown as the solid curve in Fig. 5. The calibration using a modified prior estimate of the concentration in 1800 gives the dashed curve for $b(t)$. This corre-

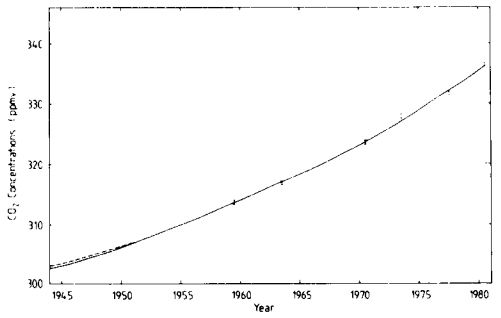

Fig. 6. Enlargement of Fig. 5 showing the period 1950–1980. The error bars show the atmospheric CO_2 data $m_i \pm u_i$ that were fitted in the calibration procedure.

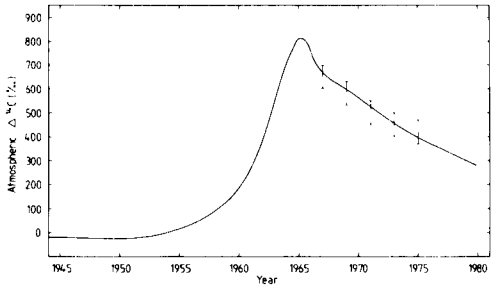

Fig. 7. Model results for $\Delta^{14}C$ of the atmosphere for the period 1950–1980. The error bars show the $\Delta^{14}C$ data that were fitted in the model calibration.

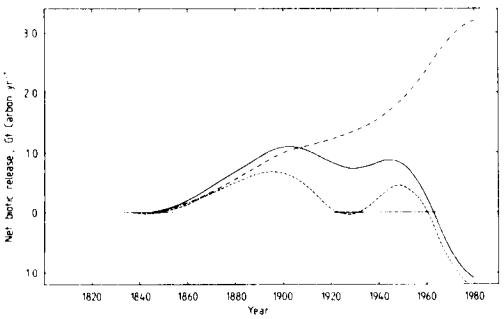

Fig. 8. Biospheric release history $b(t)$ corresponding to the two concentration histories in Fig. 5. The prior estimate is shown as the chain curve.

sponds to the concentration history given by the dashed curve in Fig. 5. It must be remembered that these are low resolution descriptions of $b(t)$ since almost 200 years of possible change are represented using only 7 parameters. The common feature is that the current value of $b(t)$ is relatively small and each of the curves has a current negative gradient of about -0.07 Gt yr^{-2}. Similar gradients occur in the other cases considered by Enting and Pearman (1986) when they incorporate various other possible data items into the calibration. This result of a negative gradient disagrees with one of the conclusions of Elliott et al. (1985) who claim to show that $b(t)$ is currently near zero and that a decreasing $b(t)$ is highly unlikely. They analysed the CO_2 record from Mauna Loa assuming that a constant proportion of the total fossil plus biospheric release remains in the atmosphere. Laurmann and Spreiter (1983) considered this assumption of a constant airborne fraction in detail and showed it to be appropriate for assessing the impact of increasing releases of fossil carbon. However, Elliott et al. neglect two important qualifications in the work of Laurmann and Spreiter. Firstly, the constant airborne fraction assumption breaks down for releases that grow more slowly than a 2% per annum exponential increase. Laurmann and Spreiter consider this to be unimportant for their study because such slower growth rates would have less serious consequences than the faster exponential increases and so accuracy was less important. Secondly, Laurmann and Spreiter specifically emphasize that their results do not apply to interpreting the current state of the global carbon cycle.

We show the extent of possible violation of the assumption of a constant proportion of CO_2 release remaining in the atmosphere. The relevant quantity is

$$\Phi = \frac{\partial c}{\partial t} \Big/ (b(t) + f(t)), \tag{5.1}$$

where Φ is denoted F by Elliott et al. (1985). Fig. 9 shows a smoothed approximation to the quantity Φ for the two CO_2 histories shown in Fig. 5. The smoothing consists of taking the concentration change over a 10-year period and dividing by the integrated source over the period. Following Elliott et al., this ratio is divided by 0.471 ppmv Gt^{-1} to give the approximation to Φ in dimensionless terms. It will be seen that Φ has ranged from 0.54 to 0.74 in these dimensionless units. The degree of variability of Φ shows how the uptake of CO_2 exhibits a strong "memory" effect so that the uptake at any time can be strongly influenced by imbalances due to past

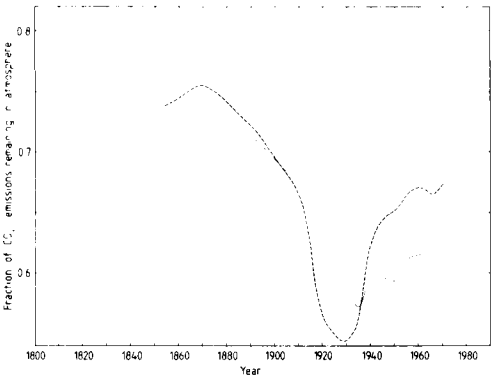

Fig. 9. True airborne fraction, i.e., $\partial c/\partial t/(b(t) + f(t))$ in dimensionless terms, calculated for the two CO_2 concentration histories of Fig. 5.

sources. General mathematical relations describing this influence were given by Oeschger and Heimann (1983) and Enting (1985b). The examples presented here show that these "memory" effects can cause Φ to change much more rapidly than the 0.001 yr^{-1} change considered (on the basis of changing the buffer factor) by Elliott et al. (1985).

In connection with the biosphere release $b(t)$, it is of interest to recall two of the results presented by Enting and Pearman (1986), as part of their study of the utility of possible additional data. In exploring the utility of measurements of atmospheric $\delta^{13}C$ and of determinations of the current biospheric release $b(t)$ they found that the model calibration could not incorporate values of d/dt $\delta^{13}C$ that differed greatly from $-0.03‰$ yr^{-1} or positive values for the current biospheric release. The conclusion is that the additional $\delta^{13}C$ information is redundant in that, within the context of the model with ^{14}C used in the calibration, it duplicates other information already available but with lower precision. Keeling et al. (1980) reached a similar conclusion regarding the utility of measurements of d/dt $\delta^{13}C$ on rather different grounds.

There are a number of possible explanations for the discrepancy between our estimated function $b(t)$ and the calculations of Moore et al. (1981). Moore et al. only considered biomass changes associated with land-use changes; any biomass changes due to imbalance between uptake and release in undisturbed ecosystems would not appear in their results. One possible cause of such an imbalance would be enhanced biotic growth due to increasing CO_2 concentrations. This type of effect was considered in the model of Oeschger et al. (1975). Alternatively, enhanced biotic growth might be associated with climatic changes.

The results in Tables 2 and 3 indicate two other minor discrepancies that are not directly related to the rôle of the biosphere. The first is associated with the factor Λ which is the scaling factor to convert from estimated effective energy yields to ^{14}C production. Thus a possible explanation of our low estimate of Λ is that the corrected energy yields in Fig. 4 are overestimates.

The other discrepancy concerns the warm ocean surface layer where the model predicts too high a surface concentration, $[\sum CO_2]$. Our interpretation for this is that our ocean circulation scheme is unrealistic in forcing about half of the global upwelling to pass through the warm ocean surface rather than having a return flow at intermediate levels. The excess upwelling of water with high $[\sum CO_2]$ raises the surface concentration to an unreasonably large extent. Inspection of the parameter estimates in Table 2 indicate that the calibration procedure is trying to compensate for this by increasing u^*_{warm} (to increase the rate of equilibration with the atmosphere) and reducing the upwelling velocities.

6. Conclusions

This paper has presented an account of a calibration of a global carbon cycle model using the techniques of constrained inversion. The results presented here go well beyond the preliminary applications of constrained inversion to the box-diffusion model presented by Enting (1985a). In particular the present study attempts to work with $b(t)$ as a genuinely unknown function. The results suggest that currently the biosphere release is small and negative (0 to -1 Gt yr^{-1}), i.e., a probable small net uptake of carbon by the biosphere. In addition there is a strong suggestion of a negative derivative for $b(t)$. This result must be treated with caution because the low resolution description of $b(t)$ restricts the variations on short time scales. In spite of this restriction, the range of possible functions that we have considered is much wider than has

normally been considered in carbon cycle modelling. The importance of avoiding arbitrary restrictions on the possible history of biospheric release is shown by the results presented in Fig. 9. The relatively large variation in the fraction of emissions remaining in the atmosphere reflects the large extent to which the current CO_2 uptake by the oceans can be influenced by the past history of atmospheric changes. Any discussion of the present behaviour of the carbon cycle must allow for such influences from the past (Oeschger and Heimann, 1983). Discussion cannot be validly restricted to contemporary data as assumed by Elliott et al. (1985).

Our study has shown the need for careful consideration of the variability of the history of biospheric changes. An appropriate study to test the reliability of our results would be to model $b(t)$ as a spline function with more nodes and apply a smoothing constraint to the spline. A more general study of this type would examine, in more detail, the utility of "indirect" determinations of past CO_2 histories through techniques such as ice-core analysis, tree-ring $\delta^{13}C$, archived spectroscopic data and re-examination of late 19th century measurements.

Our results are, of course, subject to the validity of our model. The model has been shown to be consistent with a diverse set of geophysical and biogeochemical information, the main exception being that we are unable to fit in the biospheric release estimates from Moore et al. (1981). We believe that the consistency which we have demonstrated between all the other data argues in favour of the validity of our model and of the conclusions that we have drawn concerning biospheric changes.

7. Acknowledgement

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8. Appendix

Notation

Latin

a	Reservoir index denoting the atmosphere
a_i	Coefficient of ith B-spline, B_i, giving size of biotic release pulse due to forest clearing
$[A]$	Ocean surface alkalinity
A_r	Area of ocean region r, i.e., either A_c or A_w
$b(t)$	Net anthropogenic flux of carbon from biosphere to atmosphere
B	Net primary production
$B_i(t)$	Quadratic B-spline over range $1790 + 25i$ to $1865 + 25i$
c	Reservoir index for cold surface layer
cn	For $n = 1$ to M reservoir index for layer n in cold region
C_0	Initial value of C_a
C_a, $\tilde{C}_a$	Atmospheric CO_2 concentration in ppmv, and atmospheres respectively
C_{rn}	Total inorganic carbon concentration in layer n of region r
d_m	depth of surface mixed layer
d_n	Thickness of layer n in oceans ($n = 0$ is mixed layer)
D^*	Total detrital carbon flux from ocean surface reservoirs to all subsurface reservoirs
f_{ab}, f_{ab}^*	Fractionation factors for transfer of ^{13}C, ^{14}C from atmosphere to biosphere
f_{as}, f_{as}^*	Fractionation factors for transfer of ^{13}C, ^{14}C from atmosphere to ocean surface
f_{sa}, f_{sa}^*	Fractionation factors for transfer of ^{13}C, ^{14}C from ocean surface to atmosphere
$f(t)$	Fossil carbon release rate (see Fig. 2)
$F_{ij;\alpha}$	Net flux from reservoir i to reservoir j due to process α
g_i^v	Generic function giving rate of change of N_i^v
i	General reservoir index
j	General reservoir index or general index for carbon cycle data
J	Number of items of carbon cycle data m_j
k	General index for model parameters
k_r	Density weighted piston velocity in region r
K	Number of free parameters in model
K_h	Horizontal eddy diffusion coefficient for surface layer
$K_v(z)$	$= K_v + zK_v'$, vertical eddy diffusion coefficient at depth z

L_{30}	Length of 30° parallels over oceans, i.e., length of boundary between regions
m_j	jth item of carbon cycle data to be fitted by model
M	Number of subsurface ocean layers. Set to 6 for all numerical calculations quoted here
n	Index for ocean layers
$n(t)$	Energy yield from nuclear tests (see Fig. 4)
N_*	Size of long-lived biosphere after forest cleaning
N_i^ν	Number of moles of carbon isotope ν in reservoir i
o	Reservoir index for old biosphere
P_r	CO_2 partial pressure in ocean region r, in atmospheres
q_k	Prior estimate of parameter x_k
Q	Rate of carbon transfer from young biosphere to old biosphere
r	General index for ocean regions, either c or w
$r(t)$	$^{13}C:(^{12}C + ^{13}C)$ ratio in fossil fuel
R_s	Standard $^{13}C:^{12}C$ ratio
$S_{i;\alpha}^\nu$	Source of carbon isotope ν in region i due to process α
t	Time
T	Temperature in degrees Celsius
u_r^*	Effective friction velocity in region r
v, v^*	Velocities parameterizing upwelling: upwelling at bottom of layer m is $v + v^*(M - m)/M$ for $m = 0$ to $M - 1$
v_j	Standard deviation of m_j
w	Reservoir index for warm ocean surface layer
w_k	Standard deviation of q_k
x_k	kth model parameter
x	K-dimensional vector of model parameters
y	Reservoir index for young biosphere
y_j	Model prediction for m_j
z_i	Depth of midpoint between ocean layers i and $i + 1$
Z	Prediction of model, especially of future CO_2 concentration

Greek

α	Process index, see Table 1
$\alpha(T)$	Solubility of CO_2
β	Lagrange multiplier
γ	Weighting factor in constrained inversion
δ_{DET}	^{13}C depletion of detritus relative to mixed layer
δ_0	Initial atmospheric $\delta^{13}C$
$\Delta_{0;45}$	Distance between 0° and 45° parallels of latitude
θ	Quantity minimized in constrained inversion calibration
κ_{eq}	Equilibrium constant for $H_2CO_3 + CO_3^= \rightleftharpoons 2HCO_3^-$
λ	Decay rate for ^{14}C
Λ	Production rate of ^{14}C, atoms per megaton yield
μ, ν	Isotope index, 12, 13 or 14
τ	Turn-over time for old biosphere
ϕ_{ar}, ϕ_{ra}	Gross air-sea fluxes per unit of surface area
Φ	Airborne fraction in dimensionless terms
ϕ	Quantity minimized in determining influence of data on predictions
ω	Rate of cosmic ray production of ^{14}C

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